# ANALYSIS OF TIME SERIES AND STOCK MARKET BEHAVIOR USING WAVELET METHODS

*Thesis*

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**BY**

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Dedicated

to

***My Beloved Family***

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# CERTIFICATE

This is to certify that the thesis entitled **"ANALYSIS OF TIME SERIES AND STOCK MARKET BEHAVIOR USING WAVELET METHODS"** is submitted in partial fulfilment of the requirements for the degree of **Doctor of Philosophy** with major in **Mathematics** and minor in **Electronics & Communication Engg.** of the College of Post-Graduate Studies, G.B. Pant University of Agriculture & Technology, Pantnagar, is a record of bona fide research carried out by **Mr. Anuj Kumar, Id. No. 30711** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

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**Pantnagar**
**May, 2008**

**(A. K. Sharma)**
Chairman
Advisory Committee

# CERTIFICATE

We, the undersigned, members of the Advisory Committee of **Mr. Anuj Kumar, Id. No. 30711**, a candidate for the degree of **Doctor of Philosophy** with major in **Mathematics** and minor in **Electronics & Communication Engg.**, agree that the thesis entitled **"ANALYSIS OF TIME SERIES AND STOCK MARKET BEHAVIOR USING WAVELET METHODS"** may be submitted in partial fulfillment of the requirements for the degree.

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Chairman
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| Member | Member | Member |

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# INTRODUCTION

# 1 Introduction

The last decade has witnessed an explosion of interest in wavelets, a subject area that has coalesced from roots in mathematics, physics, electrical engineering and other disciplines. Wavelets are mathematical tools for analyzing time series and are applicable in the fields of economics, finance and medical imaging, image processing etc. Wavelets provide a unique decomposition of time series observations which enables one to deconstruct the data in ways that are potentially revealing. Moreover, wavelets *Ø(.)* are particular types of functions that are localized both in time and frequency domain and are used to decompose a function *f(.)* which is in the form of a series, a signal, a surface etc., into more elementary functions which include information about the same function *f(.)*.

Wavelet transforms are often applied in the fields of signal and image processing. They transform a signal into different frequency bands by dilating and translating two basis functions that are derived from the spectral decomposition theorem which states that any time series can be broken down into multiple statistically independent time series—called resolutions, each representing the contribution of oscillations of different frequencies.

The lower the frequency level, the longer is the trend which a given resolution reflects. By summing up all these resolutions, one can exactly reconstruct the original data.

Many economic and financial time series are nonstationary in general and exhibit changing frequency pattern over the time. The usefulness of wavelet analysis is in its flexibility in handling a variety of nonstationary signals as wavelets are constructed over finite interval of time and are not necessarily homogeneous over time. They are localized both in time and scale. The two main interesting features of wavelet time scale decomposition for economic variable will be that, (i) the data need not to be differenced since the base scale includes nonstationarity components and (ii) the nonparametric nature of wavelets takes care of potential nonlinear relationships without losing detail.

The main advantage of wavelet analysis is its ability to decompose macroeconomic time series, and data in general, into their time scale components. Moreover, because of the translation and scale properties, nonstationarity in the data is not a problem and when using wavelets prefiltering is not needed. Moreover, unlike moving averages, wavelet decomposition does not introduce a time-delay into the signal—the temporal information of the raw data is preserved in each resolution. In other words, the oscillations in each resolution are not phase shifted relative to the original time series. Wavelet analysis is in some cases complementary to the existing analysis techniques (e.g. correlation and spectral analysis) and in cases is capable of solving problems for which little more progress has been made prior to the introduction of wavelets. Their unique time-frequency properties lend themselves to such applications.

The wavelet transform has its roots in the Fourier transform. In order to have better background information, we will briefly review the Fourier transform and see how this transform had led research to develop the Short Time Fourier Transform (STFT) which ultimately will lead to the birth of the wavelet transformation.

## 1.1 Mathematical Transforms

Mathematical transformations are applied to signals in order to obtain further information from that signal that is not readily available in the raw time series signal i.e. time-series signal in its original form. Most of the signals in practice are time-domain signals in their raw format. When we plot time-domain signals, we obtain a time-amplitude representation of the signal. This representation is not always the best representation of the signal. In many cases, the most distinguished information is hidden in the frequency content of the signal. The frequency spectrum of a signal is basically the frequency components (spectral components) of that signal. The frequency spectrum of a signal reveals that what frequencies exist in that signal.

So the question now arises to our mind is how do we measure frequency, or how do we find the frequency content of a signal? The answer is Fourier transform (FT). If the FT of a signal in time domain is taken then the frequency-amplitude representation of that signal is obtained.

### 1.1.1 Fourier Transform

FT decomposes a signal to complex exponential functions of different frequencies in the following manner:

$$F(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} f(t) dt \qquad \ldots (1.1)$$

$$f(t) = \int_{-\infty}^{+\infty} e^{j\omega t} F(\omega) d\omega \qquad \ldots (1.2)$$

In the above equations, ‘t’ stands for time, ‘$\omega$’stands for frequency, and ‘f’ denotes the signal at hand. Note that ‘f’ denotes the signal in time domain and the ‘F’ denotes the signal in frequency domain. This convention has been used to distinguish the two representations of the signal. Equation (1.1) is called the **Fourier transform** of f (t), and the equation (1.2) gives the **inverse Fourier transform** of $F(\omega)$, denoted by f (t).

***Shortcomings of the FT:***

Fourier Transform informs whether a certain frequency component exists or not. This information does not reveal the fact that at what time this component will occur. The Fourier Transform is thus suitable if the signal is stationary but is not suitable if the signal has a time varying frequency, i.e., the signal is non-stationary. It identifies all frequency components present in the signal; however it does not provide any information regarding the time localization of these components.

### 1.1.2 Short-Time Fourier Transform

To overcome the limitation of the FT, a window-version of Fourier Transform known as **Short Time Fourier Transform (STFT)** has been developed. In STFT, the signal is divided into small segments, where these segments (portions) of the signal can be assumed to be stationary. The expression for a STFT signal is given by the following equation:

$$\left(G_{\phi} f\right)(b,\xi) = \int_{-\infty}^{+\infty} e^{-i\xi t} f(t)\phi(t-b)dt \qquad \ldots (1.3)$$

***Shortcomings of the STFT:***

STFT face a time-frequency resolution problem due to the width of the window function that is being used i.e. if we use a narrow window, we get good time resolution but poor frequency resolution. Similarly for a wide window, we get good frequency resolution but poor time resolution. The solution to these problems is to use variable window size which in turn leads to the implementation of wavelet transform.

### 1.1.3 Wavelet Transform

The **wavelet transform (WT)** uses a scalable windowing technique. The adjustable window size allows us to trade off the time and frequency resolution in different ways. If we require analyzing a large region of low frequency signal, we will use long time intervals. Likewise, if we require analyzing a small region of high frequency signal, we will use short time intervals.

Wavelet analysis consists of breaking up the signals into shifted scaled versions of the original wavelet as compared to Fourier analysis, where signals are broken into sine waves of various frequencies instead. Unlike Fourier transform, we have a variety of wavelets which are used for signal analysis. Choice of a particular type of wavelet depends on the type of application in hand. It has a special ability to examine signals simultaneously both in time and frequency.

### 1.1.4 Continuous Wavelet Transform

The **continuous wavelet transform (CWT)** has been developed as an alternative approach to the short time Fourier transform to overcome the resolution problem. The wavelet analysis is done in a similar way to the STFT analysis, in the sense that the signal is multiplied with a function, {\is the wavelet}, and the transform is computed separately for different segments of the time-domain signal. CWT is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function. The result of the CWT is the continuous set of wavelet coefficients. When the wavelet coefficients are multiplied with the scaled and shifted wavelet, the constituent wavelets of the original signal are produced. Mathematically wavelet is denoted as:

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad \ldots (1.4)$$

where 'b' is the location parameter and 'a' is scaling parameter. For the function to be a wavelet, it should be time limited. For a given scaling parameter 'a', we translate the wavelet by varying the parameter 'b'. The expression for a CWT signal can be shown in the following equation:

$$W_f(a,b)=\int_t f(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad \ldots (1.5)$$

According to equation (1.5), for every (a, b), there is a wavelet transform coefficient, representing how much the scaled wavelet is similar to the function at the location t = (b/a).

In CWT, an analyzing window is shifted along the time domain to pick up information about the signal. However, this process is difficult to implement and the information that has been picked up may overlap which ultimately result in redundancy.

### 1.1.5 Discrete Wavelet Transform

The foundation of the DWT goes back to 1976 when Croiser, Esteban, and Galand devised a technique to decompose discrete time signals. Crochiere, Weber, and Flanagan did a similar work on coding of speech signals in the same year and named this analysis scheme as subband coding. In 1983, Burt defined a technique very similar to subband coding and named it **pyramidal coding** also known as **multiresolution analysis**. Later in 1989, Vetterli and Le Gall made some improvements to the subband coding scheme by removing the existing redundancy in the pyramidal coding scheme.

The basic idea is the same as it is in the CWT. A time-scale representation of a digital signal is obtained by using digital filtering techniques. The CWT is a correlation between a wavelet at different scales and the signal with the scale (or the frequency) being used as a measure of similarity. The continuous wavelet transform was computed by changing the scale of the analysis window by shifting the window in time, multiplying by the signal, and integrating over all times. In the discrete case, filters of different cutoff frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters to analyze the high frequencies, and then passed through a series of low pass filters to analyze the low frequencies.

The **Discrete Wavelet Transform (DWT)** can be viewed as sampled version of the continuous parameter wavelet transform. DWT involves choosing scales and positions based on powers of two viz. called **dyadic scales** and **translation**. The mother wavelet is rescaled or "dilated", by powers of two and translated by integers. The results are more efficient and are just as accurate. The DWT algorithm is capable of producing coefficients of fine scales for capturing high frequency information, and coefficients of coarse scales for capturing low frequency information. The DWT with respect to a mother wavelet, $\psi(t)$, is defined as:

$$f(t) = \sum_{k} s_j(k)\phi_{j_0,k}(t) + \sum_{j>j_0}\sum_{k} w_{j,k} 2^{\frac{j}{2}}\psi(2^j t - k) \quad \ldots (1.6)$$

where j is the dilation or level index, k is the translation or scaling index, $\phi_{j_0,k}$ is a scaling function of coarse scale coefficients $s_j(k)$ and $\Psi$ is the wavelet function of detail (fine scale) coefficients $w_{j,k}$. All functions of $\psi(2^j t - k)$ are orthonormal.

The DWT has many advantages in compressing a wide range of signals. With DWT technique, a very large proportion of the coefficients of the transform can be set to zero without appreciable loss of information. Besides that if more properties other than the stationary properties of a signal are desired, DWT certainly would be a better choice as compared with the existing technique of FT. The discrete wavelet transform (DWT) provides sufficient information both for analysis and synthesis of the original signal, with a significant reduction in the computation time. The DWT is considerably easier to implement when compared to the CWT.

**1.1.6 Maximal Overlap Discrete Wavelet Transform:**

The **maximal overlap discrete wavelet transform (MODWT)** is a non-orthogonal variant of the classical discrete wavelet transform that unlike the orthogonal discrete wavelet transform, is translate invariant because shifts in the signal do not change the pattern of coefficients. Applying a *j*th order nondecimated version of the orthogonal DWT, *i.e.* the maximal overlap DWT (MODWT), yields *J* vectors of wavelet filter coefficients $\tilde{W}_{j,t}$ for $j$ = 1,……., $J$ and $t=1,\ldots\ldots\ldots,\frac{N}{2^j}$, and one vector of wavelet filter coefficient $\tilde{V}_{j,t}$ through

$$\tilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l}\, f(t-l) \qquad \ldots (1.7)$$

$$\tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} \, f(t-l) \qquad \ldots (1.8)$$

where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are, respectively, the rescaled wavelet and scaling filter coefficient from a Daubechies compactly supported wavelet family.

## 1.2 Neural Networks

The working principle of neural network is based on the functioning of human brain. The brain consists of a large number (approximately $10^{11}$) of highly connected elements (almost $10^4$ connections per element) called **neurons**. This highly interconnected set of neurons facilitate us reading, breathing, motion and thinking. These neurons have three principal components: the dendrites, the cell body and the axon. The dendrites are tree-like receptive networks of nerve fibers that carry electrical signals into the cell body. The cell body effectively sums and thresholds these incoming signals. The axon is a single long fiber that carries the signal from the cell body out to other neurons. The point of contact between an axon of one cell and a dendrite of another cell is called a synapse. Some of the neural structures are defined at the time of birth. Other parts are developed later through learning, as new connections are made and others waste away. The vast neural network has an elaborate structure with very complex interconnections. This is illustrated in Figure 1.1 below:

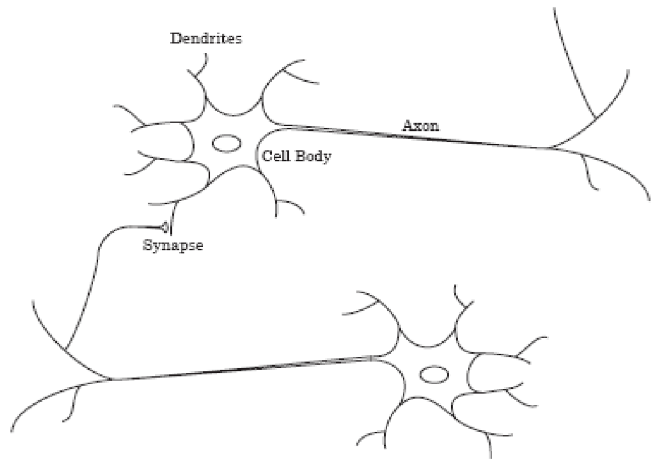


**Fig 1.1:** Elements and connectivity of Neural Network

Scientists have only just begun to understand how biological neural network operate. It is generally understood that all biological neural functions, including memory, are stored in the neurons and their inter-connections. This leads to the following question:

Is it possible to construct a small set of simple "artificial neurons" and perhaps train them to serve as a useful function? The answer is yes. The neurons that we consider here are not biological. They are extremely simple abstractions of biological neurons, realized as elements in a program or perhaps a circuit made of silicon. Network of these artificial neurons do not possess a fraction of the power of the human brain, but can be trained to perform useful functions.

So a Neural Network is a machine that is designed to model the way in which the brain performs a particular task or function of interest. More clearly a Neural Network can be viewed as a processor made up of simple processing units having a natural propensity for storing experimental knowledge and making it available for use.

It resembles the brain in two respects:

1. Knowledge is acquired by the networks from its environment through a learning process.

2. Interneuron connection strength, known as synaptic weights, is used to store the acquired knowledge.

So artificial neural networks are mathematical tools originally inspired by the way human brain processes information. The neuron receives information through a number of inputs nodes. The inputs are then multiplied by a weight denoted by W *e.g.* the input $X_1$ is multiplied by a weight of $W_1$. The same is also done for the rest of the inputs as well. Finally a weight vector comprising of all the weights is formed. The result of all the multiplication of the inputs and weights are then fed to the summer Σ where addition is executed. The output of the summer is then fed to the Linear Threshold unit. If the input to the summer is above the threshold level, an output of ‘1’ will take place, else, an output of ‘0’ will occur. All the data can thus be presented to the Network in binary (‘1’ and ‘0’) or in bipolar (‘1’ and ‘-1’) form. Figure 1.2 (Down, T., 2003) illustrates the diagram of a Linear Threshold unit (LTU).

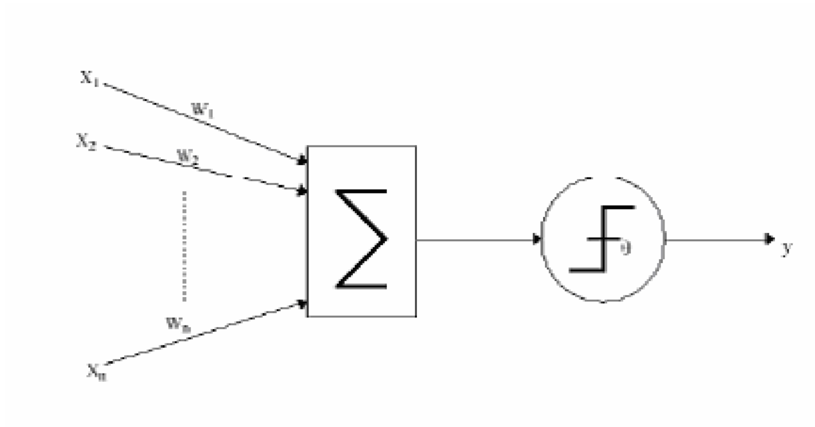


**Fig 1.2:** Linear threshold unit

## 1.3 Time Series

Time series is a set of statistical observations measured at a time interval

and arranged in chronological order, for example, the yearly demand of a commodity, weekly prices of an item, food production in a country, etc. In other words, a time series is a sequence of measured quantities $x_1$, $x_2$ ,…., $x_n$ of some physical system taken at regular intervals of time. Time series analysis provides three important specific features: prediction, modeling, and characterization. It helps in understanding the following phenomenon:

i. Knowing the real behavior of the past.

ii. Predicting the future behavior like demand, production, weather conditions, prices etc.

iii. Helps in planning the future operations.

iv. Analysis of time series also helps to compare the present accomplishments with the past performances.

## Motivation and Objectives

A general problem which occurs in wavelet analysis is the selection of a suitable wavelet according to the problem. Thus when we decide to choose a wavelet for a signal processing application at our hand, we always arrive at a multi-way junction point at which we have to select a suitable wavelet. Various signal properties and intuitions were used in the past by researchers for selecting the wavelet. Parameterization of wavelet families allows us to generate infinite number of wavelets for our selection.

There are several methods of parameterization for generating filter coefficients corresponding to compactly supported orthonormal wavelets for example polyphase matrix method, pollen-type method etc. These methods of parameterization are not suitable for scaling function filters having higher order taps. To analyze time series and for better understanding of results there is a need of a scaling function filters with higher order tabs. We shall develop a parameterization method for generating 6-tap wavelet system and then shall generate the MATLAB and MAPLE codes as per the problem requirement.

A time series is a sequence of observations associated with an ordered independent variable *'t'* (the variable *t* can assume either a discrete set of values such as integers or a continuum of values such as entire real axis. Examples of both types include time, depth or distance along a line, so a time series need not actually involve time). Wavelets are a relatively new way of analyzing time series in that the formal subject dates back to the 1980s, but in many aspects wavelets are a synthesis of older ideas with elegant mathematical tools and efficient computational algorithms.

One of the most common statistical properties violated by time series data is stationarity. The analysis of time series has often been difficult when data do not conform to well studied theoretical concepts. Wavelets provide a unique decomposition of time series observations, independent of the fact that whether time series is stationary or non-stationary. This enables one to deconstruct the data in ways that are potentially revealing. The situation is similar to, but distinct from, the insight gained from analyzing data using Fourier series. In fact as far as Fourier series are applicable to data sets under examination, the applications of Fourier analysis yield many interesting insights into dynamic relationship. For example in the GDP indices of production, we can discover that there are no simple Fourier components except in the context of non-durable goods. Only by using advanced techniques we can clarify and confirm the nature of the non-stationarity that has long been suspected.

The existence of data, such as the above defying current mathematical and statistical methods motivates researchers to develop better theories and tools to analyze them.

Another concept which arises in the analysis of financial time series is the notion of 'multiscale features.' That is, an observed time series may contain several phenomena, each occurring in different time scales (these corresponding to ranges of frequencies in the Fourier domain). Wavelet techniques possess a unique natural ability to decompose time series into several sub-series which may be associated with particular time scales. Hence, interpretation of features in complex financial time series may be alleviated by first applying a wavelet transform and subsequently interpreting each individual sub-series.

We shall analyze the financial time series and predict the results using wavelets tools. We shall take the data of Bombay Stock Exchange (BSE) and National Stock Exchange (NSE) for this purpose. We shall also revisit the issue of integration of these emerging stocks markets. The analysis will be based on the maximum overlap discreet wavelet transform (MODWT), wavelet variance, covariance and correlation coefficients.

The objectives of the proposed work are:

(i) To study the literature and material available in related fields.

(ii) To develop models for analysis of time series data.

(iii) To generate MATLAB and MAPLE codes for analysis purpose.

(iv) To evaluate the performances of developed models.

(v) To draw graphs and figures depicting the relationships between different parameters and comparison thereof with existing works.

# REVIEW OF LITERATURE

# 2 Review of Literature

This chapter provides an overview of the developments in wavelet theory and current literature referenced for the work done in this thesis.

Wavelet means 'small waves', so the wavelet analysis is all about analyzing signals with short duration finite energy function. They have several interesting features which are not exhibited by big waves like sines and cosines **(Soman [75])**. The use of wavelet techniques in data analysis has exponentially grown since its discovery and represents a synthesis of old techniques blended with robust mathematical results and efficient computational algorithms under the interest of a broad community (**Daubechies *et. al.* [18]**).

The first entrant in the field of wavelet was a Hungarian Mathematician named **Alfred Haar (1909)** who introduced the functions that are now called 'Haar wavelet'. These functions consist of simply a short positive pulse followed by a short negative pulse. **Morlet** in 1984 first, used the term wavelet. Since their "invention" in the early eighties, wavelets have drawn enormous attention of both Mathematicians and Scientists community working in applied sciences. Subsequently, **Mallat** in 1986 linked the theory of wavelets with the existing literature on subband coding and quadrature mirror filters which are the image processing community's version of wavelets.

Several monographs appeared, both on the mathematical theory of wavelets e.g. **Meyer [50], Chui [10], Mallat [48], Cohen [11]** and also on their applications **Ogden [57], Percival *et al.* [62], Thuillard [76], Soman [75]**. In particular works of **Chui [10], Percival et al. [62]** and **Soman [75]** are very useful for the theory as well as time series analysis purpose.

The construction of multidimensional wavelets is due to **Mallat [47]**, who also proposed a multivariate version of the DWT. **Cohen *et al.* [13]** introduced wavelets on the interval, i.e. wavelet bases for functions defined on an interval as opposed to the whole real line. **Geronimo *et al.* [24]** formulated multiple wavelets which use translations and dilations of more than one wavelet function.

Orthogonality is a very useful property of wavelets but does not hold universally for all wavelet classes. **Daubechies [16]** first proposed Orthonormal Bases of Compactly Supported Wavelets. **Cohen *et al.*** [12] introduced biorthogonal wavelets, where the decomposition and reconstruction steps use different non-orthogonal bases which are however, in a certain sense, mutually orthogonal. However, even non-orthogonal wavelet still provide a basis for the space into which the function is to be projected, maximum overlap discrete wavelet transform (MODWAT) and biorthogonal wavelets are examples explained in **Percival *et al.* [62]** and **Soman [75]**.

The concept of multiresolution analysis was first introduced by **Mallat [46]**. Multiresolution analysis provides a framework for examining functions at different scales: zooming in to see the fine detail and moving out to view the broader picture. It enables one to understand wavelet bases and construct new examples. **Mallat [46]** studied the properties of the operator which approximates a signal at a given resolution and shows that the difference of information between the approximation of a signal at the resolutions $2^{j+1}$ and $2^{j}$ can be extracted by decomposing this signal on a wavelet orthonormal basis of $L^2(R^n)$, which is a family of functions and is built by dilating and translating a unique function $\psi(x)$. This is known as decomposition which defines an orthogonal multiresolution representation called wavelet representation. Multiresolution representations are very very effective for analyzing the information content of images.

Functions or data sets are observed at a finite number of discrete time points. Thus a continuous representation is unsuitable but a discrete analogue is required. The discrete wavelet transform (DWT), proposed by **Mallat [47]**, connects wavelets with multiresolution analysis to provide an efficient scheme for performing a discrete, wavelet based transformation.

In the DWT, the transform proceeds from one level to the next by decomposing the smooth sequences, $c_j$. The DWT, therefore, provides a progressive analysis of the low-frequency smooths. However, the most significant information contained within a signal is frequently contained within the middle or high frequencies. Thus, an alternative decomposition which provides a suitably refined partition of these frequency bands is desirable. Such decomposition can be afforded by the wavelet packet transform introduced by **Coifman *et al.* [14]**. This transform is implemented by not only decomposing the smooth sequences, $c_j$ but also the detail sequences, $d_j$. This provides a multitude of wavelet packets.

**Coifman *et al.* [15]** introduced wavelet packets: redundant collections of linear combinations of wavelets capable of representing signals more economically than wavelets themselves.

**Herley *et al.* [29]** shown that infinite impulse response filters lead to more general wavelets of infinite support and give a complete constructive method which yields all orthogonal two channel filter banks, where the filters have rational transfer functions. It also shows how these can be used to generate orthonormal wavelet bases.

**Percival *et al.* [62]** developed the relationship between wavelets and filter banks, that is, sequence of pairs of high and low pass filters. One can approach the analysis of the properties of wavelets either through wavelet themselves or through the properties of filter banks. Many new classes of wavelets are now generated by specific properties for the filter banks.

**Selesnick [71]** described a new set of dyadic wavelet frames with two generators where the first wavelet is concentrated halfway between the spectrum of the second wavelet and the spectrum of its dilated version, whereas the second wavelet is translated by half integers rather than whole integers in the frame construction. The given arrangement leads to an expansive wavelet transform that is approximately shift invariant and has intermediate scales.

A matrix theory is developed for the noncausal polyphase representation that underlies the theory of lifted filter banks and wavelet transforms. The theory presented by **Brislawn *et al.* [6]** develops an extensive matrix algebra framework for analyzing and implementing linear phase two-channel filter banks via lifting cascade schemes. The theory benefits significantly from a number of group-theoretic structures arising in the polyphase-with-advance representation and in the lifting factorization of linear phase filter banks.

**Kumar *et al.* [35]** relaxed the usual assumptions in denoising that the data consist of a “true” signal to which normally distributed noise is added. Instead of regarding noise as the high-frequency part in the data to be removed either by a “hard” or “soft” threshold, they defined it as that part in the data which is harder to compress than the rest with the class of models considered. The lifting scheme is well known to be an efficient tool for constructing second generation wavelets and is often used to design a class of biorthogonal wavelet filter banks. For its efficiency, the lifting implementation has been adopted in the international standard JPEG2000. It is also known that the orthogonality of wavelets is an important property for many applications. **Zhang *et al.* [92]** presented how to implement a class of infinite-impulse-response (IIR) orthogonal wavelet filter banks by using the lifting scheme with two lifting steps.

A technique for linear system identification in frequency subbands by using wavelet packets was presented by **Paiva *et al.* [59]**. The wavelet-packet decomposition tree has been used to establish frequency bands where subband models are created. An algorithm is also proposed to adjust the tree structure in order to achieve a compromise between accuracy and parsimony of the model. **Huang *et al.* [31]** investigated the dependence between energy values from different subbands which may be from the same wavelet basis, or from different wavelet bases. Based on the theoretical analysis and simulation, an information-theoretic measure, mutual information, for selecting subbands for sparse representation of textures for classification was proposed.

The development of wavelet analysis using filter banks helps to clarify the relation between wavelet analysis and Fourier analysis. Because the wavelet transforms can be obtained through a cascade of low and high pass filters one can then obtain the transfer function of the filters, and if one assume stationarity in the time series, one can also determine the frequency ranges of the time series that will be captured by the filter banks. However, the validity of this interpretation depends on the assumption of stationarity of the signal. When the stationarity assumption is violated, the frequency interpretation can only be approximate and local.

Wavelets and their extensions have been applied in a multitude of areas, such as stock market analysis, signal and image processing, data compression, computer graphics, astronomy, quantum mechanics and turbulence etc. Application of wavelets in time series analysis has also seen surge in interest over the past few years, as can be seen from the review article of **Nason *et al.* [55]** and the recent monograph of **Percival *et al.* [62]**.

Several authors use wavelets in Time series analysis. **Ramsey *et al.* [66]** re-examined the U.S. stock market price index for any evidence of self-similarity or order that might be revealed at different scale using wavelet. The wavelet transform localized in time can be used to indicate how the power of projection of the signal onto the kernel varies with the scale of observation. **Rao *et al.* [67]** looked at higher order moments of wavelet transforms of nonlinear signals.

**Santoso *et al.* [70]** presented a new approach to detect, localize, and investigate the feasibility of classifying various types of power quality disturbances. The approach is based on wavelet transform analysis, particularly the dyadic-orthonormal wavelet transform. The key idea underlying in this approach is to decompose a given disturbance signals into other signals by using multiresolution signal decomposition techniques.

**Li *et al.* [42]** used wavelets to hidden periodicities and **Wong *et al.* [84]** to detect jumps. **Chiann *et al.* [9]** defined the wavelet periodogram for stationary process as a sequence of squared wavelet coefficients of the process; they also analyzed some of its properties. **Serroukh *et al.* [72]** investigated time-scale properties of time-series in various models by estimating the variance of non-decimated wavelet coefficients (so-called "wavelet variance") at different scales.

The method of **Soltani *et al.* [74]** exploited the decorrelating property of wavelets to forecast long-memory processes. **Lee *et al.* [39]** constructed a test for serial correlation.

**Whitcher [81]** proposed a method, based on wavelet packets, for simulating Gaussian process with unbounded spectra. **Wang *et al.* [80]** and **Audit *et al.* [3]**, amongst others, used wavelets to estimate the scaling exponent in self-similar process. **Bilen *et al.* [5]** proposed a model-free method for detecting outliners in time series data using wavelets.

Wavelets have been used in time series forecasting in conjunction with neural network methods (**Geva [25], Milidiu *et al.* [52], Hee *et al.* [28], Soltani [73]**). The combination of wavelet and neural network techniques were used to forecast electricity demand data (**Zhang *et al.* [89]**), financial time series (**Zhang *et al.* [90]**) and web traffic (**Aussem *et al.* [4]**).

Forecasting the future behavior of time series is, along with understanding the data generating mechanism is the main aims of time series analysis. For non-stationary Gaussian models, various more sophisticated forecasting techniques have already been developed. **Zheng *et al.* [89]** applied their SVH-ARMA (state-dependent vector hybrid ARMA) technique to the forecasting of vector time series constructed by taking the DWT of scalar time series.

**Walden *et al.* [79]** constructed multi-resolution filters for the analysis of matrix-valued time series. **Nason *et al.* [54]** used wavelet packets to model a transfer function between two non-stationary time series and **Whitcher *et al.* [82]** proposed a test for variance homogeneity in long memory process.

**Li *et al.* [41]** used wavelets (and other filter banks) to forecast seasonal patterns. The forecasting method proposed by **Wong *et al.* [85]** depends on the decomposition of the time series using wavelets into three summands: trend, harmonic and irregular components.

**Razdan [68]** applied wavelet concepts on two ‘strongly correlated’ financial time series. Apart from obtaining wavelet spectra, they also calculated wavelet correlation coefficient and shown that strong correlation or strong anti-correlation depends on the scale.

A novel approach to compute all four types of Discrete Wavelet transform (DWT) was proposed by **Liu *et al.* [44]**, by using kernel transforms and Taylor expansions. They approximated a DWT by a linear sum of discrete moments which enabled them to use computational techniques developed for computing moments to compute DWTs efficiently.

**Gallegati [22]** had applied the theory of wavelet analysis to MENA Stock Market and revisiting the issue of integration of emerging stock market with each other and with the developed markets over different time horizon using weekly stock indices data from June 1997 until March 2005 of the five major MENA equity market (Egypt, Israel, Jordan, Morocco, and Turkey) and applying the discrete wavelet decomposition analysis.

**Zhao *et al.* [93]** generalized the multiwavelet sampling theorem by reproducing a kernel that is easy to use. They also considered the general cases of the uniform non-integer and irregular sampling and finally, established a general irregular sampling theorem for multiwavelet subspaces and derived an estimate for the perturbations of uniform non-integer sampling in shift-invariant spaces.

**Chen *et al.* [7]** investigated how the seemingly chaotic behavior of stock market could be well represented using the Local Linear Wavelet Neural Network (LLWNN) technique. For this purpose, they considered the Nasdaq-100 index of NASDAQ stock market and S & P CNX NIFTY stock index and analyzed 7-year Nasdaq-100 main index values and 4-year NIFTY index values.

**Jiang *et al.* [34]** presented a new method for discussing persistence of financial risk based on volatility impulse response function. The new definition of volatility persistence and common persistence was established. Further more, they introduced corresponding topics into the area of nonlinear common persistence through wavelet neural network. The methods proposed by them are easy to be conducted, and can be extended to discuss the persistence of higher moments risk. Their empirical results have shown that the volatility persistence exists in Chinese stock markets, which can not be removed by linear combination. However, based on wavelet neural network, a nonlinear combination has been found to reduce the volatility persistence.

**Li *et al.* [40]** proposed a hybrid approach on the basis of the knowledge discovery methodology by integrating K-chart technical analysis for feature representation of stock price movements, discrete wavelet transform for feature extraction to overcome the multi-resolution obstacle, and a novel two-level self-organizing map network for the underlying forecasting model.

**Liu *et al.* [45]** have done recently technical analysis of stocks mainly focusing on the study of irregularities, which is a non-trivial task. They also presented an algorithm, PXtract to automate the recognition process of possible irregularities underlying the time series of stock data using wavelet multi-resolution analysis and radial basis function neural networks for the matching and identifying of these irregularities.

**Mitra *et al.* [53]** used the multiresolution wavelet-filtering technique to study the dynamic inter-relationships among the real and financial sectors of economy. They extracted relationships among real stock returns, inflation and real activity for two contrasting economies in the United States and the Indian economy.

**Gallegati [23]** investigated relationship between stock market returns and economic activity using signal decomposition techniques based on wavelet analysis.

The process of globalization determined through the trade and financial liberalization of the ninetees has been further intensified by the recent trends in international stock market indices to become more and more integrated. In such a context of interdependence amongst major international stock indices, equity markets of emerging countries like China and India may prove to be a profitable opportunity for international investors to invest in. It is in this context that the area of financial time series analysis and forecasting has become more prominent these days as quite a large group of big business houses are now focusing on optimizing their business processes in order to attain best estimates of production planning consistently spread over a long time period. In the present scenario, therefore, the development of new techniques and skills like wavelet analysis has become absolute necessity for the analysis and predictions based on time series.

# MATERIALS AND METHODS

# 3 Materials and Methods

The fundamental problem in many fields of research is the analysis of time series. The nature of data collected in different fields leads naturally to considerations of time series methods. In many research areas, samples are observed over a sequence of time intervals e.g. hourly, monthly or yearly basis to analyze and predict the pattern and behavior of time series which can provide as much information as is possible about the series. Recent research activities in field of wavelets have demonstrated that wavelets are capable of analyzing time series efficiently.

The classical method used to investigate features in a time series in the time domain is to compute the covariance and correlation function and in the frequency domain the frequency decomposition of the time series is achieved through Fourier analysis. In practice, we often want to consider time scale of non-stationary processes and, therefore, time resolved methods become necessary. The time resolved method for Fourier analysis is a short-time Fourier transform analysis but this method has its own limitations which makes it less desirable for the analysis of time series with certain characteristics e.g. time series with sharp spikes and discontinuities. Wavelet analysis gives a better approximation for time series with such characteristics. Additionally, wavelet analysis often extracts more information about the time series than many other classical methods of analysis.

Present chapter is organized in the following manner: the main properties and the fundamental concepts of the wavelets and the methods for the construction of parametric orthogonal wavelet are dealt in section 3.1. A brief overview of the fundamental concepts in classical time series analysis and the method of calculating the wavelet correlation coefficient through wavelet variance and covariance are discussed in section 3.2.

## 3.1 Wavelets

Wavelets are the building blocks of wavelet analysis. A function is said to be a **wavelet** if the collection of functions obtained by dyadic dilation and integral translations forms an orthonormal basis for $L^2(R)$. The precise definition of a wavelet is as follows:

**Definition 3.1.1** A function $\psi \in L^2(R)$ is said to be an **orthonormal wavelet** if the system of functions $\{\psi_{j,k} : j,k \in Z\}$ forms an orthonormal basis for $L^2(R)$, where

$$\psi_{j,k}(x) = 2^{j/2}\psi\left(2^j x - k\right), \qquad j,k \in Z. \qquad \ldots(3.1)$$

In other words, the functions $\{\psi_{j,k}\}$ are orthonormals if

$$\langle \psi_{j,k}, \psi_{j',k'} \rangle = \delta_{j,j'} \delta_{k,k'} ,$$

and if any function $f \in L^2(R)$ can be written as

$$f = \sum_{j \in Z} \sum_{k \in Z} \langle f, \psi_{j,k} \rangle \psi_{j,k} ,$$

where the convergence of the series is in the $L^2(R)$-norm. The **inner product** of two functions *g* and *h* of $L^2(R)$ is defined by

$$\langle g, h \rangle = \int_R g(x) \overline{h(x)} dx .$$

Since we shall be concerned only with orthonormal wavelets, so whenever we say that $\psi$ is a wavelet, we mean that it is an orthonormal wavelet. The first example of such a function known as **Haar wavelet** was introduced by Haar in 1909 and is defined as

$$\psi(t) = \begin{cases} 1 & 0 \le t \le \frac{1}{2} \\ -1 & \frac{1}{2} \le t \le 1 \\ 0 & elsewhere \end{cases}$$

### 3.1.2 Wavelet Coefficients

Let $f(x) \geq 0$ be a one-dimensional function taking positive or zero values. The wavelet coefficient $c_{j,k}$ of $f(x)$ at a given position *b* and scale *a* is defined

by

$$c_{j,k} = \langle f, \psi_{j,k} \rangle \qquad \text{... (3.2)}$$

i.e. if we define an integral transform $W_\psi$ on $L^2(R)$ by

$$(W_\psi f)(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x) \overline{\psi\left(\frac{x-b}{a}\right)} dx, \qquad f \in L^2(R) \qquad \text{... (3.3)}$$

then the wavelet coefficients in wavelet series

$$f(x) = \sum_{j,k=-\infty}^{\infty} c_{j,k} \psi_{j,k}(x), \qquad \text{... (3.4)}$$

become

$$c_{j,k} = (W_\psi f)\left(\frac{k}{2^j}, \frac{1}{2^j}\right) \qquad \text{... (3.5)}$$

The linear transformation $W_\psi$ is called the **"integral wavelet transform"** relative to the **"basic wavelet"** $\psi$. Hence, the $(j,k)^{th}$ wavelet coefficient of *f* is given by the integral wavelet transformation of *f* evaluated at the dyadic position $b = k/2^j$ with binary dilation $a = 2^{-j}$.

### 3.1.3 Multiresolution Analysis

Multiresolution analysis provides a framework for examining functions at different scales: zooming in to see the fine details and moving out to view the broader picture. It enables us to understand wavelet bases and construct new examples. Multiresolution analysis has also an important role in the formation of the discrete wavelet transform.

Following Mallat [47], a multiresolution analysis is defined as follows:

A **multiresolution analysis (MRA)** is a nested sequence of closed subspaces,

$V_j \subset L^2(R)$ for j$\in$ Z,

$$\ldots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \ldots$$

such that

(i) the spaces have a trivial intersection:

$$\bigcap_{j\in Z} V_j = \{0\};$$

(ii) the union is dense in $L^2$(R):

$$\overline{\bigcup_{j\in Z} V_j} = L^2(R);$$

(iii) the following two-scale relation exists:

$$f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1} \qquad \text{for all } j \in Z;$$

(iv) $f(x)\in V_0 \Leftrightarrow f(x \Leftrightarrow k)\in V_0$ for all $k \in Z$;

(v) and, finally, a scaling function $\phi \in V_0$ exists whose integer translations $\{\phi_{0,k} : k \in Z\}$ constitute an orthonormal basis of $V_0$.

Conditions (iii) and (iv) imply that $\{\phi_{j,k} : k \in Z\}$ constitutes an orthonormal basis of $V_j$. Furthermore since $V_0 \subset V_1$, the function $\phi(x)\in V_0$ can thus be represented as a linear combination of functions from $V_1$. In other words for some coefficients $\{h_k\}_{k\in Z}$, we can express

$$\phi(x)=\sum_{k\in Z} h_k \phi_{1,k}(x)=\sum_{k\in Z} h_k 2^{1/2}\phi(2x-k).$$

This equation is often called the **scaling equation**.

### 3.1.4 Wavelet decomposition

Wavelet decomposition can be considered as projection of the signal on the set of wavelet basis vectors. Each wavelet coefficient can be computed as the dot product of the signal with the corresponding basis vector. In Multiresolution analysis (MRA), wavelet functions and scaling functions are used as building blocks to decompose and construct the signal at different resolution levels. The wavelet function generates the detail version of the decomposed signal and the scaling function generates the approximated version of the decomposed signal. Mathematically it can be presented by the equation:

$$f(t) = \sum_k c_0(k)\phi(t-k) + \sum_k \sum_{j=0}^{J-1} d_j(k) 2^{j/2} \psi(2^j t - k)$$

where, $c_0$ is the '0' level scaling coefficient and $d_j$ is the wavelet coefficient at scale *j*. $\phi(t)$ and $\psi(t)$ are the scaling function and the wavelet function respectively and $k$ is the translation coefficient. The translated and scaled version of the wavelet, $\psi(2^j t - k)$**,** used in MRA will build a time-frequency picture of the decomposed signal. Multiresolution signal decomposition is used to achieve two important properties: The first is the localization property in time for any transient phenomena (This will appear by the presence of large coefficients at the time of disturbance) and the second property is the partitioning of the signal energy at different frequency bands (This gives an idea of the frequency content of the distorted signal).

### 3.1.5 Filter Banks

**A filter bank** is an array of band-pass filters that separates the input signal into several components, each one carrying a single frequency subband of the original signal. It is also desirable to design the filter bank in such a way that subbands can be recombined to recover original signal. The first process is called **analysis**, while the second is called **synthesis**. The output of analysis is referred as subband signal with as many subbands as there are filters in filter bank.

Consider a function in some approximation space $v^j$. The coefficients in terms of the scaling function basis can be written as a column matrix of values:

$$c^{j-1} = \left[c_0^j, \ldots\ldots, c_{v(j)-1}^j\right]^T$$

We create a lower resolution $c^{j-1}$ of $c^j$ with a smaller number of coefficients $v(j-1)$. This can be done by linear filtering or down sampling on the $v(j)$ entries of $c^j$.This can be expressed as a matrix equation:

$$c^{j-1} = A^j c^j$$

where $A^j$ is a constant $v(j-1)$ by $v(j)$ matrix.

Since $c^{j-1}$ contains fewer entries than $c^j$, it is clear that some details is lost in the process. The lost details $d^{j-1}$ can be computed by $d^{j-1} = B^j c^j$ where $B^j$ is a constant $w(j-1)$ by $w(j)$ matrix related to $A^j$. The pair of matrices $A^j$ and $B^j$ is called analysis filters. The process of splitting $c^i$ into $c^{i-1}$ and $d^{i-1}$ is called analysis or decomposition. The original coefficients $c^j$ can be recovered from $c^{j-1}$ and $d^{j-1}$ by using matrices, $P^i$ and $Q^i$. Recovering $c^i$ from $c^{i-1}$ and $d^{i-1}$ is called synthesis or reconstruction. $P^i$ and $Q^i$ are called synthesis filters.

$$c^j = P^j c^{j-1} + Q^j d^{j-1}$$

The analysis or decomposition equations and the reconstruction equations appear as:

$$c_k^{j-1} = \sum_l a_{l-2k} c_l^j \qquad d_k^{j-1} = \sum_l b_{l-2k} c_l^j \qquad c_k^j = \sum_l \left( p_{k-2l} c_l^{j-1} + q_{k-2l} d_l^{j-1} \right)$$

The procedure for splitting $c^j$ into a low resolution part $c^{j-1}$ and a detail part $d^{j-1}$ can be recursively applied to the lower resolution part. The original coefficients can be expressed as a hierarchy of lower resolution parts $c^0, \ldots, c^j$ and details $d^0, \ldots, d^j$ and this recursive process is called **filter bank** (Fig. 3.1).

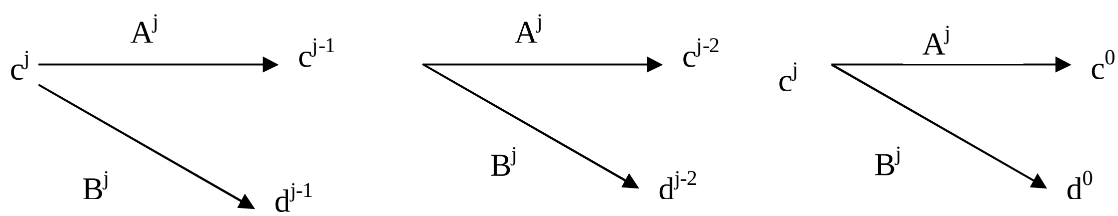


**Fig. 3.1** Filter Bank

$A^j$ and $B^j$ are formed by the matrices satisfying the relation:

$$\left[ \phi^{j-1} \middle| \psi^{j-1} \right] \left[ \frac{A^j}{B^j} \right] = \phi^j .$$

$\left[ P^j \middle| Q^j \right]$ and $\left[ A^j / B^j \right]$ are square matrices. From the above equations, we get

$$\left[ \frac{A^j}{B^j} \right] = \left[ P^j \middle| Q^j \right]^{-1} .$$

The matrices $\left[ A^j / B^j \right]$ and $\left[ P^j \middle| Q^j \right]^{-1}$ must be at least invertible.

### 3.1.6 Wavelet Packets

Wavelet packets are particular linear combinations of wavelets. As an extension of the standard wavelets, wavelet packets represent a generalization of multiresolution analysis and use the entire family of subband decompositions to generate an overcomplete representation of signals. They form bases which retain many of the orthogonality, smoothness, and localization properties of their parent wavelets. The coefficients in the linear combinations are computed by a recursive algorithm making each newly computed wavelet packet coefficient sequence the root of its own analysis tree. An orthonormal bases of $L^2(R)$ at the *j*-th resolution level generated by the wavelet has a frequency localization proportional to $2^j$.

For two sequences $(u_k)$ and $(v_k)$ the basic wavelet-packets $w_n(x)$, *n*=0,1,2,… are defined by the following recursion

$$w_{2n}(x) = \sqrt{2}\sum_{-\infty}^{\infty} u_k w_n(2x+k)$$

coupled with

$$w_{2n+1}(x) = \sqrt{2}\sum_{-\infty}^{\infty} v_k w_n(2x+k),$$

the function $w_0(x)$ belonging to $L^1(R)$ and being normalized by

$$\int_{-\infty}^{\infty} w_0(x)dx = 1$$

### 3.1.7 Orthogonality Condition and Parameterization

Like human wavelets living in families and the members of family differing in their attributes like size (filter length), shape and vanishing moments, parameterization of wavelet families allows us to generate infinite number of wavelets for us to choose from to analyze the financial time series. Parameterization method of generating the wavelets act as a secret key but there is a need of a scaling function filter with higher order taps for better analysis of time series and for better results. A brief introduction to the theory is as follows:

A function $\psi(t)$ is an **orthonormal wavelet**, if the family

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \qquad for\ j,k \in Z$$

is an orthonormal basis of $L^2(R)$.

Daubechies introduced a general method to construct wavelets. The construction is based on scaling functions satisfying a dilation equation given by a linear combination of real filter coefficient *h(K)* and dilation and translation version of the scaling function

$$\phi(t) = \sum_{K=0}^{N-1} h(k)\sqrt{2}\phi(2t-k) \qquad \ldots (3.6)$$

The associated wavelet function is given by

$$\psi(t)=\sum_{K=0}^{N-1} g(k)\sqrt{2}\phi(2t-k) \qquad \text{... (3.7)}$$

which is made orthogonal to $\phi(t)$ and its translates by the proper choice of *g(k)*. The relation between *g(k)* and *h(k)* is given by

$$g(k)=(-1)^k\ h(N-1-k)$$

where N is the number of coefficients in the filter and N must be taken as even for orthogonal wavelet system.

Imposing condition on the scaling function given via the dilation equation (3.6) put constraints on the filter coefficient. Orthogonality of translated version of scaling function implies quadratic constraints on filter coefficients. Vanishing moments on the associated wavelet function and normalization introduces linear constraints. Conditions on scaling functions are mapped into conditions on scaling filter coefficients as follows:

*Condition 1: Unit Area under scaling function*

It is necessary to impose certain condition on $\phi$ in order to uniquely determine filter coefficients. Firstly, we require $\phi$ to be normalized i.e. we set $\int\phi(t)dt=1$. The scaling relation then imposes a condition on the filter coefficients

$$\int_{-\infty}^{\infty}\phi(t)dt=\int_{-\infty}^{\infty}\sum_{k=0}^{N-1}h(k)\sqrt{2}\phi(2t-k)dt$$

$$\Rightarrow \sum_{k=o}^{N-1} h(k) \;=\; \sqrt{2}$$

*Condition 2: Orthonormaility of translates of scaling functions*

The integral translates of scaling function must be orthonormal. This requires that

$$\int \phi(t)\phi(t-k)dt = \delta_{0,k}$$

which gives

$$\sum_{l=0}^{N-1} h(l)h(l-2k) = \delta_{0,k} \qquad \left(k = 1, 2, 3, \ldots\ldots, \frac{N}{2} - 1\right)$$

In condition (2), the constraints for k = 1,2,…,N/2 – 1 is known as **double shift orthogonality constraints**.

*Condition 3: Vanishing Moments or Approximation Conditions*

In many applications, we need to approximate a signal using scaling function. The degree p of monomonials 1, t, $t^2$, ….., $t^p$ be reproduced exactly using a basis of scaling function

$$t^p = \sum_k d_k^{\,p} \phi(t-k)$$

Orthonormality of basis functions imply that

$$d_k^{\,p} = \int t^p \phi(t-k)dt$$

The scaling function should posses certain properties (suitable shape), but its shape depends on the coefficients h (k).

Now we assume that it is possible to represent exactly monomials of order up to $p$ using a given scaling function. This will impose certain conditions on h(k).

We have

$$t^p = \sum_k d_k^{\,p} \phi(t-k)$$

and projection of $t^p$ on $\psi(t)$ is

$$\int t^p \psi(t) dt = \int \sum_k d_k^{\,p} \phi(t-k) \psi(t)\, dt$$

$$\sum_k d_k^{\,p} \int \phi(t-k) \psi(t)\, dt = 0$$

since $\phi(t)$ and $\psi(t)$ are orthogonal.

That means, if $\phi(t)$ is capable to expressing monomials of order up to p, then the corresponding wavelet function must have its moments of order up to $p$ as zero. The integral

$$\int_{-\infty}^{+\infty} t^p f(t)\, dt$$

is called $p^{th}$ moment of function f(t). p vanishing moments on $\psi(t)$ implies that $H_Z(Z)$ is divisible by $\left(\frac{Z+1}{2}\right)^P$.

It should be noted that maximum value of p which we can have for a N-tap filter is N/2 and one vanishing moment is mandatory for any wavelet system to ensure that $\int \psi(t)dt = 0$.

**Parameterization:**

Consider a 4-tap wavelet system with scaling filter coefficient h(0), h(1), h(2), h(3). For an orthogonal wavelet system, corresponding scaling function and wavelet system must satisfy the following relation implying certain constraints on value of coefficients:

Normality:

$$\int_{-\infty}^{\infty} \phi(t)dt = 1 \Rightarrow \sum_{k=o}^{N-1} h(k) = \sqrt{2}$$

Orthogonality of integer translate of $\phi(t)$:

$$\int \phi(t)\phi(t-k)dt = \delta_{0,k} \Rightarrow \sum_{k=0}^{N-1} h(k)\,h(k-2n) = \delta_{0,n} \qquad n = 0,1,2,\ldots,\frac{N}{2}-1$$

for N=4 and n=0, we have

$$\sum_{k=0}^{3} h^2(k) = 1 \Rightarrow h^2(0) + h^2(1) + h^2(2) + h^2(3) = 1$$

for N=4 and n=1, we have

$$h(0)h(2) + h(1)h(3) = 0$$

orthogonality of $\phi(t)$ and $\psi(t)$:

$$\int \phi(t)\psi(t-k)dt = 0 \Rightarrow g(n) = (-1)^n h(N-1-n)$$

first vanishing moment of $\psi(t)$:

$$\int \psi(t)dt = 0 \Rightarrow \sum_n g(n) = 0 \Rightarrow \sum_n (-1)^n h(N-1-n) = 0 \Rightarrow \sum_{n-even} h(n) = \sum_{n-odd} h(n)$$

for N=4 we have

$$g(n) \equiv \{h(3), -h(2), h(1), -h(0)\}$$

Therefore

$$\sum_n g(n) = 0 \Rightarrow h(0)+h(2) = h(1)+h(3)$$

for a 4-tap wavelet system,

$$h(0)+h(2) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = h(1)+h(3)$$

in general $\sum_{n-even} h(n) = \sum_{n-odd} h(n) = \frac{1}{\sqrt{2}}$ … (3.8)

thus

$$\left[\sum_{n-even} h(n)\right]^2 + \left[\sum_{n-odd} h(n)\right]^2 = \frac{1}{2} + \frac{1}{2} = 1 \qquad \text{… (3.9)}$$

Equation (3.8) and (3.9) play the key role in parametric wavelet.

For a 4-tap wavelet system if we set

$$h(0) + h(2) = \cos(\alpha_1 + \alpha_2)$$

and $h(1)+h(3) = \sin(\alpha_1 + \alpha_2)$

and if $\alpha_1 + \alpha_2 = \pi/4$ then the equations (3.8) and (3.9) are satisfied. On expanding $\cos(\alpha_1 + \alpha_2)$ and $\sin(\alpha_1 + \alpha_2)$ and then assigning each term to h(k)'s i.e.

$$\cos(\alpha_1 + \alpha_2) = \cos\alpha_1 \cos\alpha_2 - \sin\alpha_1 \sin\alpha_2 = h(0) + h(2)$$

$$\sin(\alpha_1 + \alpha_2) = \sin\alpha_1 \cos\alpha_2 + \cos\alpha_1 \sin\alpha_2 = h(1) + h(3)$$

therefore we obtain

$$h(0) = \cos\alpha_1 \cos\alpha_2$$

$$h(2) = -\sin\alpha_1 \sin\alpha_2$$

$$h(1) = \sin\alpha_1 \cos\alpha_2$$

$$h(3) = \cos\alpha_1 \sin\alpha_2$$

Now by choosing a value of $\alpha_1$ randomly and taking $\alpha_2 = \pi/4 - \alpha_1$ we can generate scaling function coefficients. By choosing $\alpha_1 = \pi/3$ and $\alpha_2 = \pi/4 - \alpha_1$, we get the Daubechies 4-tap scaling filter coefficients

| | |
|---|---|
| h(0)=0.482963 | h(2)= -0.12941 |
| h(1)=0.836516 | h(3)= 0.224144 |

Generalizing this idea, we get the relations:

$$\sum_{n-even} h(n) = \cos(\alpha_1 + \alpha_2 + \ldots\ldots\ldots + \alpha_{N/2}) = \frac{1}{\sqrt{2}} \quad \ldots (3.10)$$

$$\sum_{n-odd} h(n) = \sin(\alpha_1 + \alpha_2 + \ldots\ldots\ldots + \alpha_{N/2}) = \frac{1}{\sqrt{2}} \quad \ldots (3.11)$$

Equation (3.10) implies that

$$\sum_{i=1}^{N/2} \alpha_i = \pi/4$$

Expanding cos(.) and sin(.) functions and redistributing the trigonometric monomials to various filter coefficients is a tedious task for scaling function filter with high order taps like 6-tap and 8-tap and so on.

## 3.2 Time-Series

A random process $\{X_t : t = 1, 2, \ldots\}$, is called a **discrete time series** if it is an ordered sequence of values of a variable at equally spaced time intervals. The time series $\{X_t : t = 1, 2, \ldots\}$ is **univariate**. If we have a pair of random process $\{X_t, Y_t\}$, we say it is a **bivariate time series**.

Classically in time series analysis, in order to make a problem statistically treatable, the hypothesis of stationarity is frequently assumed. The stationarity assumption means that the statistical properties of the time series remain unchanged over time. There are different degrees of stationarity, although, second order stationarity is the most commonly used in application. This kind of stationarity requires that the mean and variance must be constant. Given a real-valued stationary time series $\{X_t\}$, the mean is defined by

$$E(X_t)=\mu \qquad \text{… (3.12)}$$

and the *kth* lag auto-covariance sequence is given by

$$\gamma(k)=E[(X_t-\mu)(X_{t+k}-\mu)] \qquad \text{… (3.13)}$$

where **E** denotes the expectation value of the population mean and $\gamma(k)$ denotes the expectation of the auto-covariance sequence between two variables, of time *lag k*. For a stationary time series (k) depends on k only. The auto-correlation sequence, like the auto-covariance sequence, is a function of the time *lag k*:

$$\rho(k)=\frac{\gamma(k)}{\gamma(0)} \qquad for\ k=0,1,....,N \qquad \text{… (3.14)}$$

where $\mu$ is the mean about which the time series fluctuates and $\gamma(0)$ is the variance. For studying time series, two approaches have been used for the characterization of the time series, one is in the time domain and the other is in the frequency domain. In time domain the auto-correlation sequences are the main statistical functions used to analyze time series data, while in the frequency domain frequency spectrum are used widely.

Some examples of time series are given below:

In Figure 3.2 the oscillations are not uniform, but show no tendency to change their amplitude significantly overtime and the series appear to oscillate around a central value. A time series which exhibits this type of behavior is said to be **stationary** both in mean and variance. In Figure 3.3 the time series, changes with the fixed level show an overall upward trend. The variance evolves as the level of the time series increases. A time series with these characteristics is said to be **non-stationary** in both mean and variance. It is an example of a non-stationary time series. In Figure 3.4 the time series tends to exhibit repetitive behavior. The repetitive cycles can be observed clearly. We are interested in the periodic behavior of the time series because the underlying process of interest may be regular and the frequency of oscillation describing the behavior of the underlying series will help us to identify these regularities. The repetitive cycles may be due to seasonal variations. This is an example of a **seasonal time series**.

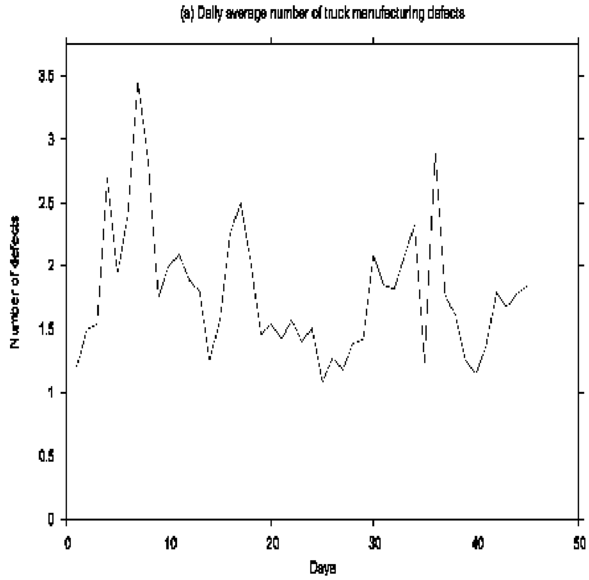


**Fig. 3.2** Stationary time series

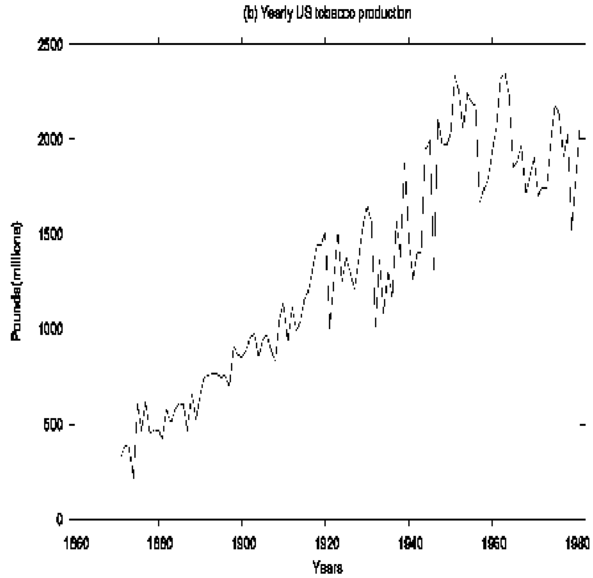


**Fig.3.3** Non-stationary time series

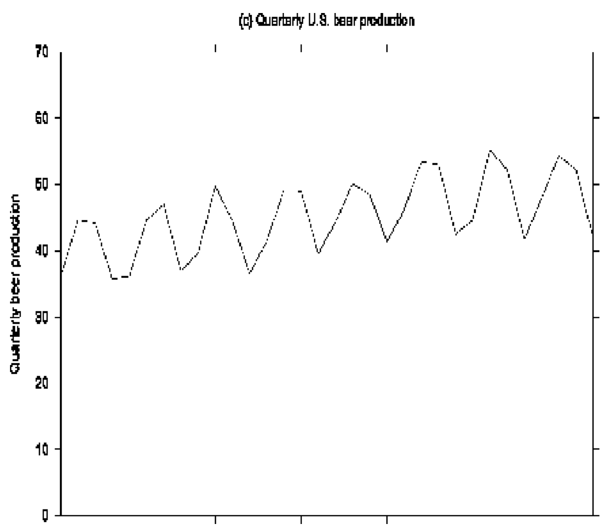


**Fig. 3.4** Seasonal time series

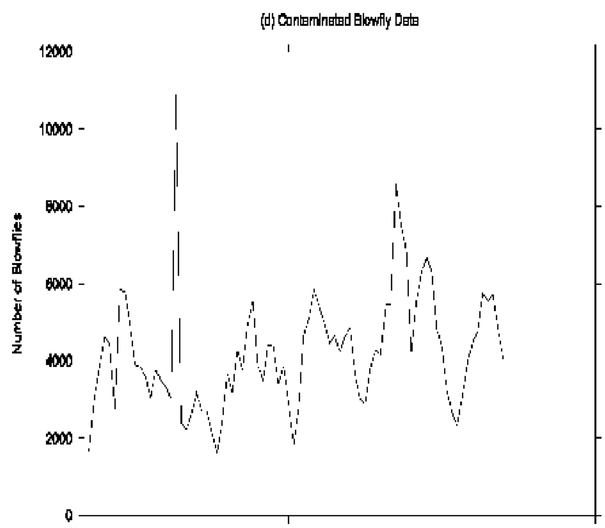


**Fig. 3.5** Time series with Non-stationarity due to external intervention

### 3.2.1 BSE and NSE of India

Bombay Stock Exchange is the oldest stock exchange in Asia with a rich heritage, now spanning over three centuries since 1875 in its total of 133 years of existence. What is now popularly known as BSE was established as "The Native Share and Stock Broker Association" in 1875. BSE is the first stock exchange in the country which obtained permanent recognition in 1956 from the Government of India under the Securities Contracts (Regulation) Act 1956. BSE's pivotal and pre-eminent role in the development of the Indian capital market is widely recognized. It migrated from the open outcry system to an online screen-based order driven trading system in 1995. Over the past 133 years, BSE has facilitated the growth of the Indian corporate sector by providing it with an efficient access to resources. There is perhaps no major corporate in India which has not sourced BSE's services in raising resources from the capital market. Today, BSE is the world's number 1 exchange in terms of the number of listed companies and the world's 5th in transaction numbers. The market capitalization as on December 31, 2007 stood up at US$ 1.79 trillion. An investor can choose from more than 4,700 listed companies, which for easy reference, are classified into A, B, S, T and Z groups. BSE provides an efficient and transparent market for trading in equity, debt instruments and derivatives. It has a nation-wide reach with a presence in more than 450 cities and towns of India. BSE has always been at par with the international standards. The systems and processes are designed to safeguard market integrity and enhance transparency in operations. BSE is the first exchange in India and the second in the world to obtain an ISO 9001:2000 certification. It is also the first exchange in the country and second in the world to receive Information Security Management System Standard BS 7799-2-2002 certification for its BSE On-line Trading system (BOLT).

The BSE Index, SENSEX, is India's first stock market index that enjoys an iconic stature, and is tracked worldwide. It is an index of 30 stocks representing 12 major sectors. The SENSEX is constructed on a 'free-float' methodology, and is sensitive to market sentiments and market realities. Apart from the SENSEX, BSE also offers 21 indices, including 12 sectoral indices.

The National Stock Exchange (NSE), located in Bombay, is India's first debt market. It was set up in 1993 to encourage stock exchange reform through system modernization and competition. It was started for trading in the mid-of 1994 and is the largest stock exchange in India in terms of daily turnover and number of trades both for equities and derivative trading. NSE is mutually-owned by a set of leading financial institutions, banks, insurance companies and other financial intermediaries in India but its ownership and management operate as separate entity. Up to 2006, the NSE VSAT terminals, 2799 in total, cover more than 1500 cities across India. In October 2007, the equity market capitalization of the companies listed on the NSE was US$ 1.46 trillion, thus making it as the second largest stock exchange in South Asia. NSE is the third largest Stock Exchange in the world in terms of the number of trades in equities. It is the second fastest growing stock exchange in the world with a record growth of 16.6%. Table (3.1) and (3.2) shows the monthly average of BSE and NSE indexes from April 1990 to March 2006, based on daily closing index.

| Year/Month | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1990-91 | 780.18 | 785.57 | 802.45 | 938 | 1116.19 | 1307.87 | 1354.02 | 1306.09 | 1161.87 | 996.45 | 1100.78 | 1180.7 |
| 1991-92 | 1255.25 | 1291.74 | 1295.15 | 1440.72 | 1723.82 | 1833.34 | 1789.5 | 1890.09 | 1872.31 | 2073.6 | 2464.74 | 3487.19 |
| 1992-93 | 4131.01 | 3366.55 | 3088.59 | 2797.27 | 2829.96 | 3243.19 | 3075.28 | 2618.2 | 2535.64 | 2532.86 | 2708.72 | 2398.27 |
| 1993-94 | 2205.37 | 2248.01 | 2281.95 | 2190.34 | 2556.16 | 2708.39 | 2688.51 | 2850.35 | 3301.85 | 3813.74 | 4039.42 | 3811.25 |
| 1994-95 | 3824.75 | 3756.1 | 4135.67 | 4106.95 | 4407.4 | 4511.34 | 4351.16 | 4139.06 | 3949.78 | 3651.59 | 3474.92 | 3408.29 |
| 1995-96 | 3359.29 | 3206.86 | 3336.46 | 3334.86 | 3402.81 | 3396.37 | 3528.1 | 3172.02 | 3060.05 | 2979.3 | 3405.56 | 3327.33 |
| 1996-97 | 3599.66 | 3732.2 | 3906.72 | 3668.21 | 3449.17 | 3390.11 | 3159.79 | 3044.28 | 2918.68 | 3410.3 | 3453.24 | 3762.52 |
| 1997-98 | 3681.5 | 3740.95 | 4001.47 | 4256.11 | 4276.31 | 3944.79 | 3991.75 | 3611.83 | 3515.54 | 3472.87 | 3413.49 | 3816.87 |
| 1998-99 | 4114.66 | 3911.95 | 3317.49 | 3271.73 | 2988.4 | 3089.88 | 2866.55 | 2912.39 | 2945.99 | 3275.05 | 3289.24 | 3689.42 |
| 1999-00 | 3455.05 | 3880.37 | 4066.84 | 4526.25 | 4662.84 | 4724.96 | 4835.47 | 4588.53 | 4802.02 | 5407.14 | 5650.66 | 3689.42 |
| 2000-01 | 4905.3 | 4253.11 | 4675.4 | 4647.34 | 4330.31 | 4416.61 | 3819.69 | 3928.1 | 4081.42 | 4152.39 | 4310.13 | 5261.77 |
| 2001-02 | 3480.94 | 3613.84 | 3439.01 | 3346.88 | 3304.99 | 2918.28 | 2933.55 | 3164.25 | 3314.88 | 3353.31 | 3528.58 | 3807.64 |
| 2002-03 | 3435.13 | 3302.91 | 3257.03 | 3214.87 | 3053.16 | 3085.53 | 2949.76 | 3058.19 | 3315.84 | 3327.66 | 3278.85 | 3580.73 |
| 2003-04 | 3036.66 | 3033.47 | 3386.89 | 3665.46 | 3977.86 | 4314.74 | 4742.32 | 4951.1 | 5424.67 | 5954.15 | 5826.74 | 3155.7 |
| 2004-05 | 5809.01 | 5204.65 | 4823.87 | 4972.88 | 5144.17 | 5423.27 | 5701.61 | 5960.75 | 6393.83 | 6300.76 | 6595.05 | 6679.18 |
| 2005-06 | 6379.29 | 6482.72 | 6925.86 | 7336.7 | 7726.03 | 8272.32 | 8220.45 | 8552.09 | 9162.07 | 9539.67 | 10090.08 | 10857.03 |

**Table 3.1** Monthly average of BSE Sensex

The averages are based on daily BSE Sensex closing index.

(Base: 1978-79 = 100)

| Year/Month | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1990-91 | 417.99 | 423.79 | 428.68 | 486 | 573.49 | 669.06 | 672.79 | 644.79 | 573.77 | 506.56 | 553.24 | 593.69 |
| 1991-92 | 627.72 | 641.32 | 639.17 | 703.02 | 825.33 | 872.98 | 853.77 | 898.09 | 880.5 | 960.14 | 1138.17 | 1579.04 |
| 1992-93 | 1850.94 | 1481.01 | 1351.06 | 1267.88 | 1260.28 | 1444.95 | 1376.05 | 1194.63 | 1162.92 | 1160.02 | 1221.62 | 1081.17 |
| 1993-94 | 993.63 | 1029.84 | 1057.45 | 1030.1 | 1199.31 | 1283.5 | 1292.72 | 1368.88 | 1589.25 | 1827.17 | 1945.4 | 1842.82 |
| 1994-95 | 1855.81 | 1822.25 | 1967.76 | 1947.56 | 2080.67 | 2133.49 | 2054.54 | 1968.58 | 1876.13 | 1755.38 | 1683.04 | 1658.97 |
| 1995-96 | 1631.55 | 1539.44 | 1570.48 | 1550.57 | 1568.33 | 1555.07 | 1603.84 | 1442.44 | 1406.95 | 1369.94 | 1556.09 | 1539.14 |
| 1996-97 | 1649.6 | 1701.15 | 1771.88 | 1676.6 | 1575.49 | 1522.67 | 1409.83 | 1356.4 | 1290.21 | 1502.66 | 1504.97 | 1629.43 |
| 1997-98 | 1586.13 | 1610.98 | 1716.56 | 1844.63 | 1863.62 | 1717.52 | 1722.58 | 1563.46 | 1525.78 | 1512.7 | 1467.54 | 1654.92 |
| 1998-99 | 1804.55 | 1728.93 | 1459.27 | 1437.4 | 1333.8 | 1371.49 | 1281.38 | 1298.19 | 1307.34 | 1452.71 | 1450.6 | 1620.74 |
| 1999-00 | 1506.84 | 1682.65 | 1755.07 | 1960.83 | 2075.59 | 2156.82 | 2272.13 | 2161.39 | 2429.71 | 2822.05 | 3394.88 | 3109.03 |
| 2000-01 | 2663.53 | 2120.93 | 2334.27 | 2344.29 | 2180.79 | 2249.43 | 1931.61 | 2017.59 | 2113.84 | 2140.09 | 2203.99 | 1829.32 |
| 2001-02 | 1641.89 | 1753.46 | 1661.26 | 1572.67 | 1559.95 | 1373.77 | 1357.64 | 1486.33 | 1587.92 | 1601.92 | 1711.43 | 1746.78 |
| 2002-03 | 1715.11 | 1661.21 | 1658.78 | 1623.07 | 1536.74 | 1521.96 | 1466.79 | 1510 | 1632.19 | 1642.07 | 1622.58 | 1559.54 |
| 2003-04 | 1504.62 | 1538.65 | 1729.15 | 1843.86 | 2055.64 | 2242.97 | 2423.87 | 2543.09 | 2813.58 | 3142.23 | 3003.89 | 2956.07 |
| 2004-05 | 3101.76 | 2772.81 | 2563.78 | 2653.04 | 2748.23 | 2908.81 | 3049.82 | 3188.92 | 3455.28 | 3403.25 | 3558.11 | 3595.2 |
| 2005-06 | 3431.03 | 3483.19 | 3697.22 | 3920.12 | 4139.35 | 4407.48 | 4353.07 | 4508.71 | 4825.99 | 5048.54 | 5303.59 | 5686.04 |

**Table 3.2** Monthly average of NSE Index

The averages are based on daily closing index.

(Base: 1983-84 = 100)

### 3.2.2 MODWT Based Time Scale Decomposition of BSE and NSE Indexes Financial Time Series

The past decades have witnessed different activities with regard to study of the nature of financial time series. Various new concepts and methods of both Applied Mathematics and Economics have been suitably employed to study financial time series for long range and short range studies. The classical method used to investigate features in a time series to compute the covariance and correlation functions in the time domain and by the frequency decomposition of the time series in the frequency domain has been achieved through the Fourier analysis. In practice, we sometime consider time scale of often non-stationary processes and, therefore, time resolved methods become absolutely necessary. The time resolved method for Fourier analysis is a windowed Fourier analysis but this method has its own limitations which makes it less desirable for the analysis of time series with specific characteristics like the time series with sharp spikes and discontinuities like financial time series. Wavelet analysis provides much better approximation for time series with such characteristics. Moreover, it often extracts more information about the series than any other classical approach of analysis. Figures (3.6) and (3.7) illustrate BSE and NSE indexes financial time series.

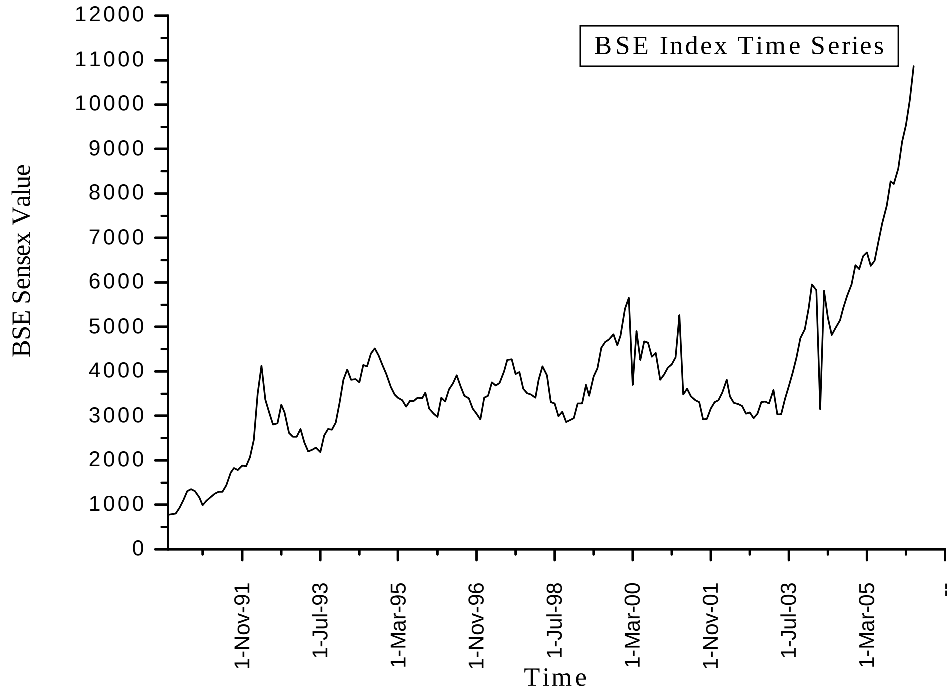


**Fig. 3.6** BSE index financial time series

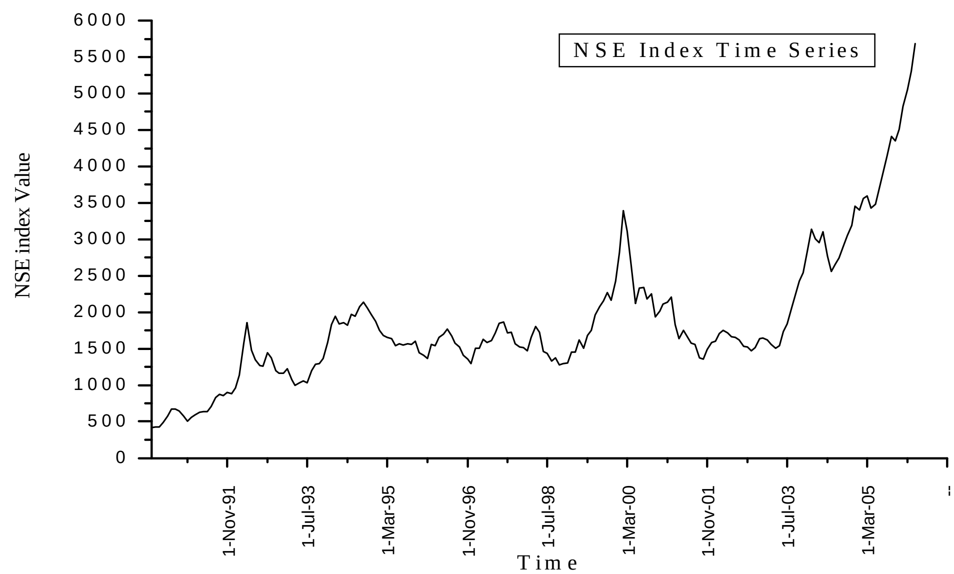


**Fig. 3.7** NSE index financial time series

## Spectral Analysis for Time Series

Spectral analysis is purely a descriptive technique. It is a tool for inspecting cyclic phenomena and highlighting lead-lag relations among time series. It provides an accurate way to define each series components and provide a reliable method by means of filtering. Cross spectral analysis allows the detailed study of the correlation among time series.

## Discrete Fourier Transform for Finite Sequences

Consider a finite time sequence $u(j), j=0,.............,N-1$ of length $T=N\delta t$, where N is the number of data and $\delta t$ is the sampling periodicity. The **discrete Fourier transform (DFT)** $U(k) of\ U(j)$ and its **inverse DFT** for finite sequences are, respectively, defined by

$$U(k)=\frac{1}{N}\sum_{j=0}^{N-1}u(j)e^{-i\frac{2\pi jk}{N}} \qquad \ldots (3.15)$$

$$U(j)=\sum_{j=-\left[\frac{N}{2}\right]}^{\left[\frac{N-1}{2}\right]} u(k)e^{i\frac{2\pi jk}{N}} \qquad \ldots (3.16)$$

where [.] denotes largest integer smaller or equal than the operand and $j=0,1,.....,N-1$. When we sample the series with finite period $\delta t$, we limit the spectrum of the study to the frequency band $\omega\in\left[-\frac{1}{2\delta t},\frac{1}{2\delta t}\right]$, where $\frac{1}{2\delta t}$

is the *Nyquist frequency* as frequency outside the range are folded inside by sampling, an effect known as *aliasing*.

**Spectral Estimation**

The Fourier decomposition is a way of separating the time series into different frequency components to give more insight into the data.

Consider a stationary time series $\{Y_t\}$ with auto-covariance sequence $\gamma(k)$. The quantity $\hat{f}(k)$ is called the **spectrum** of $\{Y_t\}$. Its estimator would be

$$f_Y(j)=\delta t \sum_{j=-(N-1)}^{(N-1)} \gamma_k e^{-i\frac{2\pi jk}{N}}$$

$$=\delta t\gamma(0)+2\delta t\sum_{k=1}^{N-1}\gamma(k)\cos\left(\frac{k2j\pi}{N}\right) \qquad \ldots (3.17)$$

where $\gamma_Y(j)=\gamma_Y(-j)=\frac{1}{N}\sum_{t=-(N-j)}^{(N-j)}(Y_t-\bar{Y})(Y_{t+j}-\bar{Y})$ is a sample estimator at *lag j* of the auto-covariance sequence and $\delta t$ is the sampling periodicity. The spectrum is a real-valued function because the series is real-valued and the auto-covariance sequence is even. The spectrum thus defined above is an asymptotically unbiased estimator of a theoretical one. To construct spectral estimator which has a small variance compared to $f_Y(j)$, we use the technique of windowing. This method is employed both in time and frequency domain. We can smooth all abrupt variations and minimize the spurious fluctuations generated every time when the series gets truncated.

The result of windowing a time series $\{Y_t\}$ with n observations is the estimated smoothed spectrum

$$\hat{f}_Y(j) = \delta t \sum_{k=-\left(\frac{M}{2}\right)}^{\left(\frac{M}{2}\right)} \omega_M(k)\hat{\gamma}_Y(k) e^{-ik\frac{2j\pi}{N}} \qquad \ldots (3.18)$$

where the auto-covariance sequence is weighted by the *lag* window $\omega_M(k)$ of width M which is equivalent to splitting the series in to n/M sub-series of length M. Alternatively, $\hat{f}_Y(k)$ can be obtained by the convolution of the expected spectrum $f_Y(k)$ with Fourier transform of $\omega_M(k)$ through

$$\hat{f}_Y(k) = \delta t \sum_{k'=-\left(\frac{M'}{2}\right)}^{\left(\frac{M'}{2}\right)} f_Y(k') W_{M'}(k-k') \qquad \ldots (3.19)$$

where $W_{M'}(k)$ is the spectral window of width $M'$. Thus the smooth spectrum at k is observed through a window opened on a convenient interval around k.

**Wavelet Analysis for Financial Time Series**

Many economic and financial time series are nonstationary and exhibit changing frequencies over time. The usefulness of wavelet analysis is in its flexibility in handling a variety of nonstationary signals. Indeed, as wavelets are constructed over finite intervals of time and are not necessarily homogeneous over time, they are localized both in time and scale. Thus, two interesting features of wavelet time scale decomposition for economic variables are that, i) since the base scale includes any non-stationary components, the data need not be differenced, and ii) the nonparametric nature of wavelets takes care of potential nonlinear relationships without losing any detail.

Roughly, wavelet analysis decomposes a given series in orthogonal components as in the case of Fourier approach but according to time scale components instead of frequency components.

Mathematically, we may note that if there are two basic wavelet functions: the father and the mother wavelets, $\phi(t)$ and $\psi(t)$ respectively then the **father wavelet** is given by the function

$$\phi_{j,k} = 2^{-\frac{j}{2}} \phi\left(\frac{t - 2^j k}{2^j}\right)$$

defined as non-zero over a finite time length support which corresponds to the given **mother wavelet**

$$\psi_{j,k} = 2^{-\frac{j}{2}} \psi\left(\frac{t - 2^j k}{2^j}\right).$$

With j = 1, ……, J we have a J-level wavelets decomposition. The former integrates to 1 and reconstructs the longest time-scale component of the series (trend), while the latter integrates to 0 (similar to sine and cosine) and is used to describe all deviations from trend. The mother wavelet, as defined above, play a role similar to sines and cosines in the Fourier decomposition. They are either compressed or dilated in time domain to generate cycles fitting to the actual data.

To compute the decomposition we have to calculate wavelet coefficients at all scales representing the projections of the time series onto the basis generated by the chosen family of wavelets i.e.

$$d_{j,k} = \int f(t)\psi_{j,k} \quad \ldots (3.20)$$

$$s_{j,k} = \int f(t)\phi_{j,k} \quad \ldots (3.21)$$

where the coefficients $d_{j,k}$ and $s_{j,k}$ are the wavelet transform coefficients representing, respectively, the projection onto mother and father wavelets.

The orthogonal wavelet series approximates to a signal or function $f(t)$ in $L^2(R)$ given by

$$f(t)=\sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \ldots\ldots\ldots + \sum_k d_{j,k}\psi_{j,k}(t) + \ldots\ldots\ldots + \sum_k d_{1,k}\psi_{1,k}(t) \quad \ldots (3.22)$$

where J is the number of multiresolution components and k ranges from 1 to the number of coefficients in the specified components. The multiresolution decomposition of the original signal f (t) is given by the sum of the smooth signal $V_J$ and the detail signals $W_J, W_{J-1}, \ldots\ldots\ldots, W_1$,

with $V_J = \sum_k s_{J,k}\phi_{j,k}(t)$ and $W_J = \sum_k d_{J,k}\psi_{j,k}(t)$ ( j = 1, 2, …….., J).

The sequence of the terms $V_J, W_J, \ldots\ldots, W_j, \ldots\ldots, W_1$ in this equation represent a set of signals components which provide representations of the signal at the different resolution levels 1 to J, and the detail signals $W_j$ provide the increments at each individual scale or resolution level.

The restrictions of DWT on sample size multiple of $2^J$ and sensitivity to circular shifts due to the downsampling approach are overcome by the maximal overlap DWT (MODWT) and applies to any sample and is translation invariant, at the cost of giving up orthogonality. The maximal overlap discrete wavelet transform (MODWT) is a non-orthogonal variant of the classical discrete wavelet transform that unlike the orthogonal discrete wavelet transform is translate invariant because shifts in the signal do not change the pattern of coefficients. Application of a $j^{th}$ order nondecimated version of the orthogonal DWT, *i.e.* the maximal overlap DWT (MODWT), yields $J$ vectors of wavelet filter coefficients $\tilde{W}_{j,t}$ for $j$ = 1,……., $J$ and $t$ = 1,…….,N/$2^j$ , and one vector of wavelet filter coefficient $\tilde{V}_{j,t}$ through

$$\tilde{W}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L_j - 1} \tilde{h}_{j,l} \; f(t-l) \qquad \ldots (3.23)$$

$$\tilde{V}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L_j - 1} \tilde{g}_{j,l} \; f(t-l) \qquad \ldots (3.24)$$

where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are, respectively, the rescaled wavelet and scaling filter coefficient from a Daubechies compactly supported wavelet family obtained by rescaling the DWT filters as follows:

$$\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}} \text{ and } \tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}.$$

Whereas DWT filters have unit energy, MODWT filters have half energy, i.e.

$$\sum_{l=0}^{L-1} \tilde{h}_{j,l}^2 = \sum_{l=0}^{L-1} \tilde{g}_{j,l}^2 = \frac{1}{2^j}.$$

Thus, MODWT provides the usual functions of DWT, such as multiresolution decomposition analysis and variance analysis based on wavelet transform coefficients, but unlike the classical DWT it

- can handle any sample size;
- is translation invariant, as a shift in the signal does not change the pattern of wavelet transform coefficients;
- provides increased resolution at coarser scales; and produces a more asymptotically efficient wavelet variance estimator than DWT.

Moreover, unlike the classical DWT which has fewer coefficients at coarser scales, MODWT has a number of coefficients equal to the sample size at each scale, and thus is over-sampled at coarse scales.

### 3.2.3 Wavelet Variance, Covariance and Correlation Analysis of Financial Time Series

In addition to multiresolution decomposition analysis, wavelet methods also provide an alternative representation of the variability and association structure of certain stochastic processes on a scale-by-scale basis. For a given stationary process *{X}* with variance $\sigma_X^2$, the wavelet variance $\sigma_{X,j}^2$ at scale *j* have the relationship:

$$\sum_{j=1}^{\infty} \sigma_{X,j}^2 = \sigma_X^2 \qquad \ldots (3.25)$$

Thus, as $\sigma_{X,j}^2$ represents the contribution of the changes at scale *j* to the overall variability of the process. The relationship implies that wavelet variance provides an exact decomposition of the variance of a time series into components that are associated to different time scales. The wavelet variance decomposes the variance of a stationary process with respect to the scale at $j^{th}$ level just as the spectral density decomposes the variance of the original series with respect to frequency *f* i.e.

$$\sum_{j=1}^{\infty} \sigma_{X,j}^2 = \sigma_X^2 = \int_{-1/2}^{1/2} S_X(f)\,df \qquad \ldots (3.26)$$

where *S(.)* denotes the spectral density function.

By definition the time independent wavelet variance at scale *j*, $\sigma_{X,j}^2$ is given by the variance of *j*-level wavelet coefficients

$\sigma^2_{X,j} = \mathrm{var}\{\tilde{W}_{j,t}\}$.

A time-independent wavelet variance may be defined not only for stationary processes but also for non-stationary processes with stationary $d^{\mathrm{th}}$ order differences with local stationarity. As the wavelet filter $\{h_l\}$ represents the difference between two generalized averages and is related to a difference operator, wavelet variance is time-independent in case of non-stationary processes with stationary $d^{\mathrm{th}}$ order differences, provided that the length $L$ of the wavelet filter is large enough. $L \geq d$ is a sufficient condition to make the wavelet coefficients $\tilde{W}_{j,t}$ of a stochastic process stationary whose $d^{\mathrm{th}}$ order backward difference is stationary.

As MODWT employs circular convolution, the coefficients generated by both beginning and ending data could be spurious. Thus, if the length of the filter is $L$, there are $(2^j - 1)(L-1)$ coefficients affected for $2^{j-1}$-scale wavelet and scaling coefficients. If $N - L_j \geq 0$, then an unbiased estimator of the wavelet variance based on the MODWT may be obtained by removing all coefficients affected by the periodic boundary conditions using

$$\tilde{\sigma}^2_{X,j} = \frac{1}{\tilde{N}_j} \sum_{t=L_j}^{N} \tilde{W}^2_{j,t},$$

where $\tilde{N}_j = N - L_j + 1$ is the number of maximal overlap coefficients at scale $j$ and $L_j = (2^j - 1)(L-1) + 1$ is the length of the wavelet filter for level $j$. Thus, the

$j^{th}$ scale wavelet variance is simply the variance of the non-boundary or interior wavelet coefficients at that level. Both DWT and MODWT can decompose the sample variance of a time series on a scale-by-scale basis via its squared wavelet coefficients, but the MODWT-based estimator has been shown to be superior to the DWT-based estimator.

To determine the magnitude of the association between two series of observations $X$ and $Y$ on a scale-by-scale basis the notion of wavelet covariance is used. The **wavelet covariance** at wavelet scale $j$ can be defined as the covariance between scale $j$ wavelet coefficients of $X$ and $Y$, i.e.

$$\tilde{\gamma}_{XY,j} = \operatorname{cov}\left[\tilde{W}^{X}_{j,t}\,\tilde{W}^{Y}_{j,t}\right] \qquad \ldots (3.27)$$

Again, after removing all wavelet coefficients affected by boundary conditions, an unbiased estimator of the wavelet covariance using MODWT will be given by

$$\tilde{\gamma}_{XY,j} = \left(\frac{N-1}{\tilde{N}_j}\right)\sum_{t=L_j-1}^{N-1}\tilde{W}^{X}_{j,t}\,\tilde{W}^{Y}_{j,t} \qquad \ldots (3.28)$$

The MODWT estimator of the wavelet cross-correlation coefficients for scale $j$ and $lag\,\tau$ may be obtained by making use of the wavelet cross-covariance $\tilde{\gamma}_{\tau,XY,j}$, and the square root of the wavelet variances $\tilde{\sigma}_{X,j}$ and $\tilde{\sigma}_{Y,j}$ by

$$\tilde{\rho}_{\tau,XY,j} = \frac{\tilde{\gamma}_{\tau,XY,j}}{\tilde{\sigma}_{X,j}\tilde{\sigma}_{Y,j}} \quad \ldots (3.29)$$

The wavelet cross-correlation coefficients $\tilde{\rho}_{\tau,XY,j}$, just as the usual unconditional cross-correlation coefficients are between 0 and 1 and provide the lead/lag relationships between the two processes on a scale-by-scale basis.

Starting from spectrum $S\omega_{X,j}$ of scale $j$ wavelet coefficients, it is possible to determine the asymptotic variance $V_j$ of the MODWT-based estimator of the wavelet variance and construct a random interval which forms a *100(1-2p)%* confidence interval.

# RESULTS AND DISCUSSION

# 4 Results and Discussion

This chapter comprises of the results and discussion of the proposed method of parameterization of orthogonal wavelet, the empirical results from Fourier and maximal overlap discrete wavelet transform analysis of BSE and NSE indexes financial time series and wavelet variance, covariance and correlation analysis of BSE and NSE indexes financial time series. In view of presenting the study in compact form, lengthy calculations have been avoided in this chapter as these can be easily computed with the help of computational algorithm. These computations have been performed by using the Origin, Timestat, WMTSA & Waveslim Matlab Package. It has been divided into the following three sections:

Section 4.1 deals with the proposed methodology for parameterization and its applicability to filters with higher number of taps. It will be shown with the help of illustrations that the proposed method is comparatively more suitable than the existing methods for generating Daubechies wavelets.

Section 4.2 will establish that MODWT based time scale decomposition analysis leads to better results than the Fourier transform based spectral analysis of BSE and NSE indexes financial time series.

In section 4.3, wavelet based variance, covariance and correlation analysis of BSE and NSE indexes financial time series have been discussed.

## 4.1 Results and Discussion for Problem 3.1.7

In this section a new method is presented for parameterization of compactly supported wavelets with arbitrary vanishing moments. In the proposed process of parameterization, the scaling function filter is represented as a product of two Laurent polynomials. The first factor ensures the requirement of vanishing moments and the second factor is parameterized and adjusted suitably to provide required length and other low pass filter requirements. Subsequently, we shall impose double shift orthogonality conditions on the resulting filter coefficients to make the filter suitable for MRA.

As an illustration, we assume that we need 6 tap scaling filters. Orthogonality and normalization constraints put the requirement of three out of the 6 possible degrees of freedom. Thus three degrees of freedom is left. Daubechies used all the three for vanishing moments and thereby achieved the so called **‘maximal flat’** filter. For parameterization, we need to forego one more degree of freedom. We then assign the remaining two for fulfilling vanishing moments requirement so that the filter can be expressed as

$$H(z) = \left(\frac{1+z}{2}\right)^2 \left[u + vz + wz^2 + (1-u-v-w)z^3\right] \qquad \text{...(4.1)}$$

The resulting filter is a low pass filter. We shall now impose double shift orthogonality to get two conditions which we subsequently solve for v and w in terms of u. Thus u is, therefore, left as a free parameter to obtain infinite number of 6-tap scaling filters with two vanishing moments.

Simplification of equation (4.1) gives

$$H(z) = \frac{1}{4}\left[1+2z+z^2\right]\left[u+vz+wz^2+(1-u-v-w)z^3\right]$$

$$= \frac{1}{4}\left[u+(2u+v)z+(u+2v+w)z^2+(1-u+w)z^3+(-2u-2v-w+2)z^4+(1-u-v-w)\right]$$

Thus the filter coefficients are

$$\left[\frac{u}{4}, \frac{2u+v}{4}, \frac{u+2v+w}{4}, \frac{1-u+w}{4}, \frac{-2u-2v-w+2}{4}, \frac{1-u-v-w}{4}\right] \qquad \text{...(4.2)}$$

Now applying double-shift orthogonality condition for N=6

$$\sum_{l=0}^{N-1} h(l)\,h(l-2k) = \delta_{0,k} \qquad k = 1,2,\ldots,\frac{N}{2}-1$$

gives that

$$h(0)h(2) + h(1)h(3) + h(2)h(4) + h(3)h(5) = 0$$

$$\left(\frac{u}{4}\right)\left(\frac{u+2v+w}{4}\right)+\left(\frac{2u+v}{4}\right)\left(\frac{1-u+w}{4}\right)+\left(\frac{u+2v+w}{4}\right)\left(\frac{-2u-2v-w+2}{4}\right)$$
$$+\left(\frac{1-u+w}{4}\right)\left(\frac{1-u-v-w}{4}\right)=0$$

$$\Rightarrow -\frac{u^2}{8}-\frac{v^2}{4}-\frac{w^2}{8}-\frac{uv}{4}-\frac{vw}{4}+\frac{u}{8}+\frac{v}{4}+\frac{w}{8}+\frac{1}{16}=0 \qquad ...(4.3)$$

and

$$h(0)h(4)+h(1)h(5)=0$$

$$\left(\frac{u}{4}\right)\left(\frac{-2u-2v-w+2}{4}\right)+\left(\frac{2u+v}{4}\right)\left(\frac{1-u-v-w}{4}\right)=0$$

$$\Rightarrow -\frac{u^2}{4}-\frac{v^2}{16}-\frac{5uv}{16}-\frac{vw}{16}-\frac{3uw}{16}+\frac{u}{4}+\frac{w}{16}=0 \qquad ...(4.4)$$

Solving equations (4.3) and (4.4) with respect to u

$$14u^2+16uv+12uw-14u-2w^2+2w+1=0 \qquad ...(4.5)$$

$$100u^4+4w^4+56u^2w^2-80u^3w-16uw^3-72u^3-8w^3+40uw^2-24u^2w-24u^2w$$
$$-80u^2-16uw-12u+4w+1=0$$

$$\ldots (4.6)$$

We thus obtain real roots for v and w from equations (4.5) and (4.6) for $-1\le u\le 2$. The above range is suitably obtained by scanning the domain [-4, 4].

The steps needed for generating filter coefficients are as follows:

(i). Generate a random number between [-1, 2] and then assign it to u.

(ii). Substitute u in (4.6) and solve it for w.

(iii). Substitute u and w in (4.5) and then solve it for v.

(iv). Substitute u, v, w in (4.2) to get filter coefficients.

This method fits well for the filters with higher number of taps. As an illustration, we shall now derive 6-tap Daubechies wavelet filter coefficients in the following steps:

For a 6-tap Daubechies wavelet filter, we have

$$H(z) = \left(\frac{1+z}{2}\right)^3 \left[u + vz + (1-u-v)z^2\right]$$

$$= \frac{1}{8}\left[1+3z+3z^2+z^3\right]\left[u+vz+(1-u-v)z^2\right]$$

$$= \frac{1}{8}\left[u+(3u+v)z+(1+2u+2v)z^2+(3-2u)z^3+(3-3u-2v)z^4+(1-u-v)z^5\right]$$

… (4.7)

Thus the filter coefficients are

$$\left[\frac{u}{8}, \frac{3u+v}{8}, \frac{1+2u+2v}{8}, \frac{3-2u}{8}, \frac{3-3u-2v}{8}, \frac{1-u-v}{8}\right] \qquad \text{...(4.8)}$$

Application of double-shift orthogonality condition

$$\sum_{l=0}^{N-1} h(l)\,h(l-2k) = \delta_{0,k} \qquad k = 1, 2, \ldots, \frac{N}{2}-1$$

leads to

$$h(0)h(2) + h(1)h(3) + h(2)h(4) + h(3)h(5) = 0$$

$$\left(\frac{u}{8}\right)\left(\frac{1+2u+2v}{8}\right)+\left(\frac{3u+v}{8}\right)\left(\frac{3-2u}{8}\right)+\left(\frac{1+2u+2v}{8}\right)\left(\frac{3-3u-2v}{8}\right)$$
$$+\left(\frac{3-2u}{8}\right)\left(\frac{1-u-v}{8}\right)=0$$

which further implies

$$-\frac{u^2}{8}-\frac{v^2}{16}-\frac{uv}{8}+\frac{u}{8}+\frac{v}{16}+\frac{3}{32}=0 \qquad \text{... (4.9)}$$

and

$$h(0)h(4)+h(1)h(5)=0$$

$$\left(\frac{u}{8}\right)\left(\frac{3-3u-2v}{8}\right)+\left(\frac{3u+v}{8}\right)\left(\frac{1-u-v}{8}\right)=0$$

giving

$$-\frac{3u^2}{32}-\frac{v^2}{64}-\frac{3uv}{32}+\frac{3u}{32}+\frac{v}{64}=0 \qquad \text{...(4.10)}$$

We shall now use one of the two possible real solutions of the equations (4.9) and (4.10), {u=1.879452; v= -1.090142} and then put it in equation (4.8) and, thereafter, we normalize it by $\sqrt{2}$ to obtain the standard 6-tap Daubechies filter coefficients as

| | |
|---|---|
| h(0)=0.3326705529 | h(3)= -0.1350110198 |
| h(1)=0.8068915088 | h(4)= -0.0854412693 |
| h(2)=0.4598776012 | h(5)= 0.0352262928 |

(Corresponding wavelet and Scaling functions have been depicted in Figs. 4.1 and 4.2 respectively).

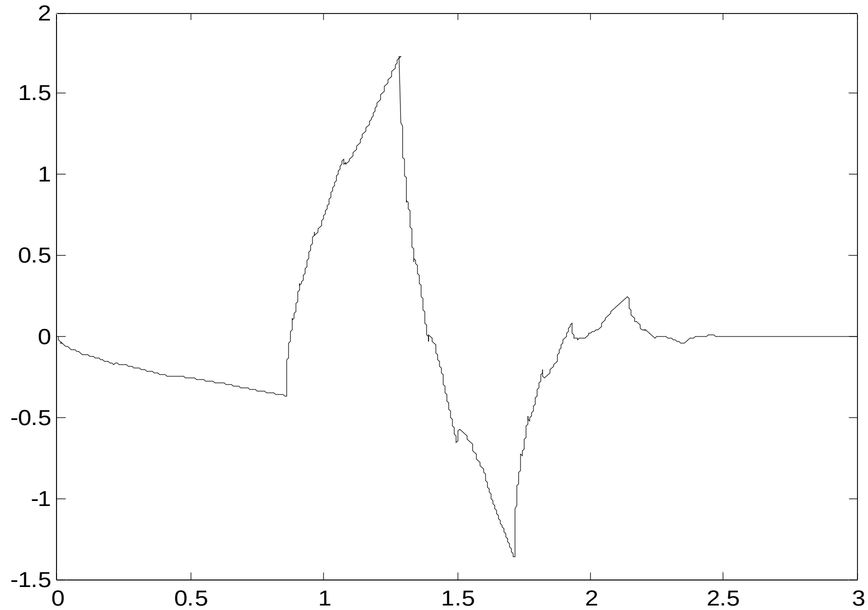


**Fig. 4.1** Daubechies 6-tap wavelet function

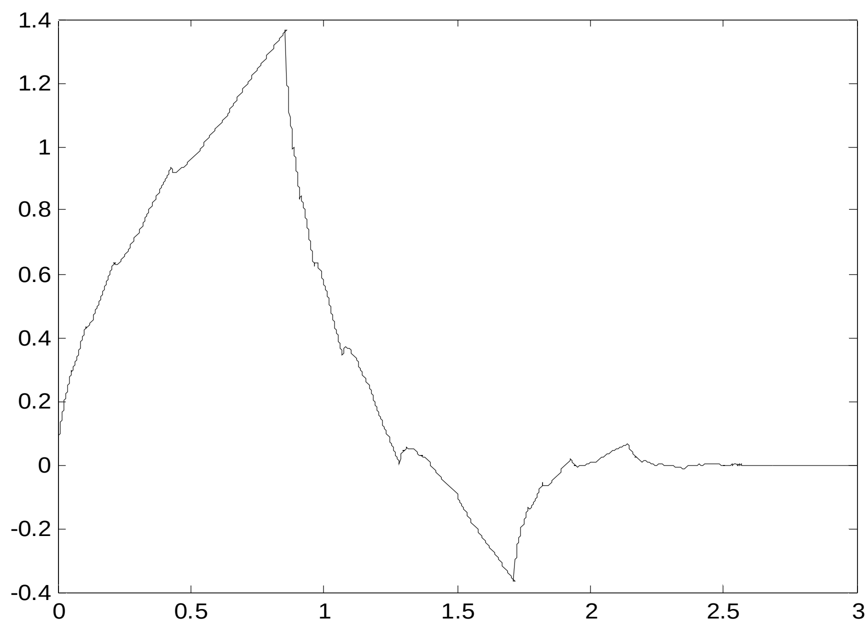


**Fig. 4.2** Daubechies 6-tap scaling function

Parameterization method of generating the wavelets act as a secret key but then there is a need of a scaling function filter with higher order taps for the better analysis of the time series and for obtaining better results. Here, we have proposed a simpler method to generate parametric families of orthogonal wavelet and then used it to generate the '6-tap Daubechies wavelet filter' in a straight forward manner. This method is applicable to filters with higher number of taps and is suitable for generating Daubechies wavelets which has applications in economics and finance for the analysis of macroeconomic and microeconomic time series and in data compression as well.

## 4.2 Results and Discussion for Problem (3.2.2)

In this section wavelet based concepts have been employed to study two 'strongly correlated' financial time series of BSE and NSE indexes using index data from April 1990 to March 2006 by decomposing index based financial time series into time-scale components using the MODWT (Maximal Overlap Discrete Wavelet Transform) analysis.

The methodologies usually employed in empirical studies may generally be stated only over a long time horizon that is only in the long-run as the time series analysis techniques may separate out in just two time scales economic time series, i.e. the short run and the long run. But the stock market sets an example of a market in which the agents involved consist of heterogeneous investors making decisions over different time horizons (from minutes to years) and operating at each moment on different time scales. In this way, the nature of the relationship between stock returns and growth rates of industrial production may well vary across time scales according to the investment horizon of the traders, as small time scales may be linked to speculative activity and coarse scales to investment activity. Thus, for example, if we think that big institutional investors have long term horizons and, consequently, follow macroeconomic fundamentals, we expect the relationship between stock returns and economic activity to be stronger at intermediate and coarsest time scales than at the finest ones.

In such a context where both the time horizons of economic decisions and the strength and direction of economic relationships between variables according to the time scale analysis may differ, the most appropriate choice of the analytical tool is wavelet analysis.

**Empirical Analysis**

**Fourier Transform Based Spectral Analysis of BSE and NSE indexes Financial Time Series**

The Fourier transform has long been applied for analysis of continuous and discrete signals and systems in many different fields. It decomposes a signal or a function into a sum of harmonic components of different frequencies via a linear combination of Fourier basic functions (sines and cosines). Fourier transform is a frequency domain representation of a signal or a function containing the same information of the original function, but summarized as a function of frequency. As a consequence, it may be interpreted as a decomposition of a signal on a frequency-by-frequency basis. Fourier transform based spectral analysis allows the detailed study of the correlation among time series.

Fig. (4.3) and (4.4) below shows the frequency spectra of BSE index financial time series in Fig. (3.6) and NSE index financial time series in Fig. (3.7).

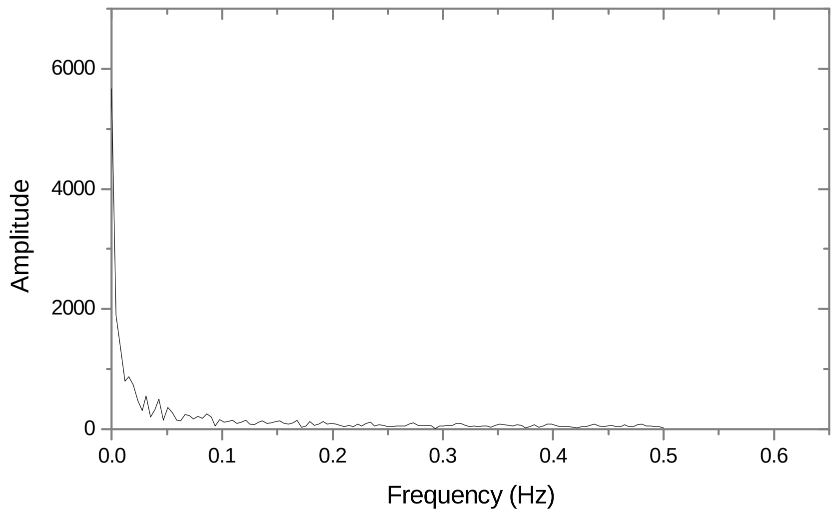


**Fig. 4.3** frequency spectra of BSE index financial time series

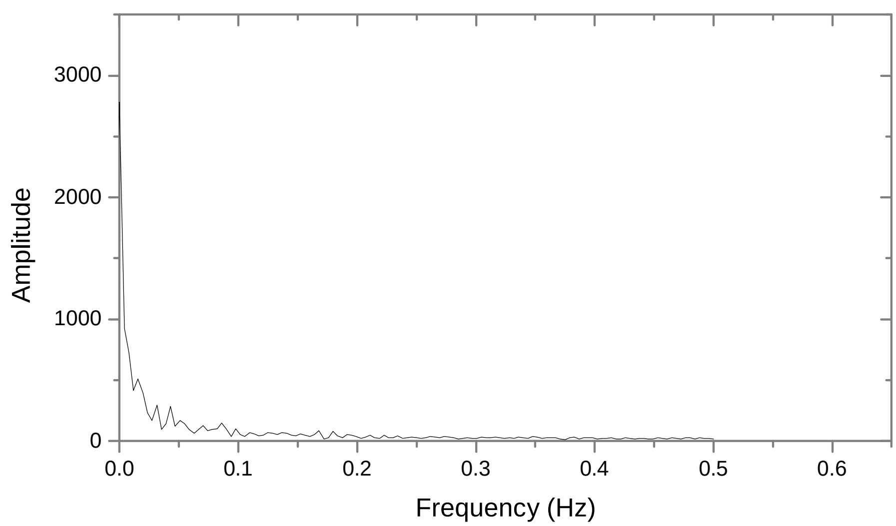


**Fig. 4.4** frequency spectra of NSE index financial time series

**MODWT Based Time Scale Decomposition Analysis of BSE and NSE indexes Financial Time Series**

The analysis has been conducted by using average of monthly data of Bombay Stock Exchange index and National Stock Exchange index of India between April 1990 and March 2006 (sources:www.sebi.gov.in) and the averages are based on daily closing index. We have decomposed the two financial time series into their time-scale components using the MODWT which is a non-orthogonal variant of the classical discrete wavelet transform that unlike the orthogonal discrete wavelet transform is translation invariant, as shifts in the signal do not change the pattern of coefficients. The wavelet filter used in the decomposition is the Daubechies least asymmetric (LA) wavelet filter of length $L = 8$, or $LA(8)$ wavelet filter, based on eight non-zero coefficients, with periodic boundary conditions. For the maximum decomposition level $J$ given by $\log_2 (N)$ we have applied the MODWT up to a level $J = 5$ which produces six wavelet and scaling filter sets of coefficients $v5$, $w5$, $w4$, $w3$, $w2$, $w1$. (Fig. 4.5).

The level of the transform defined the effective scale $\lambda_j$ of the corresponding wavelet coefficients for all families of Daubechies compactly supported wavelets, the level $j$ wavelet coefficients are associated with changes at scale $2^{j-1}$.

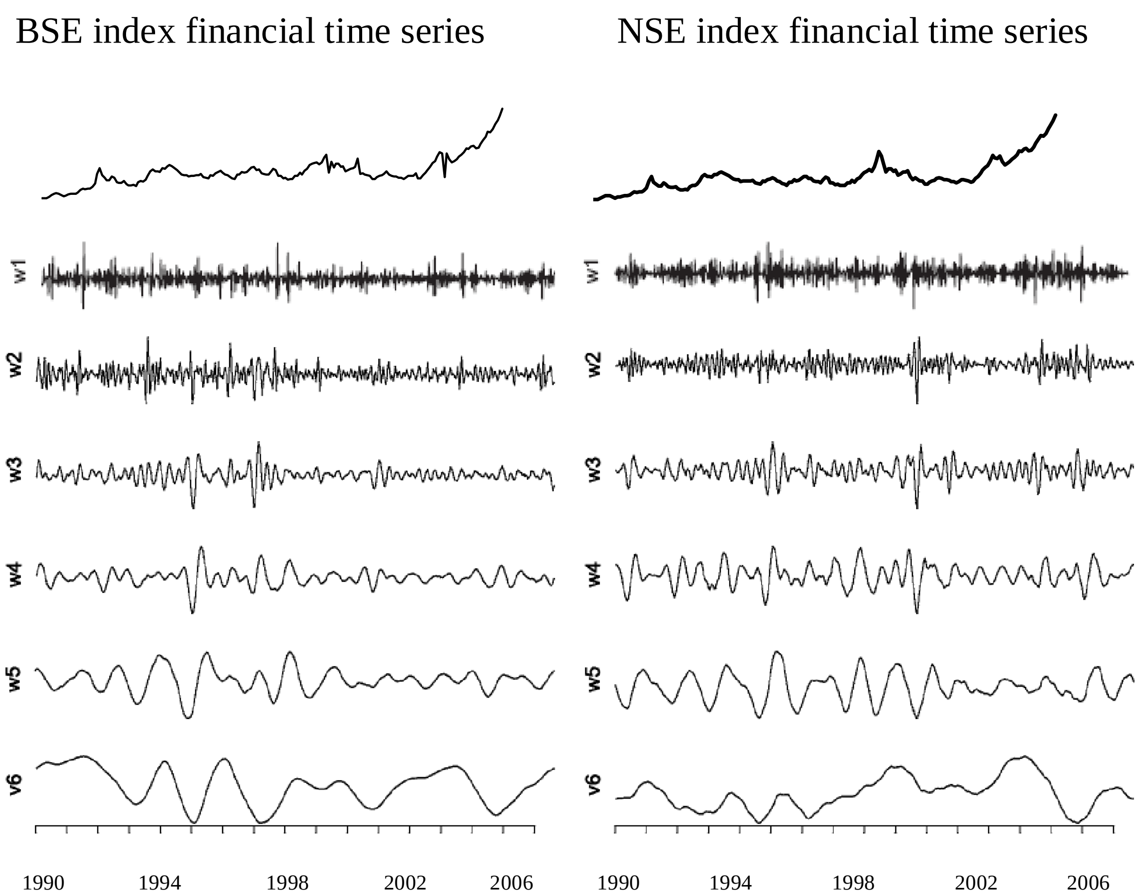


**Fig. 4.5** BSE index financial time series (left) and NSE index financial time series (right)

The frequency spectrum of BSE index financial time series and NSE index financial time series shown in Fig. (4.3) and Fig. (4.4) is dominated by higher frequency components. It does not clearly show the presence of other frequency components in BSE and NSE index financial time series so Fourier transform based spectral analysis is not the correct choice to analyze the BSE and NSE index financial time series which are non-stationary in nature. On the other hand wavelet transform (filter) depends on two parameters frequency and time, that provide the time and frequency information simultaneously. Hence it provides the so-called time-scale or time-frequency representation of the signal where the scale factor is inversely related to the frequency of the wavelet. Prior knowledge of which spectral components occur at which time interval in time series is of great importance when analyzing financial time series. So if we want to know that what spectral components occur and at which time interval it occur in a financial time series, Fourier transform based spectral analysis is not the correct transform to use thus when time localization of the spectral component is needed, wavelet transform based time scale decomposition analysis of financial time series should be adopted.

## 4.3 Results and Discussion for Problem (3.2.3)

The most commonly used measure to analyze the stock market behavior is wavelet correlation analysis. Cross-country correlations have been largely used to obtain a static estimate of the comovements of actual returns across country. With the help of wavelet analysis we can represent the variance of a process on a scale by scale basis, as the plot of $\tilde{\sigma}^2_{X,j}$ against scale $j$ indicates which scales contributes more to the process variance. Fig. (4.6) shows the MODWT-based variance of the BSE Index and NSE Index plotted on a log-log scale, where the straight line indicates the estimated wavelet variance and the "U" and "L" line the upper and lower bounds for the 95% approximate confidence interval. For the calculation of wavelet variance we apply the reflection boundary condition which reflects the original signal at its end point to produce a series of length 2N which has the same mean and variance as the original signal. In this way we may have a sufficient number of nonboundary coefficients to estimate wavelet variance up to scale 6.

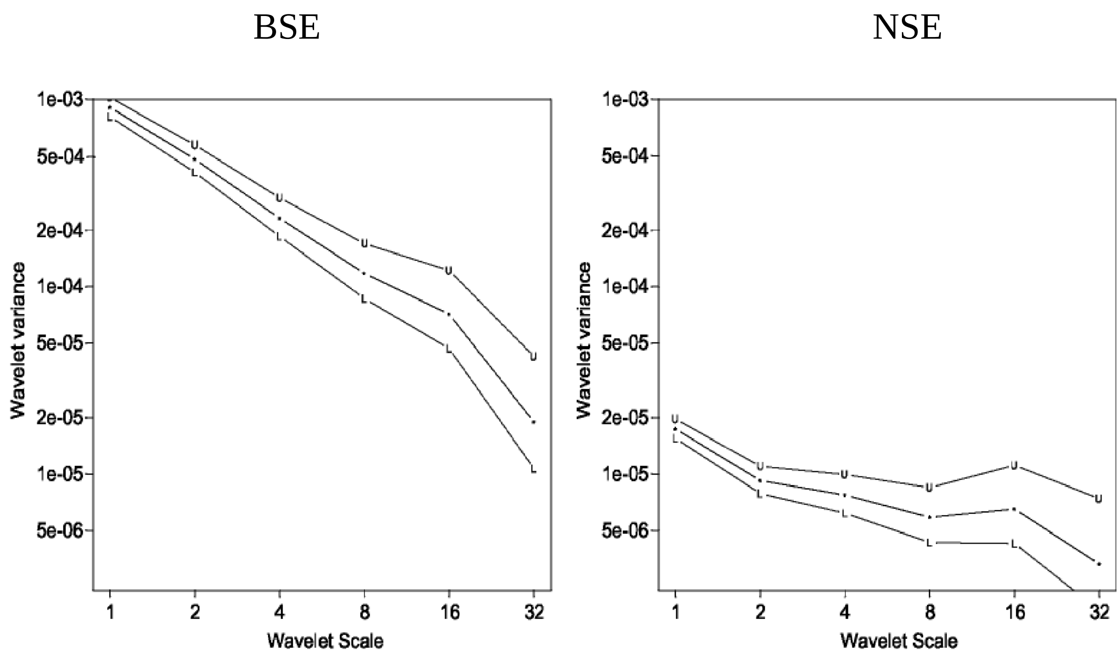


**Fig. 4.6** Wavelet variance for the BSE index and NSE index at log-log scale

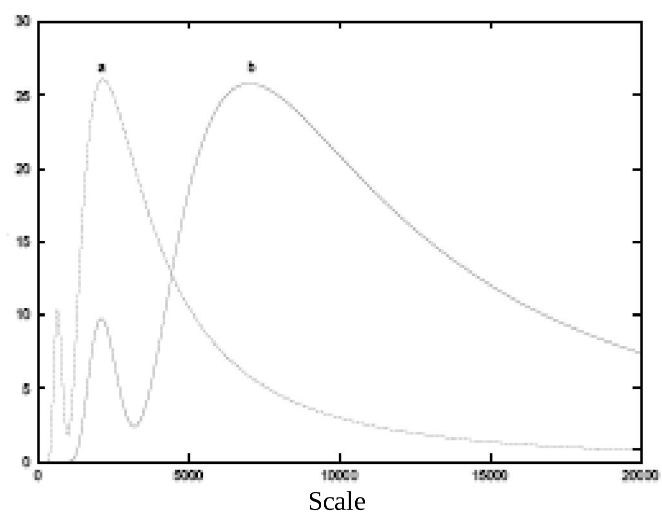


**Fig. 4.7** Wavelet spectrum of NSE (curve a) and BSE (curve b) indexes

As wavelet analysis have the ability to decompose a time series into its time scale components and thus to reveal structure at different time horizons, may be useful in analyzing situations in which the degree of association between two time series is likely to change with the time-horizon. In particular, wavelet cross-correlation analysis, the analogue of the standard time domain measure of association in the time-scale domain, may be used to determine the lead/lag relationship between two time series on a scale-by-scale basis.

Fig. (4.7) shows the Wavelet spectrum of NSE and BSE indices and Fig. (4.8) exhibits the MODWT based wavelet correlations and cross-correlation coefficients, with the corresponding approximate confidence intervals, against time leads and lags for all scales, where each scale is associated with a particular time period. For example, scale 1 is associated to 2–4 month periods, scale 2 to 4–8 month periods, scale 3 to 8–16 month periods, and so on.

At the shortest scales, i.e. scales 1 to 2, the magnitude of the association between the two variables is generally close to zero at all leads and lags, while on the other hand at coarsest scales, particularly at scales 4 and 5, such relationship become stronger.

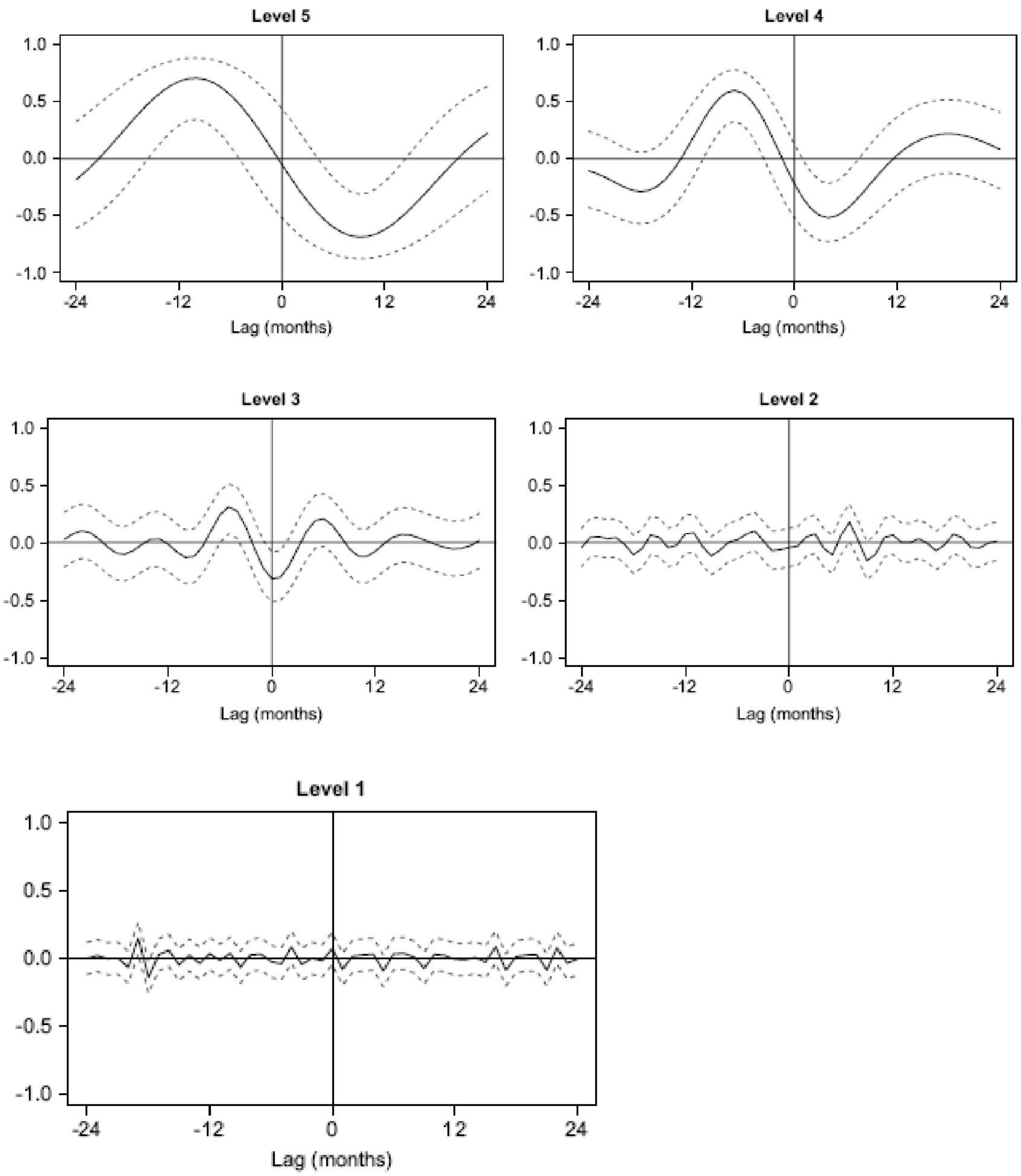


**Fig. 4.8** Wavelet cross-correlations between BSE and NSE indexes

As regards the contemporaneous time scale correlation between BSE and NSE indexes, the values of the wavelet correlation coefficients at lag 0 indicate a generally low magnitude of association between the two at scales 3 and 4. On the other hand, the cross-correlation wavelet coefficients reveal that at the coarsest scales there is a high positive leading relationship between BSE and NSE indexes (0.5 and 0.7 at scales 4 and 5, respectively), with the leading period increasing as the time scale increases (the largest cross-correlation coefficients occurring at leads 6 for wavelet scale 4, that is 16–32 month periods, and 10 for wavelet scale 5, that is 32–64 month periods).

So it is clear from the above discussion that correlation between BSE and NSE indices is scale dependent.

# SUMMARY AND CONCLUSION

# 5 Summary and Conclusion

In the thesis entitled **"Analysis of time series and stock market behavior using wavelet methods"**, we have studied the properties of wavelet transform and their uses in the analysis of time series. Wavelet means 'small wave' so wavelet analysis is all about analyzing signal or series with short duration finite energy functions. They transform the series under investigation into another form which is more compatible and have the special ability to examine signal simultaneously both in term of time and frequency domains. The development of wavelets has been fairly recent in applied mathematics, but wavelets have already made remarkable impact in other fields of research. A large number of researchers are now engaged in applying wavelets to different situations, and all are seem to report favorable results. Current physical applications of wavelets include a wide variety such as climate analysis, financial time series analysis, heart monitoring, condition monitoring of rotating machinery, seismic signal denoising, denoising of astronomical images, crack surface characterization, characterization of turbulent intermittency, audio and video compression, compression of medical and thump impression records, fast solution of partial differential equations, computer graphics etc. etc.

The thesis work has been divided into five major chapters.

Time-series analysis based on wavelet methods is not straight forward. One has to understand the foundation of wavelet transform. The chapter-1, Introduction, is purely introductory in nature and is aimed to fulfill the basic needs of introducing the various concepts and foundation needed for the analysis of time series.

Chapter-2, Review of Literature, accommodates majority of available past research works directly related with the present work.

Chapter-3, Materials and Methods, covers the theory and practices currently being used and also needed for the present study for time series analysis. This chapter has been divided into two sections. Section-3.1 of this chapter deals with the main properties and the fundamental concepts of the wavelets and the methods for the construction of parametric orthogonal wavelet while section-3.2 presents a brief overview of the fundamental concepts in classical time series analysis and the method of calculating the wavelet correlation coefficient through wavelet variance and covariance.

In sub-sections 3.1.1 to 3.1.6, concepts like wavelet coefficients, MRA, wavelet decomposition, filter banks, wavelet packets have been discussed. Sub-section 3.1.7 of the chapter deals with the orthogonality conditions and method of parameterization for generating Daubechies 4-tap scaling and wavelet filter coefficients.

Sub-section 3.2.1 presents a brief overview of the BSE and NSE of India and sub-section 3.2.2 discusses the methods which shall used for the MODWT based time scale decomposition analysis of BSE and NSE indexes financial time series.

Sub-section 3.2.3 deals with the methods for calculating wavelet variance, covariance and these methods have been used for correlation analysis of BSE and NSE indexes financial time series.

The chapter-4 comprises of the Results and Discussion of the problems discussed in chapter-3.

In Section 4.1, we have proposed a simpler method to generate parametric families of orthogonal wavelet and used it to generate the '6-tap Daubechies wavelet filter' in a straight forward manner. This method benefits significantly in a number of applications where parametric wavelet have been used.

In Section 4.2, wavelet based concepts have been employed to study two 'strongly correlated' financial time series of BSE and NSE indexes by decomposing index based financial time series into time-scale components using the MODWT (Maximal Overlap Discrete Wavelet Transform) analysis.

In section 4.3, wavelet correlation analysis of BSE and NSE indexes financial time series using index data from April 1990 to March 2006 have been performed. The analysis shows that correlation between BSE and NSE indices is **scale dependent**.

In the end, Literature Cited and the list of references, coding which have been used in this study are given.

The work carried out in this thesis finds its applications in new emerging fields like financial markets, signal processing, image processing etc. It can also be of use to young researchers involved in analyzing time-series models and the predictions based on wavelets.

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# APPENDICES

*Appendix A*

# CODING

# Coding

```
*******************************************************
Matlab coding for the plotting of Daubechies scaling function
*******************************************************
h0 = (1 + sqrt(3))/(4*sqrt(2));
h1 = (3 + sqrt(3))/(4*sqrt(2));
h2 = (3 - sqrt(3))/(4*sqrt(2));
h3 = (1 - sqrt(3))/(4*sqrt(2));

h = sqrt(2)*[h0 h1 h2 h3];
phi = 1;
n = 10;
for i = 1:n
    phi = conv(h,phi);
    if i<n
        phi=upsample(phi,2);
    end
end
x = linspace(0,3,length(phi));
plot(x,phi);
```

```
*******************************************************
Matlab coding for the plotting of Daudechies wavelet function
*******************************************************
h0 = (1 + sqrt(3))/(4*sqrt(2));
h1 = (3 + sqrt(3))/(4*sqrt(2));
h2 = (3 - sqrt(3))/(4*sqrt(2));
h3 = (1 - sqrt(3))/(4*sqrt(2));

g0 = h3;
g1 = -h2;
g2 = h1;
g3 = -h0;

h = sqrt(2)*[h0 h1 h2 h3];
g = sqrt(2)*[g0 g1 g2 g3];

psi = 1;
psi = conv(psi,g);
n = 10;
```

```
psi = upsample(psi,2);
for i = 1:n
    psi = conv(h,psi);
    if i<n
        psi=upsample(psi,2);
    end
end
x = linspace(0,3,length(psi));
plot(x,psi);
```

***************************************************************************

Matlab coding for the MODWT analysis of BSE

***************************************************************************

```
% Calculate and plot MODWT MRA of BSE index financial time series %

% Load the data
[X, x_att] = wmtsa_data('bse');

% Compute MODWT coefficients
wtfname = 'la8';
J0 = 6;
boundary = 'reflection';
[WJt, VJt, att] = modwt(X, wtfname, J0, boundary);

% Compute inverse MODWT MRA reconstruction.
[DJt, SJt] = imodwt_mra(WJt, VJt, att);

% Setup x-axis for plotting.
delta_t = 1 / 180;   % sampling interval in seconds
time_offset = 0.31;  % seconds
xaxis = (1:length(X)) * delta_t + time_offset;
xlabel_str = 't (sec)';

% Setup plotting parameters
title_str = {};
title_str(1) = {'\bfFigure \rm MODWT multiresolution analysis of BSE index financial
time series'};
title_str(2) = {['Wavelet: ' wtfname ',  NLevels: ', int2str(J0), ...
                 ',  Boundary: ', boundary]};

XplotAxesProp.XLim       = [0 12];
XplotAxesProp.XTick      = [0:1:12];
% XplotAxesProp.XTickLabel = [0:2:12];
% XplotAxesProp.XMinorTick = 'off';
```

```
XplotAxesProp.YLim      = [-1.5 1.5];
XplotAxesProp.YTick     = [-1.5 -.5 .5 1.5];
XplotAxesProp.YTickLabel = [-1.5 -.5 .5 1.5];

% Plot MRA coefficients

[hMRAplotAxes, hXplotAxes] = plot_imodwt_mra(DJt, SJt, X, att, ...
          title_str, xaxis, xlabel_str, ...
          [], XplotAxesProp);

figure_datestamp(mfilename, gcf);
```

*****************************************************

Matlab coding for the MODWT analysis of NSE

*****************************************************

```
% Calculate and plot MODWT MRA of NSE index financial time series %

% Load the data
[X, x_att] = wmtsa_data('nse');

% Compute MODWT coefficients
wtfname = 'la8';
J0 = 6;
boundary = 'reflection';
[WJt, VJt, att] = modwt(X, wtfname, J0, boundary);

% Compute inverse MODWT MRA reconstruction.
[DJt, SJt] = imodwt_mra(WJt, VJt, att);

% Setup x-axis for plotting.
delta_t = 1 / 180;   % sampling interval in seconds
time_offset = 0.31;  % seconds
xaxis = (1:length(X)) * delta_t + time_offset;
xlabel_str = 't (sec)';

% Setup plotting parameters
title_str = {};
title_str(1) = {'\bfFigure \rm MODWT multiresolution analysis of NSE index financial
time series'};
title_str(2) = {['Wavelet: ' wtfname ',  NLevels: ', int2str(J0), ...
           ',  Boundary: ', boundary]};

XplotAxesProp.XLim      = [0 12];
XplotAxesProp.XTick     = [0:1:12];
```

```
% XplotAxesProp.XTickLabel = [0:2:12];
% XplotAxesProp.XMinorTick = 'off';

XplotAxesProp.YLim      = [-1.5 1.5];
XplotAxesProp.YTick     = [-1.5 -.5 .5 1.5];
XplotAxesProp.YTickLabel = [-1.5 -.5 .5 1.5];

% Plot MRA coefficients

[hMRAplotAxes, hXplotAxes] = plot_imodwt_mra(DJt, SJt, X, att, ...
        title_str, xaxis, xlabel_str, ...
        [], XplotAxesProp);

figure_datestamp(mfilename, gcf);
```

********************************************************

Matlab coding for wavelet correlation analysis

********************************************************

```
% Estimated LA(8) wavelet variances for levels j = (1...5)
%

% Load the data
[X, x_att] = wmtsa_data('bse');

base_depth = 350.0;
delta_depth = 0.1;

depth = base_depth + delta_depth * ([0:1:length(X)-1]);
depth = depth(:);

J0 = 9;
wavelet = 'la8';
boundary = 'reflection';

% Calculate MODWT
[WJt, VJt, w_att] = modwt(X, wavelet, J0, boundary);

% Circularly advance MODWT to align with orignal time series
[TWJt, TVJt] = modwt_cir_shift(WJt, VJt, wtfname, J0);

% Compute wavelet variance
% using N = 257 samples and stepping each time point
indices = find(depth >= 400 & depth <= 500);
depth_range = depth(indices);
```

```
Tau_j = 2.^([1:J0]-1);
Tau_j = Tau_j(:);
depth_Tau_j = Tau_j * delta_depth;

[rwvar, CI_rwvar] = modwt_running_wvar(TWJt, indices, 1, 257, ...
                                        'chi2eta3', 'biased', wtfname);


% Plot wavelet variances for levels j = 1...5.
iplot = 0;
for (j = 5:-1:2)
  iplot = iplot + 1;
  subplot(2, 2, iplot);

  semilogy(depth_range, rwvar(:, j), 'b-');
  hold on;
  semilogy(depth_range, CI_rwvar(:, j, 1), 'r--');
  hold on;
  semilogy(depth_range, CI_rwvar(:, j, 2), 'r--');

  XLim = [400, 500];
  YLim = [10^-4, 10^-1];
  set(gca, 'XLim', XLim);
  set(gca, 'YLim', YLim);

  XTick = [400, 450, 500];
  XTickLabel = XTick;
  set(gca, 'XTick', XTick);
  set(gca, 'XTickLabel', XTickLabel);

  YTick = [1E10-4, 1E10-3, 1E10-2, 1E10-1];
  YTickLabel = YTick;
  set(gca, 'YTick', YTick);
  set(gca, 'YTickLabel', YTickLabel);

  if (iplot >= 3)
    xlabel('lag (months)');
  end

  title_str = [sprintf('%0.2f', lag_Tau_j(j)), ' m'];
  title(title_str);

end

title_str = ...
    {    ['Running wavelet variance with confidence intervals'], ...
```

```
    ['for running spans of 257 MODWT coefficients, levels = 1..5']};

suptitle(title_str);

figure_datestamp(mfilename, gcf);
```

*******************************************************
Octave coding for the plotting of wavelet spectrum
*******************************************************

This program performs the wavelet spectra of the input time series y.
It plots the series in normalized form
and displays the modulus (amplitude) of the wavelet spectra in the time-period space.
The period is expressed in unit of time.
What you simply have to do is:
(1) rename your time series y
(2) type DWT

```
gset nokey;
load y
y=reshape(y,length(y), 1);
clear aa period yyyy yyyylab  x1  x2  wave scale f x scale; ny=length(y);
ny2= round (ny/2);
exp1=0;
exp2=round(log2 (ny2)) + 1;
inter=20;
j =0;
 k0= 5. 4;
for m=exp1:exp2-1;
jj =inter- 1;
for n=0:jj;
a = 2(m +n/ inter);
j=j+1;
aa(j ) =a;
end;
end;
```

$a=2^{exp2}$

```
aa(j+1)=a;
omega0= 1/2* (k0./aa+sqrt(2+k0*k0) ./aa);
```

$period = 1./\omega_0 * 2 * \pi$

```
aa=aa'; period=period';
y=y';
y= (y-mean (y)) /std(y);
k0= 5. 4;
dt= 1;
n1=length(y);
```

```
 base2=fix(log(n1)/log(2)+0.4999);
 if(2^base2-n1|0) base2=base2+1;
end;
x= [y,zeros(1,2^base2-n1)];
y=y';
n=length(x);
 k= [1:fix(n/2)];
 k=k.*((2)/(n*dt));
k=[0., k, -k(fix((n-1)/2):-1:1)];
f=fft(x);
scale=aa;
J=length(aa);
 wave=zeros(J,n);
wave=wave+i*wave;
nn=length(k);
for a1=1:J;
expnt=-(scale(a1).*k - k0)^2/2.*(k 0.);

daughter=norm*exp(expnt);
 daughter=daughter.* (k0.);
 wave(a1,:)=ifft(f.*daughter)/ (scale(a1));
 end;
wave=wave(1:J,1:n1);
for k=1:exp2+1;
exponent=k-1;
 [x1, x2]=min(brol);
 yyyy(k)=x2;
yyyylab(k)=2^exponent;
 end;
gset yrange [50:120];
z=abs (wave);
mesh(z);
gset terminal postscript;
 gset output "dwt.eps";
 replot;
gset output;
```

*****************************************************
Maple coding for Groebner basis
*****************************************************

\> *with*(*Groebner*) :

$$P := [M^2 - 2, h0 + h1 + h2 + h3 - M, h0 \cdot h2 + h1 \cdot h3, h0 - h2 + 2 \cdot h3, h1 + 3 \cdot h3 - 2 \cdot h2];$$

$$\left[M^2-2, h0+h1+h2+h3-M, h0\,h2+h1\,h3, h0-h2+2\,h3, h1+3\,h3-2\,h2\right]$$

$G := gbasis(P, plex(h0, h1, h2, h3, M));$

```
Warning, Groebner[gbasis] is deprecated. Please, use Groebner[Basis].
```

$$\left[M^2-2, -1-4\,h3\,M+16\,h3^2, -M-4\,h3+4\,h2, -M+2\,h3+2\,h1, -M+4\,h3+4\,h0\right]$$

$solvefor[h3, h2, h1, h0]\left(-1-4\cdot h3\cdot \mathrm{sqrt}(2)+16\,h3^2, -\mathrm{sqrt}(2)-4\,h3+4\,h2, -\mathrm{sqrt}(2)+2\,h3+2\,h1, -\mathrm{sqrt}(2)+4\,h3+4\,h0\right);$

$$\left[\left\{h2=\frac{3}{8}\sqrt{2}-\frac{1}{8}\sqrt{6}, h1=\frac{3}{8}\sqrt{2}+\frac{1}{8}\sqrt{6}, h0=\frac{1}{8}\sqrt{2}+\frac{1}{8}\sqrt{6}, h3=\frac{1}{8}\sqrt{2}-\frac{1}{8}\sqrt{6}\right\}, \left\{h2=\frac{3}{8}\sqrt{2}+\frac{1}{8}\sqrt{6}, h1=\frac{3}{8}\sqrt{2}-\frac{1}{8}\sqrt{6}, h0=\frac{1}{8}\sqrt{2}-\frac{1}{8}\sqrt{6}, h3=\frac{1}{8}\sqrt{2}+\frac{1}{8}\sqrt{6}\right\}\right]$$

> $restart : P := \left(\left(\frac{(1+z)}{2}\right)^2\cdot\left(u+\frac{(1-u)}{2}\cdot z+\frac{(1-u)}{2}\cdot z^{-1}\right)\right):$

$P := collect(expand(P), z):$
$ldp := ldegree(P):$
$P := simplify\left(P\cdot z^{-ldp}\right):$
$P := collect(P, z):$
$Cp := PolynomialTools{:}\text{-}CoefficientList(P, z):$

$$Q := \left(\left(\frac{(1+z)}{2}\right)^2\cdot\left(v+\frac{w}{2}\cdot z+\frac{w}{2}\cdot z^{-1}+\frac{(1-v-w)}{2}\cdot z^2+\frac{(1-v-w)}{2}\cdot z^{-2}\right)\right):$$

$Q := collect(expand(Q), z):$
$ldq := ldegree(Q):$
$Q := simplify\left(Q\cdot z^{-ldq}\right):$
$Q := collect(Q, z):$
$Cq := PolynomialTools{:}\text{-}CoefficientList(Q, z):$

$Cp; Cq;$

$$\left[\frac{1}{8} - \frac{1}{8}\,a,\ \frac{1}{4},\ \frac{1}{4} + \frac{1}{4}\,a,\ \frac{1}{4},\ \frac{1}{8} - \frac{1}{8}\,a\right]$$

$$\left[-\frac{1}{8}\,c + \frac{1}{8} - \frac{1}{8}\,b,\ -\frac{1}{8}\,c - \frac{1}{4}\,b + \frac{1}{4},\ \frac{1}{8}\,c + \frac{1}{8}\,b + \frac{1}{8},\ \frac{1}{2}\,b + \frac{1}{4}\,c,\ \frac{1}{8}\,c + \frac{1}{8}\,b + \frac{1}{8},\ -\frac{1}{8}\,c - \frac{1}{4}\,b + \frac{1}{4},\ -\frac{1}{8}\,c + \frac{1}{8} - \frac{1}{8}\,b\right]$$

$C1 := 0 : C2 := 0 : Np := nops(Cp) : Nq := nops(Cq) :$

**for** i **from** 1 **to** 4 **do** C1 := C1 + expand($Cq[Nq - 4 + i] \cdot Cp[i]$) :
**end do**;

$$\frac{1}{16}\,b - \frac{1}{16}\,b\,a + \frac{1}{32}\,c - \frac{1}{32}\,c\,a$$

$$\frac{3}{32}\,b - \frac{1}{16}\,b\,a + \frac{1}{16}\,c - \frac{1}{32}\,c\,a + \frac{1}{32}$$

$$\frac{1}{32}\,b - \frac{1}{8}\,b\,a + \frac{1}{32}\,c - \frac{1}{16}\,c\,a + \frac{3}{32} + \frac{1}{16}\,a$$

$$-\frac{1}{8}\,b\,a - \frac{1}{16}\,c\,a + \frac{1}{8} + \frac{1}{16}\,a$$

**for** i **from** 1 **to** 2 **do** C2 := C2 + expand($Cq[Nq - 2 + i] \cdot Cp[i]$) :
**end do**;

$$-\frac{1}{64}\,c + \frac{1}{64}\,c\,a - \frac{1}{32}\,b + \frac{1}{32}\,b\,a + \frac{1}{32} - \frac{1}{32}\,a$$

$$-\frac{3}{64}\,c + \frac{1}{64}\,c\,a - \frac{1}{16}\,b + \frac{1}{32}\,b\,a + \frac{1}{16} - \frac{1}{32}\,a$$

$S := solvefor[b, c](C1, C2)$

$$\left\{c = -\frac{4 - 4\,a + a^2}{a},\ b = \frac{1}{2}\,\frac{6 - 3\,a + a^2}{a}\right\}$$

$Z := eval\left(S, a = \frac{7}{4}\right)$

$$\{c = 0, b = 1\}$$

$eval\left(Cp, a = \frac{7}{4}\right)$

$$\left[-\frac{1}{8}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, -\frac{1}{8}\right]$$

$eval(Cq, Z)$

$$\left[0, 0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0, 0\right]$$

# *VITA*

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# ABSTRACT

Name: Anuj Kumar Id. No. 30711
Semester and Year of admission: I$^{st}$, 2005-06 Degree: Ph. D
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Thesis title: **"ANALYSIS OF TIME SERIES AND STOCK MARKET BEHAVIOR USING WAVELET METHODS"**

Advisor: Dr. A. K. Sharma

Post liberalization, the development of new techniques and ideas in econometrics have been rapidly growing over the last few years. These developments are now being applied to a wide variety of fields and in analyzing the stock market behavior. Specially the area of analysis of financial time series is catching the attention of big business houses as they are now contemplating on optimizing the business process in order to achieve best estimate of production planning over the long span of time and also to sustain the growth momentum. Economic and financial time series are nonstationary in nature and exhibits changing frequency patterns over the time. Wavelet analysis is one such tool for analyzing non-stationary data.

Parameterization of wavelet families allows one to generate infinite number of wavelets for the different choices of selection for analyzing the financial time series. In present work, **"Analysis of time series and stock market behavior using wavelet methods"** we have proposed a simpler method to generate parametric families of orthogonal wavelet and used it to generate the **'6-tap Daubechies wavelet filter'** in a straight forward manner. The wavelet based concepts have then been employed to study BSE and NSE indexes financial time series using index data from April 1990 to March 2006 by decomposing index based financial time series into time-scale components using the MODWT (Maximal Overlap Discrete Wavelet Transform) analysis. The most commonly used measure to analyze the stock market behavior is the wavelet correlation analysis, we have used it for the analysis of BSE and NSE indexes financial time series.

The work embodied in this thesis serves as intuitive guide for analyzing the time series and can be of use for the predictions based on it by using the newly developed wavelet technique.

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