OPTIMAL DESIGN OF GRAVITY-FED TRUNK SEWER LINES

Thesis

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Deepak Singh

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June, 2015 **Author**

CERTIFICATE – I

This is to certify that the thesis entitled "**OPTIMAL DESIGN OF GRAVITY-FED TRUNK SEWER LINES"** submitted in partial fulfilment of the requirements for the degree of **Master of Technology** in **Civil Engineering** with major in **Hydraulic Engineering** of the College of Post Graduate Studies, G. B. Pant University of Agriculture & Technology, Pantnagar, is a record of *bona fide* research carried out by **Mr. Deepak Singh**, **Id No. 45791** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been acknowledged.

Pantnagar
June, 2015

(P. S. Mahar)
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CERTIFICATE – II

We, the undersigned, member of Advisory Committee of **Mr. Deepak Singh, Id. No. 45791**, a candidate for the degree of Master of Technology in Civil Engineering with major in Hydraulic Engineering agree that the thesis entitled **“OPTIMAL DESIGN OF GRAVITY- FED TRUNK SEWER LINES”** may be submitted in partial fulfilment of the requirements for the degree.

(P. S. Mahar)
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A	Area of cross section of sewer pipe (m^2)
C_e	Excavation cost per unit volume of the sewer system between two Manholes ($/$m^3$)
C_{ijk}	Cost per meter length of the i^{th} sewer link laid between j^{th} upstream node and k^{th} downstream node
C_m	Manhole cost taken per unit depth at upstream node of the link ($/m)
C_p	Pipe cost per unit length ($/m)
d	Inside diameter of pipe (m)
d_i	Decision variable at different stage i
d_n	Decision variable at stage n
d_s	Effective diameter of suspended grains (m)
E_c	Excavation cost ($)
f	Friction factor usually taken as 0.03 for sewer pipes
$f_n^*(d_i)$	Minimum cost for any specified value of the input state sewer system,
G	Specific gravity of suspended solids
g	Acceleration due to gravity (m/s^2)
ΔH_{ijk}	Elevation difference between j^{th} upstream node and k^{th} downstream Node of i^{th} Link
i	Link number
j	Upstream node number for i^{th} link
K	Dimensional constant
k	Downstream node number for i^{th} link
L	Length of pipe between two consecutive manholes
L_i	Horizontal length between two manholes connected by link i
M_c	Manhole cost ($)

M_i	Manhole no. (i=1, 2, 3,.....)
N	Manning's roughness coefficient
n	No. of stages
nd(i)	Total number of downstream nodes in the i^{th} link
nl	Total number of links in the sewer system
nu(i)	Total number of upstream nodes in the i^{th} link
P	Wetted perimeter (m)
P_c	Pipe cost ($)
Q	Peak discharge of sewer flow (m^3/s)
R	Hydraulic mean radius of channel (m)
R_n	Optimal return at stage n
S_0	Longitudinal slope of the channel bottom/sewer pipe between two sections/consecutive manholes.
S_n	State variable or no. of states
T	Total cost ($)
T_n	Stage transition
V	Velocity of flow in the pipe in m/s
V_e	Volume of excavation of the sewer system between two manholes (or two Nodes)
V_s	Self-cleansing velocity (m/s)
X_{ijk}	Length of i^{th} sewer link laid between j^{th} upstream node and k^{th} downstream node
Y	Depth of manhole at upstream node for each link
Z	Total cost of the sewer system
α_{ijk}	Angle in degree
2θ	Angle (in radian) subtended at the centre of the pipe

Introduction

1.1 General

A sewerage system consists of a network of sewer pipes laid in order to carry the sewerage from individual homes to the sewage treatment plant. The sewer system is generally considered as an integrated urban drainage system. This network of sewers may consist of house sewers (or individual house connections); main sewers (generally called trunk sewers); out fall sewer (i.e. the sewer which transports sewage to the point of treatment); etc. Manholes are provided in every sewer line at a suitable interval to facilitate their cleaning and inspection. In the sewers which carry the sewage and the storm water, inlets called catch basins are provided to permit entrance of storm water from street gutters.

The cost of the sewerage system is the major portion of the cost of a wastewater system. The sewer pipes carry sewage under gravity or as open channel flow and therefore are laid at continuous gradient in the downward direction up to the outfall point from where it may be lifted up, treated and disposed off. In the design of a sewerage system the sewer line is the basic unit. Any savings during the design of this unit will affect the overall cost of the sewerage system. A survey of the literature indicates that various optimisation techniques are being applied for least-cost solutions. Several assumptions and constraints are made in the development of optimization models which are in accordance with engineering practice. These include the assumptions of gravity flow, minimum and maximum permissible velocities, minimum cover, non-decreasing pipe sizes in the downstream direction, and a set of cost equation for pipes and manholes.

In the design of a sewerage system, a sewer line is a basic unit occurring repeatedly in the design-process and finally the combinations of these basic units formulate the complete sewer system. However, the branch sewer lines, main sewers, trunk sewers, pumping stations, treatment plant and outfall sewers are in general the main components of an urban wastewater collection, treatment and disposal systems.

In general these approaches provide continuous pipe sizes, which are converted to closest commercial sizes for adoption, which would heavily dilute the optimal outcome. Search methods are also adopted to obtain cost-effective design solutions using directly commercial pipe sizes, which are computationally expensive. The linear programming, using piecewise linearization of the objective function and constraints in every cycle, requires substantial computer time. On the other hand, dynamic programming algorithms are subjected to the “curse of dimensionality,” thus requiring large amounts of computer memory.

1.2 Sewerage System

The water supply pipes carry water without containing any kind of solid particles, either organic or inorganic in nature: on the other hand sewer lines contain such particle in suspension and the heavier of these particles may settle down as and when the flow velocity reduces thus ultimately resulting in the clogging of the sewers (Garg, 2005). In order to avoid such clogging or silting of sewers, it is necessary that the sewer pipes be of such a size and laid at such a gradient as to generate self cleansing velocity at different possible values of discharge.

The sewer must be designed such that the size is adequate to avoid their overflow and subsequent damages to properties, and health hazards. In order to provide economically adequate sized sewers, it is necessary that the likely sewage discharge be estimated as correctly as possible. The sewer system should then be designed to be laid on a slope that will permit reasonable velocity of flow. Some limitations are described in the succeeding subsection (IS: 1712-1973).

1.2.1 Design limitations

1. A circular sanitary sewer is designed to carry the maximum flow sewage while flowing 70% full.
2. From consideration of ventilation in the waste water flows, sewer should not be designing to run full.
3. Sewer up to 400 mm dia. may be designed to at half depth sewers, between 400 to 900 mm dia. may be designed for 2/3rd depth of sewers and larger sewers may be designed for 3/4th depth at ultimate design peak flow.
4. Self cleansing velocity is maintained at present peak flow. Self cleansing velocity 0.45 m/s at minimum discharge can also be ensured by keeping the minimum design velocity to a value as high as 0.8 m/s.
5. The maximum permissible velocity of sewer line depends on the pipe material used in case of cement concrete pipe the maximum permissible limit is 2.5 m/s to 3 m/s.

1.2.2 Manhole

A manhole is the top opening of the sewer line or any underground utility constructed at suitable intervals along the sewer lines for providing access point for making connections or help in joining sewer length, and also helps in their inspection, cleaning and maintenance on underground and buried public utility and other. Manhole closings are protected by a manhole cover, a flat plug designed to prevent accidental or unauthorized access to the manhole. Those plugs are traditionally made of metal, but may be constructed from precast concrete, glass reinforced plastic or other composite material. Manholes are usually outfitted with metal, polypropylene, or fibreglass steps installed in the inner side of the wall to allow easy descent into the utility space. Manholes are generally provided at every bend, junction, change of gradient or change of sewer dia.

The spacing between manholes depends mainly upon the size of the sewer line, the larger is the diameter of the sewer, and the greater will be the spacing between the manholes (IS: 1742-1982).

1.3 Objective of the Thesis

The main aim of this study is to develop optimization models for designing gravity-fed sewer lines as connecting links between different manholes by minimizing the cost of system. The specific objectives of this study are:

- To develop a linear programming model for the design of gravity-fed trunk sewer lines.
- To develop a backward recursive dynamic programming model for the design of gravity-fed trunk sewer lines.
- To illustrate the applicability of the developed optimal design models.

1.4 Organization of the Thesis

This thesis contains five chapters including this introductory chapter. In Chapter II, review of relevant literature dealing with the design of sewer lines is presented. In Chapter III details of the design considerations related to sewer system are presented. Then the developed linear programming model as well as backward recursive dynamic programming model for designing the trunk sewer lines is presented in this chapter. In Chapter IV the application of the developed optimal design models are presented with the help of an example. The last chapter contains the summary and conclusions of the thesis. The recommendations for further work are also included in chapter V.

Review of Literature

2.1 General

The sewer system is used to carry the sewerage from the individual houses to the treatment plants. The sewer system is considered a part of the integrated urban drainage system. The concept of an integrated urban drainage has gained increasing importance in the last few decades aiming at the optimization of the drainage system. The integrated consideration of all parts of the system simultaneously allows better optimization possibilities.

Generally, the sewerage or urban drainage system has two types: the combined sewerage system and the separate sewerage system. When the drainage is taken along with sewage, it is called combined sewerage system. In the combined sewerage system, the same pipeline carries both storm-water as well as sewage (also called dry weather flow) directly to the wastewater treatment plant. In case of separate sewerage system, the drainage and sewage are taken independently of each other through two different sets of conduits. Sometimes a part of drainage water especially that originating from roofs or courtyards of building is allowed to be admitted into the drains, the resulting system is called partially separate system. In economic point of view, it is difficult to treat a large amount of storm-water at the plant.

The sewer system is therefore designed to transport only a part of the storm-water (normally 2-3 times the dry weather flow). In case of heavy rain the excess water therefore, to be discharged in to the nearest receiving water bodies because the storm-water in the sewer will exceed the designed capacity of sewer system.

The overall wastewater collection and treatment systems consist of house sewers, trunk/main sewers, sometimes combined sewer systems, pumping stations, treatment plant and outfall point. Now days, wastewater reuse and recycling components are also becoming integral part of the urban wastewater systems. However, in the design of a sewerage system the sewer line is the basic unit occurring repeatedly in the design-process and finally the combinations of these basic units formulate the complete design of sewer system.

2.2 Sewer System Design

Camp (1946) was the first to present a method for the hydraulic design of the sewer networks and emphasised considering the maximum discharge for which it is designed and the transport suspended solids. Since then a large number of research workers contributed to the subject of sewer design. The concept of modern sewer systems was born in the 19th century due to hygienic reasons.

Swamee et al. (1987) presented a direct and simple method for determination of sewer geometry of circular and noncircular shapes for partly full-flowing conditions with the known variables being discharge, bed slope, and Manning's roughness coefficient. Methodology has been developed for satisfying the minimum and maximum velocity constraints in a design procedure. A criterion of time of existence of self-cleaning velocity has been suggested for checking against self-cleaning velocity. The design method determines sewer geometry and checks for velocity constraints for circular, and various non-circulars, sewer shapes.

Some of the researchers employed heuristic methodologies **(Liebman 1967; Mays and Yen 1975; Cook and Lockwood 1977; Lornie 1982; Dasher and Davis 1986; Miles and Heaney 1988; and Charalambous and Eliman 1989 and Elimam and Charalambous,1990)**. **Argaman et al. (1973)** used dynamic programming for the design of sewer lines. All these approaches used the Manning equation or Hazen-Williams equation for resistance description. The Manning equation is applicable for a limited bandwidth, **0.004–0.04**, of relative roughness **(Christensen 1984).** ASCE (1963) has recommended the use of the Darcy-Weisbach equation for open-channel resistance. On the other hand, in a detailed study **Liou (1998)** strongly discouraged the use of the Hazen-Williams equation

The researchers in the recent past have given equal importance to economic considerations and hydraulics of the flow as opposed to conventional design method based on only self cleaning velocity concepts.

2.3 Optimal Design of Sewer Networks Using Non-linear/Linear Programming

Many researchers addressed the problem of sewer line design on the basis of minimum cost. Most of methods developed by these investigators assumed a linear or nonlinear cost equation for the objective function that can be solved using standard available mathematical algorithms. **Holland (1966)** assumed a nonlinear cost function and used a combination of separable programming and random sampling as a means to solve for the optimum design. Several researchers have developed mathematical programming models for the optimal design of sewer networks. **Deininger (1970)** proposed a linear programming model, assuming the excavation and sewer costs to be linear. **Dajani et al. (1972)** addressed the problem of understanding the nature of cost functions of waste water collection networks using a convex separable programming model based upon the work of **Holland (1966)** in their linear programming model. **Deininger (1970)** as well as **Dajani et al. (1972)** assumed sewer pipes to be available in any theoretical sizes. **Dajani and Hasti (1974)** solved the drainage networks design problem by sequential linear programming methods and minimised the cost of the drainage networks. **Gupta et al. (1976)** used Powell's method of conjugate directions to optimize the cost function.

Elimam et al. (1989) presented a combined linear programming heuristic approach based on hydraulic and mathematical model for the optimum design of large gravity sewer networks. In their model the modified Hazen-Williams hydraulic model at part-full conditions was used, along with a newly developed universal expression to compute the coefficient of roughness based on pipe material. **Sukhwan and Larry (1990)** evaluated the risk of each element by applying risk and uncertainty in the developed Urban Storm Sewer Optimal Layout Design **(**USSOD) model which considers the lag time of the peak flow by way of optimization design in the urban sewer system, and develops the model which estimates the most economical pipe size, slope, elevation and return cost of setup as result of the evaluation of risk bearing cost. USSOD model that considers a risk model looked for a design which is optimized through a risk analysis -that has a least cost as an objection function by construction expense.

Yuri A. Ermolin (1998) presented an algorithm for the optimized control of a branching urban head-and-gravity sewer network is proposed. The control consists in redistribution of the sewage flows between the network structural components. The algorithm is based on a mathematical model and presupposes the use of a computer. The mathematical model consists of a set of algebraic equations with the structure transporting capacities captured as constraints. The mathematical task of control is stated and solved as an optimization problem. The objective function is the minimization of total electric energy consumption over all the network pumping stations. The assumptions substantiated by practice would reduce the problem to the known problem of linear programming. The proposed control method obtained in the sub-network of Moscow's sewer network.

Swamee (2001) formulated the sewer line design problem as a minimization of the cost function and solved by using a dimensionally consistent resistance equation and iterative application of the Lagrange-multiplier method. **Neelakantan *et al.* (2008)** conducted their study to develop an optimisation model that optimises the total cost of installation of a water distribution system and maintenance of the system for its service life (say 50 years), and to demonstrate the advantages of this model over the model that does not include maintenance cost in optimisation. The importance of incorporating break-repair costs and pipe-replacement costs in optimal design of a water distribution network is highlighted and demonstrated with a hypothetical network. Deterioration due to ageing of pipes requires expensive maintenance and causes inconvenience. The number of breaks generally increases exponentially with pipe age and small diameter pipes are more likely to break than large-diameter pipes.

Afshar and Rohani (2012) proposed cellular automata based hybrid method for the optimal design of sewer networks and compared their performance with some of the common heuristic search methods. The problem of optimal design of sewer networks was first decomposed into two sub-optimization problems which were solved iteratively in a two stage manner. In the first stage, the pipe diameters of the network were assumed fixed and the nodal cover depths of the network were determined by solving a nonlinear sub-optimization problem. In the second stage, the nodal cover depths calculated from the first stage were fixed and the pipe diameters were calculated by solving a second nonlinear sub-optimization problem.

Swamee and Sharma (2013) addressed the optimal design of the sewer system presented applying linear programming (LP) optimisation technique for the estimation of pipe diameters and sewer depths, using the Darcy–Weisbach equation as the resistance equation and commercially available pipe diameters directly in the problem formulation. Their approach eliminates the problem of rounding off the estimated pipe sizes to the nearest commercial sizes.

2.4 Optimal Design of Sewer Networks Using Dynamic Programming

Alan *et al.* (1969) addressed the waste water collection estimation as a dynamic programming model. The model was indented as a long range planning tool to estimate the present worth of investment associated with installing and maintaining sewer subsystems.

Meredith (1972) developed a dynamic programming model to determine the components of minimum cost non-branching sewer systems. **Merritt and Bogan (1973)** presented the computer based dynamic programming model for determining the least cost design of sewer system. **Walsh and Brown** (**1975)** developed a dynamic programming model for determining the least cost design of sewer system.

Mays and Yen (1975) proposed the methodologies for optimal design of branched sewer system based on minimizing the costs of the sewer system. They used traditional dynamic programming (DP) as well as a discrete differential dynamic programming (DDDP). The branched system is decomposed into equivalent serial subsystem, which were then solved sequentially. In the development of the optimisation techniques they assumed the flow occurs in the sewer system due to gravity, and pumping stations and pressurized sewers were not considered.

Mays and Wenzel (1976) formulated the optimal design of multi-level branching sewer systems as discrete differential dynamic programming. The model considered the sewer network as a serial optimization problem such that the tree- shaped structure can be handled more easily. **Dajani et al. (1977)** designed a sewage collection network using separable, dynamic, and geometric programming approaches for very small networks. Their approach provides nonstandard pipe diameters.

Joneja, et al. (1978) developed a simple design optimization approach based on the Manning's formula for various conditions like (Full, Partially filled). They noted that further improvement was needed to incorporate variable roughness with depth of flow. **Gupta et al. (1983)** used a modified dynamic programming method that is only suitable for medium-sized networks and does not guarantee global optimality.

Kulkarni and Khanna (1985) presented a Dynamic programming **(DP)** optimization algorithm for cost trade-off analysis of "**Gravity-Cum-Pumped**" systems to enable minimal cost designs of sewer networks. A modified Hazen-William's hydraulic equation has been used in this DP-based approach and application of DP to sewer design has been plagued with problems of dimensionality.

Michael and Mays (1985) presented an optimization model that determines the minimum cost detention and drainage channel system for a watershed. The model determines the location and size of detention basins, the size, type, and number of outlet structures in addition to the design of downstream channel modifications. This new model was based upon dynamic programming for non-serial systems. As evidenced by the hypothetical application to Brays Bayou, the model is capable of considering the cost interactions of the detention basins, the outlet structures, and the channels connecting the basins. Additionally, the hydraulic constraints of a given system can be satisfied and first order approximations of land and facility costs can be calculated.

2.5 Motivation for Present Work

The literature reviewed in the preceding sections establishes the fact that various researchers have worked on the design of sewer systems. In the last few decades the application of optimization methods for designing the sewer systems has increased significantly. Optimization methods result in best solutions satisfying the conditions related to the design parameters such as velocity of flow (maximum and minimum limits), depth of flow in sewer pipes, slope and discharge through the sewer system. Most of the researchers/designers are utilizing the tool of optimization by using a new method for sewer system design.

Generally minimization of cost is the objective function of the optimization models used. In this study, the total cost of the system is considered in the objective function, in order to obtain optimal pipe sizes, elevation at different nodes and optimal path of sewer line between different manholes. The linear programming and dynamic programming based optimization models are simple to use and produce least cost design satisfying the requirements of hydraulic parameters of sewer system design. The results from the proposed linear programming optimization model and backward recursive dynamic programming model are compared with the dynamic programming model which was solved earlier by Mays and Tung (1992).

2.5 Conclusion

From this chapter it is clear that a lot of optimization models have been developed for the design of sewer systems in urban area. Many of the optimization models developed use non- linear and dynamic programming algorithms. Only few researchers have used linear programming for the minimization of the cost of sewer system. This literature review clearly shows that there is a need for the development of optimization based models for the design of sewer system.

Materials & Methods

3.1 General

The main objective of this study is to develop optimization models for designing trunk sewer lines. A linear programming model as well as a dynamic programming model has been developed for minimizing the total cost of the sewer system satisfying the requirements of discharge, minimum and maximum velocities, length between two successive manholes. The models select the optimal pipe sizes from the available diameters in the market. The objective of the optimization models is to minimize the capital cost of the sewer system and select the optimal path for the sewer system design. The decision variables of the optimization model are different diameters of available pipes in market. The length between the two manholes is assumed fixed. The constraints for recommended minimum and maximum values of velocity in the pipes are also imposed in the model. In this chapter the design considerations for the design of trunk sewer lines are first discussed. Then the linear programming model and backward recursive dynamic programming model are presented.

3.2 Materials Used

In this study, a linear programming model and a backward recursive dynamic programming model are developed for designing the sewer systems. The sewer line between two successive manholes is called as a link. The linear programming model involves minimization of total cost of the sewer system in all links satisfying the requirement of the length, diameter and slope in different links. The capital cost of the cement concrete sewer pipes of different diameters has been taken from Mays and Tung (1992).

The linear programming model has been solved using software LINGO8.0 which is a comprehensive tool designed to make building and solving mathematical optimization models easier and more efficient. LINGO provides a completely integrated package that includes a powerful language for expressing optimization models, a full-featured environment for building and editing problems, and a set of fast built-in solvers capable of efficiently solving most classes of optimization models. LINGO's primary features include Algebraic Modelling Language, Convenient Data Options, Model Interactively or Create

Turnkey Applications. It allows building models that pull information directly from databases and spreadsheets. One can build and solve models within LINGO, or one can call LINGO directly from an application written by user. For building turn-key solutions, LINGO comes with callable DLL and OLE interfaces that can be called from user written applications. LINGO can also be called directly from an Excel macro or database application. LINGO currently includes programming examples for C/C++, FORTRAN, Java, C#.NET, VB.NET, ASP.NET, Visual Basic, Delphi, and Excel. LINGO provides all of the tools one will need to get up and running quickly. LINGO is available with a comprehensive set of fast, built-in solvers for linear, nonlinear (convex & non-convex), quadratic, quadratically constrained, and integer optimization. One never has to specify or load a separate solver, because LINGO reads the formulation and automatically selects the appropriate one.

3.3 Methodology

In this study, two optimization models one linear programming based and the other dynamic programming based have been developed for designing the gravity-fed sewer lines between different manholes for specified discharge values. The developed optimization models use the hydraulics of flow in sewer systems and the total costs involving cost of pipe, cost of excavation for laying the pipe and cost of excavation at manholes. The requirements of minimum and maximum permissible velocities are also used in the optimization models. In developing the proposed optimization models, many assumptions have been made considering the engineering practice related to sewers. The flow is assumed to occur under gravity in sewer line of known length. Further, a minimum cover over the sewers, non-decreasing pipe sizes in the downstream direction are also assumed. In the minimum cost design of a storm sewer system the trade-off between pipe cost and excavation cost is considered. To convey a specified quantity of runoff, using a steeper pipe slope, the required pipe size is smaller, hence a lower pipe cost. A methodology is demonstrated in this section that can be used to determine the least-cost combination of sizes and slopes of the sewers and the depth of manholes for a sewer network to collect and drain the storm water runoff from an urban drainage basin. Because sewer slope depends on the end elevations of the sewer, design variables are the diameters and the upstream and downstream crown elevations of sewers, and the depths of manholes. The methodology considers that the network's layout connecting manholes is predetermined. The related hydraulics estimation and optimization models are described in the succeeding subsections.

3.3.1 Hydraulics of flow and design considerations

The sewer and drains are generally designed as open channels except when it is especially required to design them as flowing under pressure, e.g. in the case of inverted siphons, and discharge lines from sewage pumping stations. Various empirical formulas have been suggested for determining the gradients necessary to obtain design velocities of flow in sewers. Detailed discussion on different formulae for calculation of average velocity in sewers/channels is available in (Subramanya, 2010).

The most commonly used formula for computation of velocity in an open channel is due to Manning as:

$$V = \frac{1}{N} \times R^{\frac{2}{3}} \times S_0^{\frac{1}{2}} \tag{3.1}$$

where, V is the velocity of flow in the channel in m/s, N is the manning's roughness coefficient (IS: Code 1712-1973), R is the hydraulic mean radius of channel (m) and S_0 is the longitudinal slope of the channel bottom/sewer pipe between two sections/consecutive manholes. In order to avoid clogging or silting of sewers, the velocity should be such that neither the suspended materials in sewage get silted up nor the pipe material gets scoured out. Therefore, it is necessary to design the sewer system such that the velocity is always more than a specified minimum value and less than a specified maximum value. The minimum velocity should be so high that would not permit the solids to be settled down; i.e. it should cause self-cleansing effect. Such self cleansing velocity which will even scour the deposited particles of a given size must be developed in the sewers, at least once a day, so as not to allow any deposition in the sewers. The self cleansing velocity can be calculated as (S.K. Garg, 2010):

$$V_s = \sqrt{\frac{8\,g}{f} K d_s\,(G-1)} \tag{3.2}$$

where, V_s is the self-cleansing velocity (m/s), f is the friction factor usually taken as 0.03 for sewer pipes, g is the gravitational acceleration (m/s^2), d_s is the effective diameter of suspended grains (m), G is the specific gravity of suspended solids and K is a dimensional constant, indicating an important characteristic of sediments (solids) present in sewage. Its value usually varies from 0.04 (minimum) applicable to start of scouring of clean grit, to about 0.8 applicable for removal of sticky grit. For relatively clean inorganic and organic matters present in sewage, its values are taken at 0.04 and 0.06 respectively. Self cleansing velocity is maintained at present peak flow. Self cleansing velocity 0.45 m/s at minimum discharge can also be ensured by keeping the minimum design velocity to a value as high as 0.8 m/s. The maximum permissible velocity of sewer line depends on the pipe material used in case of cement concrete pipe the maximum permissible limit is 2.5 m/s to 3 m/s.

From consideration of ventilation in the waste water flows, sewer should not be designed to run full. Sewers up to 400 mm dia. may be designed to half full, larger may be designed for 2/3rd depth to 3/4th depth at ultimate design peak flow.

The approximate gradients which give this velocity for the sizes of pipes likely to be used in building drainage and the corresponding discharges when flowing partially-full are given in Table 3.1 (IS: Code 1742-1983).

Table 3.1 Gradients for different diameters and discharges in sewers

Diameter(mm)	Gradients	Discharge (m^3/s)
100	1 in 57	0.18
150	1 in 100	0.42
200	1 in 145	0.73
230	1 in 175	0.93
250	1 in 195	1.10
300	1 in 250	1.70

In cases, where it is practically not possible to conform to the ruling gradients, a flatter gradient may be used but the minimum velocity in such cases shall on no account be less than average self-cleansing velocity (0.61 m/s).

The discharge through a sewer pipe is calculated by using Manning's equation as:

$$Q = V \times A \tag{3.3}$$

where, Q is the peak discharge of sewer flow (m^3/s) and A is the area of cross section of sewer pipe (m^2). The hydraulic radius is calculated as ratio of area of cross section to wetted perimeter, P (m). These values are calculated for any depth of flow as:

$$A = \frac{d^2}{8}(2\theta - Sin2\theta) \tag{3.4}$$

and

$$P = d \times \theta \tag{3.5}$$

where, d is inside diameter of pipe (m) and 2θ is the angle (in radian) subtended at the centre of the pipe by water surface as shown in Figure 3.1. From equations 3.3, 3.4 and 3.5 with depth of flow equal to 0.75 times the diameter and $\theta = 2.094395$, the diameter can be calculated as:

$$d = \left(3.5184 \times \frac{n}{\sqrt{S_0}} \times Q \right)^{3/8} \tag{3.6}$$

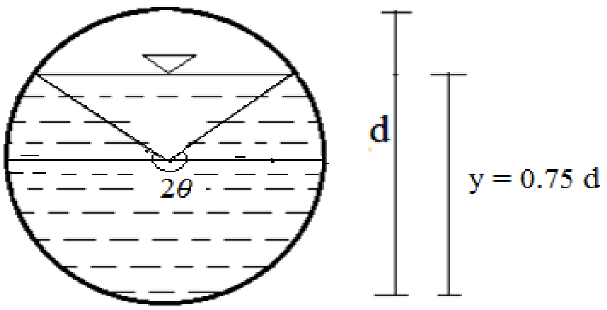

Figure 3.1: Circular channel running partially full

3.3.2 Cost Estimation

The cost of the gravity-fed trunk sewer lines can be estimated as a sum of cost of piping cost of excavation and cost of manholes as described in the following subsections. Cost estimate for foot rest and PCC has not been considered.

3.3.2.1 Pipe Cost

The cost of a pipe line varies with diameter nonlinearly and can be calculated on the basis of diameter and length of sewer pipe, according commercial pipe sizes available in the market and can be represented as:

$$P_c = C_p \times L \quad (3.7)$$

where, P_c is the pipe cost ($), C_p is the pipe cost per unit length ($/m), L is the length in (m) of pipe between two consecutive manholes.

3.3.2.2 Excavation Cost

The excavation cost can be calculated on the basis of the depth of average excavation (adding pipe size with minimum safe excavation cover), average width of excavation, and hence the volume of excavation. The excavation cost can be represented as:

$$E_c = C_e \times V_e \quad (3.8)$$

where, E_c is the excavation cost ($), C_e is the excavation cost per unit volume of the sewer system between two manholes ($/m^3) and V_e is the volume of excavation (m^3) of the sewer system between two manholes (or two nodes).

3.3.2.3 Manhole Cost

A manhole is the top opening of the sewer line or any underground utility constructed at suitable intervals along the sewer lines for providing access point for making connections or help in joining sewer length, and also helps in their inspection, cleaning and maintenance on underground and buried public utility and other. Manholes are generally provided at every bend, junction, change of gradient or change of sewer diameter.

Manhole cost varies linearly with depth of excavation and can be represented as:

$$M_c = C_m \times Y \quad (3.9)$$

where, M_c is the manhole cost ($), C_m is the manhole cost taken per unit depth at upstream node of the link ($/m) and Y is the depth (m)of manhole at upstream node for each link.

3.3.2.4 Total Cost

The total cost of sewer system is dependent on the diameter and length of sewer pipe, depth of excavation and manhole size. Total cost of the sewer system between two consecutive manholes (link) can be calculated as:

$$T = P_c + E_c + M_c \tag{3.10}$$

where, T is total cost ($).

3.3.3 Linear programming model

The linear programming based optimization model for designing a sewer system involves minimization of total cost of the sewer system in all links satisfying the requirement of the length, diameter and slope in different links. The links of a sewer pipe network are defined by the upstream node, j and the downstream node, k respectively. The decision variables are given by the sewer pipe length, which are used in each link between upstream node j and downstream node k.

3.3.3.1 Objective function

The objective of the linear programming model is to minimize the cost of total length of the sewer line including all links. The objective function can be written as:

$$Min[Z] = \sum_{i=1}^{nl} \sum_{j=1}^{nu(i)} \sum_{k=1}^{nd(i)} C_{ijk} \quad X_{ijk} \tag{3.11}$$

where Z is total cost of the sewer system, i is link number, nl is total number of links in the sewer system, j is upstream node number for i^{th} link, nu(i) is total number of upstream nodes in the i^{th} link, k is downstream node number for i^{th} link, nd(i) is total number of downstream nodes in the i^{th} link, C_{ijk} is the cost per meter length of the i^{th} sewer link laid between j^{th} upstream node and k^{th} downstream node, X_{ijk} is the length of i^{th} sewer link laid between j^{th} upstream node and k^{th} downstream node. For each link the combination of upstream and downstream nodes is chosen in such a way that downstream node is at a lower elevation than the upstream node.

3.3.3.2 Constraints The constraints imposed in the linear programming based model are related to the lengths of different links.

$$X_{ijk} \geq 0 \qquad \forall\, i, j, k \tag{3.12}$$

$$\sum_{j=1}^{nu(i)} \sum_{k=1}^{nd(i)} X_{ijk}\, Cos(\alpha_{ijk}) = L_i \qquad \forall, \; i = 1, nl \tag{3.13}$$

where, L_i is the horizontal length between two manholes connected by link i and α_{ijk} is the angle in degree for X_{ijk} such that:

$$tan(\alpha_{ijk}) = \frac{\Delta H_{ijk}}{L_i} \qquad (3.14)$$

where, ΔH_{ijk} is the elevation difference between jth upstream node and kth downstream node of ith link,

3.4 Dynamic Programming Model

In dynamic programming (DP), the optimum solution of a multivariable problem is determined by decomposing it into stages, each stage comprising a sub problem having single variable (Taha, 2007). Dynamic programming transforms a sequential or multistage decision problem that may contain many decision variables into series of single-stage problem, each containing only one or a few variables. It is a mathematical technique well suited for the optimization of multistage decision problems. It is a very useful technique in solving large complex problems by decomposing a problem into a series of a smaller sub-problems and then entire model composition. An advantage of the decomposition is that the optimization process at each stage involves only one variable. This simplifies the computational task than dealing with all the variables simultaneously. A DP model is basically a recursive equation linking different stages of the problem in a manner which guarantees that each stage’s optimal feasible solution is also optimal feasible solution for the entire problem.

3.4.1 Basic feature of the dynamic programming can be summarized as

1. In dynamic programming, decisions regarding a certain problem are typically optimized at subsequent stages rather than simultaneously. This implies that if a program is to be solved using dynamic programming, it must be separated into n sub problems.
2. DP deals with problems in which choices, or decisions, are to be made at each stage. The set of all possible choices is reflected and/or governed by the state at each stage.

3. Associated with each decision at each stage is a return function that evaluates the choice made at each decision in terms of the contributions that the decision can make to the overall objective (Maximization or Minimization).
4. Each stage n the total decision process is related to its adjoining stages by a quantitative relationship called a transition function. This transition function can either reflect discrete quantities or continuous quantities depending on the nature of the problem.
5. Given the current state, at an optimal policy for the remaining stages in terms of a possible input state is independent of the policy adopted in previous stages.
6. The solution procedure always proceeds by finding the optimal policy for each possible input state at the present stage.
7. By using the recursive relation, the solution procedure moves from stage to stage – each time finding an optimal policy for each state at that state-unit the optimal policy for the last stage found. Once the n-stage optimal policy has been discovered, the n-components decision vector can be recovered by tracing back through the n-stage transition functions. (Ravindran,1987).

Different terms in dynamic programming pertaining to sewer line design are described as:

Stages (n) are the points of the problem where decisions are to be made. Therefore, if a problem can be decomposed into **n** number of sub problems it will have **n** stages in the dynamic programming formulation. In the problem of design of pipe diameter for a sewer line, the pipe connected between the upstream M_i^{th} node (or manhole), to the downstream $M_{(i+1)}^{th}$ node will be treated as a stage of the problem. where i = 1, 2, 3,…..,M_n , (total number of manholes) thus total number of stages is equal to M_n - 1.

State or State variable (S_n) The state variables are variables describing the state of a system at any stage. A state variable of the system in a dynamic programming model has the function of linking succeeding stages so that, when each stage is optimized separately, the resulting decision is automatically feasible for the remaining stages without having to check the effect of future decisions for decisions previously made. In this case, the states are taken as the elevation of the pipe at the upstream end (manhole M_i) and downstream end (manhole M_{i+1}). At any stage the input states $S_{M_i, M_{i+1}}$ for a pipe connecting M_i and

M_{i+1} manhole will be the elevation of the pipe at the upstream end and output states will be the elevation of the pipe at the downstream end.

Decision variables (d_n) are courses of action to be taken for each stage. The number of decision variables d_n, in each stage is not necessarily to be same. The decision at each stage is selected on the basis of the minimum cost and optimal diameter of the sewer system, which also specifies the length and slope of the preceding stages.

Stage return (R_n) is the measure of the effectiveness of decision making in each stage. It is a function of the input state, the output state, and the decision variables of a particular stage.

$$R_i = r_i\ (S_{i+1}, d_i) \tag{3.15}$$

Stage transformation or state transition (T_n) is a single valued transformation which expresses the relationships between the input state, the output state, and the decision. In general, through the stage transformation the output state at any stage can be expressed as the function of the input state and the decision variable. (S. Rao, 2003).

$$S_n = \ T_n(S_n, d_n) \tag{3.16}$$

For a single stage, the above components are shown in Figure 3.2. as: (Ravindran 1987)

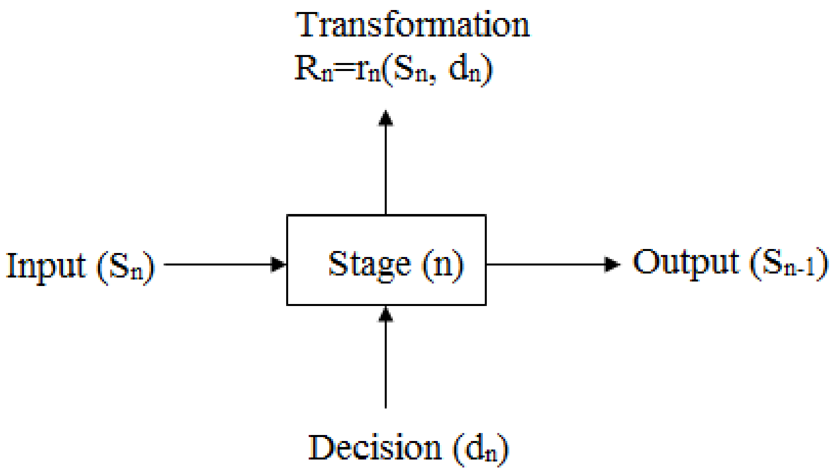

Figure 3.2: Line diagram of single stage

3.4.2 Objective function

The dynamic programming based optimization model for designing a sewer system involves minimization of total cost of the sewer system in all stages satisfying the requirement of the length, diameter and slope in different possible combinations between two states. To have separately of the objective function, we must be able to represent the objective function as the composition of the individual stage returns. This requirement is satisfied for additive functions, the objective function can be written as:

$$f_n^*(d_i) = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n} r_i\,(S_{i+1}, d_i) \qquad (3.17)$$

where, $f_n^*(d_i)$ is the minimized cost for any specified value of the input state sewer system, d_i is the decision variable at different stage i, R_i is the optimal return at the each stage, S_{i+1} is the state transformation and n is the total stages.

3.4.3 Design procedure (backward algorithm)

The design procedure using backward recursive algorithm for a sewer system comprising of three stages (links) is described and explained in this section. The three stages are shown in Figure 3.3.

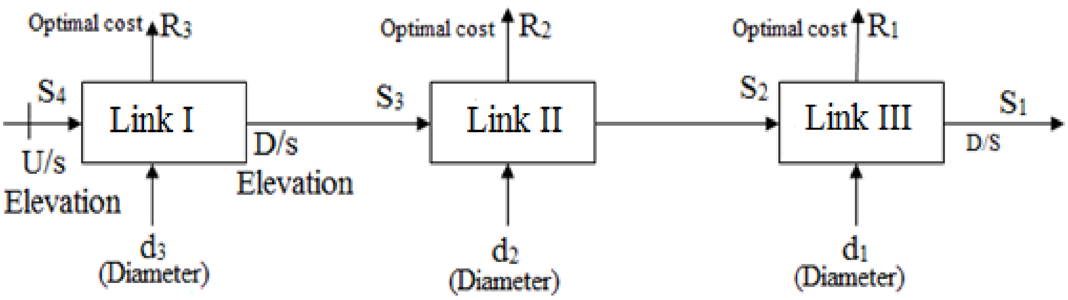

Figure 3.3: Various stages of sub-optimization

Consider the first sub-problem by starting at the last stage in backward direction i=1. If input at this stage S_2 is specified, then according to principle of optimality, d_1 must be selected to optimize R_1. Irrespective of what happens to the other stages, d_1 must be selected such that $R_1\,(S_2, d_1)$ is an optimum for the input S_2, Therefore, the optimum value is calculated as:

$$f_1^*(S_2) = opti_{d_1}\ [R_1(S_2, d_1)] \quad (3.18)$$

Next, consider the second sub-problem by grouping the last two stages together. If f_2^* denotes the optimum objective value of the second sub-problem for a specified value of input S_3, then the optimal values at this stage can be calculated as:

$$f_2^*(S_3) = opti_{d_2,d_1}\ [R_2(S_3, d_2) + R_1(S_2, d_1)] \quad (3.19)$$

The principle of optimality requires that d_1 be selected so as to optimize R_1 for a given S_2. Since S_2 can be obtained once d_1 and S_3 are specified, equation 3.16 can be written as.

$$f_2^*(S_3) = opti_{d_2}\ [R_2(S_3, d_2) + f_1^*(S_2)\ ] \quad (3.20)$$

Next, consider the third sub-problem by grouping the combination of last three stages together. If f_3^* is the optimum objective value of the third sub-problem for specified value of input S_4, then optimal values at this stage can be calculated as:

$$f_3^*(S_4) = opti_{d_3 d_2,d_1}\ [R_3(S_4, d_3)\ + R_2(S_3, d_2) + R_1(S_2, d_1)] \quad (3.21)$$

The principle of optimality requires that d_2 be selected so as to optimize R_2 for a given S_3. Since S_3 can be obtained once d_2 and S_4 are specified, equation 3.18 can be written as:

$$f_3^*(S_4) = opti_{d_3}\ [R_3(S_4, d_3) + f_2^*(S_2)\ ] \quad (3.22)$$

The above procedure can be generalized and the recursive equation to minimize the n-stage objective function $f_i^*(S_{i+1})$ which is the sum of the individual stage returns or can expand above equation as:

$$[f_n^*(S_{n+1})] = opti_{d_n ,\dots\ d_2,d_1}[R_n(d_n, S_{n+1}) + R_{n-1}(\ d_{n-1}, S_n) + \cdots + R_1(d_1, S_2)] \quad (3.23)$$

Or,

$$f_i^*(S_{i+1}) = opti_{d_i}\ [R_i(S_{i+1}, d_i) + f_{i-1}^*(S_i)\ ] \quad (3.24)$$

where, f_{i-1}^* is the optimal value of the objective function corresponding to the last stages, and S_i is the input to the stage i-1. The state and decision variables are related as:

$$S_i = T_i(S_{i+1}, d_i), \qquad i = 1, 2, 3 \ldots\ldots, n \qquad (3.25)$$

3.5 Conclusion

As we know that the traditional method of designing the sewer lines is tedious, time consuming and may not be economical. To overcome these problems, two optimization based models are developed for minimizing the cost of sewer system satisfying the maximum and minimum velocity requirements and discharge requirement between different manholes. LP and DP based models are used to design the sewer system with the objective of minimizing the cost of sewer system. The application of these models is presented in next chapter.

Results & Discussion

4.1 General

In Chapter III, the linear programming based and backward recursive dynamic programming optimal methods for design of the gravity-fed sewer lines were presented. The design requirements for gravity-fed sewer lines were also discussed in Chapter III. In the present chapter, the application of design methods is illustrated through design examples. The results obtained from the solution of the optimization models in terms of the optimal diameters and optimal path of sewer lines laid between different manholes are presented and discussed in the following sections.

4.2 Design of Trunk Sewer Lines Using Linear Programming

The linear programming (LP) has been extensively applied to the optimal design of water distribution networks. However, only few researchers have made its application in the design of sewer systems without linearization of objective function. In the proposed method using LP, the whole system is designed as single entity and not as individual pipe link. The algorithm terminates in limited number of iterations depending upon the number of sizes of available commercial pipes used in the optimization problem for formulation. With the commercial pipe sizes used directly in the sewer line design methodology, the conversion of continuous estimated pipe diameters to nearest commercial pipe sizes could be avoided which otherwise would lose the optimality of the whole system (Swamee and Sharma, 2013).

4.3 Design Example

An example is used to illustrate the application of proposed LP model to determine the minimum cost of sewer system. The problem is taken from Mays and Tung (1992) for Goodwin Avenue sewer. The data relating to ground elevations, sewer length and peak design inflows are provided in Table 4.1. In this table the sewer links are named as I, II and III instead of 5.1, 6.1 and 7.1, respectively. Similarly, the manholes are named as M_1, M_2, M_3 and M_4, respectively from u/s to d/s. A minimum cover depth over the sewer pipe of 1.0668 m is required. At a manhole the crown elevation of the downstream pipe draining the manhole must be equal to or less than the crown elevation of the upstream pipe draining into the manhole. In all, three possible elevations were considered for both u/s and d/s ends of a sewer link between two consecutive manholes. Thus total number of possible paths for each sewer link will be nine. The difference between two consecutive possible elevations of u/s (or d/s) end of the sewer link was taken as 0.3048 m.

Table 4.1 Information for Goodwin Avenue

Sewer Link	Manhole number		Ground elevation at upstream (m)	Sewer length (m)	Peak Inflow (m^3/s)
	u/s	d/s			
I	M_1	M_2	219.83	70.104	1.0423
II	M_2	M_3	218.88	48.768	1.1896
III	M_3	M_4	218.05	76.505	1.3312
	M_4				

The unit cost for excavation is 6.00$/m^3 and the manhole cost is $100.00/m depth. A 1.524 m (5ft) Excavation width is assumed for all pipes. The unit cost for excavation is 6.00$/m^3 and the manhole cost is $100.00 /m depth. The unit cost ($/m) of available commercial pipe sizes are given in Table 4.2.

Table 4.2 Unit cost of different pipe sizes

Diameter (cm)	30.48	38.1	45.72	53.34	60.96	68.58	76.2	91.44	106.68	121.92	137.16
Unit cost ($/m)	11.155	14.600	19.357	24.279	30.184	36.254	46.588	62.5	82.021	127.461	129.43

The plan of the sewer system at ground level is shown in Figure 4.1. The sewer system consists of 4 manholes denoted by circles and 3 links denoted by lines the arrow denotes the direction of flow. The elevation of the sewer system is shown in Figure 4.2 in which the possible paths for different links are denoted by lines between u/s and d/s endpoints. In Figure 4.2, the elevations of all assumed u/s and d/s endpoints of different links are also shown.

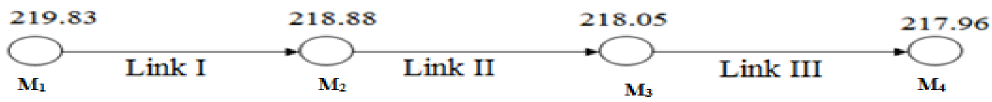

Figure 4.1: Schematic line diagram for the plan of sewer system

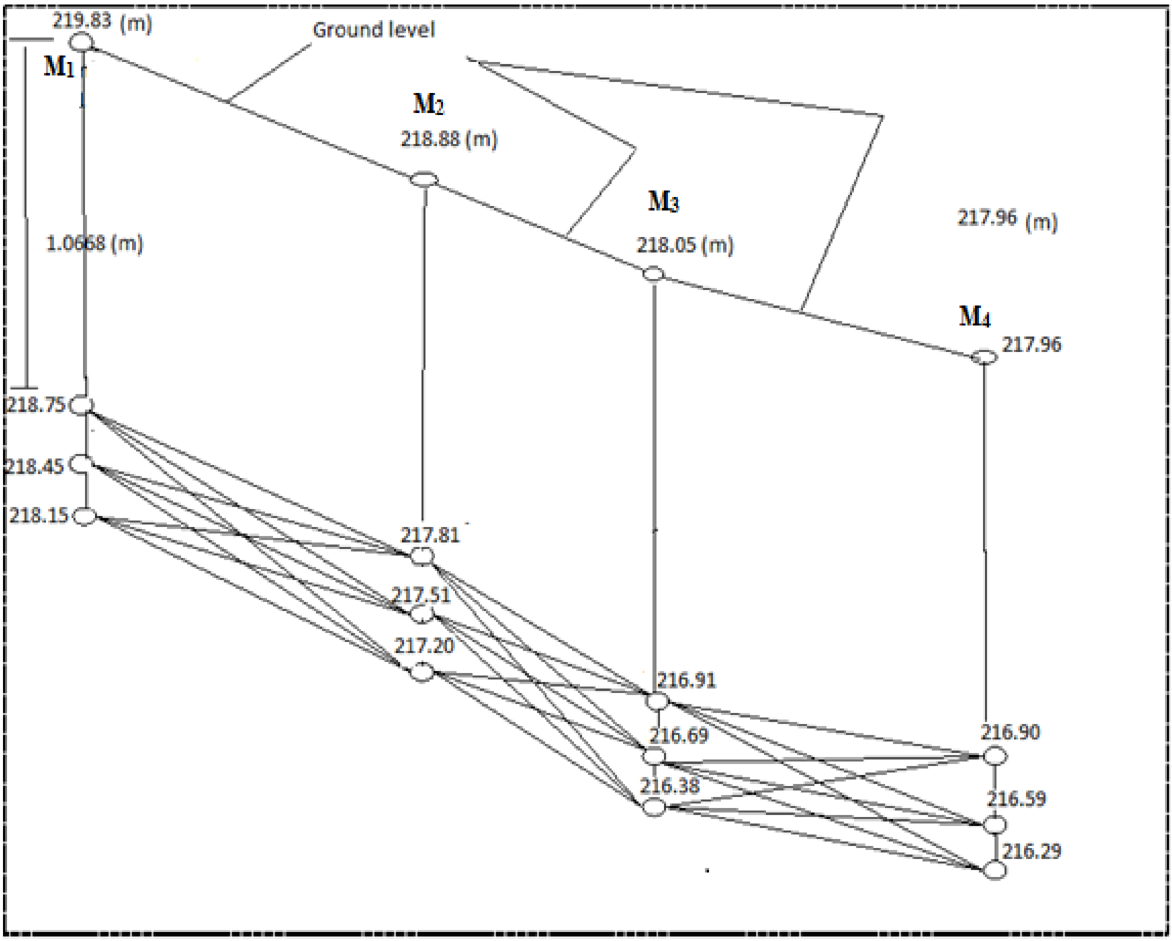

Figure 4.2: Schematic line diagram for possible paths of sewer links.

SOLUTION: Firstly the slope was calculated on the basis of elevation difference between the upstream and the downstream ends of every sewer link. Manning's coefficient was taken as 0.013 from the (IS: Code 1712-1973) for all pipes to compute the value of the diameter for a given peak inflow discharge by using equation 3.6. Then an appropriate commercial pipe size equal to or just larger available size was selected as the desired pipe size. For the selected pipe size, the pipe cost was taken from Table 4.2. The excavation cost was calculated as product of volume of excavation and the cost of unit volume of excavation. To calculate volume of excavation, average width of excavation of 1.52 m was used and the depth of excavation was used as a sum of pipe diameter and minimum safe excavation cover. The total cost was calculated as sum of pipe cost, the excavation cost and the manhole cost at upstream node by using equations 3.7 to 3.10. The above computations are shown in Table 4.3.

Table 4.3 Computation of cost of sewer system

Link No.	**U/s manhole**	**D/s manhole**	**U/p Elevation**	**Crown elevation u/s (m)**	**D/s Elevation**	**Crown elevation d/s (m)**	**Slope m/m**	**Dia. Meter (cm)**	**Pipe cost(P) ($)**	**Excavation cost (E) at u/s manhole ($)**	**U/s manhole cost (M) ($)**	**Total cost (T) ($)**	**Cost/ length**
I	M_1	M_2	1	218.75	1	217.81	0.01341	76.20	3266.00	1533.60	600.00	5399.60	77.0227
			1	218.75	2	217.51	0.01768	68.58	2541.50	1597.22	575.00	4713.72	67.2390
			1	218.75	3	217.20	0.02211	68.58	2541.50	1735.00	575.00	4841.50	69.0617
			2	218.45	1	217.81	0.00913	91.44	4381.50	1788.89	750.00	6920.39	98.7205
			2	218.45	2	217.51	0.01341	76.20	3266.00	1788.89	700.00	5754.89	82.0906
			2	218.45	3	217.20	0.01768	68.58	2541.50	1852.78	675.00	5069.28	72.3109
			3	218.15	1	217.81	0.00485	91.44	4881.50	1916.66	850.00	7148.16	101.9619
			3	218.15	2	217.51	0.00913	91.44	4381.50	2044.44	850.00	7275.94	103.7878
			3	218.15	3	217.20	0.01355	76.20	3266.00	2044.44	800.00	6110.40	87.1620
II	M_2	M_3	1	217.81	1	216.99	0.01671	76.20	2286.20	1073.33	600.00	3959.53	80.6871
			1	217.81	2	216.69	0.02282	68.58	1779.05	1118.06	575.00	3472.11	70.7543
			1	217.81	3	216.38	0.02914	68.58	1779.05	1207.50	575.00	3561.55	72.5769
			2	217.51	1	216.99	0.01060	91.44	3067.05	1252.22	750.00	5069.27	103.3010
			2	217.51	2	216.69	0.01671	76.20	2286.20	1252.22	700.00	4238.42	86.3701
			2	217.51	3	216.38	0.02303	68.58	1779.05	1296.94	675.00	3750.99	76.4373
			3	217.20	1	216.99	0.00428	106.68	4025.00	1431.11	900.00	6356.11	129.5241
			3	217.20	2	216.69	0.01040	91.44	3067.05	1431.11	850.00	5348.16	108.9844
			3	217.20	3	216.38	0.01671	76.20	2286.20	1431.11	800.00	4517.31	92.05303
III	M_3	M_4	1	216.99	1	216.90	0.00118	137.16	9901.95	2231.12	800.00	12933.07	169.0491
			1	216.99	2	216.59	0.00523	106.68	6275.00	2091.66	700.00	9066.66	118.5110
			1	216.99	3	216.29	0.00915	91.44	4781.55	2091.66	650.00	7523.21	98.3365
			2	216.69	1	216.90	-----	----	10^20	10^20	10^20	3*10^20	----
			2	216.69	2	216.59	0.00131	137.16	9901.95	2510.00	900.00	13311.95	174.0015
			2	216.69	3	216.29	0.00523	106.68	6275.00	2370.56	800.00	9445.56	123.4636
			3	216.38	1	216.90	--------	----	10^20	10^20	10^20	3*10^20	----
			3	216.38	2	216.59	--------	----	10^20	10^20	10^20	3*10^20	----
			3	216.38	3	216.29	0.00118	137.16	9901.95	2788.88	1000.00	13690.33	178.9475

4.3.1 Optimal design

The data pertaining to the cost and the lengths were used in the linear programming model as presented in section 3.3.3.1. The resulting linear programming model is:

Minimize [Z] = 77.0227*X1 + 67.2390*X2 + 69.0617*X3 + 98.7205*X4 + 82.0906*X5 + 72.31086*X6 + 101.9619*X7 + 103.7878*X8 + 87.1625*X9 + 80.6871*X10 + 70.7543*X11 + 72.5769*X12 + 103.3010*X13 + 86.3701*X14 + 76.4373*X15 + 129.5241*X16 + 108.9844*X17 + 92.05303*X18 + 169.0491*X19 + 118.5110*X20 + 98.3365*X21 + 3.92*10^18*X22 + 174.0015*X23 + 123.4636*X24 + 3.92*10^18*X25 + 3.92*10^18*X26 + 178.9475*X27;

Subject to:

X1*0.9999 + X2 *0.9998 + X3*0.99989 + X4*0.9999 + X5*0.9998+ X6*0.9997 + X7*0.9999 + X8*0.9998 + X9*0.9998 = 70.104;

X10*0.9998 + X11*0.9997 + X12*0.9996 + X13*0.9999 + X14*0.9998 + X15*0.9997 + X16*0.9999 + X17*0.9999 + X18*0.9998 = 49.0728;

X19*0.9999 + X20*0.9999 + X21*0.9999 + X22*0 + X23*0.9999 + X24*0.9999 + X25*0 + X26*0 + X27*0.9999 = 76.5048;

X1, X2, X3, X4, X5, X6, X7, X8, X9 > = 0;

X10, X11, X12, X13, X14, X15, X16, X17, X18 >= 0

X19, X20, X21, X22, X23, X24, X25, X26, X27 >= 0

The above linear programming model was solved by LINGO 8.0. The optimal cost of sewer system was obtained as $ 15712.25 and optimal lengths for the first, second and third links were obtained as X2 = 70.115 m, X11 = 49.086 m and X21 = 76.508 m, respectively. The corresponding paths for link I, link II and link III are 1-2, 1-2 and 1-3 respectively and the corresponding optimal diameters of the sewer pipes are 68.52 cm (27 inch), 68.58 cm(27 inch) and 91.44 cm (36 inch), respectively. The optimal design results are presented in Table 4.4.

Table 4.4 Optimal results from linear programming model

Link No.	Optimal length (m)	Optimal path	Optimal diameter (cm)	Optimal cost ($)
I	70.115	1-2	68.58	15712.25
II	49.086	1-2	68.58	
III	76.508	1-3	91.44	

The results obtained by linear programming model in the present study were compared with those obtained by Mays and Tung (1992) by using the forward recursive dynamic programming optimization technique. It was found that the linear programming based optimization model results in more economical sewer system than the one obtained by dynamic optimization model. The results from linear programming based model and forward recursive dynamic programming model are shown in Table 4.5.

Table 4.5 Comparison of results from linear optimization model and dynamic programming model (Mays and Tung, 1992)

S.N.	Optimal results from linear programming model				Optimal results from dynamic programming model			
Link No.	Optimal length (m)	Optimal path	Optimal Diameter (cm)	Optimal cost ($)	Optimal length (m)	Optimal path	Optimal Diameter (cm)	Optimal cost ($)
I	70.115	1-2	68.58	15712.25	70.104	1-2	68.58	16882.35
II	49.086	1-2	68.58		49.073	2-3	68.58	
III	76.508	1-3	91.44		76.104	1-3	91.44	

From Table 4.5, it can be seen that the optimal diameter is obtained by both the models are the same, and paths are also same for link I and link III. However, there is difference in paths for link II. Due to this path difference for link II the excavation cost is different, therefore the total cost obtained by linear programming model is less than the forward recursive dynamic programming model.

4.4 Sensitivity Analysis

To study the effect of slope of the sewer line on the optimal design, the linear programming model was solved for two more sets of slopes in addition to those used in the above solution. Thus, the optimal design of the sewer network was obtained for three cases (case I, case II and case III) corresponding to three different sets of slopes of the links. These different sets of slopes for all links are given in Table 4.6. For case I, the optimal solution has already been presented in the preceding sections. For case II, the crown elevation of each link at downstream end is taken as one foot (0.3048m) lower than the values for case I. The details for case II are presented schematically in Figure 4.3.

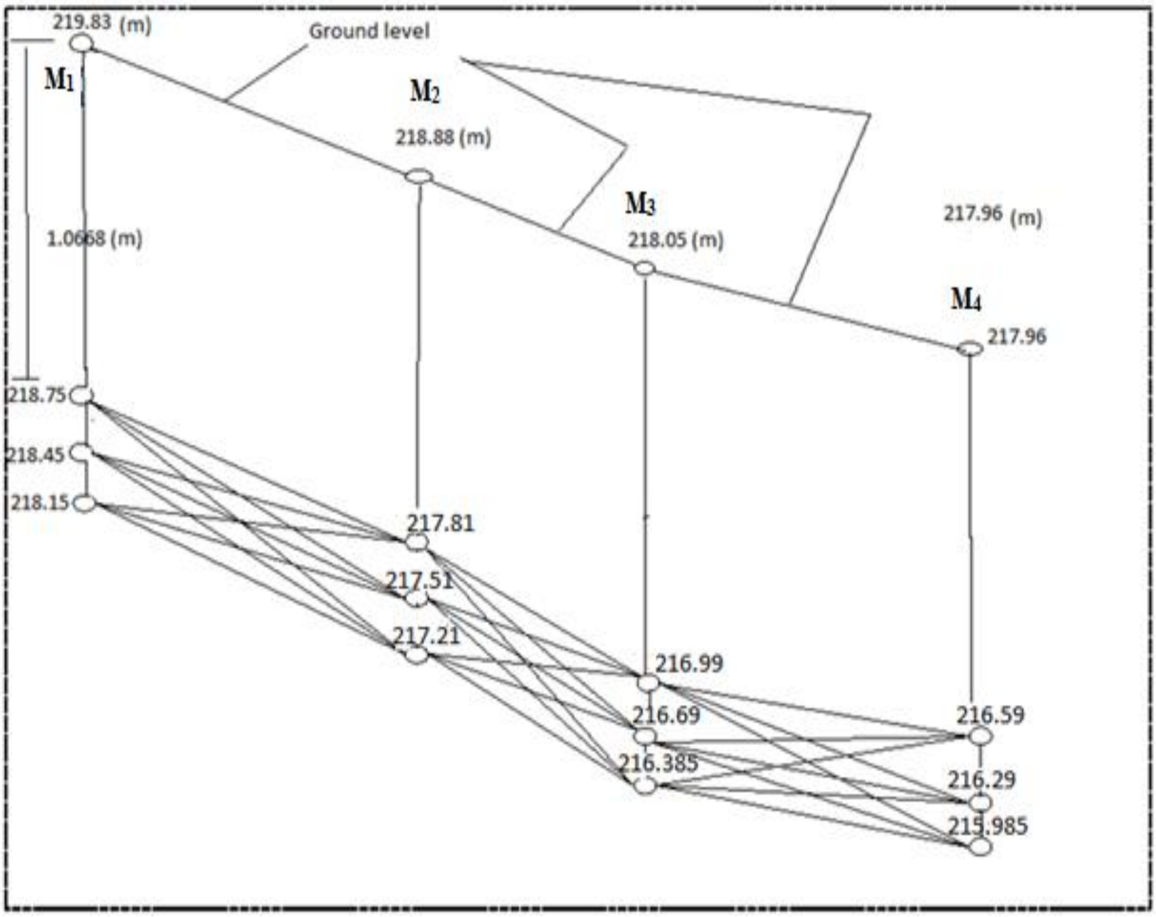

Figure 4.3: Schematic line diagram for possible paths of sewer links in case II.

The cost of the sewer system for case II was calculated as described in section 4.3 and the values are given in Table 4.7.

Table 4.6 Different sets of slopes

CASE	Path	Link I		Link II		Link III	
		U/S Elevation	D/S Elevation	U/S Elevation	D/S Elevation	U/S Elevation	D/S Elevation
I	1-1	218.75	217.81	217.81	216.99	216.99	216.90
	1-2	218.75	217.51	217.81	216.69	216.99	216.59
	1-3	218.75	217.20	217.81	216.38	216.99	216.29
	2-1	218.45	217.81	217.51	216.99	216.69	216.90
	2-2	218.45	217.51	217.51	216.69	216.69	216.59
	2-3	218.45	217.20	217.51	216.38	216.69	216.29
	3-1	218.15	217.81	217.20	216.99	216.38	216.90
	3-2	218.15	217.51	217.20	216.69	216.38	216.59
	3-3	218.15	217.20	217.20	216.38	216.38	216.29
II	1-1	218.75	217.51	217.51	216.68	216.68	216.59
	1-2	218.75	217.20	217.51	216.38	216.68	216.29
	1-3	218.75	216.90	217.51	216.07	216.68	215.98
	2-1	218.45	217.51	217.20	216.68	216.38	216.59
	2-2	218.45	217.20	217.20	216.38	216.38	216.29
	2-3	218.45	216.90	217.20	216.07	216.38	215.98
	3-1	218.15	217.51	216.90	216.68	216.07	216.59
	3-2	218.15	217.20	216.90	216.38	216.07	216.29
	3-3	218.15	216.90	216.90	216.07	216.07	215.98
III	1-1	218.75	217.66	217.66	216.84	216.84	216.90
	1-2	218.75	217.35	217.66	216.53	216.84	216.59
	1-3	218.75	217.05	217.66	216.23	216.84	216.29
	2-1	218.45	217.66	217.35	216.84	216.53	216.90
	2-2	218.45	217.35	217.35	216.53	216.53	216.59
	2-3	218.45	217.05	217.35	216.23	216.53	216.29
	3-1	218.15	217.66	217.05	216.84	216.23	216.90
	3-2	218.15	217.35	217.05	216.53	216.23	216.59
	3-3	218.15	217.05	217.05	216.23	216.23	216.29

Table 4.7 Computation of cost of sewer system for case II

Link No.	U/s manhole	D/s manhole	U/p Elevation	Crown elevation u/s (m)	D/s Elevation	Crown elevation d/s (m)	Slope m/m	Dia. Meter (cm)	Pipe cost(P) ($)	Excavation cost (E) at u/s manhole ($)	U/s manhole cost (M) ($)	Total cost (T) ($)	Cost/ length
I	M_1	M_2	1	218.75	1	217.51	0.01769	68.58	2541.50	1597.26	575.00	4713.52	67.2256
			1	218.75	2	217.20	0.02211	68.58	2541.50	1981.04	575.00	5097.57	72.6968
			1	218.75	3	216.90	0.02625	68.58	2541.50	1853.72	575.00	4970.22	70.8735
			2	218.45	1	217.51	0.01341	76.20	3266.00	1788.05	700.00	5754.05	82.0717
			2	218.45	2	217.20	0.01783	68.58	2541.50	1853.07	675.00	50769.57	72.3036
			2	218.45	3	216.90	0.02211	68.58	2541.50	1981.036	675.00	5197.54	74.1224
			3	218.15	1	217.51	0.00913	91.44	4381.68	2044.54	850.00	7276.036	103.7849
			3	218.15	2	217.20	0.01355	76.20	3266.00	2044.62	800.00	6110.62	87.1576
			3	218.15	3	216.90	0.01783	68.58	2541.5	2108.66	775.00	5425.16	77.3752
2	M_2	M_3	1	217.51	1	216.69	0.01692	76.20	2286.20	1252.40	700.00	4238.60	86.3611
			1	217.51	2	216.38	0.02303	68.58	1779.05	1297.29	675.00	3751.16	76.4202
			1	217.51	3	216.07	0.02934	68.58	1779.05	1386.98	675.00	3841.03	78.2383
			2	217.20	1	216.68	0.01060	91.44	3067.05	1431.19	850.00	5348.24	108.9788
			2	217.20	2	216.38	0.01671	76.20	2286.20	1431.32	800.00	4517.52	92.0441
			2	217.20	3	216.07	0.02303	68.58	1779.05	1476.23	775.00	4030.28	82.1283
			3	216.90	1	216.68	0.00448	106.68	4025.00	1610.02	1000.00	6635.016	135.1988
			3	216.90	2	216.38	0.01060	91.44	3067.05	1610.09	950.00	5633.14	114.7841
			3	216.90	3	216.07	0.01692	76.20	2286.20	1610.23	900.00	4796.43	97.7268
3	M_3	M_4	1	216.68	1	216.59	0.00118	121.92	7743.35	2370.556	850.00	10963.906	143.3100
			1	216.68	2	216.29	0.00510	106.68	6275.08	2370.588	900.00	11013.94	143.9543
			1	216.68	3	215.98	0.00915	91.44	4781.55	2370.66	850.00	8002.21	104.45904
			2	216.38	1	216.59	-----	----	10^20	10^20	10^20	----	---
			2	216.38	2	216.29	0.00118	121.92	7743.35	2649.44	950.00	11342.79	148.2625
			2	216.38	3	215.98	0.00523	106.68	6275.08	2649.48	1000.00	11392.83	148.9064
			3	216.07	1	216.59	--------	----	10^20	10^20	10^20	----	----
			3	216.07	2	216.29	--------	----	10^20	10^20	10^20	----	-----
			3	216.07	3	215.98	0.00118	121.92	7743.35	2928.33	1050.00	11721.68	153.2019

4.4.1 Optimal design for case II

The data pertaining to the cost and the lengths were used in the linear programming model as presented in section 3.3.3.1. The resulting linear programming model is presented as:

Minimize [Z] = 67.2256*X1 + 72.6968*X2 + 70.8735*X3 + 82.0717*X4 + 72.3036*X5 + 74.1224*X6 + 103.7849*X7 + 87.1576*X8 + 77.3752*X9 + 86.3611*X10 + 76.4202*X11 + 78.2386*X12 + 108.9788*X13 + 92.0441*X14 + 82.1283*X15 + 135.1988*X16 + 114.7841*X17 + 97.7268*X18 + 175.3196*X19 + 143.9543*X20 + 104.45904*X21 + 3.92*10^18*X22 + 180.2724*X23 + 148.9064*X24 + 3.92*10^18*X25 + 3.92*10^18*X26 + 185.2253*X27;

Subject to:

X1*0.9998 + X2*0.9997 + X3*0.9997 + X4*0.9999 + X5*0.9998 + X6*0.9997 + X7*0.9999 + X8*0.9998 + X9*0.9998 = 70.104;

X10*0.9998 + X11*0.9997 + X12*0.9996 + X13*0.9999 + X14*0.9998 + X15*0.9997 + X16*0.9999 + X17*0.9999 + X18*0.9998 = 49.0728;

X19*0.9999 + X20*0.9999 + X21*0.9999 + X22*0 + X23*0.9999 + X24*0.9999 + X25*0 + X26*0 + X27*0.9999 = 76.5048;

X1, X2, X3, X4, X5, X6, X7, X8, X9 > = 0;

X10, X11, X12, X13, X14, X15, X16, X17, X18 >= 0;

X19, X20, X21, X22, X23, X24, X25, X26, X27 >= 0;

The above linear programming model was solved by LINGO 8.0 The optimal cost of sewer system was obtained as $ 16457.42 and optimal lengths for the first, second and third links were obtained as X2 = 70.118 m, X11 = 49.088 m and X21 = 76.528 m, respectively, and corresponding paths were, 1-2, 1-2 and 1-3 respectively. The corresponding optimal diameters of the sewer pipes are 68.52 cm (27 inch), 68.58 cm (27 inch) and 91.44cm (36 inch), respectively. The optimal design results are given in Table 4.8.

Table 4.8 Optimal results from linear programming model for case II

Link No.	Optimal length (m)	Optimal path	Optimal diameter (cm)	Optimal cost ($)
I	70.118	1-2	68.58	16457.42
II	49.088	1-2	68.58	
III	76.528	1-3	91.44	

For case III, the crown elevation of each link at downstream end is taken as half foot (0.1524m) lower than the values for case I that is at midpoint of the case I and case II. The details for case III are presented schematically in Figure 4.4 and the computation of costs is given in Table 4.9.

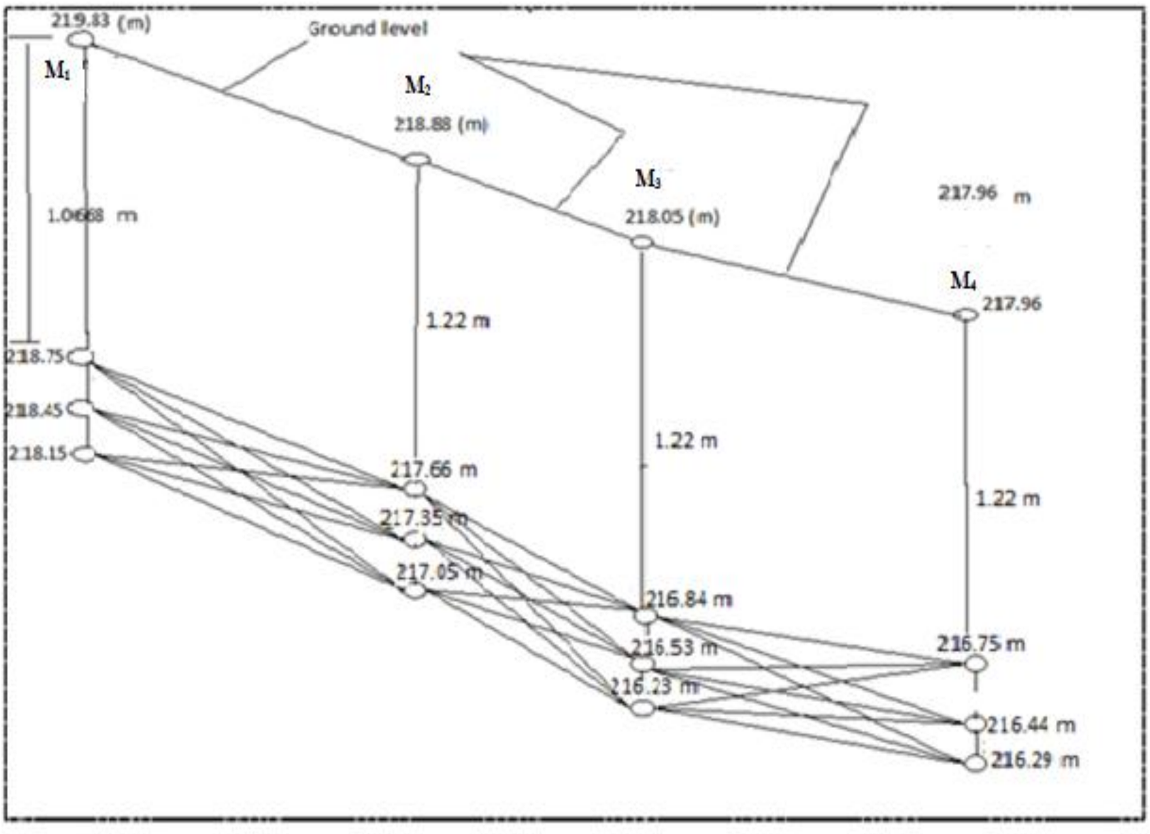

Figure 4.4: Schematic line diagram for possible paths of sewer links in case III.

Table 4.9 Computation of cost of sewer system for case III

Link No.	U/s manhole	D/s manhole	U/p Elevation	Crown elevation u/s (m)	D/s Elevation	Crown elevation d/s (m)	Slope m/m	Dia. Meter (cm)	Pipe cost(P) ($)	Excavation cost (E) at u/s manhole ($)	U/s manhole cost (M) ($)	Total cost (T) ($)	Cost/ length
I	M_1	M_2	1	218.75	1	217.66	0.01555	76.20	3266.00	1597.22	600.00	5463.22	77.930
			1	218.75	2	217.35	0.01998	68.58	2541.50	1661.11	575.00	4777.61	68.151
			1	218.75	3	217.05	0.02425	68.58	2541.50	1788.89	575.00	4905.39	69.973
			2	218.45	1	217.66	0.01127	76.20	3266.00	1725.00	700.00	5691.00	81.179
			2	218.45	2	217.35	0.01569	76.20	3266.00	1852.78	700.00	5818.78	83.002
			2	218.45	3	217.05	0.01997	68.58	2541.50	1916.67	675.00	5133.17	73.222
			3	218.15	1	217.66	0.00699	91.44	4881.50	1980.56	850.00	7712.06	110.009
			3	218.15	2	217.35	0.01141	76.20	3266.00	1980.56	850.00	6096.56	86.965
			3	218.15	3	217.05	0.01569	76.20	3266.00	2108.33	800.00	6174.33	88.074
II	M_2	M_3	1	217.66	1	216.84	0.01671	76.20	2286.20	1162.78	650.00	4098.98	83.529
			1	217.66	2	216.53	0.02303	68.58	1779.05	1207.50	625.00	3611.55	73.596
			1	217.66	3	216.23	0.02914	68.58	1779.05	1296.94	625.00	3640.99	74.196
			2	217.35	1	216.84	0.01039	76.20	3067.05	1252.22	750.00	5069.27	103.301
			2	217.35	2	216.53	0.01671	76.20	2286.20	1341.67	750.00	4377.87	89.117
			2	217.35	3	216.23	0.02283	68.58	1779.05	1386.39	725.00	3890.44	79.279
			3	217.05	1	216.84	0.00428	91.44	3067.05	1431.11	900.00	5398.16	110.003
			3	217.05	2	216.53	0.01060	76.20	2286.20	1431.11	850.00	4567.31	93.072
			3	217.05	3	216.23	0.01671	76.20	2286.20	1520.56	850.00	4656.76	94.895
III	M_3	M_4	1	216.84	1	216.90	0.00131	121.92	7743.35	2231.11	800.00	10774.46	140.834
			1	216.84	2	216.59	0.00523	91.44	4781.55	2091.67	700.00	7573.22	98.991
			1	216.84	3	216.29	0.00928	91.44	4781.55	2231.11	700.00	7712.66	100.813
			2	216.53	1	216.90	-----	----	10^20	10^20	10^20	3*10^20	------
			2	216.53	2	216.59	0.00118	121.92	7743.35	2510.00	900.00	11153.35	145.786
			2	216.53	3	216.29	0.00523	91.44	4781.55	2370.56	750.00	7902.11	103.289
			3	216.23	1	216.90	--------	----	10^20	10^20	10^20	3*10^20	------
			3	216.23	2	216.59	--------	----	10^20	10^20	10^20	3*10^20	-----
			3	216.23	3	216.29	0.00131	121.92	7743.35	2788.89	1000.00	11532.24	150.739

4.4.2 Optimal design for case III

The data pertaining to the cost and the lengths were used in the linear programming model as presented in section 3.3.3.1. The resulting linear programming model is presented as:

Minimize [Z] = 77.930*X1 + 68.151*X2 + 69.973*X3 + 81.179*X4 + 83.002*X5 + 73.222*X6 + 110.009*X7 + 86.965*X8 + 88.074*X9 + 83.529*X10 + 73.596*X11 + 74.196*X12 + 103.301*X13 + 89.117*X14 + 79.279*X15 + 110.003*X16 + 93.072*X17 + 94.895*X18 + 140.834*X19 + 98.991*X20 + 100.813*X21 + 3.92*10^18*X22 + 145.786*X23 + 103.289*X24 + 3.92*10^18*X25 + 3.92*10^18*X26 + 150.739*X27;

Subject to:

X1*0.9998 + X2*0.9997 + X3*0.9997 + X4*0.9999 + X5*0.9998 + X6*0.9997 + X7*0.9999 + X8*0.9999 + X9*0.9998 = 70.104;

X10*0.9998 + X11*0.9997 + X12*0.9996 + X13*0.9999 + X14*0.9998 + X15*0.9997 + X16*0.9999 + X17*0.9999 + X18*0.9998 = 49.0728;

X19*0.9999 + X20*0.9999 + X21*0.9999 + X22*0.9999 + X23*0.9999 + X24*0.9999 + X25*0.9999 + X26*0.9999 + X27*0.9999 = 76.5048;

X1, X2, X3, X4, X5, X6, X7, X8, X9 > = 0;

X10, X11, X12, X13, X14, X15, X16, X17, X18 >= 0;

X19, X20, X21, X22, X23, X24, X25, X26, X27 >= 0;

The above linear programming model was solved by LINGO 8.0 The optimal cost of sewer system was obtained as $ 15965.78 and optimal lengths for the first, second and third links were obtained as X2 = 70.12504 m, X11 = 49.08753 m and X20 = 76.51245 m, respectively and corresponding paths were, 1-2, 1-2 and 1-2, respectively. The corresponding optimal diameters of the sewer pipes are 68.58 cm (27 inch), 68.58 cm (27 inch) and 91.44 cm (36 inch), respectively. The optimal design results for this case are given in Table 4.10.

Table 4.10 Optimal results from linear programming model for case III

Link No.	Optimal length (m)	Optimal path	Optimal diameter (cm)	Optimal cost ($)
I	70.12504	1-2	68.58	15965.78
II	49.08753	1-2	68.58	
III	76.51245	1-2	91.44	

Comparison of the optimal design results for three cases based on the difference in slope of the sewer line can be seen from Table 4.11.

Table 4.11 Comparison of optimal results obtained for different sets of the slope

Case	Link	Diameter(m)	Paths	U/S Elevation	D/S Elevation	Length(m)	Total cost ($)
I	I	68.58	1-2	218.75	217.51	70.115	15712.25
	II	68.58	1-2	217.81	216.69	49.086	
	III	91.44	1-3	216.99	216.29	76.508	
II	I	68.58	1-2	218.75	217.20	70.118	16457.42
	II	68.58	1-2	217.51	216.38	49.088	
	III	91.44	1-3	216.68	215.98	76.528	
III	I	68.58	1-2	218.75	217.35	70.12504	15965.78
	II	68.58	1-2	217.66	216.53	49.08753	
	III	91.44	1-2	216.84	216.59	76.51245	

It is clear from Table 4.11 that the optimal cost of the sewer system is maximum for case II and minimum for case I, thus it is found to increase with increase in the slope of the pipes. The optimal paths for case I and case II are found same, but for case III, there is difference for link III. The optimal diameters are found same for all cases. The increase in the cost of the sewer system with increase in the slope only is due to increase in the cost of depth/volume of excavation.

4.5 Design of Trunk Sewer Lines Using Dynamic Programming

The dynamic programming (DP) optimization method with backward algorithm has been applied to determine the optimal design of the sewer systems. The dynamic programming based optimization model for designing a sewer system involves minimization of total cost of the sewer system in all stages satisfying the requirement of the length, diameter and slope in different possible combinations between two states. The algorithm terminates in limited number of iterations depending upon the minimum and maximum sizes of commercial pipes used in the optimization problem for formulation. With the commercial pipe sizes used directly in the sewer line design methodology, the conversion of continuous estimated pipe diameters to nearest commercial pipe sizes could be avoided which otherwise would lose the optimality of the whole system (Mays and Tung, 1992).

4.5.1 Optimal design by backward recursive dynamic programming

Cost can be calculated as earlier in Table 4.3. Here backward algorithm is applied to optimize the given problem. Computation in dynamic programming (DP) is done recursively, so that the optimum solution of each sub-problem is used as input to the next sub-problem. By the time the last sub-problem is solved the optimum solution for the entire problem is at hand. The manner in which the recursive computations are carried out depends on how we decompose the original problem. In particular, the sub-problems are normally linked by common constraints as we move from one sub-problem to the next, the feasibility of these common constraints must be maintained.

The design of the trunk sewer lines can be represented as multi stage decision process between the different manholes. The decision variables d_1, d_2, and d_3 represent the diameter for link I, II and III, respectively. Thus the variable d_1 can take nine discrete values, each corresponding to a particular feasible slope between two successive manholes for link I, out of these nine feasible slope the model select only one suitable slope, corresponding this suitable slope select the diameter of the sewer pipe, similarly selects the variables d_2 and d_3 for link II and link III, respectively.

Sub optimization of Stage 1 (Link I)

For the sub-optimization of stage 1, minimize the cost R_1 (S_2, d_1) for any specified value of input state S_2 to obtained $f_1^*(S_2)$ as:

$$f_1^*(S_2) = min_{d_1}\ [R_1(S_2, d_1)] \quad (4.1)$$

The results of sub-optimization of stage 1 are given in Table 4.12

Table 4.12 Results of sub-optimization at 1st stage, for link III

S_2(U/S) elevation	(D/S) elevation (m)	Diameter(cm) Corresponding (slope)	Optimal dia. for min. cost (d_1^*)(cm)	Cost for minimum dia. (f_1^*) ($)	Optimal cost for optimal dia.($)	Corresponding value of S_1=(D/S) elevation	Optimal path
216.99	216.90	137.16		12933.07			
	216.59	106.68		9066.66			
	216.29	91.44	91.44	7523.21	7523.21	216.29	1-3
216.68	216.90	----		3*10^20			
	216.59	137.16		13311.95			
	216.29	106.68	106.68	9445.56	9445.56	216.29	2-3
216.38	216.90	----		3*10^20			
	216.59	----		3*10^20			
	216.29	137.16	137.16	13690.33	13690.33	216.29	3-3

From Table 4.12, it can be seen that the optimal diameter of the sewer lines 91.44 cm, 106.68 cm and 137.16 cm, respectively for link I, II and III, and corresponding optimal paths are 1-3, 2-3, and 3-3.

Sub-optimization of Stages2 and Stage1 (between link II & link I)

In this stage we combine the link II and link I and use the optimal results obtained from the sub-optimization stage 1 and minimize the cost (R_1+R_2) for any specified value of the input S_3 state to obtained $f_2^*(S_3)$ as

$$f_2^*(S_3) = min_{d_2,d_1}\ [R_2(S_3, d_2) + R_1(S_2, d_1)] = min_{d_2}\ [R_2(S_3, d_2) + f_1^*(S_2)\ ] \quad (4.2)$$

The results of sub-optimization of stage 2 and stage 1 are given in Table 4.13

Table 4.13 Shows sub-optimization at 2nd stage, (for link III and link I)

S_3(U/S) elevation	(D/S) elevation (m)	Dia. (cm) Corresponding (slope) d_2	Cost of link2 R_2 ($)	S_2 (D/S) elevation (m)	Optimal cost of link3 f_1^* ($)	Combined cost at stage 2 $R_2+f_1^*$ ($)	Optimal cost up to stage 2 (f_2^*) ($)	Optimal dia. for min. cost (d_2^*) (cm)	Optimal path
217.81	216.99	76.20	3959.53	216.99	7523.21	11482.74	11482.74	76.20	1-1
	216.69	68.58	3041.53	216.69	9445.56	12487.09			
	216.38	68.58	3561.55	216.38	13690.33	17251.88			
217.51	216.99	91.44	5069.27	216.99	7523.21	12592.48	12592.48	91.44	2-1
	216.69	76.20	4238.42	216.69	9445.56	13683.98			
	216.38	68.58	3750.99	216.38	13690.33	17441.32			
217.20	216.99	106.68	6356.11	216.99	7523.21	13879.32	13879.32	106.68	3-1
	216.69	91.44	5348.16	216.69	9445.56	14793.72			
	216.38	76.20	4517.31	216.38	13690.33	18207.64			

From Table 4.13, it can be seen that the optimal diameter of the sewer lines 76.20 cm, 91.44 cm and 106.68 cm, respectively for link I, II and III, and corresponding optimal paths are 1-1, 2-1, and 3-1.

Now desired quantities (f_2^* and d_2^*) corresponding to the various discrete values of S_3 can be summarized as follows in Table 4.14.

Table 4.14 Show optimal values at 2nd stage

S_3(U/S) elevation	Optimal dia. For min. cost (d_2^*) (cm)	Optimal cost up to stage 2 (f_2^*) ($)	S_2 (D/S) Elevation (m)
217.81	76.20	11482.74	216..99
217.51	91.44	12592.48	216..99
217.20	106.68	13879.32	216..99

Sub-optimization of Stages3, Stage2 and Stage1 (among link III, link II & link I)

In this case we consider the all three stages 3, 2 and 1, and use the optimal results obtained from the sub-optimization stage 2 and minimize the cost ($R_3+R_2+R_1$) for any specified value of the input S_4 to obtained $f_3^*(S_4)$ as:

$$f_3^*(S_4) = min_{d_3 d_2, d_1}[R_3(S_4, d_3) + R_2(S_3, d_2) + R_1(S_2, d_1)] = min_{d_3}\ [R_3(S_4, d_3) + f_2^*(S_2)\ ] \qquad (4.3)$$

The results of sub-optimization of stage 3, 2 and 1 are given in Table 4.15

Table 4.15 Shows sub-optimization at 3rd stage, (for link III, link II and link I)

S_4(U/S) elevation	(D/S) elevation (m)	Dia. (cm) Corresponding (slope) d_3	Cost of link1 R_3 ($)	S_3 (D/S) elevation (m)	Optimal cost at stage2 f_2^* ($)	Combined cost at stage 2 $R_3 + f_2^*$ ($)	Optimal cost up to stage 3 (f_3^*) ($)	Optimal dia. for min. cost (d_3^*)(cm)	Optimal path
218.75	217.81	76.20	5399.60	217.81	11482.74	16882.34	16882.34	76.20	1-1
	217.51	68.58	4713.72	217.51	12592.48	17306.20			
	217.20	68.58	4841.50	217.20	13879.32	18720.82			
218.45	217.81	91.44	6920.39	217.81	11482.74	18403.13			
	217.51	76.20	5754.89	217.51	12592.48	18347.37	18347.37	76.20	2-2
	217.20	68.58	5069.28	217.20	13879.32	18948.60			
218.15	217.81	91.44	7148.16	217.81	11482.74	18630.90	18630.90	91.44	3-1
	217.51	91.44	7275.94	217.51	12592.48	19868.45			
	217.20	76.20	6110.40	217.20	13879.32	19989.72			

From Table 4.15, it can be seen that the optimal diameter of the sewer lines 76.20 cm, 76.20 cm and 91.44 cm, respectively for link I, II and III, and corresponding optimal paths are 1-1, 2-2, and 3-1.

Now desired quantities (f_3^* and d_3^*) corresponding to the various discrete values of S_4 can be summarized as follows in Table 4.16.

Table 4.16 Show optimal values at 3rd stage

S_4(U/S) elevation	Optimal dia. for min. cost (d_3^*) (cm)	Optimal cost up to stage 3 (f_3^*) ($)	S_3 (D/S) elevation (m)
218.75	76.20	16882.34	217.81
218.45	76.20	18347.37	217.20
218.15	91.44	18630.90	217.81

From Table 4.16, it can be seen that the minimum cost of the sewer system is 16882.34 $.

Finally optimal cost of sewer system is obtained with the help of dynamic optimization model 16882.34 $. corresponding optimal values of decision variables d_1^*, $d_2^*$and d_3^* are 76.20 cm, 76.20 cm and 91.44 cm, respectively for link I, II and III, and optimal path is obtained crown elevation joining 1-3, 1-1 to 1-1 in backward direction for link I, II and III, respectively.

Now finely we compared the results obtained by the linear programming model, backward recursive dynamic model and forward recursive dynamic model were illustrated by Mays and Yen 1992. The comparison is given in Table 4.17

Table 4.17 Comparison of the results obtained by in this study

S. N.	**Optimal results from linear programming model**			**Optimal results from dynamic programming model (Mays and Tung, 1992)**			**Optimal results from dynamic programming backward model developed in this study**		
Link No.	Optimal path	Optimal Diameter (cm)	Optimal cost ($)	Optimal path	Optimal Diameter (cm)	Optimal cost ($)	Optimal path	Optimal diameter	Optimal cost ($)
I	1-2	68.58	15712.25	1-2	68.58	16882.35	1-1	76.20	16882.34
II	1-2	68.58		2-3	68.58		1-1	76.20	
III	1-3	91.44		1-3	91.44		1-3	91.44	

From Table 4.17, it is clear that the linear programming optimization based model is more economical than the dynamic programming method.

4.6 Conclusion

One design example was solved in this chapter. The design example is solved by both linear programming as well as dynamic programming methods. First, the design example is solved by the linear programming based optimization method developed in this study. To compute the optimal cost of the sewer system. Then the sensitivity analysis is applied to find out the changes in the optimal cost of the design example. After then the dynamic programming (Backward Recursive Algorithm) method is applied to determine the optimal cost of sewer system. On comparing the results obtained from both the optimization models illustrated. From the results it is clear that the linear programming optimization based model is more economical than the dynamic programming method.

Summary & Conclusions

5.1 Summary

The sewer system is generally considered as an integrated urban drainage system. This network of sewers may consist of house sewers (or individual house connections); main sewers (generally called trunk sewers); out fall sewer (i.e. the sewer which transports sewage to the point of treatment); etc. Manholes are provided in every sewer line at a suitable interval to facilitate their cleaning and inspection. In the sewers which carry the sewage and the storm water, inlets called catch basins are provided to permit entrance of storm water from street gutters.

The sewer system design is an important work for development of industrial, urban and rural area. The design of sewer system in an urban area is important for disposing the domestic sewage such that the velocity remains within the permissible limits. The traditional method is a trial and error method which is time consuming and may not be economical. In this study an attempt has been made to develop a linear programming based optimization method and a dynamic programming (backward recursive) optimization method to design a sewer system in urban area. Both the optimization models have the objective of minimizing the total cost of the sewer system meeting the requirement of velocity and slopes being within limits of minimum and maximum permitted values.

The inputs to the linear programming optimization model are: unit cost of different pipe sizes for which velocity is within permissible limits, unit costs of excavation and unit cost of manhole. The outputs of the linear programming optimization model are: optimal diameters selected for different lengths and minimum cost of the sewer system. The optimal design model developed in this study presents the advantage that it select the optimal length or optimal roots between two successive manholes on the basis of optimal length the optimal diameters of pipes selected from the list of available pipe sizes, which would be time consuming and inaccurate by traditional method.

The performance of the developed linear programming based optimization model is evaluated for two other sets of slopes. The results were obtained satisfactorily better than the dynamic programming based optimization model.

5.2 Conclusions

From this study, the following conclusions can be made.

1. The developed linear programming based optimization model can be used to achieve minimum cost design of gravity-fed trunk sewer lines considering the requirements of minimum and maximum velocity through the pipes.
2. The developed dynamic programming (backward recursive algorithm) based optimization model can be used to design of gravity-fed trunk sewer lines.
3. Comparison of the results from linear optimization model with those obtained from dynamic programming based optimization models (both forward recursive as well as backward recursive) establishes that the developed linear programming based optimization model results in more economical design of sewer system in urban area.
4. The effect of slope on the performance of the linear programming model showed that the cost of sewer system increased with increase in the slope of the pipes. The optimal cost increased 1.613% and 4.742% when the slope in each section was increased by lowering the pipe 15 cm and 30 cm in each section, respectively.

5.3 Recommendation for Further Work

The possible extension of this study could be to apply this developed linear programming based optimization model as well as dynamic programming based optimization model for economical design of sewer system for any industrial areas, urban and rural areas.

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VITA

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ABSTRACT

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A sewerage system consists of a network of sewer pipes laid in order to carry the sewage from individual homes to the sewage treatment plant. The design of sewer system for disposing the domestic sewage should be such that the velocity remains within the permissible limits. The traditional method is a trial and error method which is time consuming and may not be economical. Many researchers addressed the problem of sewer line design on the basis of minimizing the cost. Most of methods developed by these investigators assumed a linear or nonlinear cost equation for the objective function that can be solved using standard available mathematical algorithms.

The main objective of this study is to develop optimization models for designing trunk sewer lines. A linear programming model and a dynamic programming model has been developed for minimizing the total cost of the sewer system satisfying the requirements of discharge, minimum and maximum velocities, length between two successive manholes. The objective of the optimization models is to minimize the capital cost of the sewer system and select the optimal depth of laying the sewer system. The total costs involve the cost of pipe, cost of excavation for laying the pipe and cost of excavation at manholes. The cost per unit length of each link of sewer lines is taken as the input variable for this study. These models select that link which has the minimum cost per unit length. On the basis of minimum cost per unit length of links, the optimal pipes sizes are selected from the available diameters in the market. The developed optimization models have been applied to a design example. The performance of the developed linear programming based optimization model is evaluated for two other sets of slopes. The results obtained in this study shows that the optimal cost of the designed sewer network increases with the increase of slope of the pipes. The linear programming model resulting in more economical design than the dynamic programming model.

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संक्षेप

नाम : **दीपक सिंह** **परिचयांक** : **४५७६१**

सत्र एवं प्रवेश वर्ष : **प्रथम, २०१३–२०१४** **उपाधि** : **प्रौद्योगिकी में स्नातकोत्तर**

मुख्य विषय : **द्रव्यचालित अभियांत्रिकी** **विभाग** : **जनपत अभियांत्रिकी**

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शोध का शीर्षक : **गुरुत्वाकर्शण प्रवाह द्वारा सीवर पाईप का इष्टतम डिजाइन**

सलाहकार : **डॉ0 पी0 एस0 महर**

सीवरेज प्रणाली एक पाइप नेटवर्क होता है, जो अलग–अलग मकानों से निकलने वाले मलजल को उपचार सयंत्रों तक ले जाने का कार्य करता है। सीवरेज प्रणाली का डिजाइन घरेलू सीवेज को ले जाने के लिए इस प्रकार होना चाहिए ताकि मलजल वेग उच्चतम तथा निम्नतम अनुमेय सीमा तक रहे। सीवर लाइन डिजाइन की पारम्परिक विधि एक परीक्षण एवं त्रुटिपूर्ण (अनुमानिक एवं अपूर्ण) विधि है, जो कि डिजाइन में अधिक समय लेती है और किफायती भी नहीं है। इस परीक्षण एवं त्रुटिपूर्ण(अनुमानिक एवं अपूर्ण) विधि को उच्चतम डिजाइन म परिवर्तन करने के लिए कई शोधकर्ताओं ने लागत को कम करने के आधार पर सावर लाईन डिजाइन की समस्या में शोध के प्रयास किये। इन शोधकर्ताओं द्वारा विकसित विधियों में यह कहा गया है कि एक रैखीय या आरैखीय कीमत समीकरण मानक उपलब्ध गणितीय एल्गोरिदम द्वारा हल किया जा सकता है।

इस अध्ययन का मुख्य उद्देश्य ट्रंक सीवर लाईन का डिजाइन करने के लिए अनुकूल मॉडल विकसित करना है। इस अध्ययन में एक रैखिक प्रोग्रामिंग अनुकूल मॉडल और एक गतिशील प्रोग्रामिंग अनुकूल मॉडल विकसित किये गए हैं, जिससे सीवर प्रणाली की कीमत घट सके और दो मैनहोल के बीच न्यूनतम और अधिकतम मलजल वेग अनुमेय सीमा तक रहे तथा मैनहोल के बीच धारा प्रवाह की आवश्यकता, बीच की लम्बाई तथा ढ़लान आदि की आवश्यता को संतुष्ट कर सके। बेहतर मॉडल का उद्देश्य पूंजीगत लागत को न्यूनतम करना और सीवर प्रणाली का बेहतर निर्णय करना है। सीवर लाईन डिजाइन की पूर्णलागत में पाईप की कीमत, पाईप बिछाने के लिए खुदाई की कीमत और मैनहोल के उत्खनन की कीमत आदि का समायोजन है। इस अध्ययन में सीवर लाईन की ईकाई, लम्बाई प्रति लागत की कड़ी को चर इनपुट लिया गया है। इस अध्ययन में माना गया है कि बाजार में जिस पाईप की ईकाई, लम्बाई प्रति लागत न्यमनतम हो, उसी व्यास के पाईप को अध्ययन में बेहतर माना गया है, और डिजाईन में उसी को चुना गया है। विकसित बेहतरीन मॉडल का उपयोग एक उदाहरण के द्वारा प्रस्तुत किया गया है, जिसमें रैखिक प्राग्रामिंग अनुकूल मॉडल का उपयोग तीन अलग–अलग ढलानों के लिए किया गया है और जिसमें यह देखा गया है कि ढलान के बढ़ने के साथ सीवर प्रणाली की लागत बढ़ रही है। इस अध्ययन से यह भी स्पष्ट होता है कि हमारा रैखिक प्रोग्रामिंग प्रारूप में सीवर प्रणाली की पूँजीगत लागत सबसे न्यूनतम है अतः यह सबसे सस्ता और सरल मॉडल है।

(पी0 एस0 महर)
परामर्शदाता

(दीपक सिंह)
लेखक