A COMPARATIVE STUDY OF DIGITAL INTEGRATORS AND APPLICATION IN DESIGNING OF DISCRETE FILTERS FOR HIGH SPEED CONTROL SYSTEMS

Thesis

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By

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CERTIFICATE

This is to certify that the thesis entitled **"A COMAPARATIVE STUDY OF DIGITAL INTEGRATORS AND APPLICATION IN DESIGNING OF DISCRETE FILTERS FOR HIGH SPEED CONTROL SYSTEMS"**, submitted in partial fulfilment of the requirements for the degree of **Master of Technology** with major in **Electronics and Communication Engineering** and minor in **Information Technology** of the College of Post-Graduate Studies, G. B. Pant University of Agriculture & Technology, Pantnagar, is a record of *bona fide* research carried out by **Mr. Rohitashwa Rastogi,** Id. No. **42647** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation and source of literature have been duly acknowledged.

03-08-2013

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We, the undersigned, Members of the Advisory Committee of **Mr. Rohitashwa Rastogi,** Id. No.**42647**, a candidate for the degree of Master of Technology with major in Electronics and Communication Engineering and minor in Information Technology, agree that the thesis entitled **"A COMPARATIVE STUDY OF DIGITAL INTEGRATORS AND APPLICATION IN DESIGNING OF DISCRETE FILTERS FOR HIGH SPEED CONTROL SYSTEMS"** may be submitted in partial fulfillment of the requirements for the degree.

03/08/2013

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f_c	Center frequency
f_{pass}	Pass frequencies
f_{stop}	Stop frequencies
ω_0	Free running frequency
ζ	Damping co-efficient

LIST OF ABBREVIATIONS

BPF	Band Pass Filter
BWE	Backward Euler Integration
FIR	Finite Impulse response
FWE	Forward Euler Integration
HPF	High Pass Filter
IIR	Infinite Impulse Response
LPF	Low Pass Filter
TRAP	Trapezoidal Integration

Chapter 1

Introduction

CHAPTER-1 INTRODUCTION

In the present age of Information Technology, the Information has a precious role in our daily life. We are nothing without the information. For example, weather forecasting, internet, mobile communication, wireless communication, Seismology etc. These all are the important part of our life. The transmission of information is performed by many means like wireless communication, optical fiber etc. They offer a medium to the information signal, but they often introduce some unwanted signal in the original signal, which is called noise. This noise is additive in nature and it can corrupt the original information signal. So it is not so much easy to receive the original signal. To retrieve the original signal we need a device that can acknowledge the main signal or the unwanted noise signal. Such type of systems need to process on the received signal in such a manner so that they can remove the noise signal or they can increase the level of main signal with respect to the noise signal and the main signal can be identified easily at the receiver. Such types of systems are called filters. The filters are nothing but the electronic systems that process on the input to remove the unwanted signals.

Generally, a filter is a system whose gain is a function of input frequency. The frequency band for which the gain is larger, called pass band and the frequency band for which the gain of the system is very smaller than that of the pass band, is called the stop band or reject band. Generally, the difference between the pass band gain and stop band gain is measured in dB and its typical value is 3-dB. The frequency, at which the gain is 3-dB less than the pass band gain, is called the 3-dB frequency. For example a filter magnitude H (f) can be explained as a piecewise defined function of frequency.

$$\text{Magnitude } |H(f)| = \begin{cases} \text{less than -3dB} & ; \quad 0 < f < 10 \text{ kHz} \\ 0 \text{ dB} & ; \quad 10\text{kHz} < f < 15\text{kHz} \\ \text{less than -3dB} & ; \quad f > 15 \text{ kHz} \end{cases}$$

A filter can be classified in analog type and discrete type. Analog filter can be designed by using the various passive devices like resistance, capacitance, inductance or their combination with active devices like BJT, MOS, OP AMP.

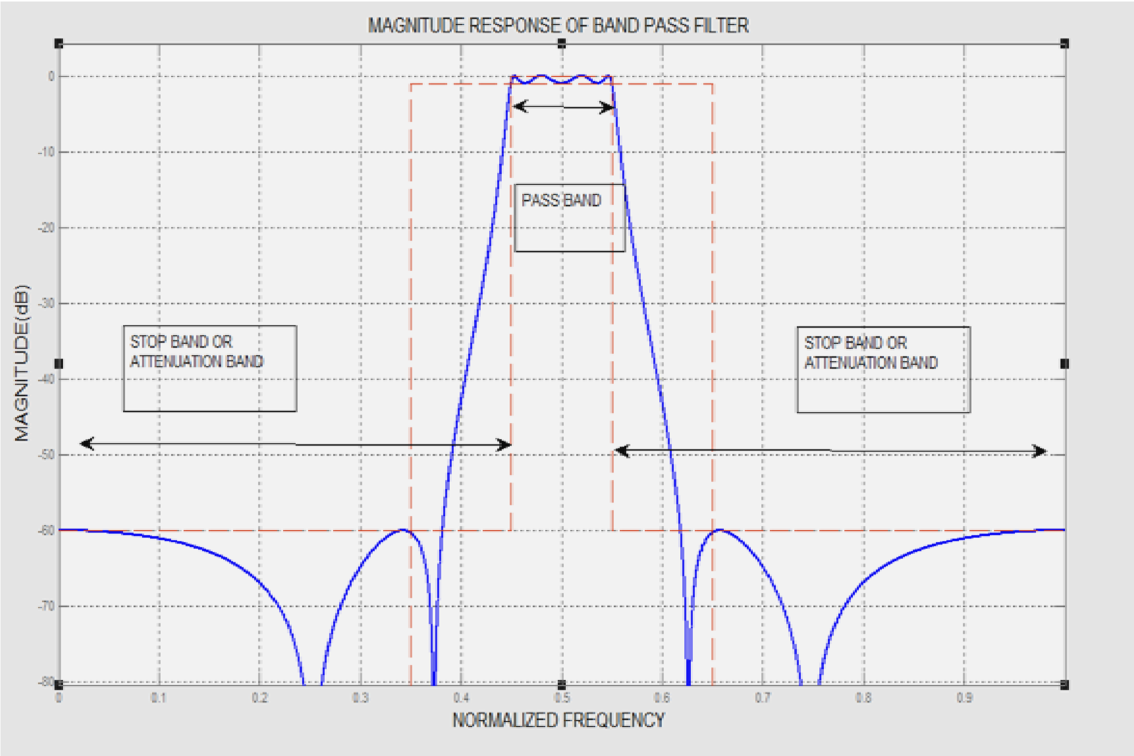


Figure: 1.1 Example of filter with pass-band and stop-band

Digital filters can be designed by using delay element and amplification units. Hence, the digital filter can be defined as a system that performs the mathematical operations on the sampled, discrete signals to achieve the desired bandwidth. A digital filter is characterized by its transfer function or a difference equation. Difference equation and the transfer function are both can be converted into each other by using the mathematical tool called Z-transform. When the unit impulse signal is applied to the filter, the response is called unit impulse response. On the basis of the impulse response the digital filters can be classified into two categories, one is IIR and other is FIR. IIR stands for infinite impulse response while FIR for finite impulse response, means the response of applying the impulse signal as input to the filter is infinite for IIR and finite for FIR. On the basis of their frequency response, the filters can be classified into few these types. The gain of filter is measured in dB (decibel). Frequency at which the gain is -3dB than the pass band gain is called cut-off frequency $f_{cut\text{-}off}$ .Figure 1.1 shows the description of all types of filters with the pass band and stop band limits. Here f_c represents the cut-off frequency.

Table 1.1 : Classification of filters

TYPE OF FILTER	PASS BAND	STOP BAND
Low pass filter	$f_{pass} < f_c$	$f_{stop} < f_c$
High pass filter	$f_{pass} > f_c$	$f_{stop} < f_c$
Band pass Filter	$f_{c1} < f_{pass} < f_{c2}$; where $f_{c1} < f_{c2}$	$f_{stop} > f_{c2}$ & $< f_{c1}$
Band reject Filter	$f_{pass} > f_{c2}$ & $< f_{c1}$	$f_{c1} < f_{stop} < f_{c2}$; where $f_{c1} < f_{c2}$

1.1 Filter Characteristics

An ideal filter does have some characteristics, which are important while designing the filter for required operation. They are defined as follows:

A. Causality

The causality refers to a property of any system which ensures that the system is physically realizable, meaning that the system exists in mathematical equations as well as in real world also.

B. Time Invariance

The relation between input and output is a major property of the system. In filters it varies as the function of frequency. Means for pass band frequencies the gain is large as compare to stop band. But this relationship should not vary with the time. Means it should be constant with respect to time. For example $y(t)=2.x(t)$ and $z(t)=t.x(t)$ here $y(t)$ is a time invariant system. $z(t)$ is time variant system. Because the relationship of z & x depending on the time.

C. Linear Phase Response

The system gain which is responsible to all frequencies in its pass band, introduces a phase delay in all frequencies. The relationship between the produced phase shift and frequency is called the phase response. For an ideal filter the phase response should be linear so that it can provide the constant phase shift to all frequencies in pass band.

D. Phase delay

Phase delay of a digital filter is defined as

$$\tau_p = -\frac{\phi(\omega)}{\omega} \; ;$$

E. Group delay

The group delay is defined as the delayed response of filter as a function of frequency to a signal. It is defined mathematically as

$$\tau_g = -\frac{d\phi(\omega)}{d\omega} \; ;$$

The filters for which the phase delay and the group delay are constant i.e. dependent of frequency, are called linear phase filters.

F. Stability

If the system is physically realizable, it does not indicate that the system is stable. Stability is a property of the system that ensures the system will be stable for a long time. Often so many systems are physically realizable but they are not stable.

For example $H_1(s) = \frac{1}{s+a}$ and $H_2(s) = \frac{1}{s-a}$ (a>0) ; the $H_1(s)$ has a pole at s=-a, $H_2(s)$ has the pole at s=+a ; $h_1(t)=e^{-t}$; $h_2(t) = e^{+t}$. Here first system is stable while second is unstable due to its pole in right hand s-plane.

G. Low convergence time

When the impulse signal or the step signal is applied to the filter, the output is called impulse response or step response respectively. If it is stable then its output should reach to a constant finite value other than infinity. Now the time required to reach that fix value of output response is called as Settling Time or Convergence Time. For a faster response the filter’s convergence time should be as least as possible.

H. Flat magnitude response

There are so many mathematical models regarding the filters like Butterworth, Chebyshev, Elliptical filters etc. They have their specific magnitude response. In all the Butterworth filter has a special property that it has a flat magnitude response which is very close to ideal.

1.2 Plan of Proposed Work

By analyzing, study and discussion of former work done we get basic ground to design the digital filters for high speed control system. Proposed Design of Discrete Filter is focusing toward the flat magnitude response without any spike in pass band as well as stop band and very short convergence time. The proposed plane of work is as follows.

1. Study of Integrators and Differentiators.
2. Estimation of different Integrators and Differentiators deviation from Ideal one.
3. Design of discrete Filters with help of these integrators.
4. Calculation of error with respect to the Butterworth Filter Response.
5. Study of Step Response for different discretization methods.
6. Design and implementation of digital filter using MATLAB.

1.3 Design Methods for Digital Filter

There are many methods for the design of FIR and IIR.

Design methods for FIR are

1. Fourier series method
2. Frequency sampling methods
3. Window techniques

Design of IIR filters the main methods include

1. Impulse invariance method
2. Step invariance method
3. Euler integration
4. Trapezoidal integration

All the IIR design methods are explained in chapter 3. Besides of these methods several Integrators and Differentiators have been developed for the IIR filter design. These differentiators are combination of 2nd order or 1st order digital differentiators. They are combined to get the minimum deviation from ideal properties. For example Al-Aloie differentiator is designed by mixing the backward Euler differentiator and trapezoidal differentiator in proper ratio. Besides of them there are fractional order differentiators also.

1.4 Organization of Thesis

The thesis is divided into 4 chapters. They are as follows.

Chapter 2 Review of Literature Summarizes the literature that contains all important research work regarding the design of digital filters. The literature cited here is a basic layer to implement the proposed design of discrete filters for high speed control systems.

Chapter 3 Material and Methods explains types of methods for design of the digital IIR filters. It gives the analysis of digital integrators and differentiators. The analysis for the higher order integrator is described. Later in the chapter the condition for flatness of the magnitude response is evaluated. After that the design constraint for HPF is evaluated.

Chapter 4 Results and Discussion contains an analysis of all the types of digital integrators and gives an overview of higher order integrators. This chapter does also contain all the work regarding the Digital filter design for high speed control systems.

Chapter 5 Summary and Conclusion shows the summary of the work and conclusion for the proposed work.

Chapter 2

Review of Literature

For the design of digital filters with flat magnitude response and without any resonance peak for high speed discrete control systems following literature have been studied.

Zhang, X. and Iwakura, H. (1999) have proposed a method for IIR digital all pass filter with an Equi-ripple phase response that can be proven to be optimum in the Chebyshev Sense. The proposed procedure is based on the formulation of an Eigen value problem by using the Remez Exchange Algorithm. The design algorithm proposed in this correspondence not only retains the speed inherent but also simplifies the interpolation step.

Blas, Y.C., and Vinagre, M. (2002) have proposed a new infinite impulse response (IIR)-type digital fractional order differentiator (DFOD) using a new family of 2^{nd} order digital differentiators expressed in the second-order IIR filter form. The integer 2^{nd} order digital differentiators are obtained by the stable inversion of the weighted sum of Simpson integration rule and the trapezoidal integration rule. The distinguishing point of the proposed DFOD lies in an additional tuning knob to compromise the high-frequency approximation accuracy.

Roy, S.C. Dutta (2005) has given simple method for deriving the spectral transformations of infinite-impulse response (IIR) filters, which can be used to transform a prototype low-pass (LP) digital filter to another LP, high-pass (HP), band pass (BP), or band-stop (BS) digital filter with prescribed pass band edge(s) and the same tolerances as those of the prototype. The method is based on a combination of bilinear transformation with the analog frequency transformation and is simpler conceptually, as well as from the calculation point of view than the conventional method based on all-pass transformation functions.

Tseng, C.C. (2005) have investigated the design of digital differentiator. First, he has established a relation between fractional delay filter and first-order differentiator such that the differentiator can be obtained from fractional delay filter by using the computation of limit. Then, conventional finite-impulse response (FIR), all pass and Narrow-based fractional delay filters are directly applied to design first-order differentiator. Next, he has extended this technique to design higher order differentiators. Finally, he has illustrated several design examples to demonstrate the effectiveness of this new design approach.

Vazquez, A. F., and Dolecek, G.J. (2006) have presented a new direct design of infinite-impulse response (IIR) filters with a flat magnitude response in both Pass band and stop band (Butterworth filters). The design specifications are pass band and stop band frequencies and pass band droop and stop band attenuation. The approach is based on an all pass filter with flatness at frequency points $\omega = 0$ and $\omega = \pi$. Depending on the parity of the IIR filter order, the all pass filter is either real or complex.

Al-Alaoui, M.A. (2007) has proposed a novel approach to design approximately linear phase infinite-impulse-response (IIR) digital filters in the pass band region. The proposed approach has yield digital IIR filters whose numerators represent linear phase finite-impulse-response (FIR) filters. As an example he introduced a low-pass IIR differentiators. The range and high-frequency suppression of the proposed low-pass differentiators are comparable to those obtained by higher order FIR low-pass differentiators.

Tseng, C.C. (2007) has used the numerical integration rules and fractional sample delays to obtain the closed-form design of infinite-impulse response (IIR) digital integrators. He investigated two types of numerical integration rules. One is Newton–Cotes Quadrature Rule; the other is Gauss–Legendre Integration Rule. Although the proposed IIR digital integrators will involve the implementation of fractional sample delays, this problem is easily solved by applying well-documented design techniques of the finite-impulse response Lagrange and IIR all-pass fractional delay filters.

Al-Alaoui, M.A. (2007) has explained how the α-approximation for discretization of analog systems, is exactly the same as the parameterized Al-Alaoui operator.

Al-Alaoui, M.A. (2007) has proposed new transformation polynomials for discretization of analog systems. He has proposed that the discretization of $\left(\frac{1}{s}\right)^n$ should be done independently rather than by raising the discrete representation of $1/s$ to the power n. Several examples are given in to back this idea. He has also shown that the new transformation polynomial for discretization of analogue systems is exactly the same as the parameterized Al-Alaoui operator.

Krishna, B. T. and Reddy, K.V.V.S. (2008) have designed digital differentiators and integrators of order (1/2). First, the rational approximation for the fractional operator $(s)^{\pm 1/2}$ is calculated. Next, using s to z transforms it is digitized. It has been shown that results hence obtained are very closer to ideal characteristics.

Tseng, C.C. & Lee, S. (2008) have investigated a new design of digital integrator. First he applied the trapezoidal integration rule and differential equation to derive the transfer function of the digital integrator. The Richardson extrapolation is then used to generate high-accuracy results while using low-order formulas. Next, the conventional Lagrange finite-impulse response fractional delay filter is directly applied to implement the designed integrator. Here two implementation structures are studied: direct substitution and poly-phase decomposition. Finally, numerical comparisons with conventional digital integrators are made to demonstrate the effectiveness of this new design approach.

Sondur, V.V. et al. (2008) have observed that the weighted least-square (WLS) technique, including the modifications, results in Equiripple error curve. The resultant error as a percent of the ideal value is highly non-uniformly distributed over the range of frequencies for which the differentiator is designed. They have proposed a modification to the technique so that the optimization procedure results in lower maximum relative error compared to the ideal values.

Radisavljevic, V. (2008) has shown that the well-known class of first-order digital differentiators and integrators can be simply derived from a classical continuous-time approximate differentiator by using the bilinear transformation. The result obtained represents a simple and alternative method of deriving the Al-Alaoui operator (also known as Al-Alaoui's differentiator, integrator, transform).

Tahmasbi, A. & Shokouhi, S.B.(2010) have designed a suitable IIR filter for approximating Parks- McClellan Low pass differentiator using modified Al-Alaoui's method, and then denominator polynomial coefficients of resulting transfer function optimized by Genetic algorithm. He also defined a suitable fitness function optimize both magnitude and phase responses; moreover, appropriate weighting coefficients and GA parameters are reported for several cut-off frequencies. He has presented that the order-4 proposed low-pass differentiators yield a frequency response which is almost equal to order-30 Parks-McClellan low-pass differentiators. Furthermore, they has found steep roll-off properties, small magnitude error and almost linear phase in the pass-band; the percentage error of magnitude response is less than 0.5%.

Zhang, X. (2009) has designed a Fractional Delay (FD) IIR filter with max-flat frequency response in frequency domain fractional delay. First He described the flatness condition of FD filters at an arbitrarily specified frequency point, and then derived a system of linear equations from the flatness condition. Hence by solving these equations, He finds a set of filter coefficients. For a special case in which the frequency response is required to be max flat at ω=0 or pi, a closed-form expression for its filter coefficients He derives them solving a linear system of Vander-Monde equations. He also shows that the existing max-flat FD finite-impulse-response (FIR) and IIR filters are special cases of the FD IIR filters proposed in this paper.

Reshmin, B.I. (2010) has studied the errors in the gain frequency characteristics of a low pass filter with different methods of numerical integration. And hence he has suggested a simple method to correct the filter parameters taking into account the quantization step and the method of numerical integration, which ensure the development of a filter with optimal properties for high speed control systems.

Auger F. (2010) has worked on three classes of s-to-z transforms. For each class, a closed-form expression is proposed, and a particular element, having an imaginary part of its frequency response very close to the ideal value, is presented. These s-to-z transforms are then related to all-pass infinite-impulse-response fractional delay filters, allowing another way to choose their degree of freedom.

Dai, C. & Chen W. (2010) have designed a seeker-optimization-algorithm (SOA)-based evolutionary method is proposed for digital IIR filter design. SOA is based on the concept of simulating the act of human searching in which the search direction is based on the empirical gradient by evaluating the response to the position changes and the step length is based on uncertainty reasoning by using a simple fuzzy rule. The algorithm's performance is studied with comparison of three versions of differential evolution algorithms, four versions of particle swarm optimization algorithms, and genetic algorithm.

The simulation results show that SOA is superior or comparable to the other algorithms for the employed examples and can be efficiently used for IIR filter design.

The study and analysis of the methods described in the literature cited above provide an approach to design the discrete filters, differentiators, integrators. Now in the following chapter materials and methods the complete approach towards the comparative study of digital integrators and design of discrete filters for high speed control system is being shown.

Chapter 3

Materials & Methods

CHAPTER-3 MATERIALS AND METHODS

After studying and analyzing the last research on digital integrators/differentiators the design of max flat digital filter for high speed control systems can be start. The following chapter shows the different methods to design the IIR, digital integrators and their analysis and finally the design method to design the proposed filter is described.

3.1 Design Methods for IIR Filters

IIR filters are those filters whose response towards the unit impulse signal is not ending at some finite time. If we are given a specific data to design the IIR digital filter then there are two types of methods for the design. In first type the pole-zero transformation from the equivalent analog filter into digital domain takes place or in other words we can say that direct sampling of filter's impulse or step response is done to get the expression for equivalent digital filter. In this a direct relationship between the poles and zeroes for analog and digital filter is established. They are explained in later sections. The main methods in this category are

- Impulse Invariance Method
- Step Invariance Method

In the second type of methods, the characteristic of the analog filter is approximated to get the desired digital filter's response. In this we consider any one out of two properties of the analog filter's magnitude response.

- Area under the curve (magnitude response)
- Slope of the curve (magnitude response)

Approximation of slope and area of the magnitude response give the Euler integration method and Trapezoidal integration method (also known as Bilinear transform) respectively. After analyzing their derivations and conclusions further higher order integration methods can be used to discretized the analog systems.

3.2 Impulse Invariance Method

In this method the sampling is done for the impulse response of the equivalent analog filter. In time domain if the analog filter has an impulse response $H_a(t)$.Then the equivalent response of digital filter is

$$H_d(t) = H_a(nT),$$

Here T is the sampling time interval and n is the integer value.Suppose the analog filter has poles at s= p_i (i=0, 1, 2,….n) then the impulse response of analog filter is

$$H_a(s) = \sum_{i=1}^{n} \frac{R_i}{s-p_i}; \qquad \text{---- (3.1)}$$

Here R_i are the residues at the poles. Taking Inverse Laplace Transform on both the sides

$$H_a(t) = \sum_{i=1}^{n} R_i\, e^{p_i t} u_a(t); \qquad \text{---- (3.2)}$$

Here $u_a(t)$ is the unit step function. Here sampling can be done by replacing the time variable ‘t’ by ‘nT’. It gives the time domain response of the equivalent digital filter.

$$H_d(n) = H_a(nT) = \sum_{i=1}^{n} R_i\, e^{p_i nT} u_a(nT); \qquad \text{----(3.3)}$$

Taking Z-transform on both sides we get the required expression for the digital filter.

$$H_d(z) = Z\left\{\sum_{i=1}^{n} R_i\, e^{p_i nT} u_a(t)\right\} = \sum_{n=0}^{\infty}\left(\sum_{i=1}^{n} R_i\, e^{p_i nT} u_a(t)\right) z^{-n}$$

Interchanging the order of summation we have

$$H_d(z) = \sum_{i=1}^{n}\left(\sum_{n=0}^{\infty} R_i\, e^{p_i nT} u_a(t)\right) z^{-n} = \sum_{i=1}^{n} \frac{R_i}{1-e^{T.p_i} z^{-1}};$$

Hence the digital filter thus obtained has poles at z=$e^{T.p_i}$. If the pole p_i is negative then the analog filter is stable also the digital pole $z = e^{T.p_i}$ is lying in the unit circle. Thus if the analog filter is stable the equivalent digital system will also be stable. Hence IIR filter can be designed by using these steps.

1. Find the poles of the analog system. and calculate the residues at each pole.

2. For each pole there will be a part like $\frac{1}{s-p}$; Replace each pole by its digital equivalent pole that is $\frac{1}{1-z^{-1}e^{pT}}$. Add the entire digital equivalent module to get the digital filter.

3.3 Euler Methods

Figure 3.1 shows the curve y=f(t) with its tangent T and chord C at P(t ,y). If Q(t+Δt,y+Δy) tends towards the P(t,y), chord coincides with the tangent.

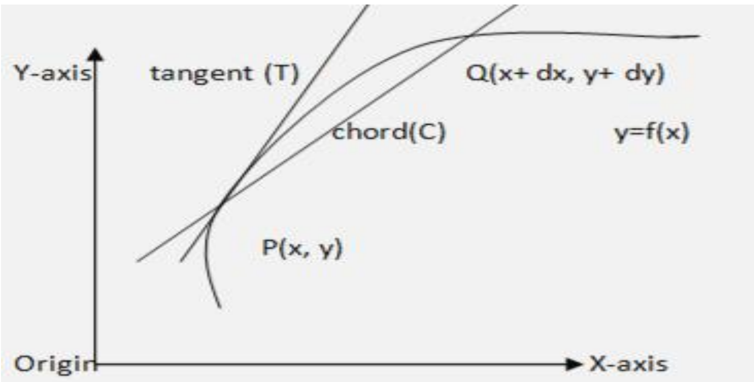


Figure: 3.1 Tangent to a curve

In this method the slope of the analog filter's magnitude response is considered for the approximation. At some point t=T the slope is defined in continuous manner is

$$\frac{\partial y}{\partial t} = \lim_{h\to 0}\frac{y(T+h)-y(T)}{h}$$

In discrete sense it can be defined as

$$\frac{y(nT+T)-y(nT)}{T} \quad \text{(Forward direction) Or} \quad \frac{y(nT)-y(nT-T)}{T} \qquad \text{(Backward direction)}$$

Taking Z-transform in discrete domain we have

Slope = $\frac{z-1}{T}y(z)$ or $\frac{1-z^{-1}}{Tz^{-1}}y(z)$ (Forward direction)

or Slope = $\frac{1-z^{-1}}{T}y(z)$ (Backward direction).

Now comparing both the operator of analog and digital differentiators we have

$s \rightarrow \frac{z-1}{T}$ [Forward Euler differentiator] and

$s \rightarrow \frac{z-1}{Tz}$ [Backward Euler differentiator]

'*s*' is operator for differentiation in analog domain. Hence we have established a relationship in analog differentiator and digital differentiator. If any analog filter is to be converted in IIR digital domain then directly the 's' operator can be substituted by its digital equivalent.

3.4 Trapezoidal Method

In last section the slope of curve is approximated and the two Euler methods for discrete filter design generated. Similarly, if the area of analog filter's magnitude response is approximated then the Trapezoidal Integrator (also called the bilinear transform) is generated, which is explained as follows.

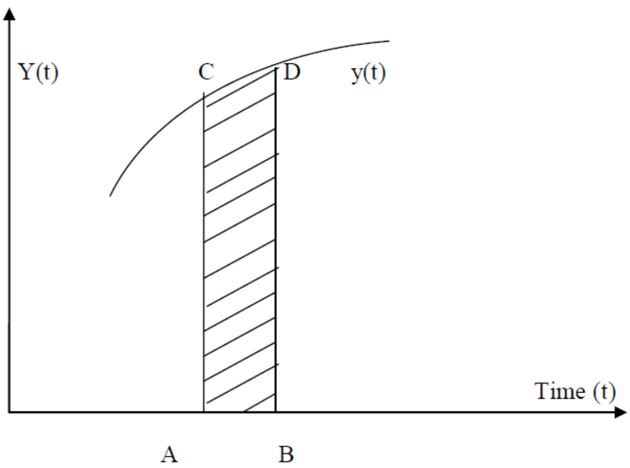


Figure: 3.2 Approximation of area

Figure 3.2 shows the curve y (t) with respect to't'. Section ABCD is a trapezium whose area is given by $\int_a^b y(t)dt$ .But in case of discrete signal the area can be calculated using geometry. "ABCD" is a parallelogram whose area can be calculated as

Area=$\frac{1}{2}$*(sum of two parallel sides)*distance between parallel sides

Area =$\frac{1}{2}$ *[y(n-1)+y(n)]*T [here y(n)=y(nT)]

Hence the area can be approximated as

$$\int_{(n-1)T}^{nT} y(t)dt = \frac{T}{2}[y(n-1) + y(n)] \qquad \text{----(3.4)}$$

Let us assume a simple analog filter $H(s) = \frac{b}{s+a} \Rightarrow \frac{Y(s)}{X(s)} = \frac{b}{s+a}$;

$sY(s) + aY(s) = bX(s);$

Y(s) is the output of the system if the applied input is X(s). Taking inverse Laplace transform on both the sides

$$\frac{dY(t)}{dt} + a.Y(t) = b.X(t)$$

Integrating on both sides from (n-1)T to nT

$$\int_{(n-1)T}^{nT} dY(t) + a\int_{(n-1)T}^{nT} Y(t)dt = b\int_{(n-1)T}^{nT} X(t)dt$$

$$Y(nT) - Y\big((n-1)T\big) + a\frac{T}{2}\Big[Y(nT) + Y\big((n-1)T\big)\Big] = b\frac{T}{2}\Big[X(nT) + X\big((n-1)T\big)\Big]$$

Taking Z transform on both the sides we have

$$Y(z) - Y(z)z^{-1} + \frac{aT}{2}[Y(z) + Y(z).z^{-1}] = \frac{bT}{2}[X(z) + X(z).z^{-1}]$$

$$Y(z)[1 - z^{-1}] + a\frac{T}{2}Y(z)[1 + z^{-1}] = b\frac{T}{2}X(z)[1 + z^{-1}]$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{b}{a + \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]}$$

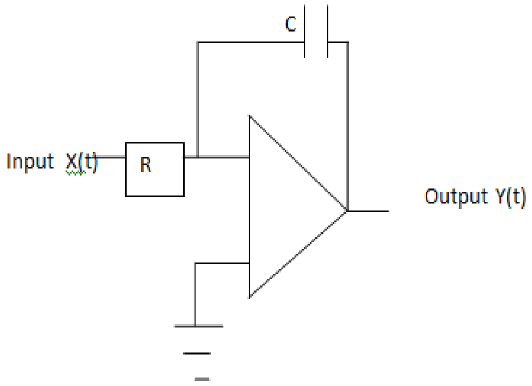
Hence comparing both the transfer functions H (s) and H (z) it can be approximated that

$$s \rightarrow \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]$$

It is called **bilinear transformation** or **Trapezoidal differentiator**. As in analog domain the operator ‘s’ works as a differentiator and ‘1/s’ works as an integrator. This is approximated for digital differentiator/integrator design.

3.5 Integrators

Integrator is a mathematical tool which integrates the input signal within the prescribed limits. Analog integrator can be designed by using proper combination of passive devices. For example the figure shows an analog integrator.



$$y(t) = -\frac{1}{RC}\int_0^t x(t)dt$$

Figure: 3.3 Analog integrator

But in digital domain the design is not so simple. As in the previous section the operator ‘s’(called analog differentiator) is approximated into digital domain. Similarly ‘1/s’ can be approximated to design the digital integrator. Digital integrator can be designed by various methods; these are explained in later section.

3.5.1 Types of Digital Integrator

As per above discussion the digital integrators can be designed in various manners. The digital integrator can be designed by methods of approximation of analog integrator by using the previous discussed method. They are explained in later sections.

3.5.2 Backward Euler Integrator

It is defined as H(z) $=\frac{z-1}{Tz}$ or $\frac{1-z^{-1}}{T}$ here T is sampling time and z=exp (jωT). ω is the digital frequency.

3.5.3 Forward Euler Integrator

It is defined as H (z) $=\frac{z-1}{T}$ or $\frac{1-z^{-1}}{z^{-1}}$; here T is sampling time and z=exp (jωT). ω is the digital frequency.

3.5.4 Trapezoidal Integrator

It is defined as H (z) = $\frac{2(z-1)}{T(z+1)}$; here T is sampling time and z=exp (jωT). ω is the digital frequency.

3.5.5 Higher Order Integrators

As we have seen the integrators with order one. They are called first order integrators. If we observe the higher order numerical integration techniques, we can find other higher order integrators. These can be derived in a combined manner with the help of Cote's Method. This is explained as follows.

Cote's Method

In this method the Lagrange's interpolation formula is used. In the Lagrange's Formula the continuous relation in y=f(x) is approximated into discrete steps. Applying Lagrange's Formula for approximation of f(x) Let us assume $x_i=x_0+nh$.

$$f \approx p(x) = \sum_{k=0}^{n} L_k(x).y_k$$

Figure 3.4 shows the curve y=f(x) , which is sampled at some finite intervals x_0, x_1,x_2,---- x_n.

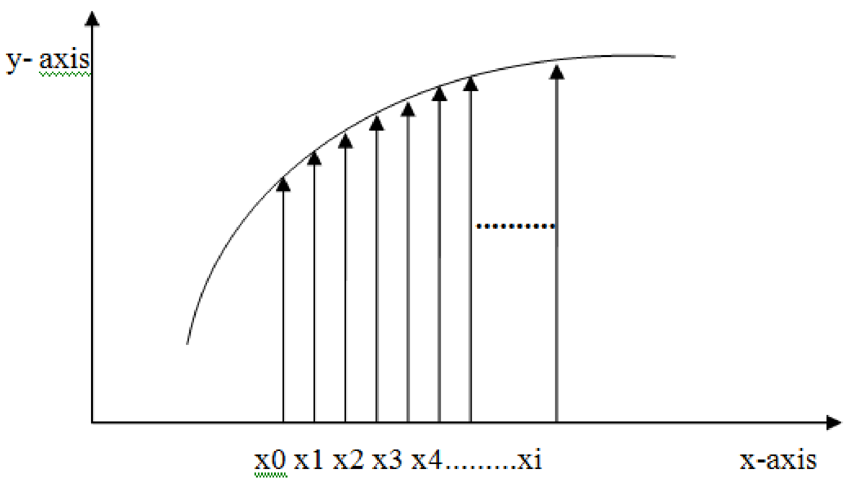


Figure: 3.4 Cote's Method

Where $L_k(x) = \frac{\prod_{j=0}^{n}(x-x_j)}{\prod_{j=0}^{n}(x_k-x_j)}$

Now $\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} p(x)dx = \int_{x_0}^{x_0+nh} [\sum_{k=0}^{n} L_k(x).y_k]dx$

$$= y_0 \int_{x_0}^{x_0+nh} L_0\,(x)dx + y_1 \int_{x_0}^{x_0+nh} L_1\,(x)dx + \cdots + y_n \int_{x_0}^{x_0+nh} L_n\,(x)dx$$

$$= \sum_{k=0}^{n} \left[y_k \int_{x_0}^{x_0+nh} L_k\,(x)dx \right]$$

Putting $x = x_0 + m\lambda \quad hence\;\; dx = md\lambda$

$$\sum_{k=0}^{n} \left[y_k \int_{x_0}^{x_0+nh} L_k\,(x_0 + m\lambda)dx \right] = mn \sum_{k=0}^{n} y_k \frac{1}{n} \int_0^n L_k\; d\lambda$$

Now putting the value of $L_k\,(x)$ from the equation (3)

$$L_k(x_0 + m\lambda) = \frac{\prod_{j=0}^{n}(m+j)}{\prod_{j=0}^{n}(k-j)}\;; \qquad \text{----(3.5)}$$

Say $\frac{1}{n}\int_0^n L_k\; d\lambda\; = C_k^n$ where C_k^n is called as Cote'S number. Table 3.1 shows the values of Cote's Number for some values of n.

Hence the integral is approximated as $\int_a^b y(x)dx = (b-a)\sum_{k=0}^{n} y_k\,.\,C_k^n$

This is called Cote's Formula for Numerical integration. If it is applied in the analysis of previous section, we get

$$\mathrm{H(z)} = \frac{1}{\mathrm{kT}}\left[\frac{\mathrm{z^k - 1}}{\sum_{\mathrm{L=0}}^{\mathrm{n}} \mathrm{z^L . C_k^n}}\right] \qquad \text{----(3.6)}$$

Table 3.1 : Cote's numbers for various 'n'

	C_0^n	C_1^n	C_2^n	C_3^n	C_4^n
n=1	$\frac{1}{2}$	$\frac{1}{2}$			
n=2	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$		
n=3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	
n=4	$\frac{7}{90}$	$\frac{32}{90}$	$\frac{12}{90}$	$\frac{32}{90}$	$\frac{7}{90}$

Few properties of the Cote's number are listed below

1) $C_k^n = C_{n-k}^n$
2) $\sum_{k=0}^{n} C_k^n = 1$

And hence we have got a generalized expression for the Higher order Digital Integrator. The higher order digital integrators are summarized in the table 3.2.

Table 3.2 : Digital Integrator's expression for higher integer order

Order of Integrator	**Integrator expression**	**Name of integrator**
n=1	$\frac{2}{T}\left[\frac{z-1}{z+1}\right]$	Trapezoidal integrator
n=2	$\frac{3}{T}\left[\frac{z^2-1}{z^2+4z+1}\right]$	Simpson's 1/3 integrator
n=3	$\frac{8}{3T}\left[\frac{z^3-1}{z^3+3z^2+3z+1}\right]$	Simpson's 3/8 integrator

3.6 Analysis of Digital Integrators

3.6 Analysis of Digital Integrators

Different integrators are analysed for their frequency response and compare with the ideal integrator. The analysis is as follows.

3.6.1 Backward and Forward Euler Integrator

The expression for the Backward and Forward Euler Integrator is given by

$$H_{bwe}(z) = \frac{Tz}{z-1}\ ; \qquad H_{fwe}(z) = \frac{T}{z-1}\ ;$$

Clearly it can be stated that $|H_{bwe}(z)| = |H_{fwe}(z)|$

Here $z = e^{j\omega T}$ Figure 3.5 shows the magnitude response of the backward Euler integrator compared to ideal integrator.

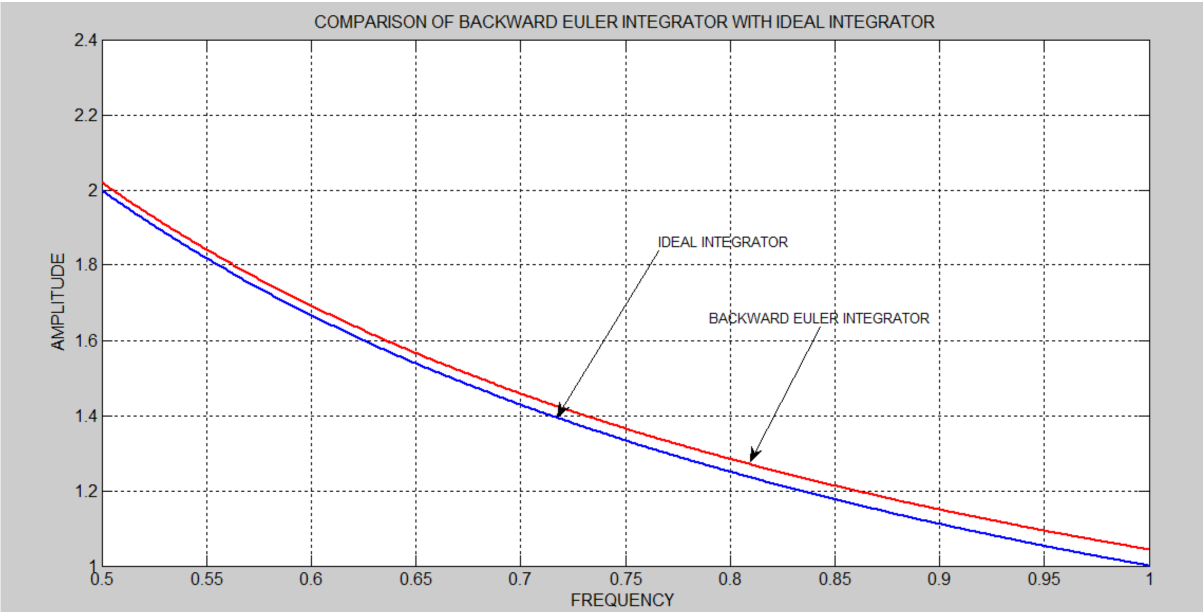


Figure: 3.5 Frequency response of Backward Euler Integrator

3.6.2 Trapezoidal Integrator

The expression for the Trapezoidal Integrator is given by $H_{trap}(z) = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$; Figure 3.6 shows the magnitude response of the trapezoidal integrator compared to the ideal integrator.

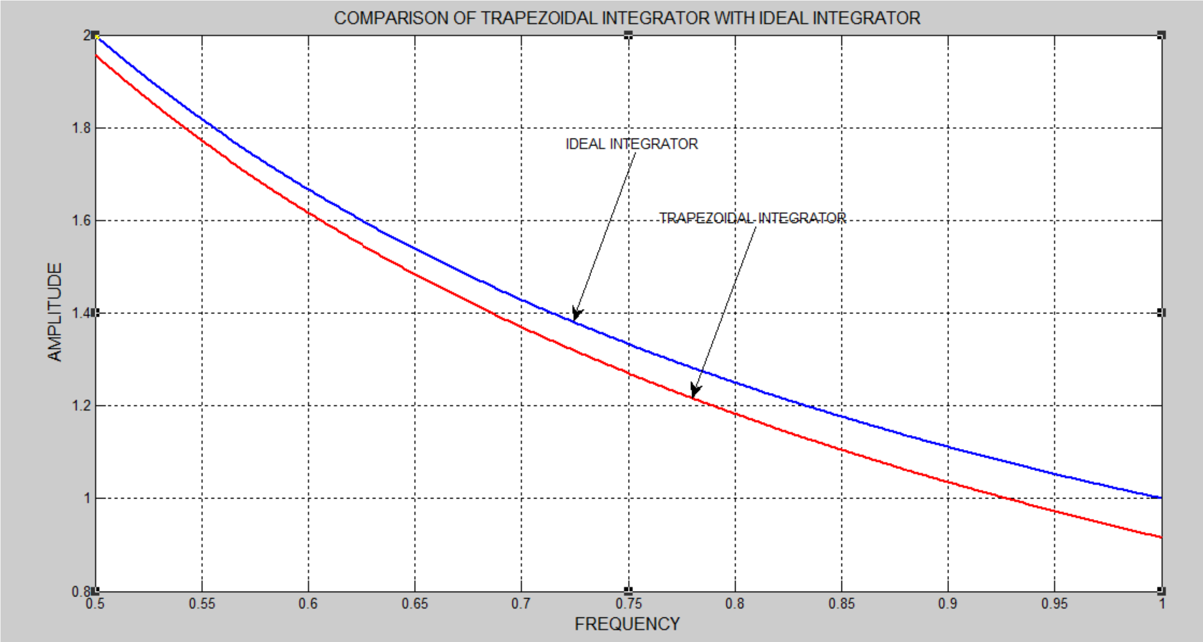


Figure: 3.6 Frequency response of Trapezoidal integrator

3.6.3 Simpson's 1/3 Integrator:

The expression for the Simpson's 1/3 Integrator is given by

$H_{simp}(z) = \frac{3}{T}\left[\frac{z^2-1}{z^2+4z+1}\right]$; Figure 3.7 shows the magnitude response of the trapezoidal integrator compared to the ideal integrator

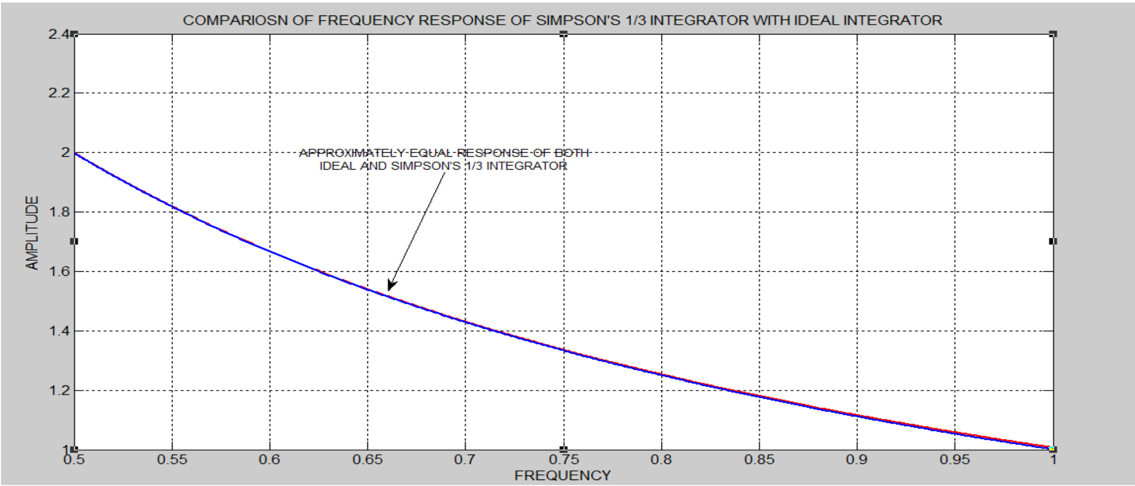


Figure: 3.7 Frequency response of Simpson's 1/3 integrator

3.6.4 Analysis of Error:

For each integrator the magnitude response has plotted and it is observed that behaviour of Simpson's 1/3 integrator is very close to ideal integrator. Figure 4.4 shows the value of error in all above integrator. The Backward Euler integrator and Forward Euler integrator shows the positive error with respect to the ideal integrator. While the Trapezoidal integrator shows the negative error with respect to ideal integrator.

Besides of them the Simpson's integrator which has a higher order than the two discussed, shows approximately ideal response.

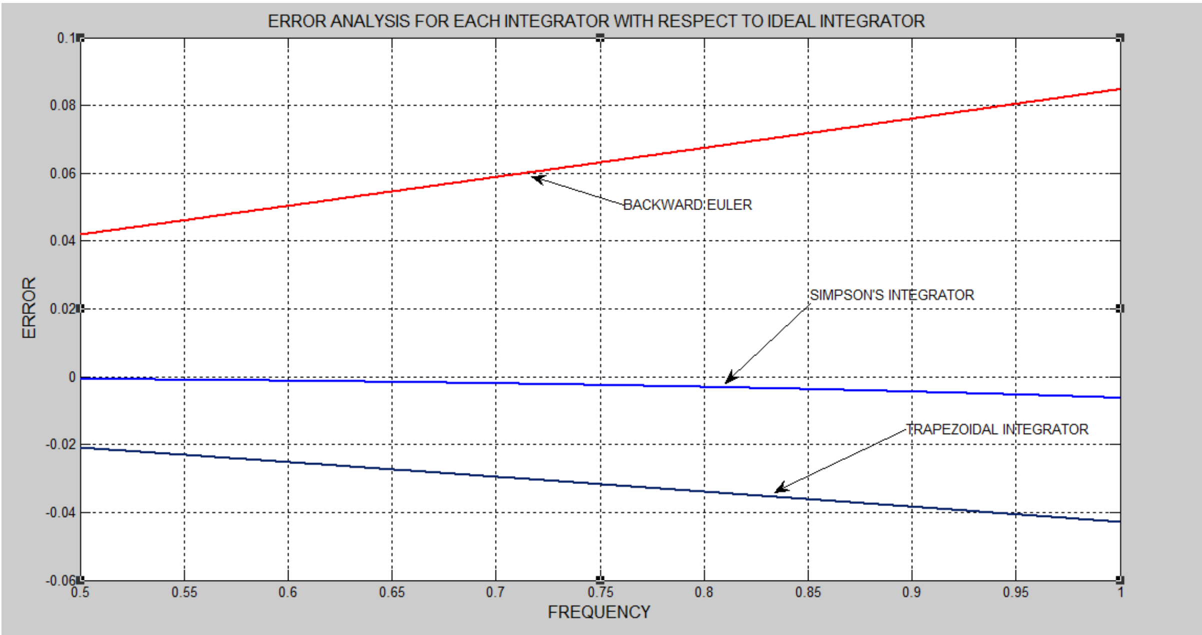


Figure: 3.8 Error Analysis of digital integrators

3.7 Magnitude Response of a filter

Magnitude response of a filter is a plot of magnitude of unit impulse response vs frequency. In analog domain if H(s) is a filter expression, its magnitude response is given as $M(\Omega) = \text{abs}\big(H(j\Omega)\big)$, here the Ω is the analog angular frequency and $(-\infty \leq \Omega \leq \infty)$.

But in digital domain the magnitude response is given by $M(\omega)$ =abs(H(exp(j ω)). An interesting fact about the digital filter's magnitude response is that it is periodic in nature and Its period is 2π. Hence the magnitude response of the digital filter is considered for one period only [0,2π] or [-π,+ π].

3.7.1 Condition for flat magnitude response without response peaks

If H (z) defines an expression for the digital IIR filter. Its magnitude response can be obtained by calculating the absolute value of the H (z). This is defined as

$$\mathrm{abs}(\mathrm{H(z)}) = \sqrt{\mathrm{Re}(\mathrm{H(z)})^2 + \mathrm{Im}(\mathrm{H(z)})^2} \qquad \text{----(3.6)}$$

As the abs (H (z)) will be a function of ω. Here $z=e^{j\omega T}$; where T is sampling time period.

Taking M (ω) = $\mathrm{abs}(\mathrm{H(z)})$;

M(ω) will be an even function of ω due to the term of cos(ω). If the curve of M (ω) is flat in some region about any point (value of ω) and there is no spike in the region then it is required for flatness that all the derivatives of M (ω) should be zero at that point. Hence the constraint for the flatness of M (ω) is given by

$$\lim_{\omega\to\omega_0} \frac{\partial^n}{\partial\omega^n} \mathrm{M}\ (\omega) = 0 \qquad \text{----(3.7)}$$

By above equation we can get the required constraint for the flatness.

Now let $M(\omega) = \frac{N(\omega)}{D(\omega)}$ ----(3.8)

Where $N(\omega)$ & $D(\omega)$ are the numerator and denominator of $M(\omega)$. And also

M (ω)*D (ω) = N (ω)

Differentiating it 'n' times by using Leibnitz theorem we get

$$C_0^n\ \mathrm{M}(\omega)\mathrm{D}_n(\omega) + C_1^n\ \mathrm{M}_1(\omega)\mathrm{D}_{n-1}(\omega) + \text{-------} + C_n^n\ \mathrm{M}_n(\omega)\mathrm{D}(\omega) = \mathrm{N}_n(\omega) \qquad \text{----(3.9)}$$

For the purpose of easiness first finding the constraint about the point $\omega = 0$ .We have to apply limit $\omega \to 0$. One another interesting point is that if a function is even, continuous and differentiable at ω=0, then certainly the all odd derivative will be zero at origin. And M (ω), N (ω) & D (ω) are even function of ω .So all odd derivatives of each of them will be zero. Applying the limit $\omega \to 0$

$$\lim_{\omega\to 0} M(\omega).D_n(\omega) = \lim_{\omega\to 0} N_n(\omega) \qquad \text{---(3.10)}$$

Assuming the normalized value of M (ω) to be unity.

$$\lim_{\omega \to 0} D_n(\omega) = \lim_{\omega \to 0} N_n(\omega)$$

Now assuming the general expression for digital filter

$$\mathrm{H(z)} = \frac{\mathrm{B_n z^n + B_{n-1} z^{n-1} + \cdots + B_0}}{\mathrm{A_m z^m + A_{m-1} z^{m-1} + \cdots + A_0}}$$

N (ω) $= \mathrm{abs}(\mathrm{B_n z^n + B_{n-1} z^{n-1} + \cdots + B_0})$

And D (ω) $= \mathrm{abs}(\mathrm{A_m z^m + A_{m-1} z^{m-1} + \cdots + A_0})$

Applying above values to the equation (10) and applying limit $\omega \to 0$

$A_m[\, m^k A_0 + (m-1)^k A_1 + \text{---} + A_{m-1}] + A_{m-1}[\, (m-1)^k A_0 + (m-2)^k A_1 + \text{---} + A_{m-2}] + \text{---} + A_1 A_0 =$

$B_n[\, n^k B_0 + (n-1)^k B_1 + \text{---} + B_{n-1}] + B_{n-1}[\, (n-1)^k B_0 + (n-2)^k B_1 + \text{---} + B_{n-2}] + \text{---} + B_1 B_0$ ----(3.11)

Applying m=2 and n=2, for second order filter

$B_2(4B_0+B_1) + B_1B_0 = A_2(4A_0+A_1) + A_1 A_0$ ----(3.12)

The LPF can be defined as a filter that passes the frequencies less than a cut off frequency. Hence the max-flat condition is same as derived in previous section.

If the H(z) represents a LPF,

$$\mathrm{H(z)} = \frac{B_2 z^2 + B_1 z + B_0}{A_2 z^2 + A_1 z + A_0} \;;$$ ----

(3.13)

Condition for max-flat magnitude response is

$$\boxed{B_2(4B_0+B_1) + B_1B_0 = A_2(4A_0+A_1) + A_1}$$ ----(3.14)

3.7.2 Design Of Max-Flat Digital Low Pass Filter For High Speed Control Systems (Reshmin B. I., 2010)

In previous section it has been shown that a certain relationship among the coefficients of digital IIR filter for max-flat response can be established. Here it is required to design the digital LPF with max-flat response and without any resonance peak for minimum settling time. Speed of discrete system is considered as the convergence time of its step response. As we know that the response of the Butterworth analog filter is nearly flat in the pass band region. While the Chebyshev and Elliptical filters are not flat in the prescribed region. Hence to design the digital IIR LPF with a flat magnitude response it is required to use the analog Butter worth filter for approximation. The prototype for the analog low pass filter is given by

$$H(s) = \frac{1}{B.T_1 .s^2+B.s+1} = \frac{\omega_0^2}{s^2+2\xi\omega_0+\omega_0^2}; \qquad \text{----(3.15)}$$

Where ω_0 is the natural frequency and the ξ is damping ratio defined as in terms of coefficients B & T_1.

$$\omega_0 = \sqrt{\frac{1}{BT_1}} \quad and\ \xi = \sqrt{\frac{B}{4T_1}} \qquad \text{---(3.16)}$$

Now to design the digital low pass filter for flat magnitude and having no peaks, the author applied different methods in the prototype of LPF. After applying different methods the author applied the constraints for the flat magnitude response and corrected the parameters of LPF. Thus he got various LPF expressions by various methods. Then he compares their step-responses for their convergence time. Along with the study of step response the author did found the error at cut-off frequency for each case. His results are as discussed here.

(i).Backward Euler Integrator:

The expression for backward Euler Integrator is given by $H_{bwe}(z) = \frac{z-1}{Tz}$ hence replacing 's' by "$\frac{z-1}{Tz}$" and comparing with the standard equation 3.13 and then applying desired condition for flatness equation 3.14 we get $B=2T_1-T$. Consider a case when T=1 s and T_1=1 s. We have B=1; Figure 3.5 and 3.6 show the magnitude plot and step response for $H_{bwe}(z)$.

$$H_{bwe}(z) = \frac{z^2}{3z^2-3z+1};$$

As it is clear from figure the response is flat in pass band the magnitude response is flat. And also the magnitude response does not have any resonance peak.

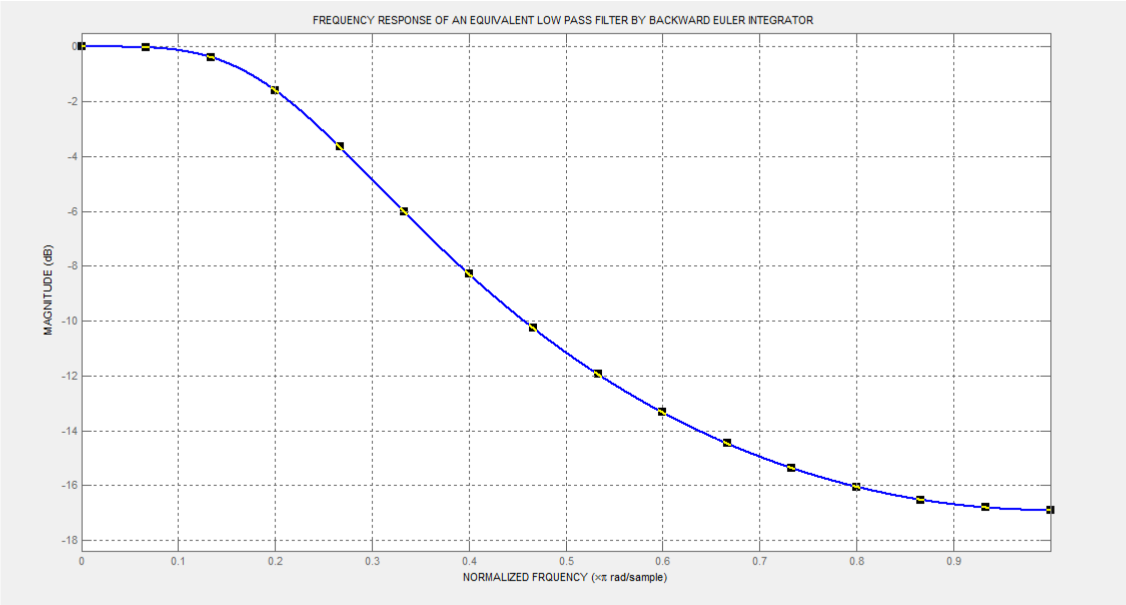


Figure: 3.9 Frequency response of LPF designed by backward Euler integrator

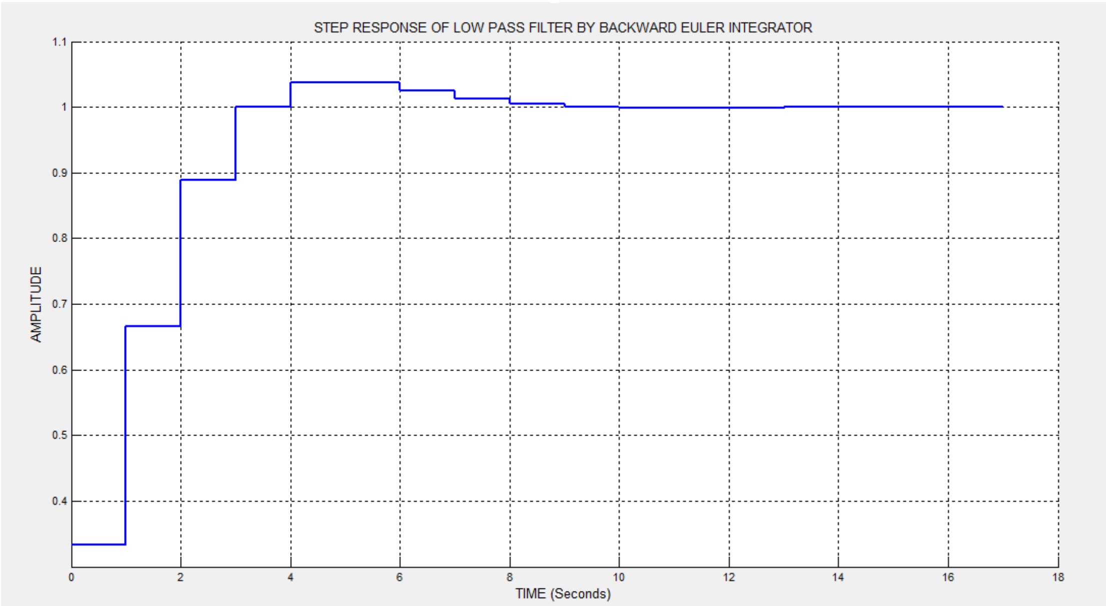


Figure: 3.10 Step response of LPF designed by backward Euler integrator (Settling time = 07 Seconds)

Figure shows the step response of LPF mentioned above. As the figure indicates that the step response is tending toward the constant value 1. Thus it is stable filter.

(ii) Forward Euler Integrator:

The expression for backward Euler Integrator is given by $H_{fwe}(z) = \frac{z-1}{T}$ .Hence replacing 's' by "$\frac{z-1}{T}$" and Comparing with the standard equation 3.13 and then applying desired condition for flatness equation 3.14 we have $B=2T_1+T$;Consider a case when T=1 s and T_1=1 s; we get B=3. Figure: 3.7 , 3.8 shows the magnitude plot & step response for $H_{fwe}(z)$.

$$H_{fwe}(z) = \frac{1}{2z^2-3z+1};$$

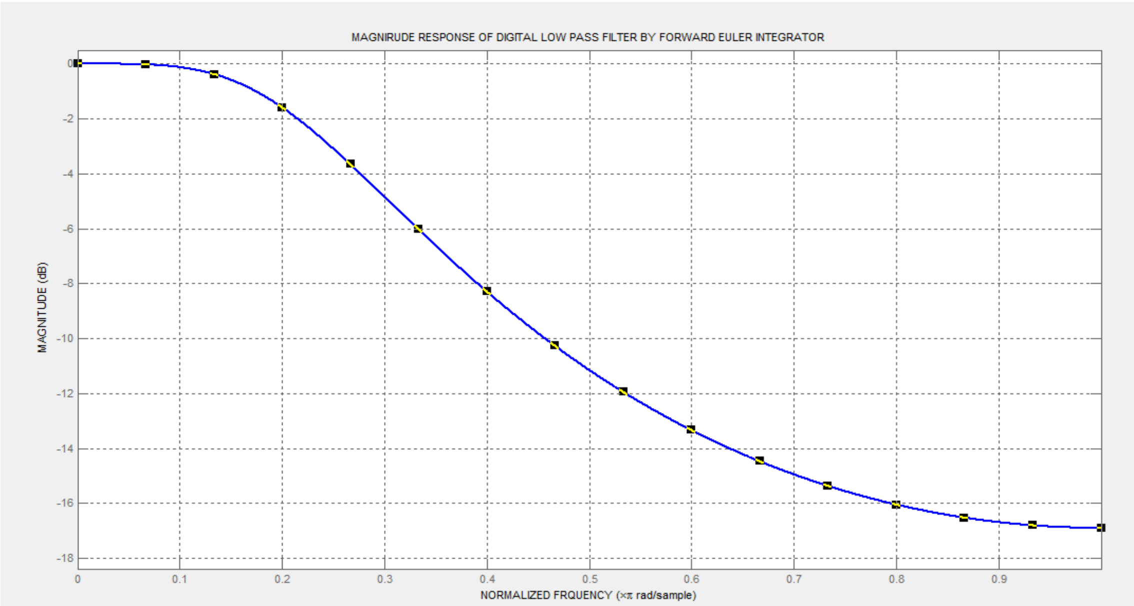


Figure: 3.11 Frequency response of LPF by forward Euler integrator

The figure shows that the magnitude response of the filter by Forward Euler Method has a flat zone in its pass band and does not contain any resonance peak in anywhere else.

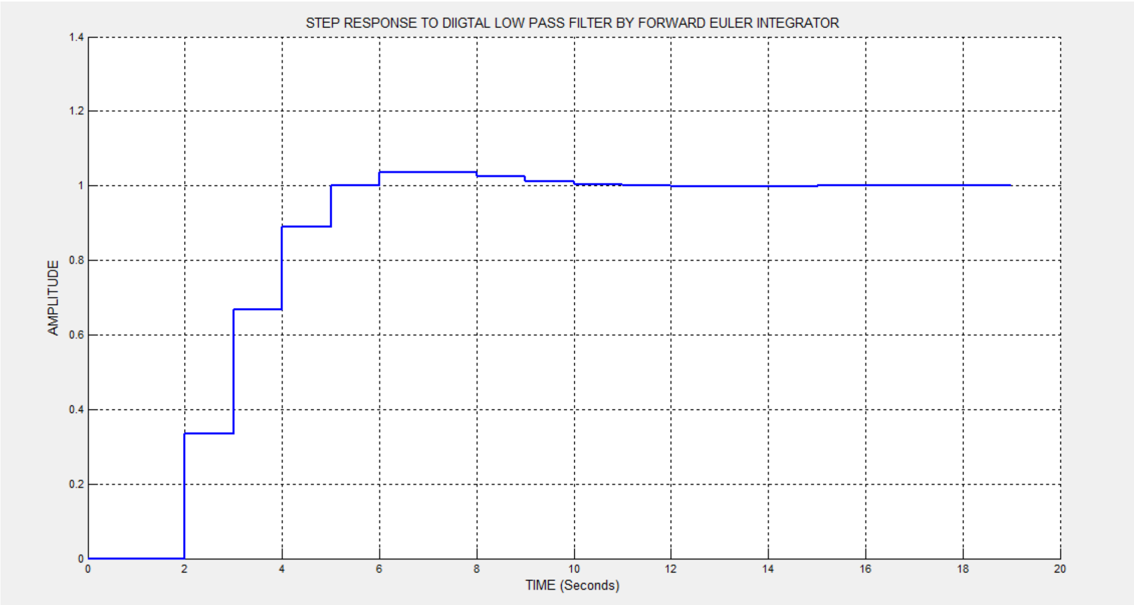


Figure: 3.12 Step response of LPF designed by forward Euler integrator

Figure shows the step response of LPF mentioned above. As the figure indicates that the step response is tending toward the constant value 1. Thus it is stable filter.

(iii) Trapezoidal Integrator:

The expression for Trapezoidal Integrator is given by $H_{trap}(z) = \frac{2(z-1)}{T(z+1)}$ hence replacing 's' by "$\frac{2(z-1)}{T(z+1)}$" and Comparing with the standard equation 3.13 and then applying desired condition for flatness equation 3.14 we have $B=2T_1$; Consider a case when T=1 s and T_1=1 s gives B=2. Figure 3.9 and 3.10 show the magnitude response and step response for $H_{trap}(z)$.

$$H_{trap}(z) = \frac{z^2+2z+1}{13z^2-14z+5};$$

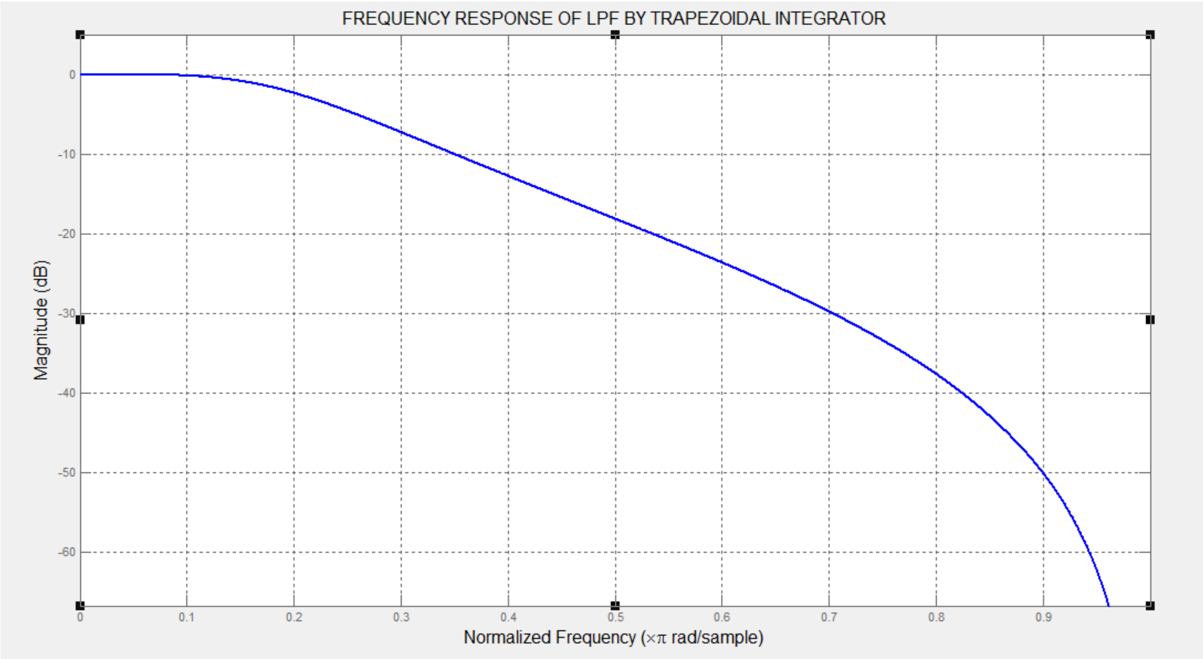


Figure: 3.13 Frequency response of LPF by trapezoidal integrator

The figure shows that the magnitude response of the filter by Forward Euler Method has a flat zone in its pass band and does not contain any resonance peak in anywhere else.

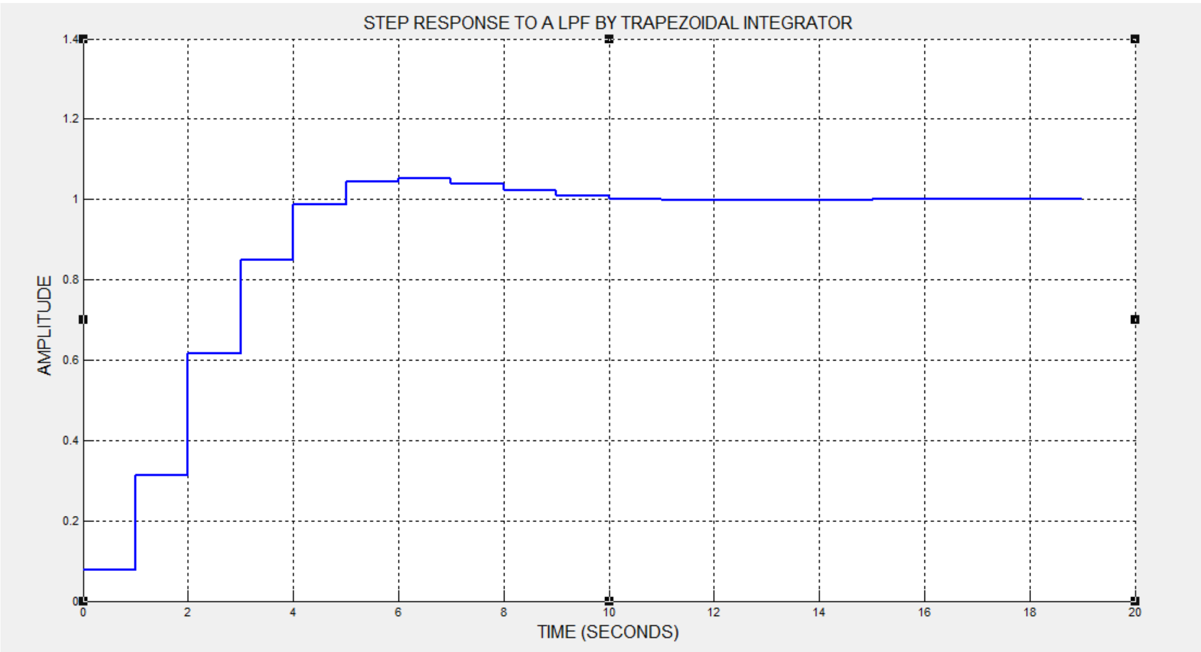


Figure: 3.14 Step response of LPF by Trapezoidal integrator (Settling time = 09 sec.)

Figure shows the step response of LPF mentioned above. As the figure indicates that the step response is tending toward the constant value 1. Thus it is stable filter.

3.7.3 Results of comparison of magnitude response of the digital filter obtained by different methods

If the digital filters hence designed by each method and the results of comparison of them are shown in the following table 3.3

Table 3.3 : Comparison of different LPF for settling time

Name of method applied	**Expression for filter (T=1,T_1=1)**	**Settling time (in seconds)**
Backward Euler Method	$\frac{z^2}{3z^2 - 3z + 1}$	07 seconds
Forward Euler Method	$\frac{1}{2z^2 - 3z + 1}$	09 seconds
Trapezium Method	$\frac{z^2 + 2z + 1}{13z^2 - 14z + 5}$	09 seconds

Hence it can be concluded that for the purpose of Least convergence time the backward Euler method is the best method to design the LPF with flat magnitude response and without any peak in the magnitude response.

3.8 Application of method of impulse invariance method

As in previous section 3.5 we have seen that Reshmin B. I. (2010) designed a low pass filter with max-flat magnitude response. He observed that the LPF constructed by Backward Euler method gives the smallest convergence time. Hence it can be employed for high speed control systems. In the method of Impulse Invariance the poles of the analog filter are find first. After that the poles are transformed into digital poles to construct the digital filter. The prototype for the analog filter is given by

$$H(s)= \frac{1}{BT_1 s^2+Bs+1} \ ;$$

Poles of above LPF are

$$S = \frac{-1}{2T_1} \pm \sqrt{\left[\frac{1}{4T_1^2} - \frac{1}{BT_1}\right]}$$

Let the poles are a & b where

$$a = \frac{-1}{2T_1} + \sqrt{\left[\frac{1}{4T_1^2} - \frac{1}{BT_1}\right]} \quad and\ b = \frac{-1}{2T_1} - \sqrt{\left[\frac{1}{4T_1^2} - \frac{1}{BT_1}\right]}$$

$$H(s) = \frac{1}{BT_1(s-a)(s-b)} = \frac{1}{BT_1}\left[\frac{1}{s-a} - \frac{1}{s-b}\right]\frac{1}{(a-b)} = \frac{1}{\sqrt{B^2 - 4BT_1}}\left[\frac{1}{s-a} - \frac{1}{s-b}\right]$$

Applying pole-zero transform

$$\frac{1}{s-p} \to \frac{1}{1 - e^{pT}z^{-1}};\ H(\mathrm{z}) = \frac{1}{\sqrt{B^2 - 4BT_1}}\left[\frac{1}{1 - e^{aT}z^{-1}} - \frac{1}{1 - e^{bT}z^{-1}}\right]$$

$$= \frac{1}{\sqrt{B^2 - 4BT_1}}\left[\frac{z}{z - e^{aT}} - \frac{z}{z - e^{bT}}\right] = \frac{1}{\sqrt{B^2 - 4BT_1}}\left[\frac{z\left(e^{aT} - e^{bT}\right)}{z^2 - z(e^{aT} + e^{bT}) + e^{(a+b)T}}\right]$$

Applying condition for flatness B_2 $(4B_0+B_1)$ + B_1B_0=A_2 $(4A_0+A_1)$ + A_1 A_0 and putting T=1sec and T_1=1 sec. We have

$$(4e^{(a+b)T} - (e^{aT} + e^{bT})) - e^{(a+b)T}\left(e^{aT} + e^{bT}\right) = 0$$

$$\text{here } a, b = -0.5 \pm \sqrt{0.25 - \tfrac{1}{B}};$$

$$\Rightarrow e^{-0.5}\left[e^{\sqrt{0.25 - \frac{1}{B}}} + e^{-\sqrt{0.25 - \frac{1}{B}}}\right] = \frac{4e^{-0.1}}{1 + e^{-0.1}};$$

Hence $e^{\sqrt{0.25 - \frac{1}{B}}}$ = 0.998+j 0.126 .

It shows that value of B is complex, which is undesirable. Hence our I^st assumption was wrong. Taking II^nd assumption that is

$$a = \frac{-1}{2T_1} + j\sqrt{\left[\frac{1}{BT_1} - \frac{1}{4T_1^2}\right]} \quad b = \frac{-1}{2T_1} - j\sqrt{\left[\frac{1}{BT_1} - \frac{1}{4T_1^2}\right]}$$

(Both a , b are complex numbers lying in left half s-plane). Let us assume $a, b = m \pm jn$

$$m = \frac{-1}{2T_1} \; and \; n = \sqrt{\left[\frac{1}{BT_1} - \frac{1}{4T_1^2}\right]}$$

Now the digital filter is

$$H(z) = \frac{1}{(a-b)}\left[\frac{z(e^{aT} - e^{bT})}{z^2 - z(e^{aT} + e^{bT}) + e^{(a+b)T}}\right]$$

$$H(z) = \frac{1}{(a-b)}\left[\frac{z(e^{mT})\left(e^{jnT} - e^{jnT}\right)}{z^2 - z(e^{mT})(e^{jnT} + e^{jnT}) + e^{2mT}}\right]$$

Applying $e^{jnT} - e^{jnT} = 2jsin(nT) \; and \; e^{jnT} + e^{jnT} = 2\cos(nT)$

$$H(z) = \frac{\frac{z.e^{mT}\sin(nT)}{n}}{z^2 - 2e^{mT}\, z.cos\,(nT) + e^{2mT}} \;;$$

Applying constraint for flatness; $B_2(4B_0+B_1) + B_1B_0 = A_2(4A_0+A_1) + A_1 A_0$

We have cos(nT)= $\frac{2e^{mT}}{1+e^{2mT}}$ putting T=1 sec and T_1=1 sec. Hence Cos(nT) =0.887 or nT= $\cos^{-1}(0.887)$ =0.48 radians where $n = \left[\frac{1}{BT_1} - \frac{1}{4T_1^2}\right]^{1/2}$ Hence we get B=2.080 (approx) now the digital filter is

$$H_{imp}(z) = \frac{0.584z}{z^2 - 1.076z + 0.3678}$$

In the next chapter Results and Discussion the Magnitude response and the Step response are given. And the comparison of results is indicated in table.

3.9 Design Of Max-Flat Digital High Pass Filter With High Speed

Now in similar fashion the design of HPF is done. The complete analysis is following. At first the condition for flatness is derived for HPF. And then all the methods are employed for a IIR HPF design.

A. Condition of max-flat digital HIGH PASS FILTER

The HPF is a filter that passes the frequencies greater than a cut-off frequency. It is clear that the magnitude response of HPF can be obtained by simply applying a shift of π.

Hence let us assume the H(z) represents a HPF.

$$H(z) = \frac{B_2 z^2 + B_1 z + B_0}{A_2 z^2 + A_1 z + A_0} ;$$

$$H(j\omega) = \frac{B_2 e^{2j\omega T} + B_1 e^{j\omega T} + B_0}{A_2 e^{2j\omega T} + A_1 e^{j\omega T} + A_0} ;$$

Now replacing $\omega T = \omega' T + \pi$, we have

$$H(j\omega') = \frac{B_2 e^{2j(\omega' \mathrm{T}+\pi)} + B_1 e^{j(\omega' \mathrm{T}+\pi)} + B_0}{A_2 e^{2j(\omega' \mathrm{T}+\pi)} + A_1 e^{j(\omega' \mathrm{T}+\pi)} + A_0} ;$$

$$H(j\omega') = \frac{B_2' e^{2j\omega T} + B_1' e^{j\omega T} + B_0'}{A_2' e^{2j\omega T} + A_1' e^{j\omega T} + A_0'} ;$$

Comparing we get

$$B_2' = B_2 . e^{2j\pi} \; ; \; B_1' = B_1 . e^{j\pi} \; ; \; B_0' = B_0$$
$$A_2' = A_2 . e^{2j\pi} \; ; \; A_1' = A_1 . e^{j\pi} \; ; \; A_0' = A_0$$

In special case putting T=1

$$B_2' = B_2 \; ; \; B_1' = -B_1 \; ; \; B_0' = B_0$$
$$A_2' = A_2 \; ; \; A_1' = -A_1 \; ; \; A_0' = A_0$$

Hence the condition for max-flat response is

$$\boxed{B_2 (4B_0 - B_1) - B_1 B_0 = A_2 (4A_0 - A_1) - A_1} \qquad \text{-------(3.17)}$$

B. Design of high speed digital HPF:

To design the digital IIR HPF with a flat magnitude response it is required to use the analog Butter worth filter for approximation. The prototype for the analog high pass filter is given by

$$\mathrm{H(s)} = \frac{\mathrm{s}^2}{\mathrm{B.T_1}\, .s^2 + \mathrm{B.s} + 1} = \frac{\mathrm{s}^2}{s^2 + 2\xi\omega_0 + \omega_0^2}$$

(i). Backward Euler Integrator:

The expression for backward Euler Integrator is given by $H_{bwe}(z) = \frac{z-1}{Tz}$ hence replacing 's' by "$\frac{z-1}{Tz}$" we have

$$H(z) \quad = \frac{(\frac{z-1}{Tz})^2}{BT_1(\frac{z-1}{Tz})^2 + B\left(\frac{z-1}{Tz}\right) + 1}$$

$$= \frac{z^2 - 2z + 1}{BT_1(z-1)^2 + BTz(z-1) + T^2z^2}$$

Now taking a case when $T_1 = 1$ sec and $T = 1$sec

$$H(z) = \frac{z^2 - 2z + 1}{B(z-1)^2 + Bz(z-1) + z^2} = \frac{z^2 - 2z + 1}{z^2(2B+1) - 3Bz + B}$$

Comparing with the standard equation

$$H(z) = \frac{B_2z^2 + B_1z + B_0}{A_2z^2 + A_1z + A_0}$$

And applying condition for max-flat magnitude response

$$B_2(4B_0 - B_1) - B_1B_0 = A_2(4A_0 - A_1) - A_1A_0$$

We get $(2B+1)[4B+3B]+3B^2=8$ or $7B^2+7B-8=0$ or $B = 0.51$ (taking only + ve value)

$$H(z) = \frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5}$$

From denominator $2z^2 - 1.5z + 0.5 = 0$ or $z = \frac{3}{8} \pm j\frac{\sqrt{7}}{8}$;

$\because |z| = \frac{1}{2}$ (< 1) Hence the filter is stable. The magnitude response and the Step response are discussed in nest chapter Results and Discussion.

(ii). Forward Euler Integrator:

The expression for backward Euler Integrator is given by $H_{fwe}(z) = \frac{z-1}{T}$ hence replacing 's' by "$\frac{z-1}{T}$" we have

$$H(z) \quad = \frac{(\frac{z-1}{T})^2}{BT_1(\frac{z-1}{T})^2 + B\left(\frac{z-1}{T}\right) + 1}$$

$$= \frac{z^2 - 2z + 1}{BT_1(z-1)^2 + BT(z-1) + T^2}$$

Now taking a special case when $T_1 = 1$ and $T = 1$

$$H(z) = \frac{z^2 - 2z + 1}{B(z-1)^2 + B(z-1) + z^2} = \frac{z^2 - 2z + 1}{Bz^2 - Bz + 1}$$

Comparing with the standard equation

$$H(z) = \frac{B_2 z^2 + B_1 z + B_0}{A_2 z^2 + A_1 z + A_0}$$

And applying condition for max-flat magnitude response

$$B_2 (4B_0 - B_1) - B_1 B_0 = A_2 (4A_0 - A_1) - A_1 A_0$$

We get $B(4+B)+B = 8$ or $B^2+5B-8=0$ or $B=1.28$ (taking only +ve value)

$$H(z) = \frac{z^2 - 2z + 1}{1.28z^2 - 1.28z + 1}$$

From denominator $1.28z^2 - 1.28z + 1 = 0$

$$z = 0.5 \pm 0.73j \ ;$$

$$\because |z| < 1$$

Hence the filter is stable. Figure show the magnitude plot and step response of the filter. The magnitude response and the Step response are discussed in nest chapter Results and Discussion.

(iii). Trapezoidal Integrator

The expression for backward Euler Integrator is given by $H_{trap}(z) = \frac{z-1}{T}$ hence replacing ‘s’ by "$\frac{z-1}{T}$" we have

$$H(z) \quad = \quad \frac{\left[\frac{2(z-1)}{T(z+1)}\right]^2}{BT_1\left[\frac{2(z-1)}{T(z+1)}\right]^2 + B\left[\frac{2(z-1)}{T(z+1)}\right] + 1}$$

$$= \frac{4(z-1)^2}{4BT_1(z-1)^2 + 2BT(z-1)(z+1) + T^2(z+1)^2}$$

Now taking a special case when $T_1 = 1$ and $T = 1$

$$H(z) = \frac{4(z^2-2z+1)}{4B(z-1)^2 + 2B(z+1)(z-1) + (z+1)^2} = \frac{4z^2-8z+4}{(6B+1)z^2 + (2-8B)z + (2B+1)}$$

Comparing with the standard equation

$$H(z) = \frac{B_2z^2+B_1z+B_0}{A_2z^2+A_1z+A_0}$$

And applying condition for max-flat magnitude response

$B_2(4B_0-B_1)-B_1B_0=A_2(4A_0-A_1)-A_1A_0$ We get

$(6B+1)(8B+4+8B-2) + (2B+1)(8B-2)=128$ or $112B^2+32B-128=0$

$B=0.9375$ (approx.) Hence the filter is

$$H(z) = \frac{4z^2-8z+4}{6.625z^2-5.5z+2.875}$$

From denominator $6.625z^2 - 5.5z + 2.875 = 0$

$$z = 0.42 \pm 0.512j$$
$$|z| = 0.663 \ (< 1)$$

Hence the filter is stable. The magnitude response and the Step response are discussed in nest chapter Results and Discussion. In next chapter Results and Discussion the final comparison is done for the Low convergence time and flat magnitude response.

Chapter 4

Results & Discussion

As last chapter indicates the analysis of digital integrators/differentiators and all mathematical model of LPF and HPF designed for the optimum condition that is Flat magnitude response without any resonance peak. In the present chapter the results of previous works are discussed. They are explained as follows.

4.1 Results for Analysis of Digital Low Pass Filter Design

As we have seen that the digital LPF designed by Impulse Invariant method for fulfilling the constraint for Flat magnitude response and no resonance peak is

$$H_{imp}(z) = \frac{0.584\, z}{z^2 - 1.076z + 0.3678}$$

Figure 4.1 and 4.2 show the magnitude response and the step response for the filter $H_{imp}(z)$.

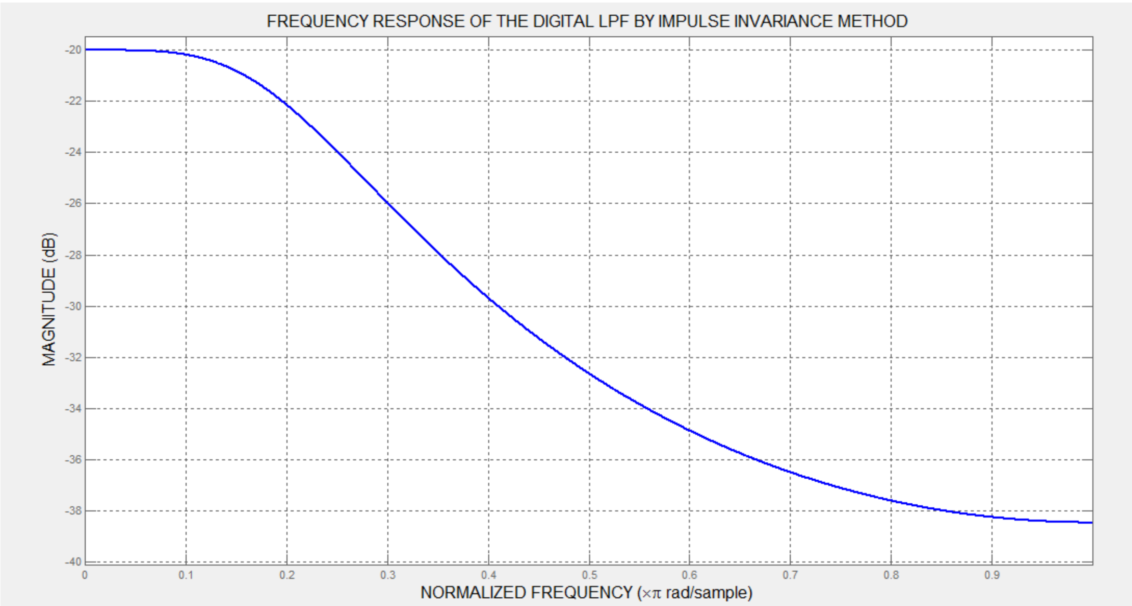


Figure: 4.1 Magnitude response of digital LPF designed by Impulse Invariance method

As it is clear from figure that the filter designed by the Impulse invariance method has a flat magnitude response in pass band and it does not have any resonance peak .

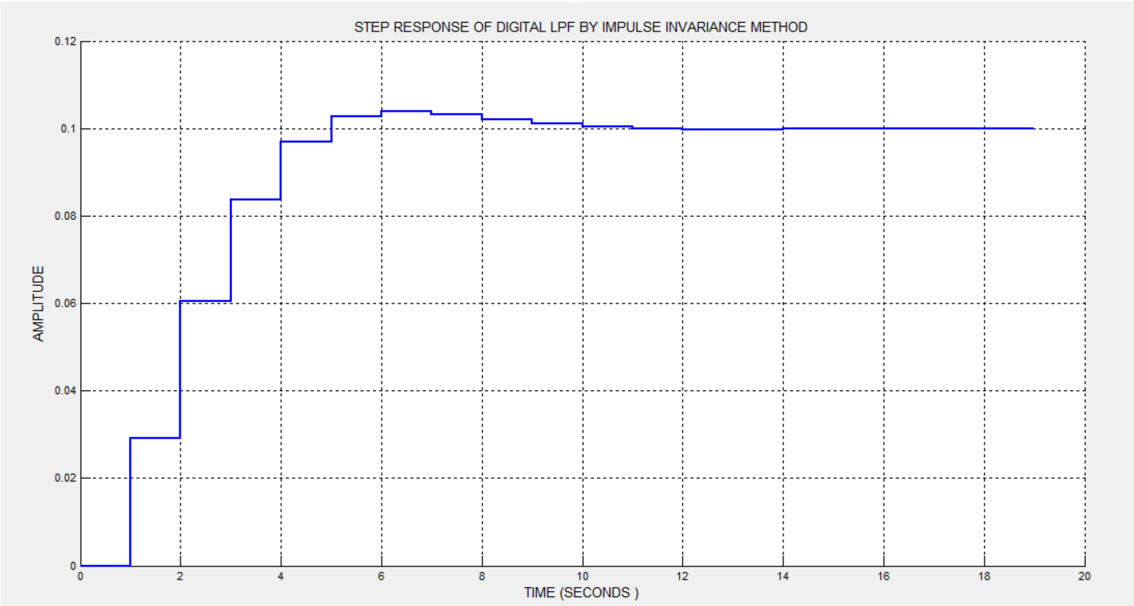


Figure: 4.2 Step response of digital LPF designed by Impulse Invariant method (settling time = 09 seconds)

As it is clear from figure the step response of the LPF is tending toward a finite value that is 0.1 hence the filter is stable.

4.2 Comparison of Different Design Techniques For LPF Design

By analyzing the convergence time that is 0.8 seconds. It is obvious that Backward Euler Method is the best method for high speed discrete systems design.

Table: 4.1 Comparison of different methods for LPF design

Applied method	**Digital filter expression (T=0.1 s & T_1=0.1 s)**	**Settling time (in Seconds)**
Backward Euler Integration	$\frac{z^2}{3z^2 - 3z + 1}$	07 seconds
Forward Euler Integration	$\frac{1}{3z^2 - 3z + 1}$	09 seconds
Trapezoidal Integration	$\frac{z^2 + 2z + 1}{13z^2 - 14z + 5}$	09 seconds
Impulse Invariance Method	$\frac{0.28\, z}{9.6\, z^2 - 10.33z + 3.532}$	08 seconds

Table 4.1 shows the results of different methods application in designing of digital LPF. By studying all the methods, it can be said that the filters generated by Backward Euler integration method gives a faster convergence time as compared to other.

4.3 Results for Design of Max Flat Digital High Pass Filter with Minimum Convergence Time

In the last chapter, the constraint for flat magnitude response is find by using the graph shifting property. And hence the constraint for general digital HPF filter is

$$B_2(4B_0-B_1)-B_1B_0=A_2(4A_0-A_1)-A_1A_0$$

By using the above constraint the filter is designed. The results for the designs are as follows.

A. Backward Euler Method

The filter expression designed by this method is

$$H(z) = \frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5}$$

The magnitude response and step response are as follows

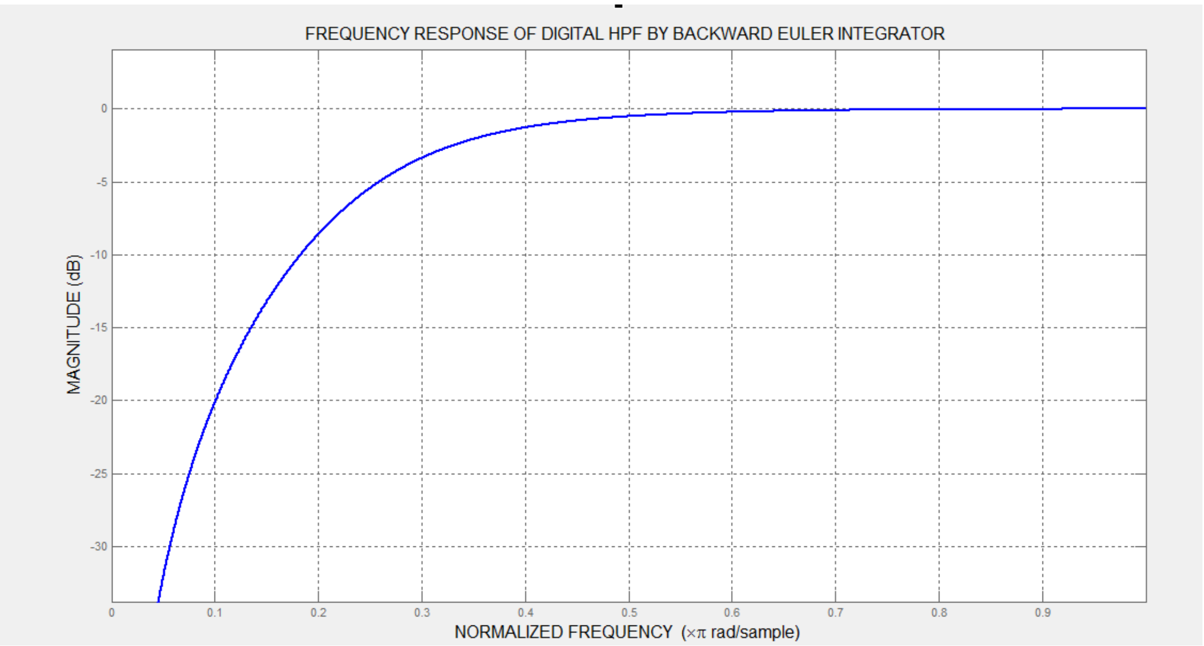


Figure: 4.3 Frequency response of HPF designed by backward Euler integrator

As the figure indicated the magnitude response of the HPF designed by the Backward Euler method has a flat zone in its pass band and also it does not contain any resonance peak. Hence we can see the constraint designed by our assumption is correct.

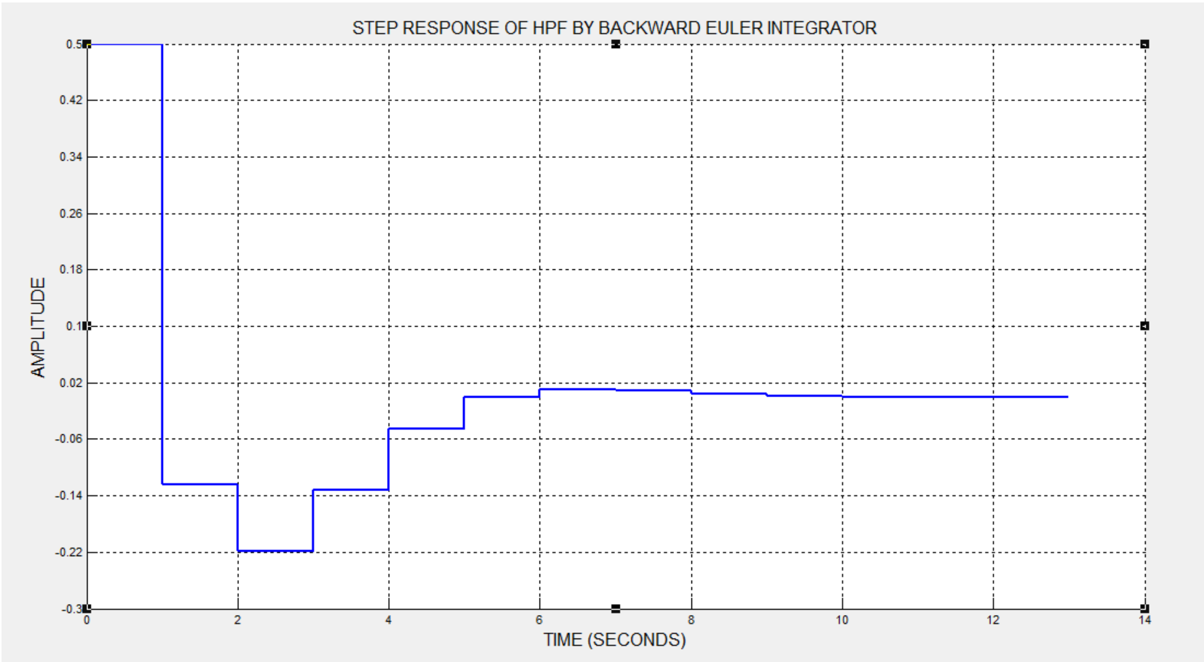


Figure: 4.4 Step response of HPF designed by backward Euler integrator

As it is clear from figure the step response of the HPF is tending toward a finite value that is 0. Hence the filter is stable.

B. Forward Euler Method

The filter expression designed by this method is

$$H(z) = \frac{z^2 - 2z + 1}{1.28z^2 - 1.28z + 1}$$

The magnitude response and step response are as follows. As the figure indicated the magnitude response of the HPF designed by the Forward Euler method has a flat zone in its pass band and also it does not contain any resonance peak.

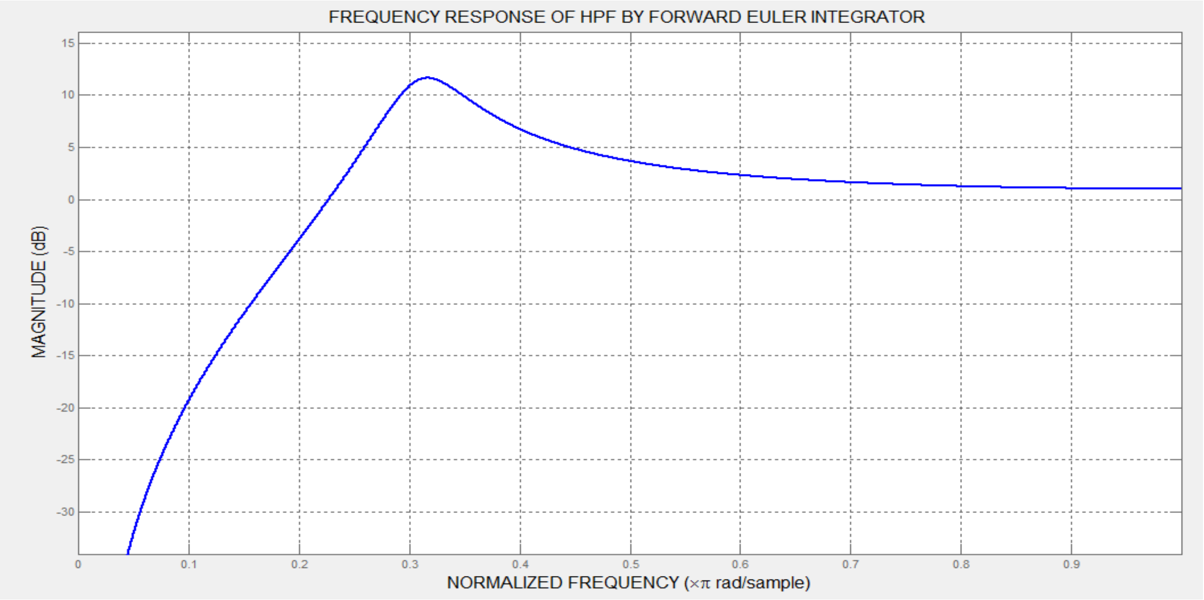


Figure: 4.5 Frequency response of HPF designed by forward Euler integrator

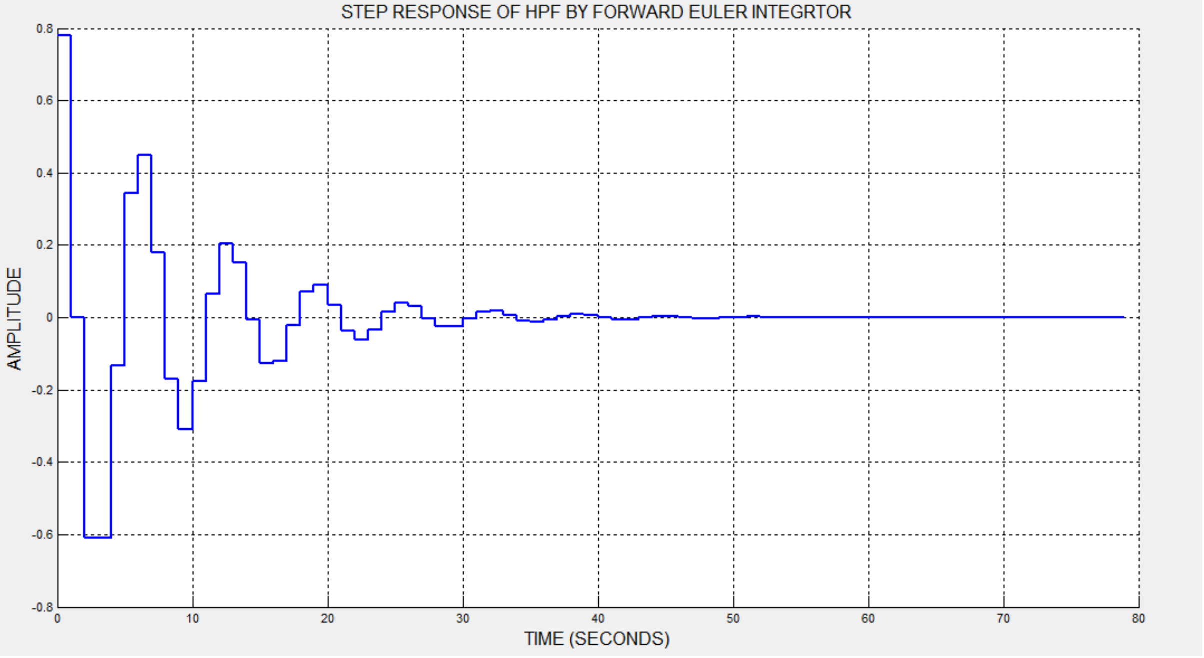


Figure: 4.6 Step response of HPF designed by forward Euler integrator

As it is clear from figure the step response of the HPF is tending toward a finite value that is 0. Hence the filter is stable.

C. Trapezium Method

The filter expression designed by this method is

$$H(z) = \frac{4z^2 - 8z + 4}{6.625z^2 - 5.5z + 2.875}$$

The magnitude response and step response are as follows

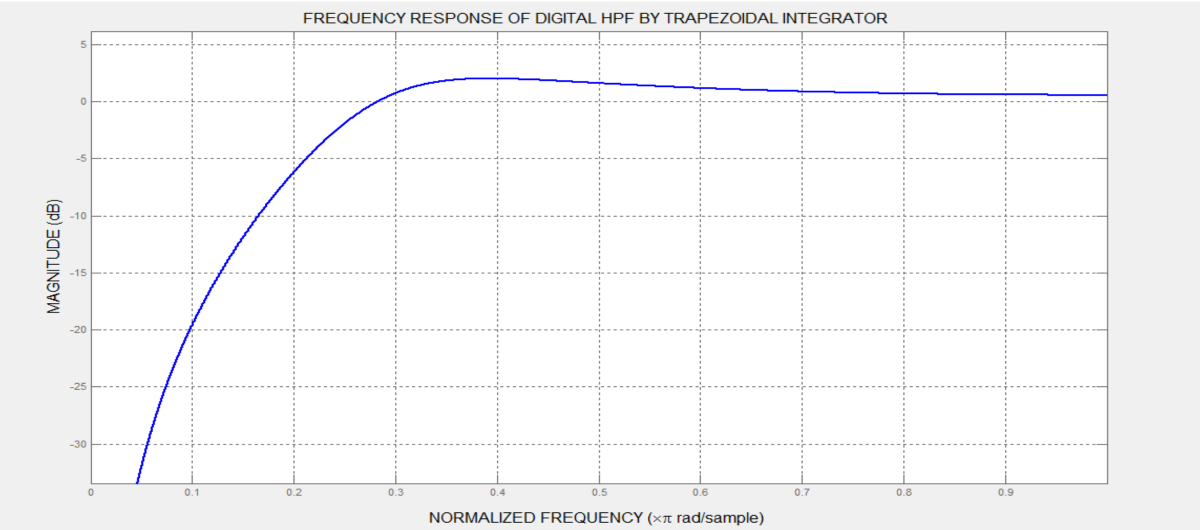


Figure: 4.7 Frequency response of HPF designed by Trapezoidal integrator

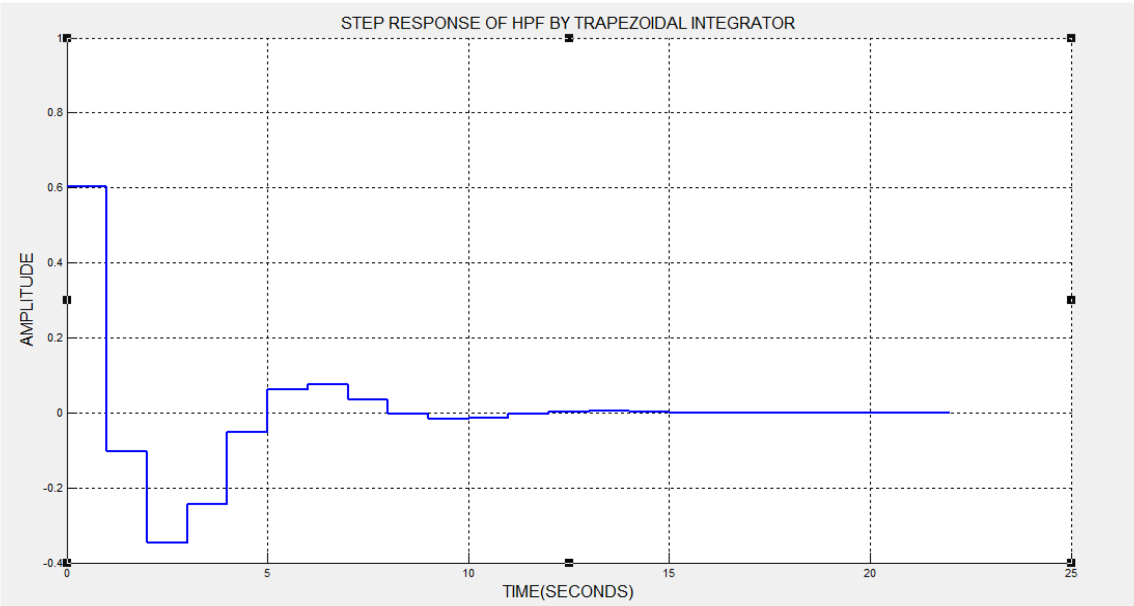


Figure: 4.8 Step response of HPF designed by Trapezoidal integrator

As it is clear from figure the step response of the HPF is tending toward a finite value that is 0. Hence the filter is stable.

4.4 Comparison of Different Design Techniques for HPF Design for the settling time

Table 4.2 shows the results of comparison of different design techniques for HPF in the sense of settling time.

NAME OF METHOD	EXPRESSION OF FILTER	SETTLING TIME (IN Seconds)
BACKWARD EULER METHOD	$\frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5}$	07 seconds
FORWARD EULER METHOD	$\frac{z^2 - 2z + 1}{1.28z^2 - 1.28z + 1}$	33 seconds
TRAPEZOIDAL METHOD	$\frac{4z^2 - 8z + 4}{6.625z^2 - 5.5z + 2.875}$	11 seconds

4.5 Comparison of Integration Methods By Using Discrete Control System

To compare the HPF designed by different methods we need to design a control system which can show the response of individual filter. Figure 4.9 shows the control system for comparison of settling time. The figure 4.10 shows the results of simulation of the digital control system shown in figure 4.9, as the figure indicates the two responses the yellow response shows the response of the section with Trapezoidal Integrator and pink graph shows the response of section with the Backward Euler method. The section with the filter designed by Backward Euler has a faster response than that of due to Trapezoidal integrator.

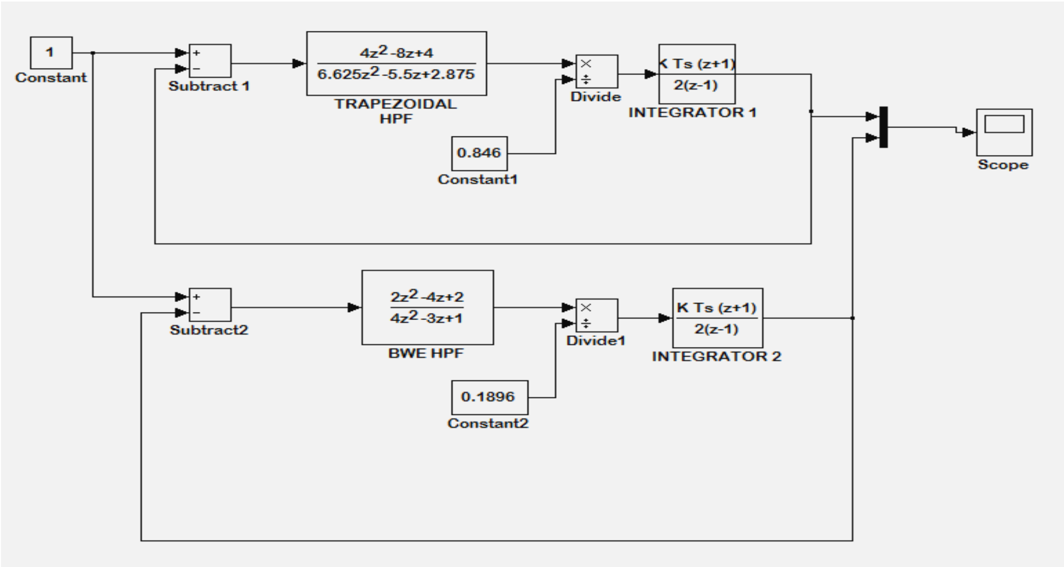


Figure: 4.9 Digital Control System Model in MATLAB SIMULINK

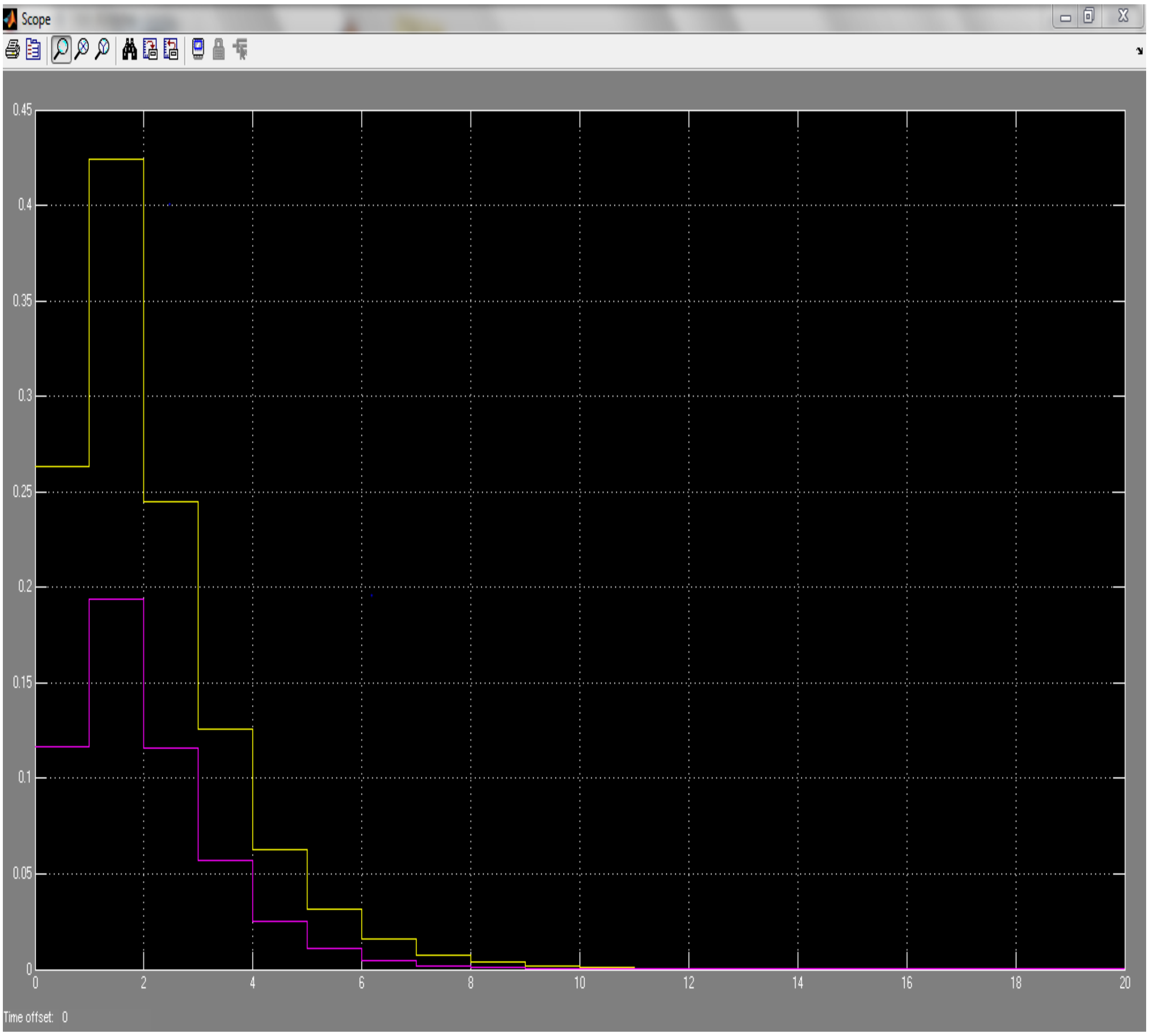


Figure: 4.10 Results of Simulation of Digital Control System

Chapter 5

Summary and Conclusions

The filters are the electronic systems whose gain is a function of input frequency. The filter can be categorized in two types , 1^{st} is analog type and other is digital type. The analog type filters are proper combination of capacitors and inductors with the resistance. The active devices like the Op-Amp, BJT are employed for the gain device. But digital filters are designed by combination of multiplication and delay elements. Digital fitters can be categorized into two types on the basis of impulse response, that is IIR and FIR. IIR stands for Infinite Impulse response and FIR stands for Finite Impulse Response.

Design methods of IIR filters are based on the sampling of magnitude response of equivalent analog filter's magnitude response and the approximation of any of its properties like slope and area. The operator 's' is differentiator in analog domain. If it is approximated in digital domain, the expression for digital differentiators can be design. These digital differentiators can be directly use to design the equivalent digital filter.

There are so many differentiators available like Euler differentiators, Bilinear differentiators, Simpson's 1/3 differentiators, Simpson's 3/8 differentiators etc. Many differentiators can be realize by combining these simple differentiators in a proper ratio to get the ideal differentiator's properties. To design the digital filters , these differentiators can be employed. The digital filter for high speed control system should be such that they have the least convergence time or called settling time. The digital filters by many methods are designed and after applying the constraint for the maximum flat magnitude response without peak near to pass band, they are simulated in MATLAB and their settling times are compared. By comparing all the filters this designed, it is concluded that the filters designed by Backward Euler Method gives the least settling time. Now to check the behaviour of them toward the digital control system, a digital control system is designed and simulated in the MATLAB simulink tool. And the results of simulation verified the results.

Backward Euler method is the best method that can design the digital filters for high speed control systems. In fact the method bilinear trans-formation, gives the good results for flatness but the convergence time of the filters designed by this method is not so much fast as compare to the Backward Euler Method.

Finally, it can be concluded that if the differentiators and integrators are compared to the ideal response respectively, the Simpson's differentiator and integrator gives the best matching with the ideal properties. But for the point of view of settling time the backward Euler method is the best method that provide the least settling time. Hence the digital filters designed by this method are suitable for the high speed discrete control systems.

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Major	**:**	**Electronics and Communication Engineering**			
Minor	**:**	**Information Technology**			
Thesis Title	**:**	**A COMARATIVE STUDY OF DIGITAL INTEGRATORS AND APPLICATION IN DESIGNING OF DISCRETE FILTERS FOR HIGH SPEED CONTROL SYSTEMS**			
Advisor	**:**	**Dr. PARAS**			

In the modern age of Information technology, the digital communication has a very important role. In the digital communication system to process on the discrete signals we require the discrete control systems. Digital filters are the main part of such discrete control systems. Digital filters are those electronic circuits whose gain is a function of input frequency. The frequency range for which the gain is large is called the pass band. In fact the filters are used to filter-out the desired frequencies in transmitters and receivers.

The IIR filters can be designed by various methods. In modern digital communication system those filters are required whose magnitude plot is flat and it does not have any resonance peak. To design such filters the study of digital differentiators and integrators has been done and their response and deviation from the ideal differentiators and integrators is calculated. By studying different types of differentiators and applied to the low pass filter's, high pass filter's prototype with application of respective mathematical constraint for flat and resonance peak less magnitude response. This generated the different digital filters for different differentiators.

The digital filters designed hence, compared with the analog filter for the settling time and it is observed that the Backward Euler method generates the fast settling digital filters out of all methods.

By studying and analyzing the different descretization methods, it is concluded that a Backward Euler method can generate the digital filters with flat magnitude response for high speed control systems applications.

DR. PARAS
(Advisor)

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सांराश

नाम :रोहिताश्व रस्तोगी **परिचयांक :४२६४७**

अर्धवर्ष, प्रथम, २०११ **उपाधि : प्रौधोगिकी परास्नातक**

प्रवेश का बर्ष: **विभाग : ऋण कणिका एवं संचार अभियांत्रिकी**

मुख्य :ऋणकणिका एवं संचार अभियांत्रिकी

अल्प :सूचना प्रौधागिकी

शोध प्रबंध शीर्षक :डिजिटल समाकलकों का तुलनात्मक अध्यन एवं उच्च गतीय नियंत्रण निकायों के लिए विवृत छिन्नको की रचना में अनुप्रयोग

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सूचना प्रौधौगिकी के आधुनिक युग में, डिजिटल संचार की एक महत्वपूर्ण भूमिका है| डिजिटल संचार में विवृत संकेतो के क्रियान्वन हेतु हमें विवृत नियंत्रण निकायों की आवश्यकता होती है| डिजिटल छिन्नक ऐसे विवृत नियंत्रण निकायों का एक मुख्य भाग है| डिजिटल छिन्नक वे एलेक्टोनिक परिपथ है जिन का लाभ निवेशित आवृत्ति का फलन होता है| वह आवृत्ति परास जिस के लिए लाभ अधिक होता है, पास समूह कहलाता है| वास्तब में छिन्नक संप्रेषक तथा ग्राही परिपथो में वांछनीय आवृत्ति को छानने के लिए प्रोयोग होते है|

आई आई आर छिन्नक विभन्न विधियों से रचे जा सकते है| आधुनिक डिजिटल संचार निकायों में ऐसे छिन्नको की आवश्यकता होती है जिन का परिमाण वक्र चपटा हो एवं इस में कोई अनुनाद शीर्ष ना हो| ऐसे छिन्नको की रचना हेतु डिजिटल अवकलको एवं समकलको का अध्यन किया गया एवं आदर्श अवकलको एवं समकलको से उन का व्यबहार तथा विचलन निकला गया| विभन्न प्रकार के अवकलको का अध्यन कर के एवं निम्न पास छिन्नको, उच्च पास छिन्नको के प्रतिदर्शो में प्रयुक्त कर के उन में चपटे और अनुनाद शीर्ष रहित परिमाण व्यवहार के लिए सापेक्ष गणितीये प्रतिवन्धो का प्रयोग किया गया| इस से विभन्नअवकलको के लिएविभन्नडिजिटल छिन्नक उत्प्पन हुए|

इस प्रकार रचित,डिजिटल छिन्नक की तुलना एनालॉग छिन्नको से स्थिरीकरण समय के लिए की गई और यह पाया गया कि इन सभी विधियों में पश्चगामी यूलर विधि तीव्र स्थरिक्रत डिजिटल छिन्नक उत्पन्न करता है|

विव्रतीकरण की विभन्न विधियों के अध्यन एवं विशलेषण के अनुसार, यह निष्कर्षित होता है की पश्चगामी यूलर विधिद्वारा उच्च गतीय नियंत्रण निकायोंमें प्रयुक्त चपटे परिमाण वक्र वाले डिजिटल छिन्नक उत्पन्न किये जा सकते है|

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(पारस) (रोहिताश्व रस्तोगी)

सलाहकार लेखक