

**FLOW OF VISCOUS FLUIDS IN VARIOUS CHANNELS PLACED IN A
MAGNETIC FIELD**



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CHAPTER 1

INTRODUCTION

INTRODUCTION

The motion of fluids is studied under Fluid Mechanics as it a very well known branch of applied mathematics and it gives the idea how fluids interact under forces. Liquids along with gases altogether are called to be fluids in this branch of science. Further, the branch of Fluid Mechanics is sub-divided into two more specific fields. These are Fluid Dynamics and Fluid Statics which when fluids are in motion and when fluids are at rest respectively. Since, Fluid Mechanics is a completely vital subject in the analysis of the behaviour of liquids similarly when very still and when in movement. Fluid Mechanics deals with the branch of Mathematics which ponders the issues emerging out in the fluid movement through different channels set in a magnetic field and in various physical conditions existing in various segments of science, innovation and businesses. For example, aerodynamics, ship motion, power generation, petroleum industry, chemical engineering and rocket propulsion. Its applications are very wide as it holds a vital role in industries concerning steel, plastic, electric wire, glasses, oil, gases and molten metals. It also makes it easier in controlling the temperature of computer chips, vehicle engines and high power machines. In our day-to-day life, the water pipe supply, cooking gas through pipes, use of lubricants in moving objects etc also require its deep knowledge.

In the field of agriculture, its role is important as fluid carries the dissolved minerals and nutrients from soil to the various segments of the plants. In a human body, the blood carries nutrients to various organs through arteries and veins. In medical science, the purpose of fluid that flows requires a wide knowledge for the matter related with cardiology, nephrology, haematology and respiratory track. Also under different physical conditions, it eases to solve many problems existing in solar science, metrology, cosmology, water purification and flood control.

The neo and exciting developments in technology, sciences, computer programmes and other complicated mathematical techniques has given rise to great research activity in this discipline of fluid dynamics. As to overcome a great number of flow problems in viscous fluids, channel flows got much more noticeable as of their applicability in solving of problems of drainage and irrigation, etc. The rigorous works in MHD flows has received much attention and has gained momentum because of the discovery of largely flowing electric currents in some stars and the feasibility of MHD to generate energy. In today's

world, visco-elastic fluids along with dust particles help to understand environmental pollution problems and forecasting.

Since the time of Poiseuille and Couette, much attention has been discussed to the problems bearing steady and unsteady flow of visco elastic fluids under different boundary conditions. The investigations are also being carried out involving laminar visco-elastic flows in contact with fixed as well as movable boundaries.

In general, the flow occurs having many phases known as multiphase flow and the magnetic field highly affects the magnetic sensitive particles; the flow profile is very significant from the perspective of its applications. Since mostly all the fluids being utilised in industries are non-Newtonian, therefore seeing their large scale applications, this study gives light on the flow of non-Newtonian fluids through various channels situated in magnetic field.

Various physical parameters are taken under consideration, and the channels are considered to be 1) vertical channel. 2) horizontal channel.

The terms and definitions used in this thesis have been mentioned over here and also the various category of fluid which are classified as follows:

Types of fluid:

Ideal fluid:

The fluid for which $\mu = 0$ if $\tau = 0$ is termed as an ideal fluid. The ideal fluid is free and clean from solid particles.

Newtonian fluid:

“Fluids in which obey the Newton’s law of viscosity” (i.e. shear stress is linearly proportional to the velocity gradient in the direction perpendicular to the plane of shear) are known as Newtonian fluids. Common fluids like water, air and mercury are all Newtonian fluids”.

Non- Newtonian fluids:

The fluids in which the Newton’s law of viscosity is not being followed (i.e. the shear stress and the velocity gradient are not proportional) are known as non-Newtonian fluids. Fluids like coal tar, polymer and paints solutions are all non-Newtonian fluids.

Viscous fluid:

A fluid in which the shearing stresses and the normal stresses both exist is termed as viscous fluid. As all the fluids have viscosity, a viscous fluid, in everyday sense of term, is one that has high level of viscosity, hence these type of fluids move slowly depending on how much viscous they are.

Inviscid fluid:

A fluid which does not exert any shearing stress, whether at rest or in motion is termed as inviscid fluid. Hence the pressure exerted by the inviscid fluid on any surface is always along the normal to the surface at that point.

Visco-elastic Fluids:

Visco-elasticity is the property of materials that display both viscous and elastic qualities while experiencing distortion. Viscous materials, similar to honey, oppose shear flow and strain directly with time when a pressure is applied. Flexible materials strain when extended, rapidly come back to their unique state once the pressure is evacuated.

Visco-elastic materials have elements of both of these properties and, thus display time-independent strain. While elasticity is normally the consequence of bond extending along crystallographic planes in very strong, therefore viscosity is due to the diffusion of atoms in the material.

Time Dependent Fluids:

These types of fluids are characterized by the fact that the shear rate depends not only on the applied stress, but, also on the duration of the stress. Fluids which show a decrease in viscosity with time under isothermal conditions and steady shear are called thixotropic while those fluids which show increase in the viscosity are called rheopectic. The thixotropic fluids are more common than rheopectic fluids and are of great importance in industries.

The movement of time-independent fluids is featured to the sequence of molecular structural changes and the knowledge, regarding the mechanism of breaking and of a deformation of the molecular changes.

Some significant types of flow:

Multiphase flow:

In Fluid Mechanics multiphase stream flow occurs in a framework which contains numerous conditions of issue where the stages are not chemically related (e.g. dusty gases) or where in excess two stages are available (e.g. in displaying of proliferating steam blast).

Compressible flow:

Compressible flow is the area of fluid mechanics that deals with fluids in which the fluid density varies significantly in response to a change in pressure.

Incompressible flow:

In fluid mechanics, or generally continuum mechanics, the flow in which the density of the fluid does not change with respect pressure is called Incompressible flow.

Rotational flow:

In fluid mechanics, the flow in which the fluid particles rotate about their own particular axis is called rotational flow. In such type of flow, the curl of the fluid is not zero.

Irrotationalflow:

A flow in which the fluid particles do not rotate about their own axes is termed as irrotational flow. In such type of flow the curl of the fluid is zero.

Couette flow:

It is a viscous fluid flow between two parallel plates in a laminar form and the flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates.

Steady flow:

The flow is such that the properties and conditions associated with the motion of the fluid are independent of the time so that the pattern of the flow remains unchanged with the time is termed as steady flow.

Unsteady flow:

The flow which is having the properties and conditions associated with the motion of the fluid depending on the pattern of flow varies with time is termed as unsteady flow.

Uniform flow:

A flow in which the fluid particles having equal velocities at each cross-section of the pipe or channel is known as uniform flow.

Non-uniform flow:

A flow in which the fluid particles having different velocities at each cross-section of the channel or pipe is called non-uniform. These terms are usually used in the explanations of the flow in channels.

Laminar flow:

A flow in which each fluid particle traces out a definite curve and the curves traced out by any two different fluid particles do not intersect, is said to be laminar flow. In this flow the motion of the fluid is in the form of layers with one layer sliding over another. Hence there is no exchange of the fluid particle from one layer to the other which results in no transfer of lateral momentum to the adjacent layer. This type of flow is generally used in pipe flow.

Turbulent flow:

A flow in which each fluid particle undergoes irregular fluctuations, hence the layers of the moving fluid mixes with the other, therefore, the speed of the fluid continuously undergoes a change in direction as well as magnitude is said to be turbulent flow.

Magnetohydrodynamics (MHD):

Magneto-hydrodynamics (MHD) is the study of the interaction between magnetic field and hydrodynamics. The idea of MHD is that the magnetic field can induce currents in a moving conductive fluid which create forces on the fluid and also change the magnetic field itself. The set of equations which describes MHD is a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have to be solved simultaneously either analytically or numerically.

Ideal MHD equations:

In MHD, it is being assumed that fluid which has very little resistivity can be regarded as perfect conductor; if such type of condition occurs then it is termed as an ideal MHD. In, ideal MHD, Lenz' law states that the fluid is said to be attached to the magnetic field lines.

The association between fluid in perfect MHD and attractive field lines settles the topology of attractive field in the fluid.

In the following:

\mathbf{B} : magnetic field

\mathbf{E} : electric field.

\mathbf{v} : bulk plasma velocity

\mathbf{J} : current density

ρ : mass density

\mathbf{P} : liquid pressure

t : Time

The equation of continuity is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1.1)$$

The momentum equation is

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{p} \quad (1.2)$$

Lorentz force term $\mathbf{J} \times \mathbf{B}$ can be expanded to give:

$$\mathbf{J} \times \mathbf{B} = \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{\mu_0} - \nabla \left(\frac{B^2}{2\mu_0} \right) \quad (1.3)$$

Where the first term on the right side is the magnetic tension force and the second expression is the magnetic pressure force.

The Ohm's law for ideal condition for plasma is stated by:

$$E + \frac{v \times B}{c} = 0 \quad (1.4)$$

Faraday's law is

$$\frac{\partial B}{\partial t} = -\nabla \times E \quad (1.5)$$

The Ampere's law for low-frequency neglects displacement current and is stated by

$$J = \frac{c}{4\pi} \nabla \times B \quad (1.6)$$

The divergence constraint of magnetic field is

$$\nabla \cdot B = 0 \quad (1.7)$$

The equation for energy is stated by

$$\frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = 0 \quad (1.8)$$

Where $\gamma = 5/3$ is the fraction of specific heat for an adiabatic equation of state. This equation of energy is only approachable in the absence of heat conduction as it gives that the entropy of a fluid element does not change.

The dynamical behavior of fluids is determined by the mechanical fluid forces as well as the magnetic forces exerted on the particles as a result of the magnetic field induced by the motion of the charged fluid particles by the property of conductivity, hence Maxwell's electromagnetic equations which gave light on the dynamical behavior of the fluid can be described mathematically as:

$$\nabla \cdot E = \frac{4\pi}{\epsilon} q \quad (1.9)$$

$$\nabla \cdot H = 0 \quad (1.10)$$

$$\nabla \times H = 4\pi J \quad (1.11)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (1.12)$$

$$J = \sigma(E \times \mu v \times H) \quad (1.13)$$

Where E : Electric field intensity.

ε : Dielectric constant of the fluid.

H : Magnetic field intensity.

μ : Permeability of the medium.

J : Conduction current density vector.

q : Charge per unit volume.

σ : Conductivity of the medium.

Further, the three constitutive equations in the case of a viscous fluid are:

$$\frac{\partial p}{\partial t} + \rho \frac{\partial u_k}{\partial x_k} = 0, \quad k = 1,2,3 \quad (1.14)$$

$$\delta \left[\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_k}{\partial x_k} \right] = -\nabla p + J \times B \quad (1.15)$$

$$\delta \left[\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} \right] = -\nabla(Pv + Q) + (J \cdot E) \quad (1.16)$$

Moreover the energy equation for incompressible fluids becomes

$$\rho \frac{d\varepsilon}{dt} = KV^2T + \eta\varphi + \frac{J^2}{\sigma} \quad (1.17)$$

The terms which are on the right hand side represents heat conduction, viscous dissipation and joule heating effect respectively and φ being a positive definite quadratic form of the velocity derivative.

The Hartmann number $M = \mu H l_0 \left(\frac{\sigma}{\rho \nu} \right)^{1/2}$ gives the relative effect of magnetic and viscous drag, where $\nu = \frac{\eta}{\rho}$, η = conductivity of the medium and ρ = density of the fluid.

Implementation of MHD:

There are various implementations of MHD flow as in the area of Geophysics, Astrophysics and Cosmology, Engineering and Magnetic Drug Targeting etc.

Viscosity:

Viscosity is characteristic of the fluid which displays a specific resistance from modification of shape. All fluids possess the property of viscosity in the varying ranges.

Properties and behavior of viscosity:

The friction in-between the active boundaries cause the fluid to shear. The force which is essential to measure this action is known as the fluid's viscosity. The flow of this type is known as a couette flow.

Laminar shear, is the outcome of the geometry of channel through which the liquid is flowing. In common, in any flow, different layers move at different velocities and the fluid's viscosity emerges from the shear stress between the layers that eventually resist any applied force. The bond between the velocity gradient and the shear stress can be acquired by taking two plates which are very closely spaced at a distanced y , and isolated by homogeneous substances. After considering that the plates are very large, having an area A , such that the effect of the edge may be disregarded, whereas the lower plate is taken to be in a fixed condition, Now let there be an applied force F to the upper plate. In the event that this force F makes the substance between the plates experience shear stream with a speed inclination u/y

the state of matter which is known as fluid. The force applied is corresponding to the region is in proportion to the inclination in the fluid.

Dynamic (shear) viscosity:

The dynamic (shear) viscosity of a liquid communicates its protection from shearing streams, where neighbouring layers move parallel to each other with various velocities. It can be characterized through the circumstance known as a Couette flow, where a layer of liquid is caught between two even plates, one settled and one moving on a level plane at steady speed u . (The plates are kept very vast, with the aim that one need not consider what occurs close to their edges.)

The magnitude F of this force is initiated to be correlated to the speed u and the area A of each plate which is inversely correlated to their detachment y :

$$F = \mu A \frac{u}{y}.$$

The proportionality (corresponded) factor μ in this formula is the viscosity (particularly, the dynamic thickness of the liquid). The proportion u/y is known as the rate of shear distortion or shear speed, and is the subsidiary of the liquid speed toward the path opposite to the plates.

Isaac Newton expressed the viscous forces by the differential equation

$$\tau = \mu \frac{\partial u}{\partial y},$$

Where

$$\tau = F/A \text{ and}$$

$\partial u/\partial y$ is the local shearing velocity.

This given formularepresents that the stream is moving along parallel lines and the Y-axis, opposite to the stream, focuses toward greatest shear speed. This condition can be utilized

where the speed does not fluctuate directly with y , for example, in liquid coursing through a pipe.

Kinematic viscosity:

The kinematic viscosity is the relation of the dynamic viscosity μ to the density of the fluid ρ . It is usually indicated by the Greek letter nu (ν).

$$\nu = \frac{u}{\rho}$$

It is an appropriate concept when scrutinizing the Reynolds number that expresses the relation of the inertial forces to the viscous forces:

$$Re = \frac{\rho u L}{\mu} = \frac{u L}{\nu}$$

Where L is an average length scale of the system.

Fluidity:

The opposite of viscosity is known fluidity, generally symbolized by $\phi = 1 / \mu$ or $F = 1 / \mu$, ($\text{cm} \cdot \text{s} \cdot \text{g}^{-1}$), which is termed as rhe. Also, Fluidity is occasionally utilized as a part of building practice.

Dusty viscous fluid:

The viscous fluid for which the dusty particles are available and its streamlined features protection (i.e. resistance) is not as much as that of a perfect gas is called dusty viscous fluid.

Significance of Dusty Fluid Motion:

As of late, numerous issues in applied sciences related with flow of non-Newtonian liquids with in excess of one stage have come into picture as the wide variety of the fluids or gases are tainted and contain an appropriation of strong particles e.g. fluidization process, the way toward breathing in oxygen in breath, arrangement of rain drops by the blend of little beads, the origination of dusty loaded air, utilizing dust in gas-cooling framework to improve warm exchange process. Researchers and technologists have appreciated the investigation of the issues of gas-strong molecule stream happening in business. Dusty liquid occurrence is essential in sedimentation, pipe stream, gas cleaning, and transport-process. The gas molecule stream is critical in drop out of toxin in air and water. In physiological science, activity of platelets in the fluids plasma through supply routes can give imperative data for cardiovascular issue. The power age by MHD generator, as an elective wellspring of vitality, can likewise be the dusty liquid occurrences. The issues of two segments liquids affected by temperature contrast are helpful in soil science and geo-material science. The amount of strong molecule introduce in such frameworks is variable yet certainly successful.

Basic Equations of Dusty Viscous Fluid:

The basic equations of motions for a dusty incompressible fluid are:

$$\text{Div } v = 0 \quad (1.18)$$

$$(v \cdot \text{grad})v + \text{grad } P = \mu \text{curl } H \times H + NK(v - u). \quad (1.19)$$

$$\text{div}(Nu) = 0. \quad (1.20)$$

$$\text{curl } (v \times H) = 0. \quad (1.21)$$

$$\text{Div } H = 0. \quad (1.22)$$

$$m (u \text{ grad})u = K(u - v). \quad (1.23)$$

where v , u and H are the liquid velocity vector, dust velocity vector, and magnetic field, individually. p and ρ are the scalar point work known as pressure and density, N is the molecule number density, m the mass, and K the stocks protection coefficient for the dusty particles.

Equation of Continuity:

It states that for a given volume of fluid the rate of mass of generation is balanced by an equal net outflow of mass from the volume under consideration. As the Continuity Equation also states that it conserves mass, therefore, the differential form for continuity equation is:

$$\frac{\partial p}{\partial t} + \nabla(\rho u) = 0. \quad (1.24)$$

Where,

P is fluid thickness, t is time and u is regarded as fluid velocity in vector field.

Euler's Equation of Motion:

The Euler's Equation of motion is the equations governing inviscid flow. The well known equations are named after Leonhard Euler. The conditions speak to preservation of mass, force (momentum) and energy, relating to the Navier-Stokes conditions with zero consistency and warmth conduction terms. Verifiably, just the continuity condition and force (momentum) conditions have been determined by Euler. Be that as it may, liquid progression

writing frequently implies to the full set of equations including the energy condition together known as "the Euler conditions".

Aligned to the Navier–Stokes conditions, the Euler conditions are typically composed in one of two structures: "the conservation type" and the "non – conservation type". The conservation frame underscores the physical understanding of the conditions as protection laws through control volume settled in space. The non-conservation frame stresses changes to the condition of control volume as it moves with the liquid.

As, the Euler equations are all applicable to compressible as well as to incompressible flow

In differential form, the equations are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1.25)$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\mathbf{u} \otimes (\rho \mathbf{u})) + \nabla p = 0 \quad (1.26)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u} (E + p)) = 0 \quad (1.27)$$

where

ρ is the fluid

\mathbf{u} is the fluid velocity vector, with components u , v , and w ,

$E = \rho e + \frac{1}{2} \rho (u^2 + v^2 + w^2)$ is the total energy per unit volume, with e being the internal energy/ unit mass for the fluid,

p is the pressure,

\otimes denotes the tensor product, and

$\mathbf{0}$ is the zero vector.

Porosity:

Porosity is a measure of the empty (i.e., "exhaust") spaces in a material, which is a small amount of the volume of voids over the aggregate volume, which is inside the region of 0 and 1.

Porous medium:

A porous medium is a material containing pores (voids). The skeletal segment of the material is usually called the "framework" or "edge". The pores are normally loaded with a liquid (fluid or gas).

Flow through porous media:

The analysis of fluid flow in porous media is required in a large range of applications. The porous media can be naturally formed (e.g. rocks, sand beds, sponges, woods) or fabricated (e.g. pellets, wicks, insulations, catalytic). The applications of this flow are in the area of production of chemicals, environmental engineering, mechanical engineering, petroleum engineering and geology.

Heat transfer in porous media:

Heat conduction through fully-saturated matrices (i.e. a single-phase fluid occupying the porous) , as with heat conduction through any heterogeneous media, depends on the structure of the matrix and the thermal conductivity of each phase. One of the most difficult aspects of the analysis of heat conduction through porous medium is the structural modelling. This is because the representative elementary volumes are three- dimensional and have complicated structures that vary greatly among different porous media. Since the thermal conductivity of the solid phase is generally different than that of the fluid, the manner in which the solid is interconnected influences the conduction significantly. Even when dealing with non-

consolidated particles, the contact between the particles plays a significant role and as the simultaneous fluid flow and heat transfer in porous media, the role of the macroscopic and microscopic velocity fields on the temperature field needs to be examined.

Diffusion and Mass Transfer:

In nature, the flow of fluids is caused not only by temperature differences but also by concentration. For example, in atmospheric flows, there exist differences in water concentration and hence flow is affected by such concentration differences. Also in a number of engineering applications, foreign gases are injected and due to such mass transfer, it has been observed that there is a reduction in the wall shear stress, the mass transfer conductance or buoyancy forces arising due to temperature difference and those arising due to concentration differences are also important.

Blood:

Blood is especially a blend of plasma (watery fluid) and cells that buoy in it. It is a specific organic liquid that provisions basic substances and supplements, for example, sugar, oxygen, and hormones to our cells, and diverts squander from those cells, this waste in the long run flushed outside the body in pee, excrement, sweat, and lungs (carbon dioxide). Blood also contains:

- Blood cells
- Carbon dioxide
- Glucose (sugar)
- Hormones
- Proteins

Importance of blood flow:

The circulating blood keeps our immune system healthy and our heart pumping. It literally gives life. Blood circulation is essential for a healthy body. Each cell of the body needs to get oxygen and supplements. Blood is very rich in oxygen which is further sent to the body organs, tissues and cells to support them and the waste items that outcome are arranged off through a similar framework. The heart, lungs and veins cooperate to finish course. The two noteworthy pathways are cardiopulmonary and fundamental course. The three specific courses are coronary, entryway and fetal flow. The body cannot work well without a solid circulatory framework.

The Circulatory framework is an organ framework driven by the heart, to give a consistent supply of blood to the body transported by the veins. The course of blood assumes an essential part in your wellbeing as it transports grown-up foundational microorganisms, white and red platelets, hormones, plasma etc. At the slim/venule bed (microcirculation) there is a development of supplements to the cells of the body i.e. vitamins, minerals, amino acids (protein), starches like glucose, fats. There is likewise the arrival of oxygen which is given to the cells; expulsion of carbon dioxide from the cell and evacuation of metabolic acids (by results of the cell).

Steady flow in streamline coordinates:

It is helpful to pick the Frenet– Serret outline along a streamline as the arrange framework for depicting the momentum part of the Euler condition:

$$\frac{Dv}{Dt} = -\frac{1}{\rho} \nabla p, \tag{1.28}$$

Where v is termed as velocity, p is termed as pressure and ρ is the thickness, respectively.

Let $\{e_s, e_n, e_b\}$ be a Frenet-serret ortho-normal basis which consists of tangential unit vector, a normal unit vector, and a binomial unit vector the streamline, respectively. Since a streamline is a bend that is digression to the velocity vector of the flow, the left side of the above condition, the significant subordinate of velocity, can be depicted as takes after:

Bernoulli's equation came from the first equation:

$$\frac{\partial}{\partial s} \left(\left(\frac{v^2}{2} + \int \frac{\partial p}{\rho} \right) \right) = 0 \quad (1.29)$$

The second condition expresses that, for the situation the streamline is bended, there should exist a pressure slope perpendicular to the streamline on the grounds that the centripetal increasing speed of the liquid bundle is just produced by the typical weight inclination. The third condition communicates that pressure is steady along the binomial pivot.

Ohm's law:

$$J = \sigma(E + q \times B_0) \quad (1.30)$$

Where, J is current density, E is electrical field, σ is a material dependent conditions known as conductivity, B is magnetic field, μ is magnetic constant.

Bessel's Differential Equation:

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{n^2}{x^2} \right) y = 0 \quad (1.31)$$

is called Bessel's Differential equation which is of order 'n', its solution is:

$$y = PJ_n(x) + QJ_{-n}(x) \quad (1.32)$$

where

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(n+k+1)} \left(\frac{x}{2}\right)^{n+2k} \quad (1.33)$$

and

$$J_{-n}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \Gamma(-n+k+1)} \left(\frac{x}{2}\right)^{-n+2k} \quad (1.34)$$

are the Bessel's Function of the 1st kind of order n and -n respectively. P and Q are constant whereas the value for n ≠ 0. If n = 0, then the Bessel's equation is:

$$y = PJ_0(x) + QY_0(x) \quad (1.35)$$

Where

$$J_0(x) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \left(\frac{x}{2}\right)^{2k} \quad (1.36)$$

and

$$Y_0(x) = J_0 \log x + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}\right) \left(\frac{x}{2}\right)^{2k} \quad (1.37)$$

is Bessel's Function of 2nd kind of zero order.

For the Bessel's function of 2nd kind of order n i.e. for:

$$Y_n(x) = J_n(x) \int \frac{dx}{xJ_n^2(x)} \quad (1.38)$$

Hence the solution of Bessel's Equation is:

$$Y = PJ_n(x) + QY_n(x) \quad (1.39)$$

NAVIER- STOKES EQUATIONS:

Navier-stokes conditions are basically partial differentials conditions. It must contain momentum, conservation of mass, and energy. These are the three equation are called the Navier- stokes equations for the motion of a viscous compressible fluid in Cartesian coordinates.

$$\rho \frac{Du}{Dt} = \rho B_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[\mu \left\{ 2 \frac{\partial u}{\partial x} - \frac{2}{3} (\nabla \cdot q) \right\} \right] + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] \quad (1.40)$$

$$\rho \frac{Dv}{Dt} = \rho B_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left[\mu \left\{ 2 \frac{\partial v}{\partial y} - \frac{2}{3} (\nabla \cdot q) \right\} \right] + \frac{\partial}{\partial z} \left[\mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right] + \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \quad (1.41)$$

$$\rho \frac{Dw}{Dt} = \rho B_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left[\mu \left\{ 2 \frac{\partial w}{\partial z} - \frac{2}{3} (\nabla \cdot q) \right\} \right] + \frac{\partial}{\partial x} \left[\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] + \frac{\partial}{\partial y} \left[\mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right] \quad (1.42)$$

Particular case: Incompressible viscous fluid flow:

The above system of equation become further simplified in the specimen of incompressible fluids ($\rho = \text{constant}$) even if the temperature is not constant. We have $\nabla \cdot q = 0$. Further, equations of motion as for incompressible flow are:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho B_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1.43)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho B_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (1.44)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho B_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (1.45)$$

Plane 2-dimensional flow which is for incompressible viscous fluid can be given as:

Here we have $w=0$ and $\frac{\partial}{\partial z} = 0$. Then reduces to

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \rho B_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (1.46)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \rho B_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (1.47)$$

$$0 = \rho B_z \quad (1.48)$$

Dimensionless Parameters:

Reynold's Number:

The Reynold's number is defined as the ration of force of inertia to the viscous forces.

$$Re = VL/v = VL\rho/\mu \quad (\because v = \mu/\rho). \quad (1.49)$$

Where

V : Characteristic velocity

L : Characteristic length

ρ : Density of the fluid

μ : Coefficient of viscosity

Schmidt Number:

The ratio $\frac{v}{D}$ is known as Schmidt Number.

$$Sc = v/D \quad (1.50)$$

Where

D : Mass diffusivity

It involves only the physical properties of the fluid.

Prandtl Number:

Prandtl number is defined as the ratio of viscous force to thermal force.

$$Pr = \mu C_p / k \quad (1.51)$$

Where

C_p : Specific heat at constant pressure

k : conductivity which depends only on the property of the fluid

Eckert Number:

Eckert number is defined as:

$$Ec = \frac{v^2}{g C_p (\Delta T)_0} \quad (1.52)$$

Where

V : Characteristic velocity

g : Acceleration due to gravity

C_p : Specific heat at constant pressure

$(\Delta T)_0$: temperature difference between the wall and the fluid

Hartmann Number:

The magnetic field parameter M is known as Hartmann number and is defined as:

$$M = \frac{L^2 \sigma B_0^2}{\nu \rho} \quad (1.53)$$

Where,

L : Characteristic length

ν : kinematic viscosity

σ : Electrical conductivity

ρ : Density of the fluid

B_0 : Applied magnetic field.

Grashoff Number for Heat Transfer:

The Grashoff number Gr is defined as:

$$Gr = \frac{g \beta (T_1 - T_0) L^3}{\nu^2} \quad (1.54)$$

- g : Gravitational acceleration
- β : Thermal expansion
- $T_1 - T_0$: Temperature difference between the wall and fluid
- ν : Kinematic viscosity
- L : Characteristic Length.

It is observed that the fluid flow through different channels placed in a magnetic field has got an important place in industrial applications therefore a major thrust and attention for its development is absolutely necessary so that the results may be applied with greater accuracy for better industrial products.

Hence the focus in this study has been given on MHD flow of Newtonian and Non-Newtonian fluids through various channels containing porous medium placed in different positions.

Since it can be transparently seen that the motion of blood through veins and arteries like in our body the motion of any fluid through any channel may be discussed under various conditions of fluid dynamics as given above.

Here a small study of blood flow has also been included in this work.

OBJECTIVES

- To examine the impact of magnetic field on Heat and Mass transfer in the flow of a non-Newtonian fluid through two inclined plates.
- To observe the effect of magnetic field on the flow of Newtonian and Non-Newtonian fluids through parallel plates having porous medium.
- To evolve mathematical models for these flows and justify them with the physical nature of the problem under assumed parameters.

CHAPTER 2

REVIEW OF

LITERATURE

REVIEW OF LITERATURE

It is evident that many scientists, technocrats and mathematicians are taking keen interest in studying the MHD flow through channels so that they can have new results which when applied in respective fields may produce better outcomes having enhanced accuracy. So in this context, their contributions and achievements have been studied and mentioned in a chronological order so that this may be helpful in evolving new mathematical models which would be based on adopted theories and established formulae.

Rivlin (1948) studied the non-linear stress strain velocity relation in case of non-Newtonian visco-inelastic fluid flow. Constitutive equations of visco-elastic fluids was introduced by **Oldroyd (1950, 1958) and Walters (1960)**. **Mitra and Prasad (1973)** investigated the peristaltic transport of the Newtonian viscous fluid in a uniform channel considering the elasticity of the wall and reported reverse flow happens at the centre of the channel if the walls of the channel were elastic and viscous damping at the boundary was considered. The MHD flow of an incompressible viscous fluid by deformation of a plane surface was studied by **Mitra and Ram (1974)**. In **(1978) Vajravelu and Sastry** observed the effect of waviness of one of walls on the flow of heat transfer of an incompressible viscous fluid confined between two long vertical walls and set in to motion by a difference in wall temperature. The magneto hydrodynamic flow of a viscous incompressible fluid conducting between a parallel flat wall and long wavy wall was proposed and studied by **Reddy and Bathaiah (1980)**. They discussed the effects of magnetic suction and frequency parameter on velocity and temperature distribution. Mass with heat transfer on the flow past an accelerated infinite vertical plate was studied by **Raptis and Tzivanidis (1981)**. The magneto hydrodynamic flow of a dusty viscous conducting fluid between a parallel flat wall and a long wavy wall was studied by **Sakunthala and Bathaiah et al. (1982)**. They gave an expression for the velocity of fluid and dust and the coefficients of skin fraction at both the walls.

Soudalgekar and Bhat (1984) gave results on oscillating MHD channel flow and heat transfer. **Agarwal and Singhal (1985)** investigated the MHD oscillatory motion of visco-elastic Maxwell fluids between two parallel plates and between coaxial circular cylinders. **Maitra et al. (1986)** studied the MHD flow of viscous incompressible and slightly concluding fluid flowing between a parallel wall and wavy wall, and determine the velocity and temperature distribution with respect to pressure gradient. **Singh and Singh (1988)** discussed the oscillating MHD flow of visco-elastic fluid past an oscillating infinite porous plate with variable suction. **Joseph (1990)** studied the fluid dynamics of visco-elastic liquids. **Kumar and Gupta (1990)** investigated the flow of stratified fluid through porous medium between two oscillating plates.

Mittal, Mali and Chandra (1991) gave a note on vorticity and MHD flow through porous medium bounded by an oscillating porous plate. MHD fluctuating flow through porous medium past an infinite porous vertical plate with time dependent suction was given by **Kumar and Mohan (1991)**. **Kumar and Singh (1991)** studied the velocity field for both liquid and dust particles at different values of Hartmann number. **Sengupta (1992)** observed the MHD flow of two immiscible visco-elastic Rivlin-Ericksen fluids through a non-conducting rectangular channel in presence of transient pressure gradient. **Dutta et al. (1993)** obtained the solution of unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel. **Raptis et al. (1993)** studied the unsteady two dimensional flow of a viscous fluid through a porous medium bounded by an infinite porous plate with constant suction and variable temperature. **Panja et al. (1994)** investigated the unsteady hydrodynamic flow and heat transfer in visco-elastic fluid over a continuous moving porous flat surface. **Srivastava et al. (1995)** have considered a two-fluid model consisting of a core region of suspension of all the erythrocytes (particles) in plasma (fluid) assumed to be a particle-fluid mixture and a peripheral layer of cell-free plasma (Newtonian fluid). The

analytical results have been obtained in the said model for effective viscosity, velocity profiles and flow rate have been evaluated numerically for various values of the parameters. Quantitative comparison has shown that present model suitability represents blood flow at ($\leq 40\%$) and in vessels up to $70\mu\text{m}$ in diameter. **Maity et al. (1996)** studied the unsteady flow of two immiscible visco-elastic Oldroyd fluids between two inclined parallel plates. Further the study of free convection heat and mass transfer from a vertical plate embedded in a fluid saturated porous medium with constant wall temperature and concentration. Integral method of Von-Karman type is used to find the solution of the problem in the flow which was given by **Singh and Queeny (1997)**. **Kumari et al. (1997)** investigated the effect of visco-elastic parameter, the magnetic Prandtl number, the magnetic parameter, the pressure gradient parameter, the Eckert number and the fluid Prandtl number on the shear stress, the x-component of induced magnetic field and the heat transfer parameter. **Anwar (1998)** studied the MHD unsteady free convective flow past a vertical porous plate. **Sengupta and Paul (1999)** gave the unsteady hydro-magnetic flow of Maxwell fluid through a circular cylinder. **Brevdo, Laure, Dias et al. (1999)** has studied flow down an inclined plane which has several features that make it an interesting prototype for studying transition in a shear flow: the basic parallel state is an exact explicit solution of the Navier-Stokes equation; the experimentally observed transition of this flow shows many properties in common with boundary-layer transition; and it has a free surface, leading to more than one class of modes. **Kumar (1999)** discussed the thermal convection in Walter's B visco-elastic fluid permeated with suspended particles in porous medium. **Sengupta et al. (1999)** studied the unsteady MHD flow of visco-elastic Oldroydian fluids with periodic pressure gradient in a porous rectangular duct with a possible generalization. **Johari and Gupta (2000)** analyzed the problem of unsteady two dimensional free convective magneto-hydrodynamic flow of a visco-elastic fluid through a porous medium bounded by a vertical infinite porous plate of constant temperature with slip

on the plate. **Acharya, Dash and Singh (2000)** discussed the steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field. **Shehawey and Husseny (2000)** presented the work on peristaltic pumping by a sinusoidal travelling wave in porous walls of a two dimensional channel filled with a viscous incompressible fluid through a porous medium. **Reddy et al. (2000)** studied the three dimensional laminar free convective flow of a incompressible viscous fluid through a porous medium bounded by an infinite vertical flat porous surface.

Helmsy, Idriss and Kassem (2001) analyzed the problem of non-Newtonian power conducting fluid past a semi-finite plate in the presence of an external magnetic field. **Chakraborty (2001)** discussed the laminar convection flow of an electrically incompressible conducting second order visco-elastic fluid in porous medium down a parallel plate channel inclined at an angle to the horizontal surface in the presence of uniform transverse magnetic field. **Singh and Sharma (2001)** studied the coquette flow of a incompressible viscous fluid through a porous medium between two infinite horizontal parallel flat plates. **Singh et al. (2002)** examined the MHD flow of unsteady two dimensional second grade fluids and distribution with respect to magnetic flux function. **Sahoo, Dutta and Biswal (2003)** studied the MHD unsteady free convective flow past an infinite vertical plate with constant suction and heat sink. **Aziz et al. (2003)** gave the three dimensional oscillatory free convective MHD flow and heat transfer along a straight vertical porous plate. **Khare (2004)** described the MHD flow of a dusty fluid through different channels and derived expression for velocities of fluid and particle phase. **Rahman et al. (2004)** studied the unsteady MHD flow of visco-elastic Oldroyd fluid under varying body force through a rectangular channel. **Sarangi and Jose (2004)** discussed the unsteady two-dimensional free convective flow and mass transfer

through a viscous incompressible electrically conducting fluid through a porous medium bounded by a vertically infinite surface with constant suction velocity and constant heat flux in the presence of uniform magnetic field. The effect of Grashoff number, modified Grashoff number, Schmidt number, Eckert number, permeability parameter, phase angle and magnetic number on velocity and temperature profiles was discussed. **Abel, Subash and Mahantesh (2005)** studied the heat transfer in MHD visco-elastic boundary layer flow. **Varsheny and Singh (2006)** presented the effect of porous medium on the flow of non-Newtonian (Walter's liquid model-B) liquid embedded with symmetrically distributed uniform, non-conducting, solid, spherical dust particles in the presence of uniform, magnetic field applied transversely. **Kumar et.al (2006)** worked on the hydro-magnetic flow of a dusty visco-elastic incompressible (Maxwell type) fluid through a long uniform tube. **Sugunamma et al. (2006)** observed that the visco-elastic fluid through a rectangular channel. **Jadon, Jha and Yadav (2007)** discussed the problems on unsteady free convection flow of an incompressible, electrically conducting, visco-elastic (Rivlin-Ericksen) fluid through porous medium between two vertical parallel plates in the presence of uniform transverse magnetic field. **Singh, Kumar and Sharma (2008)** analyzed the unsteady flow of visco-elastic fluid through porous medium in a rectangular channel with transient pressure gradient.

Siddabasappa et al. (2008) discussed the flow of a viscous dusty fluid where velocity of dust particle was everywhere parallel to that of fluid and analyzed the variation of pressure on it.

Gireesha et al. (2008) made analytical study of Bettrami flow of dusty fluid between two parallel plates, using differential geometry and Laplace transformation; they derived expression for flow pattern. **Reddy, Murthy and Reddy (2009)** presented MHD flow and mass transfer past an accelerated infinite vertical porous plate. **Singh et al. (2009)** investigated the MHD flow of a dusty visco-elastic fluid through porous medium induced by

an impulsively started vertical plate with velocity decreasing exponential with time in the presence of uniform heat and mass flux at the plate.

Srivastava et al. (2009) have studied the problem of blood flow in a narrow catheterized artery has been investigated using a two-phase macroscopic model of blood (i.e., a suspension of red cells in plasma). It has been found that the effective viscosity and the frictional resistance increase with hematocrit. Numerical results reveal that the effective viscosity and the increased frictional resistance assume their minimal magnitude and consequently the volumetric flow rate assumes its maximal magnitude during the artery catheterization at the catheter size approximately fifty percent of the artery size. **Hazem et al. (2009)** studied the transient magneto hydrodynamics flow of a dusty incompressible electrically conducting non-Newtonian Bingham fluid through a circular pipe taking Hall Effect into consideration. **Khare (2009)** studied the Magneto hydrodynamic Flow of a Dusty Fluid through Circular Channel.

Seth, Nandkeolyar, Answari et al. (2010) studied the unsteady hydro magnetic convective flow of a viscous incompressible electrically conducting heat generating/absorbing fluid within a parallel plate rotating channel in a uniform porous medium under slip boundary conditions. Exact solution of the governing equations for fully developed flow was obtained in closed form. Expressions for skin friction due to primary and secondary flows and Nusselt number at the plate $\eta = 1$ are also derived. The numerical values of the primary and secondary velocities and fluid temperature displayed graphically versus channel width variable η for various values of pertinent flow parameters whereas numerical values of skin frictions due to primary and secondary flows and Nusselt number at the plate $\eta = 1$ are presented in tabular form for different values of pertinent flow parameters. **Khare and Singh (2010)** studied the on MHD flow of a dusty viscous incompressible fluid confined between two vertical walls with volume fraction of dust, **Agrawal et al. (2010)** worked on the effect of chemical

reaction on MHD visco-elastic boundary layer flow through porous medium with free convection past a continuous moving surface with heat source. **Singh et al. (2010)** discussed the effect of mass transfer in MHD free convection flow of a visco-elastic (Kuvshinki type) dusty gas through a porous medium with heat source. **Attia and Ewis (2010)** analyzed the unsteady magneto hydrodynamic couette flow of an electrically conducting incompressible non-Newtonian visco-elastic fluid between two parallel horizontal non-conducting porous plates. **Chamkha and Ahmed (2011)** studied the effect of unsteady MHD heat and mass transfer by mixed convective flow in the forward stagnation region of a rotating sphere in the presence of chemical reaction and heat source.

Khare and Singh (2012) studied the MHD flow of a dusty non Newtonian fluid through an inclined Hexagonal Channel. **Wahab and Salem (2012)** studied the flow of an incompressible, viscous, electrically conducting fluid in the presence of transverse magnetic field. The main result of this work was that the effect of magnetic field was to decrease the velocity profile and the flow rate. **Senapati et.al (2012)** observed mass transfer effect on MHD unsteady free convective Walter's memory flow with constant suction and heat sink. **Khare and Singh (2012)** have presented the magneto-hydrodynamic flow of dusty fluids between two inclined co-axial cylinders. **Wahab and Salem (2012)** investigated the magnetohydrodynamic blood flow in a narrow tube.

Gbadeyan and Dada (2013) proposed the model for radiation and heat transfer effects on a MHD non-Newtonian unsteady flow in a porous medium with slip conditionally. The fluid was assumed not to absorb its own emitted radiation but that of the boundaries.

Mishra (2013) investigated the transfer in MHD free convective flow of visco-elastic dusty fluid through a porous medium with constant heat source induced by motion of a semi infinite flat vertical plate moving with velocity decreasing exponentially and dust

particles. **Reddy et.al (2013)** studied the MHD heat and mass transfer flow of a visco-elastic fluid past an impulsively started infinite vertical plate. The study of heat and mass transfer in MHD oscillatory flow between two inclined porous plates with radiation, absorption and chemical reaction was given by **Goyal and Kumari (2013)**. **Rai and Khare (2013)** observed the MHD flow of a dusty non-Newtonian fluid through a rectangular channel. The study of effects of dusty visco-elastic (Walter's liquid model-B) fluid on MHD flow through porous medium. The effect of Grashoff number, Hartmann number, Porosity parameter, Dusty fluid parameter and Mass concentration of dust particle of velocity and skin friction were investigated by **Sharma and Varshney(2013)**. **Verma and Srivastava (2013)** studied the effect of Magnetic field on steady blood flow through an inclined circular tube.

Paul and Murli (2013) studied the unsteady Magneto hydrodynamic Free Convective Flow Past a Vertical Plate. **Khare and Rai (2013)** studied the MHD Flow of a non-Newtonian Fluid through a Rectangular Channel and derived the expression for velocity. **Ismail and Ganesh (2014)** analyzed the study of dust particles between two parallel plates through porous medium with constant suction in the upper plate and constant injection in the lower plate. **Nayak, Dash and Singh (2014)** worked on the effect of heat and mass transfer on free convective flow of a visco-elastic incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependant oscillatory permeability and suction in the presence of a uniform transverse magnetic field, heat source and chemical reaction. The investigation of unsteady Couette flow between two infinite parallel porous plates in an inclined Magnetic field with heat transfer was studied by **Joseph, Daniel and Joseph (2014)**.

Naser, Khare and Paul (2015) investigated the unsteady motion of a dusty fluid through a uniform pipe with sector of a circle as cross-section. **Ibrahim and Suneetha (2015)** an analytical solution of unsteady magneto hydrodynamic flow of visco-elastic fluid past as

impulsively started infinite vertical plate in presence of chemical reaction. **Sharmilaa and Gayathri (2015)** discussed the unsteady magneto hydrodynamic flow of an electrically conducting viscous, incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field. The effect of heat and mass transfer on unsteady MHD porous flow between two infinite parallel porous plates in an inclined magnetic field was observed by **Joseph, Nyitor *et al.* (2015)**. **Khathyayani and Babu (2015)** gave the study for heat and mass transfer on unsteady MHD free convective flow of second grade fluid past an infinite rotating vertical plate.

The study of Heat and mass transfer which affected the MHD natural convection flow past an impulsive moving vertical plate with ramped temperature was done by **Sheri, Suram *et al.* (2016)**. **Sasikumar and Govindarjan (2016)** investigated the effect of heat and mass transfer on MHD oscillatory flow with chemical reaction and slip conditions in asymmetric wavy channel. **Yasmin, Ahmed and Anika (2016)** observed the effect of visco-elastic fluid flow on MHD free convective and mass transfer flow with thermal and mass diffusion. The observation of thermal diffusion effect on MHD heat and mass transfer flow past a semi infinite moving vertical porous plate with heat generation and chemical reaction was done by **Mamatha, Varma *et al.* (2016)**.

The unsteady MHD free convective three phase flow through porous medium sandwiched between viscous fluids was studied by **Moses, Lawal *et al.* (2017)**. The study of heat transfer on MHD convective flow of heat Generating/Absorbing second grade fluid through porous medium in a rotating parallel plate channel was done by **Mallikarjuna and Prasad (2017)**.

Shamshuddin (2017) investigated the effect of heat and mass transfer on the unsteady MHD flow of chemically reacting micro polar fluid with radiation and joule heating. **Pattnaik (2017)** observed the effect of radiation and mass transfer on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature.

As it is evident that there is a continuous development in the field of MHD flow through various channels under different conditions and parameters so this work would be helpful in enhancing the application and scope of the subject.

CHAPTER 3

METHODOLOGY

METHODOLOGY

After going through the available literature on MHD flow and understanding the well established theories and equations of fluid dynamics, electro-dynamics and plasma mechanics and using the base of pre-existing mathematical models, an effort has been done to develop a new mathematical model for the flow of visco-elastic fluid through different channels with various parameters and hence this study can be applied in different fields of Science and Technology.

This study of MHD flow of visco-elastic fluid through various channels bearing many parameters under different physical conditions has been made here. After forming the differential equations of the flow under consideration, necessary modifications has been inserted and the set of newly formed differential equations has been solved using non-dimensional parameters, Variable Separable method and standard form of Bessel's differential equation and the solution has been formed under assumed boundary conditions and then the derived relation has been used to make the graphical and analytical study of velocity profile of the fluid by choosing a set of suitable numerical values of the parameters. After this a critical analysis of the velocity profile has been done on theoretical and mathematical basis to justify the result for the consistency for the physical nature of the phenomenon.

In this study of non-Newtonian fluids, the flow is through parallel plate channel and horizontal plate channel and a tube placed in a transversally applied magnetic field.

The effect of the following parameters on the flow pattern has been examined:

- i. Magnetic field (B_0)
- ii. Grashof number (Gr)
- iii. Distance from the base plate (y)
- iv. Frequency of oscillation (ω)
- v. Chemical Reaction parameter (J)
- vi. Schmidt number (Sc)
- vii. Angle of inclination of the tube (θ)
- viii. Radius of the tube (R_0)

- ix. Non-Newtonian factor (C)
- x. Reynolds number (Re)
- xi. Porosity factor (S)

In chapter no. 4, 5(A), 5(B) and 6 problems have been solved. Chapter 7 contains the graphical study and its analysis with results and discussion. Chapter 8 states the Summary and Conclusion, Recommendations and Limitations of the overall study.

CHAPTER 4

*Heat and Mass Transfer of an
Oscillatory Flow of Fluid between two
Inclined porous plates in Magnetic
Field.*

INTRODUCTION

The flow of visco-elastic fluids through various channels placed in a magnetic field has got importance in many industries of chemical processing, food preservation, petroleum production, polymer production and electro-static precipitation. Such flows occur in these areas as they transfer heat and mass which needs a deep knowledge of MHD flow. Several researchers have contributed their work in this field. Ganesan and Palani (2013) studied the convective flow over an inclined plate with variable heat and mass flux. Nandeppanavar et al. (2008) worked on the heat transfer in MHD visco-elastic boundary layer over a stretching sheet with thermal radiation and non-uniform heat source. Goyal M. and Kumari K. (2013) discussed the heat and mass transfer in MHD oscillatory flow between two inclined porous plates with radiation absorption and chemical reaction.

In the present chapter, the concentration is on non-Newtonian fluids flowing through parallel plate channel placed in a magnetic field. It is assumed that channel is filled with porous medium and is inclined with the axis. Different equations are formed by inserting the necessary terms of various parameters as required by the physical nature of the problem in the standard equation of continuity, momentum, energy and diffusion. They have been solved using non-dimensional parameters and the derived equation of the velocity profile has been used for graphical study and analyzing the result. The result has been found consistent in physical nature of the problem.

Formulation of the problem:

The MHD oscillatory flow of a non-Newtonian fluid between two inclined porous plates in the presence of magnetic field is considered subject to thermal radiation, absorption and chemical reaction. Let X -axis be along the lower plate and straight line perpendicular to that as the Y -axis. In this present visco-elastic fluid model, a non-Newtonian parameter has been

introduced in the momentum equation and its study has been conducted with respect to transversely applied magnetic field. Under such assumptions the momentum equation, energy equation and diffusion equation which govern the flow field are:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho} \frac{\partial P'}{\partial x'} + v_1 \frac{\partial^2 u'}{\partial y'^2} + v_2 \frac{\partial^3 u'}{\partial y'^2 \partial t'} - \frac{v_1}{K'} u' - \frac{\sigma_e B_o^2 u'}{(1 + W^2)} + g\beta_T (T' - T_o') \sin\phi$$

$$+ g\beta_c (C' - C_o') \sin\phi \quad (4.1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q}{\partial y'} - Q (T' - T_o') + Q_c (C' - C_o') \quad (4.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_r (C' - C_o') \quad (4.3)$$

Where

T' : Fluid temperature

u' : Axial velocity

U : Flow mean velocity

R_e : Reynolds number

P_e : Peclet number

S_c : Schmidt number

G_r : Grashof number

G_c : Solutal Grashof number

C_p : Specific heat at constant pressure

Q_1 : Radiation absorption parameter

B_o : Magnetic field

- k : Thermal conductivity
 E : Heat source parameter
 R : Radiation parameter
 H : Hartmann number
 J : Chemical reaction parameter
 S : Porous medium shape factor
 θ : Fluid temperature
 W : non-Newtonian parameter
 T_o, T_w : Temperature of the walls
 C' : Concentration of the fluid
 α : Mean radiation absorption coefficient

The boundary conditions are:

$$u' = 0, T' = T_w', C' = C_w' \text{ at } y = 1$$

$$u' = 0, T' = T_o', C' = C_o' \text{ at } y = 0 \quad (4.4)$$

The heat flux is given by:

$$\frac{\partial q}{\partial y'} = 4\alpha^2(T' - T_o') \quad (4.5)$$

Non-dimensional variables are:

$$x = \frac{x'}{a}, y = \frac{y'}{a}, u = \frac{u'}{U}, Re = \frac{Ua}{\nu_1}, \theta = \frac{(T' - T_o')}{(T_w' - T_o')}$$

$$C = \frac{(C' - C_o')}{(C_w' - C_o')}, Gc = \frac{g\beta_c(C_w' - C_o')a^2}{u_1U}, Gr = \frac{g\beta_r(T_w' - T_o')a^2}{u_1U},$$

$$P = \frac{aP'}{\rho u_1 U}, Da = \frac{K'}{a^2}, H^2 = \frac{a^2 \sigma_e B_0^2}{\rho \nu_1 (1+W^2)}, Q = \frac{Q_c(C_w' - C_o')a^2}{k(T_w' - T_o')},$$

$$P_e = \frac{Ua\rho C_p}{k}, R^2 = \frac{4\alpha^2 a^2}{k}, S_c = \frac{D}{aU}, S^2 = \frac{1}{Da}, E = \frac{Qa^2}{k}, J = \frac{K\rho a}{U} \quad (4.6)$$

In view of above non dimensional variables, the system of the equation (4.1) to (4.3) reduce to the following dimensionless form

$$R_e \frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \gamma \frac{\partial^3 u}{\partial y^2} - \left(S^2 + \frac{H^2}{(1+W^2)} \right) u + Gr_1 \theta + Gc_1 C \quad (4.7)$$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (R^2 - E)\theta + Q_1 \quad (4.8)$$

$$\frac{\partial C}{\partial t} = S_c \frac{\partial^2 C}{\partial y^2} - JC \quad (4.9)$$

Where $Gr_1 = Gr \sin \phi$ and $Gc_1 = Gc \sin \phi$

The corresponding boundary conditions in dimensionless form are:

$$u = 0, \theta = 1, C = 1 \text{ at } y = 1$$

$$u = 0, \theta = 0, C = 0 \text{ at } y = 0 \quad (4.10)$$

Solution of the problem

In order to solve equations (4.7) to (4.10) for purely oscillatory flow, let

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, u(y, t) = u_o(y) e^{i\omega t}, \theta(y, t) = \theta_o(y) e^{i\omega t}, C(y, t) = C_o(y) e^{i\omega t} \quad (4.11)$$

Substituting in equation (4.11) into equations (4.7) to (4.10) and obtaining the results as:

$$(1 + i\gamma\omega) \frac{d^2 u_o}{dy^2} - \frac{M_2^2}{(1+W^2)} u_o = -\lambda - Gr_1 \theta_o - Gc_1 C_o \quad (4.12)$$

$$\frac{d^2 \theta_o}{dy^2} + M_1^2 \theta_o + Q_1 C_o = 0 \quad (4.13)$$

$$\frac{d^2 C_o}{dy^2} - M_3^2 C_o = 0 \quad (4.14)$$

Where

$$L^2 = 1 + i\omega\gamma$$

$$M_1^2 = R^2 - E - i\omega P_e$$

$$M_2^2 = S^2 + \frac{H^2}{(1 + W^2)} + i\omega R_e$$

$$M_3^2 = \frac{J + i\omega}{S_c}$$

$$k_1 = \frac{Q_1}{M_1^2 + M_3^2}$$

The corresponding boundary conditions are:

$$u_o = 0, \theta_o = 1, C_o = 1 \text{ at } y = 1$$

$$u_o = 0, \theta_o = 0, C_o = 0 \text{ at } y = 0 \quad (4.15)$$

Solving equations (4.12) to (4.14) under the boundary conditions (4.15), the solution of the velocity, temperature and concentration distributions are given as follows:

$$u(y, t) = \left[Gr_1 \left\{ \left(\frac{1 + k_1}{M_1^2 L^2 + \frac{M_2^2}{(1+W^2)}} \right) \frac{\sin M_1 y}{\sin M_1} + \left(\frac{k_1}{M_3^2 L^2 + \frac{M_2^2}{(1+W^2)}} \right) \frac{\sin M_3 y}{\sin M_3} - \right. \right. \\
\left. \left(\frac{1 + k_1}{M_1^2 L^2 + \frac{M_2^2}{(1+W^2)}} + \frac{k_1}{M_3^2 L^2 + \frac{M_2^2}{(1+W^2)}} \right) \frac{\sin \frac{M_2}{L\sqrt{(1+W^2)}} y}{\sin \frac{M_2}{L\sqrt{(1+W^2)}}} \right. \\
\left. + \frac{Gc_1}{M_3^2 L^2 - \frac{M_2^2}{(1+W^2)}} \right. \\
\left. \left(\frac{\sinh \frac{M_2}{L\sqrt{(1+W^2)}} y}{\sinh \frac{M_2}{L\sqrt{(1+W^2)}}} - \frac{\sin M_3 y}{\sin M_3} \right) \right. \\
\left. + \frac{\lambda}{\frac{M_2^2}{(1+W^2)}} \left(1 - \cosh \frac{M_2}{L\sqrt{(1+W^2)}} y \right) \left(1 - \frac{\sinh \frac{M_2}{L\sqrt{(1+W^2)}} y}{\sinh \frac{M_2}{L\sqrt{(1+W^2)}}} \right) \right] e^{i\omega t} \tag{4.16}$$

$$\theta(y, t) = \left[\left(1 + \frac{Q_1}{M_1^2 + M_3^2} \right) \frac{\sin M_1 y}{\sin M_1} - \left(\frac{Q_1}{M_1^2 + M_3^2} \right) \frac{\sin M_3 y}{\sin M_3} \right] e^{i\omega t} \tag{4.17}$$

$$C(y, t) = \left(\frac{\sin M_3 y}{\sin M_3} \right) e^{i\omega t} \tag{4.18}$$

CHAPTER 5

Flow of a Newtonian and Non-Newtonian Fluid through parallel plates with porous medium.

Part A

INTRODUCTION

The flow of visco-elastic fluids through various channels placed in a magnetic field has got importance in many industries of food preservation, Geo-thermal energy, petroleum production, polymers technology and power generation engineering etc. Such flows occur in these areas as they transfer heat and mass which needs a deep knowledge of MHD flow. Several researchers have contributed their work in this field. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of Reynolds number. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field on unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite parallel porous plates with constant temperature and variable mass diffusion. Manyonge et al (2012) studied steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discover that high magnetic field strength decreases the velocity. Joseph and Daniel (2014) investigated the unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer.

In the present chapter, the effect of inclined magnetic field on two infinite parallel plates having porous medium with heat and mass transfer has been studied. Here the equation of continuity, momentum equation, and the equations which govern the flow field are solved by using non dimensional parameters and a graphical approach has been studied.

Formulation of the problem:

An electrically conducting, unsteady, viscous, incompressible, Newtonian fluid is moving between two infinite parallel plates kept at a distance of $2h$ apart which are placed in an inclined magnetic field. Consider one dimensional flow so that the axis of the channel formed by two plates is x -axis and the flow is in this direction. The equation governing the flow field is:

The equation of continuity:

$$\nabla \cdot V = 0 \quad (5.1)$$

The Navier –Stokes equation:

$$\rho \left[\left(\frac{\partial}{\partial t} + V \cdot \nabla \right) \right] V = f_B - \nabla P + \mu \nabla^2 V \quad (5.2)$$

Where

f_B : body force per unit mass of the fluid

μ : fluid viscosity

P : pressure exerted on the fluid

V : velocity of electrically conducting fluid

R_e : Reynolds number

P_r : Prandtl number

G_r : Grashof number

G_c : Solutal Grashof number

ρ : Density of the fluid

E : Electrical conductivity of the fluid

K : Thermal conductivity

C_p : Specific heat at constant pressure

B_0 : Magnetic field strength component

ν : Kinematic Viscosity

It is observed that u^* , v^* and w^* are the velocity components in x^* , y^* and z^* directions respectively. Then this implies $v^*=w^*=0$ and $u^* \neq 0$, then the continuity equation is satisfied.

Here the body force is neglected and replaced with Lorentz force and from the assumptions that the flow is one dimensional.

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{F_x}{\rho} \quad (5.3)$$

Where F_x is the component of magnetic force in the direction of x – axis.

Therefore $\frac{F_x}{\rho} = \frac{\sigma}{\rho} B_0^2 \bar{u}$; and when the angle of inclination (θ) is introduced to the equation

(5.3), it is obtained,

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2 \theta \quad (5.4)$$

Where θ is the angle between V and B . The characteristic velocity v_0 is taken due to the porosity of the lower plate which is constant to maintain the same pattern of flow. The momentum equation with heat and mass parameters is given as:

$$\frac{\partial u^*}{\partial t^*} = -v_0 \frac{\partial u^*}{\partial y^*} - \frac{\partial p^*}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2 \theta + \rho g \beta (T^* - T_\infty^*) + \rho g \beta (C^* - C_\infty^*) \quad (5.5)$$

The energy equation is:

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (5.6)$$

The boundary conditions are:

$$u^*(y, t) = 0, T^* = T_\infty \text{ at } t^* = 0,$$

$$u^*(-L, t^*) = 0, u^*(L, t^*) = \frac{v}{L}, T^* = T_w att^* > 0 \quad (5.7)$$

Non-dimensional parameters are:

$$x^* = xL, y^* = yL, p^* = p\rho \frac{v^2}{L^2}, u^* = \frac{uv}{L}, t^* = \frac{tL^2}{\nu}, Pr = \frac{\mu c_p}{k}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - T_\infty^*},$$

$$Ha^2 = \frac{\sigma L^2 B_0^2}{\mu} \quad (5.8)$$

The equations (5.5) and (5.6) are solved using the boundary condition and non-dimensional parameter and we obtain the following equations:

$$\frac{\partial u}{\partial t} = -\frac{Re}{\rho} \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + M^2 u + G_r \theta + G_c C \quad (5.9)$$

Where $M = M^* \sin \theta$ and $M^* = LB_0 \sqrt{\frac{\sigma}{\mu}}$. $G_c = \rho L^2 g \beta (T_w^* - T_\infty^*)$,

$G_c = \rho L^2 g \beta (C_w^* - C_\infty^*)$, $\frac{\partial p}{\partial x} = 0$ (assuming the flow is couette flow).

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (5.10)$$

The dimensionless boundary conditions are:

$$u(y, t) = 0, \theta(-1, t) = 0 \text{ at } t = 0$$

$$u(-1, t) = 0, u(1, t) = 1, \theta(1, t) = 1 \text{ at } t > 0 \quad (5.11)$$

Hence solving the equation (5.9) and (5.10) with the help of separation of variable technique using boundary conditions (5.11), the equation obtained is of the form:

$$u(y, t) = u(y)u(t) \quad (5.12)$$

$$\theta(y, t) = \theta(y)\theta(t) \quad (5.13)$$

Therefore the solutions of velocity profile and temperature distribution are:

$$u(y, t) = e^{-\lambda^2 t} (C_1 e^{m_1 y} + C_2 e^{m_2 y}) + \frac{G_r \theta + G_c}{M^2 - \lambda^2} \quad (5.14)$$

$$\theta(y, t) = e^{-\frac{\lambda^2}{Pr} t} (C_3 \cos \lambda y + C_4 \sin \lambda y) \quad (5.15)$$

Where $B = M^2 + \lambda^2$

$$m_1 = \frac{A + \sqrt{A^2 + 4B}}{2}$$

$$m_2 = \frac{A - \sqrt{A^2 + 4B}}{2}$$

$$C_1 = -C_2 e^{m_1 - m_2}$$

$$C_2 = \frac{e^{-\lambda^2 t}}{e^{m_1 - m_2} - e^{m_2 - m_1}}$$

$$C_3 = \frac{e^{-\frac{\lambda^2}{Pr} t}}{2 \sin \lambda}$$

$$C_4 = C_3 \tan \lambda$$

Where,

λ, m_1, m_2, A and B , are constants.

Part B

INTRODUCTION

The flow of non-Newtonian fluids through various channels placed in an inclined magnetic field has got importance in many fields like petroleum production, polymers technology and power generation engineering etc. Such flows occur in these areas as they transfer heat and mass which need a deep knowledge of MHD flow. The interaction between the conducting fluid and the magnetic field radically modifies the flow, with effects on such important flow properties as heat transfer, the detail nature of which is strongly dependent on the orientation of the magnetic field. When a fluid moves through a magnetic field, an electric field and consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. Several researchers have contributed their work in this field. Sreekala and Kesavareddy (2014) studied the Hall Effect on unsteady MHD flow of a non-Newtonian fluid through porous medium with uniform suction and injection. Joseph and Daniel (2014) investigated the unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. Ahsan (2015) discussed the effect of volume fraction on non-Newtonian flow of a dusty fluid between two oscillating parallel plates. Shivashanker and Charyulu (2015) studied the MHD flow of non-Newtonian fluid through porous medium with heat transfer using slip boundary condition.

In this chapter the effect of non-Newtonian factor ' c' ' in the presence of inclined magnetic field on two infinite parallel plates having porous medium with heat and mass transfer has been studied. Here the equation of continuity, momentum equation, and the equations which govern the flow field are solved by using non dimensional parameters.

Formulation of the problem:

A non-Newtonian, conducting and incompressible fluid moving between two infinite parallel plates kept at a distance of $2d$ apart and is placed in an inclined magnetic field. The flow is considered to be one dimensional flow so that the axis of the channel formed by two plates is x -axis and the flow is in this direction. The equation governing the flow field is as follows:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} + \frac{\partial u^*}{\partial z^*} = 0 \quad (5.16)$$

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{F_x}{\rho} \quad (5.17)$$

Where F_x is the component of magnetic force in the direction of x – axis. It is observed that u^* , v^* and w^* are the velocity components in x^* , y^* and z^* directions respectively. Then this implies $v^* = w^* = 0$ and $u^* \neq 0$, then the continuity equation is satisfied. Here the Lorentz force replaces body force and from the assumptions that the flow is one dimensional.

Therefore $\frac{F_x}{\rho} = \frac{\sigma}{\rho} B_0^2 u^*$; and when the angle of inclination is introduced then we have,

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^* \sin^2 \varphi}{\rho(1+c^2)} \quad (5.18)$$

Where φ is the angle between velocity of the conducting fluid (V) and magnetic field strength (B). The characteristic velocity v_0 is taken due to the porosity of the lower plate which is constant to maintain the same pattern of flow.

The momentum equation with heat and mass parameters is given as:

$$\frac{\partial u^*}{\partial t^*} = -v_0 \frac{\partial u^*}{\partial y^*} - \frac{\partial p^*}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^* \sin^2 \varphi}{\rho(1+c^2)} + \rho g \beta (T^* - T_\infty^*) + \rho g \beta (C^* - C_\infty^*) \quad (5.19)$$

The energy equation is:

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (5.20)$$

Where,

G_r : Grashof number

G_c : Solutal Grashof number

ρ : Density of the fluid

C_p : Specific heat at constant pressure

μ : Viscosity of the fluid

B_0 : Magnetic field strength component

R_e : Reynolds number

P_r : Prandtl number

ν : Kinematic Viscosity

c : non-Newtonian Factor

E : Electrical conductivity of the fluid

K : Thermal conductivity

The boundary conditions:

$$u^*(y, t) = 0, T^* = T_\infty \text{ at } t^* = 0,$$

$$u^*(-L, t^*) = 0, u^*(L, t^*) = \frac{\nu}{L}, T^* = T_w \text{ at } t^* > 0 \quad (5.21)$$

The non-dimensional parameters are:

$$x^* = xL, y^* = yL, p^* = p\rho \frac{\nu^2}{L^2}, P_r = \frac{\mu c_p}{k}, u^* = \frac{uv}{L}, t^* = \frac{tL^2}{\nu}, \varphi = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - T_\infty^*},$$

$$Ha^2 = \frac{\sigma L^2 B_0^2}{\mu}, G_r = \rho L^2 g \beta (T_w^* - T_\infty^*), G_c = \rho L^2 g \beta (C_w^* - C_\infty^*). \quad (5.22)$$

The equations (5.19) and (5.20) are solved using non-dimensional parameter and boundary condition, we obtain the following equations:

$$\frac{\partial u}{\partial t} = -\frac{R_e}{\rho} \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{M^2 u}{(1+c^2)} + G_r \varphi + G_c C \quad (5.23)$$

Where $M = M^* \sin \varphi$ and $M^* = LB_0 \sqrt{\frac{\sigma}{\mu}}$, (Assuming $\frac{\partial p}{\partial x} = 0$)

$$P_r \frac{\partial \varphi}{\partial t} = \frac{\partial^2 \varphi}{\partial y^2} \quad (5.24)$$

The dimensionless boundary conditions are:

$$u(y, t) = 0, \varphi(-1, t) = 0 \text{ at } t = 0$$

$$u(-1, t) = 0, u(1, t) = 1, \varphi(1, t) = 1 \text{ at } t > 0 \quad (5.25)$$

Hence solving the equation (5.23) and (5.24) with the help of variable separable technique using the above boundary conditions, we obtain:

$$u(y, t) = u(y)u(t) \quad (5.26)$$

$$\varphi(y, t) = \varphi(y)\varphi(t) \quad (5.27)$$

Hence the solutions of velocity and temperature distribution are:

$$u(y, t) = e^{-\lambda^2 t} (C_1 e^{m_1 y} + C_2 e^{m_2 y}) + \frac{G_r \varphi + G_c}{\frac{M^2}{(1+c^2)} - \lambda^2} \quad (5.28)$$

$$\varphi(y, t) = e^{-\frac{\lambda^2}{P_r} t} (C_3 \cos \lambda y + C_4 \sin \lambda y) \quad (5.29)$$

Where $B = \frac{M^2}{(1+c^2)} + \lambda^2$

$$m_1 = \frac{A + \sqrt{A^2 + 4B}}{2}$$

$$m_2 = \frac{A - \sqrt{A^2 + 4B}}{2}$$

$$C_1 = -C_2 e^{m_1 - m_2}$$

$$C_2 = \frac{e^{-\lambda^2 t}}{e^{m_1 - m_2} - e^{m_2 - m_1}}$$

$$C_3 = \frac{e^{-\frac{\lambda^2}{Pr} t}}{2 \sin \lambda}$$

$$C_4 = C_3 \tan \lambda$$

Where,

λ, m_1, m_2, A and B , are constants.

CHAPTER 6

*Influence of Magnetic Field on the
Blood Flow through an Inclined
Narrow Tube*

INTRODUCTION

As blood circulation is regarded a fundamental function of the human body hence it becomes important to study the nature of blood flow through arteries and veins under the effect of magnetic field since the study of the rheological properties of blood flow can allow a better understanding of blood circulation in a human body. This depends on many factors such as the driving force of the heart, the shape, as well as mechanical and physiological behavior of the vascular walls. Blood can be regarded as a magnetic fluid appeared in which red blood cells (RBCs), which is magnetic in nature due to iron oxide in it. The movement of blood in an externally applied magnetic field is governed by the law of magneto hydrodynamics (MHD). When the body is subjected to a magnetic field the charged particles of the blood flowing transversally to the field get deflected by the Lorentz force. Therefore the interactions between these induced currents and applied magnetic field can cause a reduction of blood flow rate. The interaction of magnetic field with blood flow is demonstrated by many researchers. Wahab and Salem (2012) studied the effect of magnetic field on the blood flow in a narrow tube. Verma and Srivastava (2013) investigated the effect of externally applied magnetic field on steady blood flow through an inclined circular tube.

In the present chapter the study of blood which behaves as a non-Newtonian fluid therefore a non-Newtonian factor ' C ' is introduced which influences the velocity profile of blood under the effect of magnetic field.

Formulation of the problem:

Now Let us consider a one-dimensional laminar steady blood flow through a uniform straight and inclined narrow tube in the presence of transverse magnetic field. The equations governing the flow are given as:

$$\nabla \cdot \mathbf{u} = 0 \quad (6.1)$$

$$-\frac{\partial p}{\partial z} + \mu \nabla^2 u + \rho g \sin \theta - \frac{\sigma B_0^2 u}{1 + C^2} = 0 \quad (6.2)$$

Where

$$\nabla^2 = \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \right) \quad (\text{Operator in cylindrical coordinate defined by the } z \text{ component})$$

u : Axial velocity

$-\frac{\partial p}{\partial z}$: Pressure gradient

R_0 : Radius of tube

B_0 : Transverse component of magnetic field

μ : Coefficient of viscosity of blood

ρ : Density of blood

θ : Inclination angle

σ : Electrical conductivity of the medium

The Boundary conditions are:

$$u = 0 \text{ at } r = R_0 \quad (6.3)$$

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \quad (6.4)$$

$$-\frac{\partial p}{\partial z} + \mu \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) \right) u + \rho g \sin \theta - \frac{\sigma B_0^2 u}{1 + C^2} = 0 \quad (6.5)$$

$$-\frac{\partial p}{\partial z} + \mu \frac{1}{r} \left(r \frac{\partial^2}{\partial r^2} + \frac{\partial}{\partial r} \cdot 1 \right) u + \rho g \sin \theta - \frac{\sigma B_0^2 u}{1 + C^2} = 0 \quad (6.6)$$

$$-\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} \right) + \rho g \sin \theta - \frac{\sigma B_0^2 u}{1 + C^2} = 0 \quad (6.7)$$

$$\mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\sigma B_0^2 u}{1 + C^2} = \frac{\partial p}{\partial z} - \rho g \sin \theta \quad (6.8)$$

Now we introduce a new transformation:

$$y = \frac{r}{R_0} \Rightarrow \frac{\partial y}{\partial r} = \frac{1}{R_0} \quad (6.9)$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} \quad (6.10)$$

$$= \frac{\partial u}{\partial y} \cdot \frac{1}{R_0} \quad \therefore \frac{\partial y}{\partial r} = \frac{1}{R_0} \quad (6.11)$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{\partial}{\partial r} \left(\frac{\partial u}{\partial r} \right) = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} \right) \frac{\partial y}{\partial r} = \frac{1}{R_0} \frac{\partial^2 u}{\partial y^2} \quad (6.12)$$

Putting these values back in equation (6.8)

$$\mu \left(\frac{1}{R_0^2} \frac{\partial^2 u}{\partial y^2} + \frac{1}{y R_0} \frac{\partial u}{\partial y} \frac{1}{R_0} \right) - \frac{\sigma B_0^2 u}{1 + C^2} = \frac{\partial p}{\partial z} - \rho g \sin \theta \quad (6.13)$$

$$\frac{\mu}{R_0^2} \left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{y} \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0^2 u}{1 + C^2} = \frac{\partial p}{\partial z} - \rho g \sin \theta \quad (6.14)$$

$$\left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{y} \frac{\partial u}{\partial y} \right) - \sigma \frac{R_0^2}{\mu} \frac{B_0^2 u}{(1 + C^2)} = \frac{R_0^2}{\mu} \left(\frac{\partial p}{\partial z} - \rho g \sin \theta \right) \quad (6.15)$$

$$\text{Now } M = B_0 R_0 \sqrt{\frac{\sigma}{\mu}}$$

Where M = Hartmann number.

$$\frac{\partial^2 u}{\partial y^2} + \frac{1}{y} \frac{\partial u}{\partial y} - \frac{M^2 u}{(1+C^2)} = \frac{R_0^2}{\mu} \left(\frac{\partial p}{\partial z} - \rho g \sin \theta \right) \quad (6.16)$$

The equation above represents a Bessel's Equation. Hence the differential equation (6.16)

represents a Bessel's equation; hence its solution can be given as:

$$u = \frac{R_0^2}{\lambda^2 \mu} (1+C^2) \left(\frac{\partial p}{\partial z} - \rho g \sin \theta \right) \left[1 - \frac{J_0(\lambda y)}{J_0(\lambda)} \right] \quad (6.17)$$

Where $\lambda^2 = -M^2$.

CHAPTER 7

Results and Discussion

CHAPTER 4

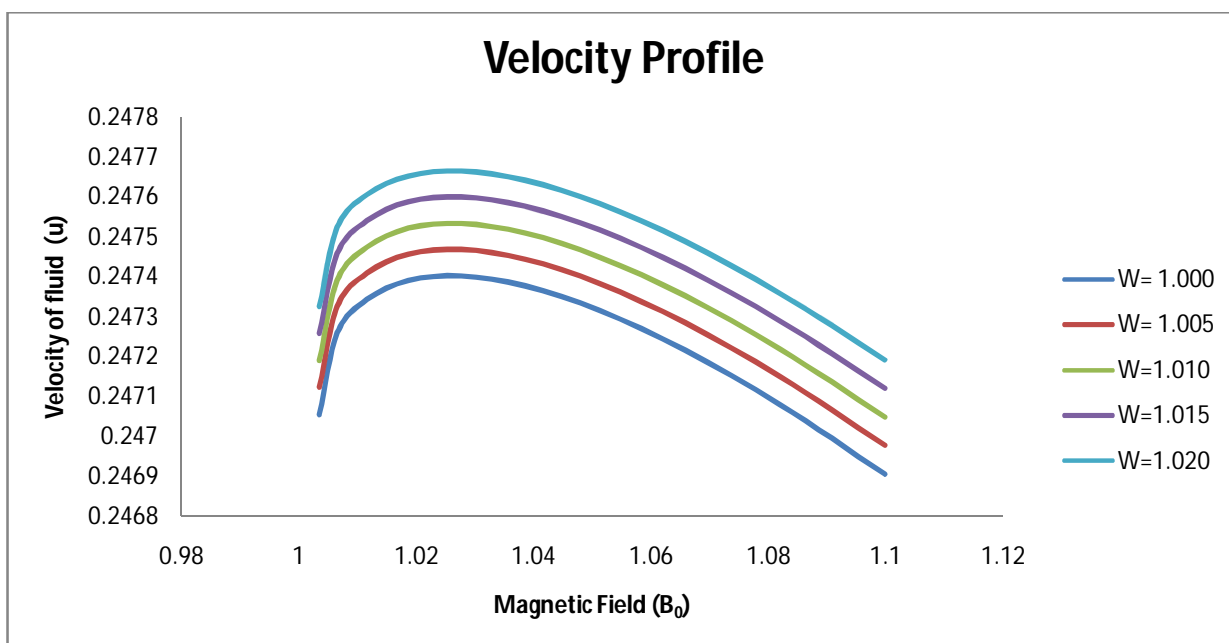
Considering the derived expression of the velocity pattern in chapter 4, suitable values are chosen and the graph has been plotted for different parameters.

Table (4.1): Velocity of the fluid and Magnetic Field

$$(R_e = 1, \rho = 1.06, \gamma = 0.2, G_c = 4, G_r = 4)$$

Sr. No.	Magnetic Field (B_0)	Velocity of fluid				
		Non-Newtonian factor (W=1.000)	Non-Newtonian factor (W=1.005)	Non-Newtonian factor (W=1.010)	Non-Newtonian factor (W=1.015)	Non-Newtonian factor (W=1.020)
1.	0.5	0.247054850	0.247123255	0.247191350	0.247259135	0.247326609
2.	1.0	0.247259395	0.247326233	0.247392771	0.247459010	0.247524949
3.	1.5	0.247342916	0.247409214	0.247475214	0.247540917	0.247606321
4.	2.0	0.247390623	0.247456726	0.247522529	0.247588035	0.247653241
5.	2.5	0.247402118	0.247468377	0.247534333	0.247599985	0.247665334
6.	3.0	0.247377040	0.247443812	0.247510273	0.247576423	0.247642262
7.	3.5	0.247315068	0.247382714	0.247450039	0.247517041	0.247583721
8.	4.0	0.247215922	0.247284809	0.247353359	0.247421571	0.247489446
9.	4.5	0.247079373	0.247149867	0.247220006	0.247289789	0.247359217
10.	5.0	0.246905237	0.246977709	0.247049803	0.247121519	0.247192858

Fig:- (4.1): Velocity Profile with respect to Magnetic field (B_0).



Results and Discussion

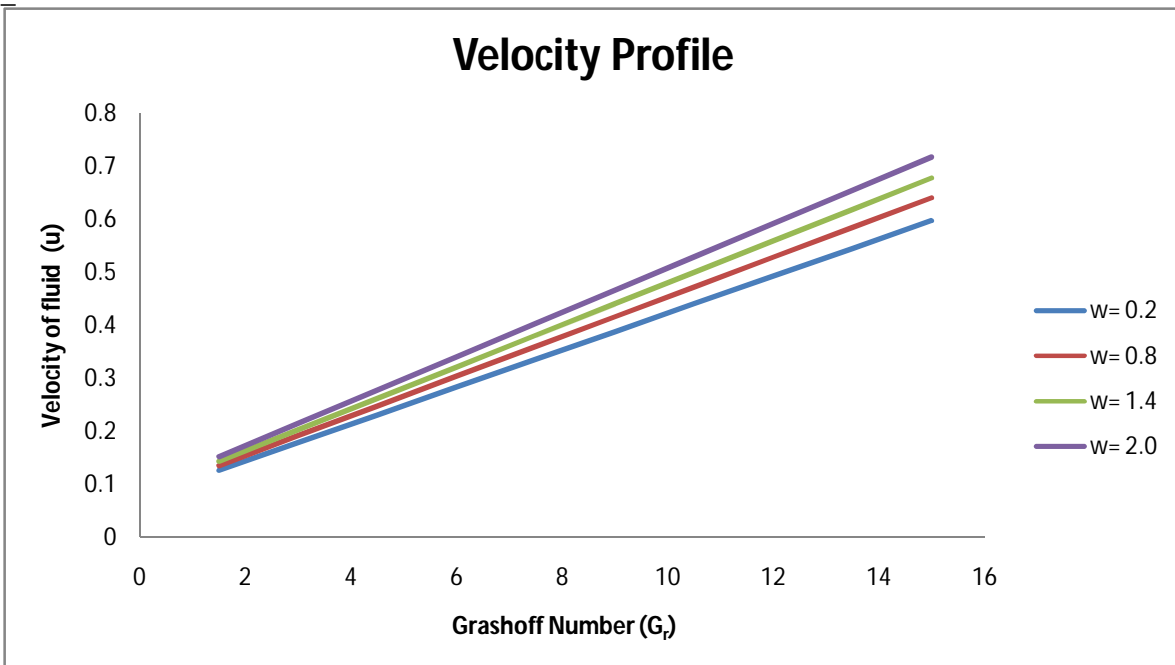
The above graph represents variation in velocity of the fluid with respect to magnetic field. It is clear that higher value of non-Newtonian factor brings more bending around maximum value of velocity. All the curves start from a fixed value of $B_0 = 1.001$ and gaining a maximum value for nearly $B_0 = 1.02$ show a decreasing trend. Therefore the graphical study indicates that the velocity increases in the beginning obtaining a maximum value, it starts decreasing for each value of non-Newtonian factor but somehow the velocity profile shows a resonance character, the curvature is highly dominated by the non-Newtonian factor. Also the derived relation for the velocity has a non-Newtonian factor as an important component in controlling the velocity. However the presence of other terms cannot be ignored. The applied magnetic field creates a resistive force due to which the particles loose the velocity, the presence of trigonometric function bearing non-Newtonian terms put the motion in hyperbolic nature and for a certain value of magnetic field, the resonance occurs. Therefore this value can be used in controlling the metallic flow in industries by applying the magnetic field.

Table (4.2): Velocity of fluid and Grashof Number

$(R_e = 1, \rho = 1.06, \gamma = 0.2, G_c = 4, B_0 = 0.5)$

Sr. No.	Grashoff Number (G_r)	Velocity of fluid			
		Non-Newtonian factor (W=0.2)	Non-Newtonian factor (W=0.8)	Non-Newtonian factor (W=1.4)	Non-Newtonian factor (W=2.0)
1.	1.5	0.125849007	0.135285608	0.143482775	0.152272120
2.	3.0	0.178242986	0.191409453	0.202843133	0.215099646
3.	4.5	0.230636964	0.247533298	0.262203492	0.277927172
4.	6.0	0.283030943	0.303657143	0.32156385	0.340754699
5.	7.5	0.335424922	0.359780988	0.380924209	0.403582225
6.	9.0	0.387818901	0.415904833	0.440284567	0.466409751
7.	10.5	0.440212880	0.472028678	0.499644926	0.529237277
8.	12.0	0.492606859	0.528152523	0.559005284	0.592064803
9.	13.5	0.545000837	0.584276368	0.618365643	0.65489233
10.	15.0	0.597394816	0.640400213	0.677726001	0.717719856

Fig:- (4.2): Velocity Profile with respect to Grashof Number (G_r).



Results and Discussion

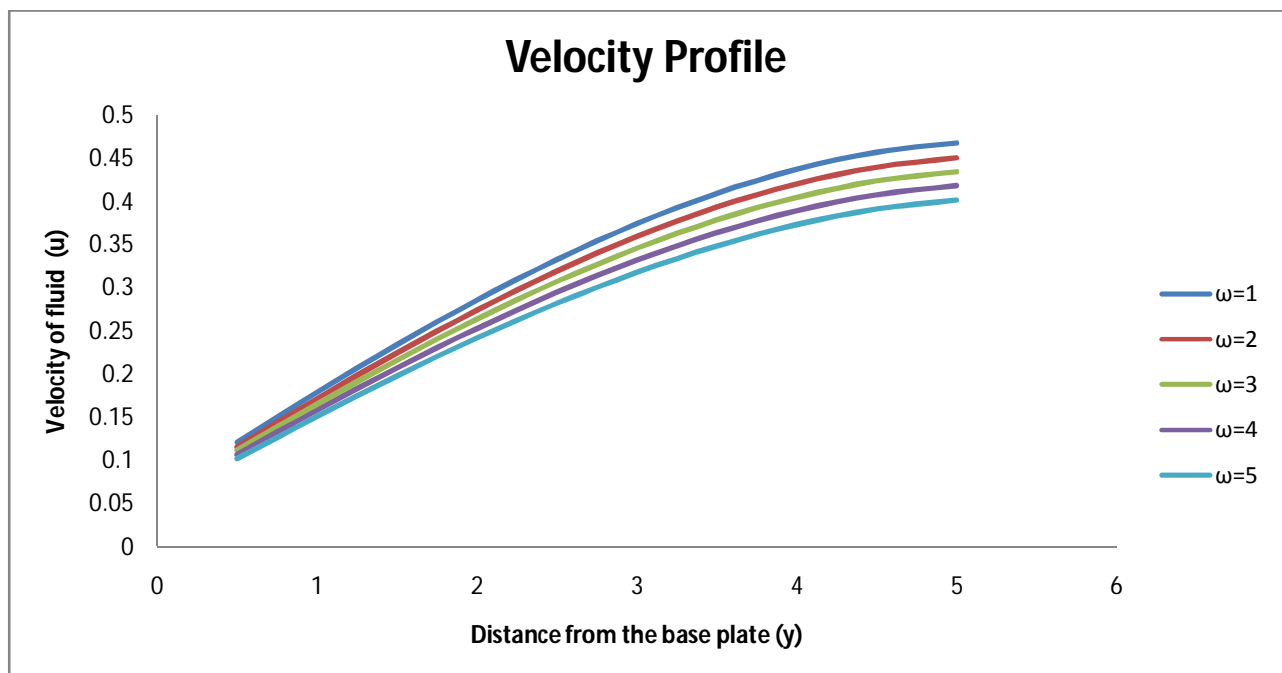
In this graph all the other parameters are kept constant and the curve has been plotted between velocity and Grashof number for different non-Newtonian factor and as the Grashof number increases, the velocity also increases, since there exist a linear relationship between them but the presence of other terms cannot be ignored as Grashof Number bears the parameters density and kinematic viscosity. All the lines start from a fixed value of $G_r = 1.5$ and gaining a maximum value for nearly $G_r = 15$ which shows a linear increasing trend therefore graphical study indicates that the velocity increases at a constant level to each increasing value of Grashoff number, and also the presence of non-Newtonian factor cannot be ignored as it highly dominates the velocity profile.

Table (4.3) : Velocity of fluid and Distance from the base plate

$$(R_e = 1, \rho = 1.06, W = 1, J = 2, G_c = 4, B_0 = 0.5)$$

Sr. No.	Distance from the base plate (y)	Velocity of fluid				
		Frequency of oscillation ($\omega=1$)	Frequency of oscillation ($\omega=2$)	Frequency of oscillation ($\omega=3$)	Frequency of oscillation ($\omega=4$)	Frequency of oscillation ($\omega=5$)
1.	0.5	0.121241	0.116306	0.111729	0.107028	0.102465
2.	1.0	0.179334	0.172056	0.165306	0.158370	0.151637
3.	1.5	0.234592	0.225119	0.216330	0.207298	0.198528
4.	2.0	0.286150	0.274675	0.264025	0.253077	0.242443
5.	2.5	0.333136	0.319895	0.307602	0.29496	0.282674
6.	3.0	0.374674	0.359943	0.346261	0.332184	0.318497
7.	3.5	0.409878	0.393971	0.379189	0.363972	0.349167
8.	4.0	0.437857	0.42112	0.405556	0.389524	0.373914
9.	4.5	0.457709	0.440513	0.424512	0.408016	0.391942
10.	5.0	0.468519	0.451260	0.435186	0.418601	0.402423

Fig:- (4.3): Velocity Profile with respect to the distance from the base plate (y).



Results and Discussion

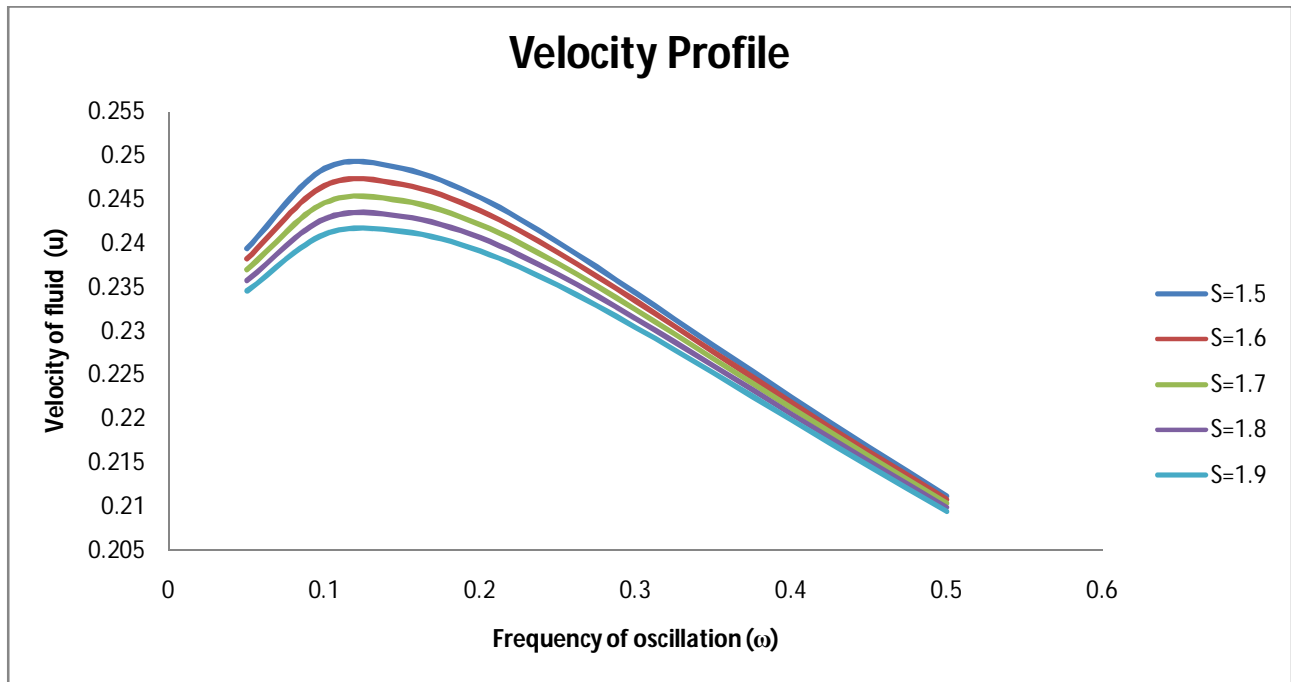
In the above graph, the velocity profile of non-Newtonian fluid is in continuously increasing trend and reaches a higher value for different distances from the base plate at different value of frequencies of oscillation therefore a resonance character has been shown due to the hyperbolic functions present in the velocity profile equation. The Graph starts in a converging pattern at ($y=0.5$) and then for higher values of distance from the base plate ($y=5$), it starts to diverge as the magnetic properties exhibited by the particles decreases and also the effect of the frequency of oscillation between the particles cannot be ignored, therefore, as the plates distance increases, the pattern of the velocity profile takes this form in the figure.

Table (4.4) : Velocity of fluid and Frequency of oscillation

$(R_e = 1, \rho = 1.06, \gamma = 0.2, \lambda = 0.1, G_c = 4, G_r = 4)$

Sr. No.	Frequency of oscillation (ω)	Velocity of fluid				
		Porous medium shape factor (S=1.5)	Porous medium shape factor (S=1.6)	Porous medium shape factor (S=1.7)	Porous medium shape factor (S=1.8)	Porous medium shape factor (S=1.9)
1.	0.05	0.239434	0.238216	0.237002	0.235794	0.234592
2.	0.10	0.248543	0.246549	0.244644	0.242819	0.241064
3.	0.15	0.248552	0.246686	0.244871	0.243105	0.241388
4.	0.20	0.245223	0.243690	0.242165	0.240651	0.239154
5.	0.25	0.240155	0.238949	0.237727	0.236494	0.235255
6.	0.30	0.234374	0.233427	0.232456	0.231464	0.230455
7.	0.35	0.228405	0.227653	0.226875	0.226074	0.225251
8.	0.40	0.222501	0.221894	0.221261	0.220605	0.219928
9.	0.45	0.216773	0.216274	0.215752	0.215208	0.214644
10.	0.50	0.211267	0.210851	0.210414	0.209957	0.209481

Fig:- (4.4): Velocity Profile with respect to the frequency of oscillation (ω).



Results and Discussion

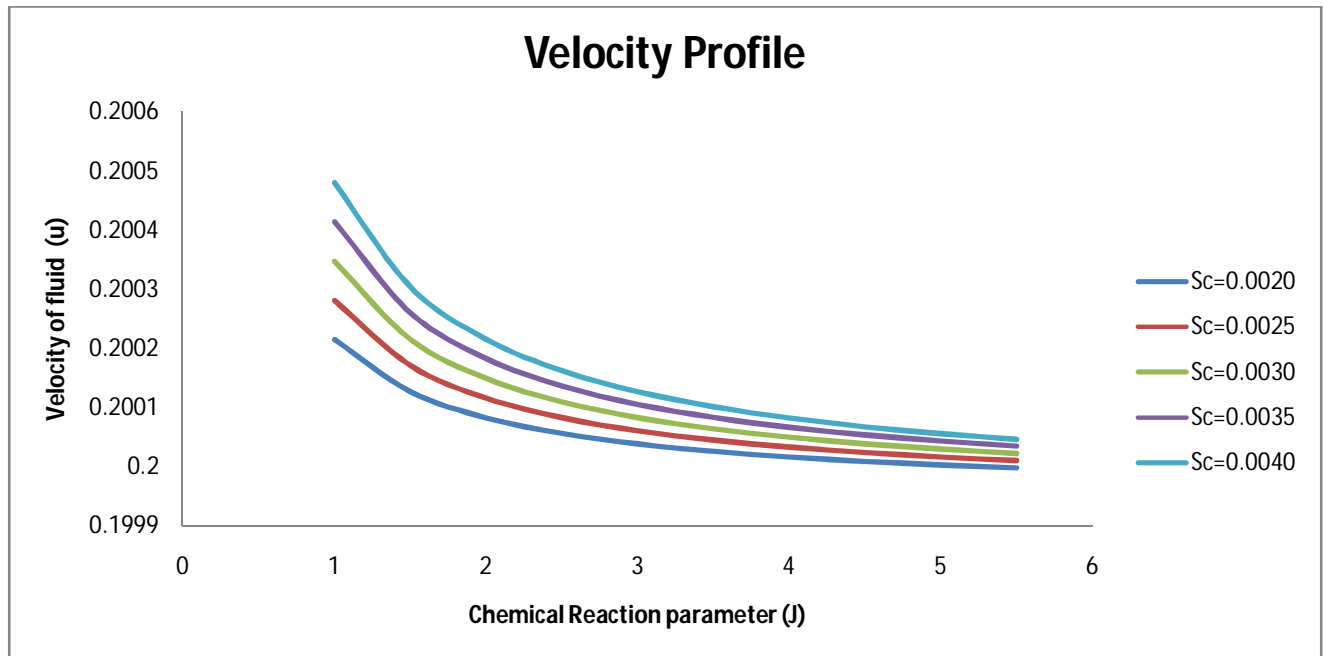
In the above graph the velocity pattern against the frequency parameter (ω) shows an increase and then, decreases in the profile after attaining a suitable height for different values of porous factors (S). Therefore the fluid profile somehow takes a resonance nature which is due to the presence of sine function in the velocity equation. The graph shows a diverging pattern in the beginning at lower values frequency of oscillation by the particles ($\omega = 0.005$) and then starts to decrease as the frequency parameter increases and the garph starts to converge for its higher values ($\omega = 0.5$)but the presence of other factors cannot be ignored.

Table (4.5) : Velocity of fluid and Chemical reaction parameter

$$(R_e = 1, \rho = 1.06, \gamma = 0.2, G_c = 4, G_r = 4)$$

Sr. No.	Chemical Reaction parameter (J)	Velocity of fluid				
		Schmidt Number ($S_c=0.0020$)	Schmidt Number ($S_c=0.0025$)	Schmidt Number ($S_c=0.0030$)	Schmidt Number ($S_c=0.0035$)	Schmidt Number ($S_c=0.0040$)
1.	1.0	0.200214	0.200281	0.200347	0.200414	0.20048
2.	1.5	0.200126	0.200171	0.200215	0.200260	0.200304
3.	2.0	0.200082	0.200116	0.200149	0.200182	0.200216
4.	2.5	0.200056	0.200082	0.200109	0.200136	0.200162
5.	3.0	0.200038	0.20006	0.200082	0.200105	0.200127
6.	3.5	0.200025	0.200044	0.200063	0.200083	0.200102
7.	4.0	0.200016	0.200032	0.200049	0.200066	0.200083
8.	4.5	0.200008	0.200023	0.200038	0.200053	0.200068
9.	5.0	0.200002	0.200016	0.200029	0.200043	0.200056
10.	5.5	0.199998	0.200010	0.200022	0.200034	0.200046

Fig:- (4.5): Velocity Profile with respect to Chemical Reaction parameter (J).



Results and Discussion

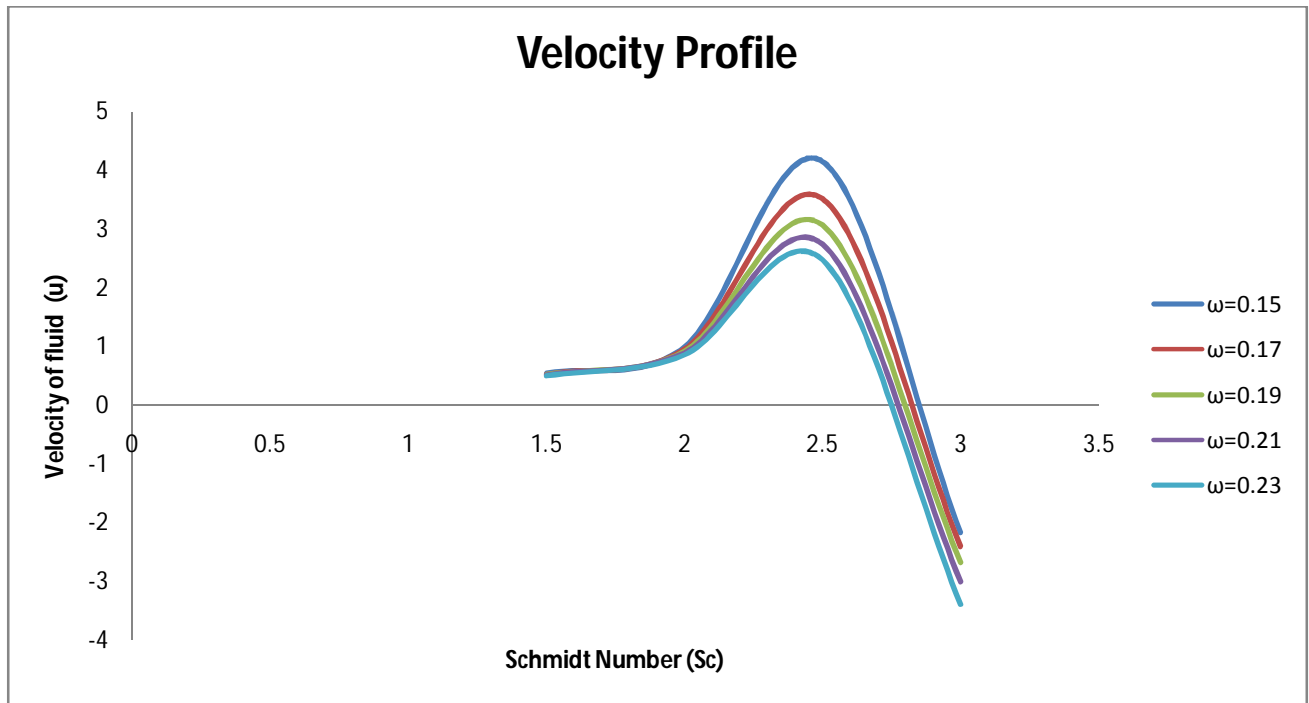
In this graph, the non-Newtonian fluid velocity decreases with the increase of Chemical reaction parameter (J) and this pattern is only due to the change in different values of Schmidt number (S_c). The graph shows a diverging pattern for lower values of Chemical reaction parameter ($J=1$) for different Schmidt number and then starts to decrease and the curvature is highly dominated by Schmidt number and further the graph starts to converge for higher values of Chemical reaction parameter ($J=5.5$), but the effect of constant magnetic field and non-Newtonian factor present in the velocity profile equation cannot be ignored.

Table (4.6) : Velocity of fluid and Schmidt Number

$(R_e = 1, \rho = 1.06, \gamma = 0.2, G_c = 4, G_r = 4)$

		Velocity of fluid				
Sr. No.	Schmidt Number (S_c)	Frequency of oscillation ($\omega = 0.15$)	Frequency of oscillation ($\omega = 0.17$)	Frequency of oscillation ($\omega = 0.19$)	Frequency of oscillation ($\omega = 0.21$)	Frequency of oscillation ($\omega = 0.23$)
1.	1.5	0.538775	0.528674	0.519254	0.510445	0.502189
2.	2.0	0.990585	0.952416	0.91838	0.887842	0.86029
3.	2.5	4.149355	3.516194	3.068909	2.736315	2.479463
4.	3.0	-2.175010	-2.409550	-2.683710	-3.007860	-3.396340

Fig:- (4.6): Velocity Profile with respect to Schmidt Number (S_c).



Results and Discussion

It is indicated clearly in the above graph that the velocity profile shows an increase and then decrease with the increase in Schmidt number (S_c) for different values of frequency of oscillation of fluid particles (ω) and it shows a wavy character which is due to the trigonometric terms occurring in the velocity profile equation. Further the graph starts at a fixed value of Schmidt number ($S_c=1.5$) increases and attains a highest value at $S_c=2.4$ and then it decreases sharply as the Schmidt number increases ($S_c=2.8$), however the frequency parameter (ω) which highly dominates the velocity profile and the presence of other terms cannot be ignored.

CHAPTER 5 A

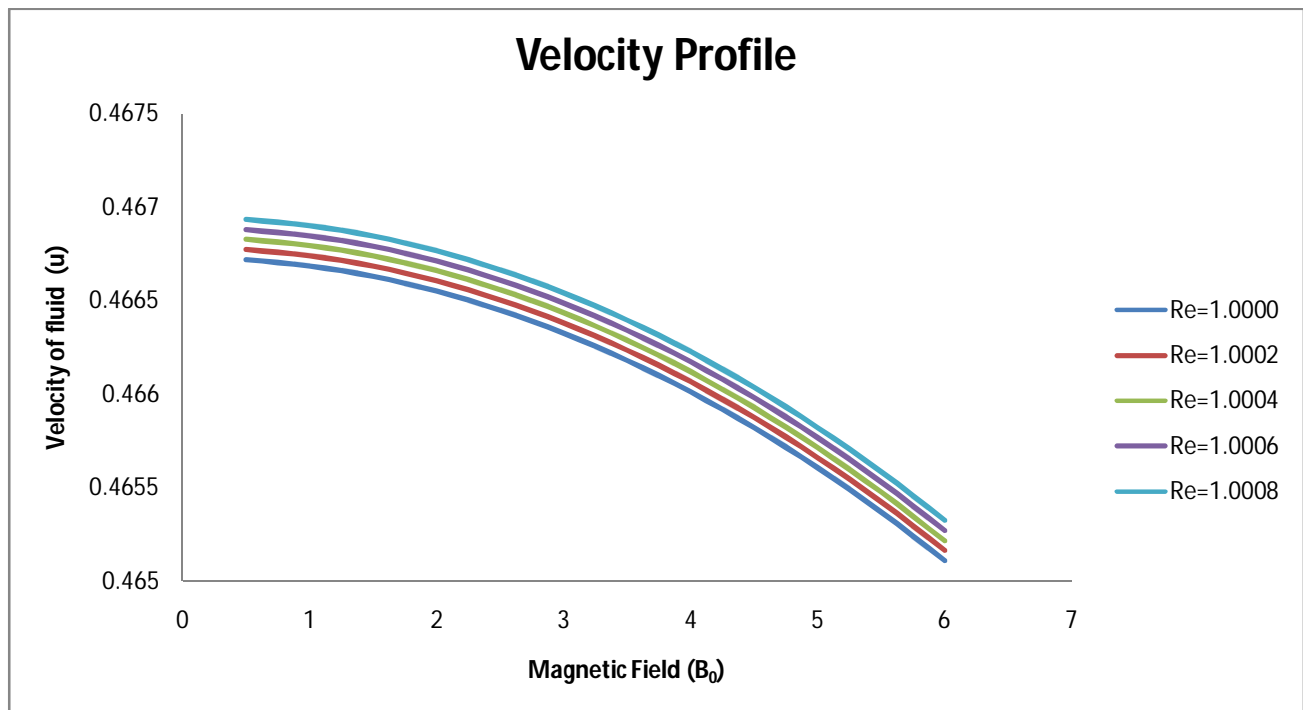
Considering the derived expression of the velocity pattern in chapter 5(A), suitable values are chosen and the graph has been plotted for different parameters.

Table (5.1) : Velocity of fluid and Magnetic Field at different Reynolds number

$$(\mu = 0.01, \rho = 1.03, G_r = 2, G_c = 2, y = 0.2)$$

		Velocity of fluid				
Sr. No.	Magnetic Field (B_0)	Reynolds number ($Re=1.0000$)	Reynolds number ($Re=1.0002$)	Reynolds number ($Re=1.0004$)	Reynolds number ($Re=1.0006$)	Reynolds number ($Re=1.0008$)
1.	0.5	0.466720395	0.466774068	0.466827745	0.466881423	0.466935104
2.	1.0	0.466686701	0.466740376	0.466794053	0.466847732	0.466901413
3.	1.5	0.466630536	0.466684211	0.466737889	0.466791569	0.466845252
4.	2.0	0.466551883	0.466605560	0.466659239	0.466712921	0.466766605
5.	2.5	0.466450722	0.466504401	0.466558082	0.466611765	0.466665451
6.	3.0	0.466327027	0.466380708	0.466434391	0.466488077	0.466541765
7.	3.5	0.466180763	0.466234447	0.466288133	0.466341821	0.466395512
8.	4.0	0.466011894	0.466065580	0.466119269	0.466172961	0.466226654
9.	4.5	0.465820372	0.465874063	0.465927755	0.465981450	0.466035147
10.	5.0	0.465606149	0.465659843	0.465713539	0.465767238	0.465820939
11.	5.5	0.465369166	0.465422865	0.465476565	0.465530268	0.465583973
12.	6.0	0.465109362	0.465163064	0.465216770	0.465270477	0.465324187

Fig:- (5.1): Velocity Profile with respect to Magnetic Field (B_0).



Results and Discussion

The figure shows the velocity profile of a Newtonian fluid flowing between two infinite porous plates which are placed in an inclined magnetic field. It appears from the graph that velocity decreases with increasing magnetic field for all values of Reynolds number. For lower value of Reynolds number the velocity adopts lower values and vice-versa and thus the pattern remains the same for all such values.

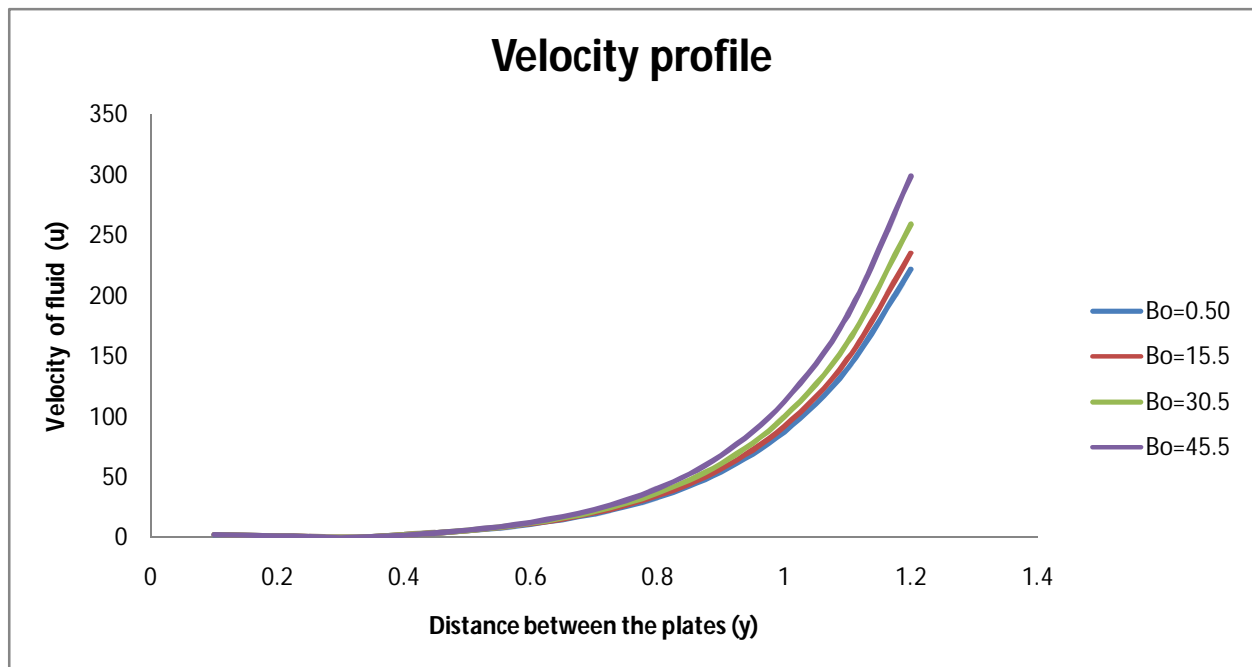
It is clear that on increasing the magnetic field, the force of attraction increases and which reduces the velocity of magnetic sensitive particles. Also from the derived relation, the magnetic field term appears in the denominator which also supports the decrement in velocity and on further increasing the magnetic field, the graph adopts hyperbolic tendency due to the presence of exponential terms in the derived relation. However the presence of other terms cannot be ignored in the velocity profile. The present study can be extended in this field and the results can be applied in various metallic industries.

Table (5.2) :Velocity of fluid and Distance from the base plate

$$(\mu = 0.01, \rho = 1.03, G_r = 2, G_c = 2)$$

Sr. No.	Distance from the base plate (y)	Velocity of fluid			
		Magnetic field ($B_0=0.50$)	Magnetic field ($B_0=15.5$)	Magnetic field ($B_0=30.5$)	Magnetic field ($B_0=45.5$)
1.	0.1	2.304611282	2.393947	2.558409	2.834757
2.	0.2	1.408127258	1.481171	1.617797	1.852704
3.	0.3	6.92948E-05	0.040597	0.121211	0.271677
4.	0.4	2.211539619	2.233013	2.260016	2.273687
5.	0.5	5.685305275	5.821409	6.048830	6.371603
6.	0.6	11.14156031	11.48493	12.07730	12.96907
7.	0.7	19.71173516	20.42361	21.66936	23.59072
8.	0.8	33.17297597	34.53146	36.93154	40.69115
9.	0.9	54.31666095	56.79776	61.21561	68.22215
10.	1.0	87.52723201	91.94050	99.85465	112.5460
11.	1.1	139.6913709	147.4060	161.3343	183.9055
12.	1.2	221.6260499	234.9469	259.1563	298.7915

Fig:- (5.2): Velocity Profile with respect to Distance from the base plate (y).



Results and Discussion

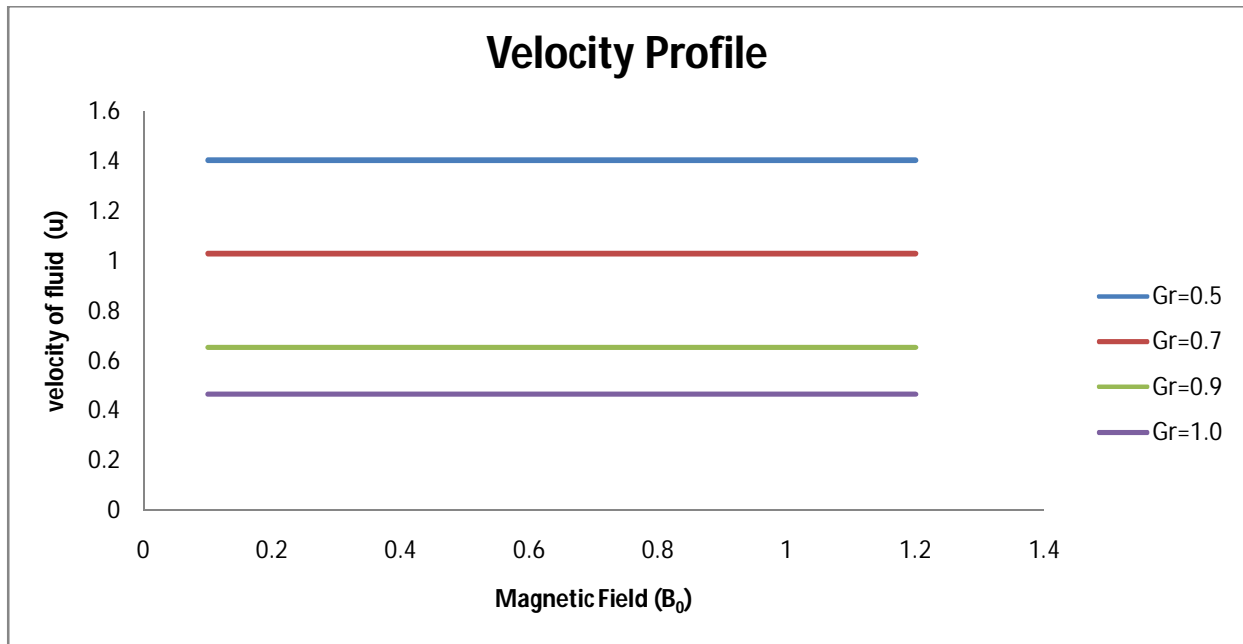
The above graph shows that the velocity profile keeps on increasing as the distance between the plate increases at different values of magnetic field which is responsible for attractiveness between the magnetic sensitive particles which gives a curvy pattern. For higher value of magnetic field, the fluid velocity adopts a higher pattern and vice-versa. Further the velocity pattern starts forming a converging nature at $y = 0.1$ and keeps on increasing for which the profile diverges and attains a highest value for $y = 1.2$. The pattern of the velocity profile is due to the presence of magnetic field and hence the results can be applied in various metallic industries.

Table (5.3) : Velocity of fluid and Magnetic field at different Grashof Number

$$(\mu = 0.01, \rho = 1.03, G_c = 2, \gamma = 0.2)$$

Sr. No.	Magnetic field (B_0)	Velocity of fluid			
		Grashof Number ($G_r = 0.5$)	Grashof Number ($G_r = 0.7$)	Grashof Number ($G_r = 0.9$)	Grashof Number ($G_r = 1.0$)
1.	0.1	1.404446	1.029446	0.654445	0.466945
2.	0.2	1.404446	1.029444	0.654443	0.466942
3.	0.3	1.404446	1.029442	0.654439	0.466937
4.	0.4	1.404445	1.029439	0.654433	0.466930
5.	0.5	1.404444	1.029435	0.654425	0.466920
6.	0.6	1.404443	1.02943	0.654416	0.466909
7.	0.7	1.404442	1.029424	0.654405	0.466896
8.	0.8	1.404441	1.029417	0.654392	0.466880
9.	0.9	1.404440	1.029409	0.654378	0.466862
10.	1.0	1.404438	1.0294.0	0.654362	0.466843
11.	1.1	1.404436	1.02939	0.654344	0.466821
12.	1.2	1.404434	1.029379	0.654324	0.466797

Fig:- (5.3): Variation in Velocity Profile with respect to Grashoff Number (Gr).



Results and Discussion

The graph shows that velocity profile of a non-Newtonian fluid follows a straight line motion for different values of Grashoff number at various values of magnetic field. This is due to the terms present in the formulation of Grashoff number i.e. the ratio of gravitational acceleration, thermal expansion constant and the temperature difference between the wall and fluid to the kinematic viscosity, hence the velocity pattern gets highly affected by the Grashoff number but the presence of magnetic field on the velocity profile dominates the flows which is in constant motion.

CHAPTER 5 B

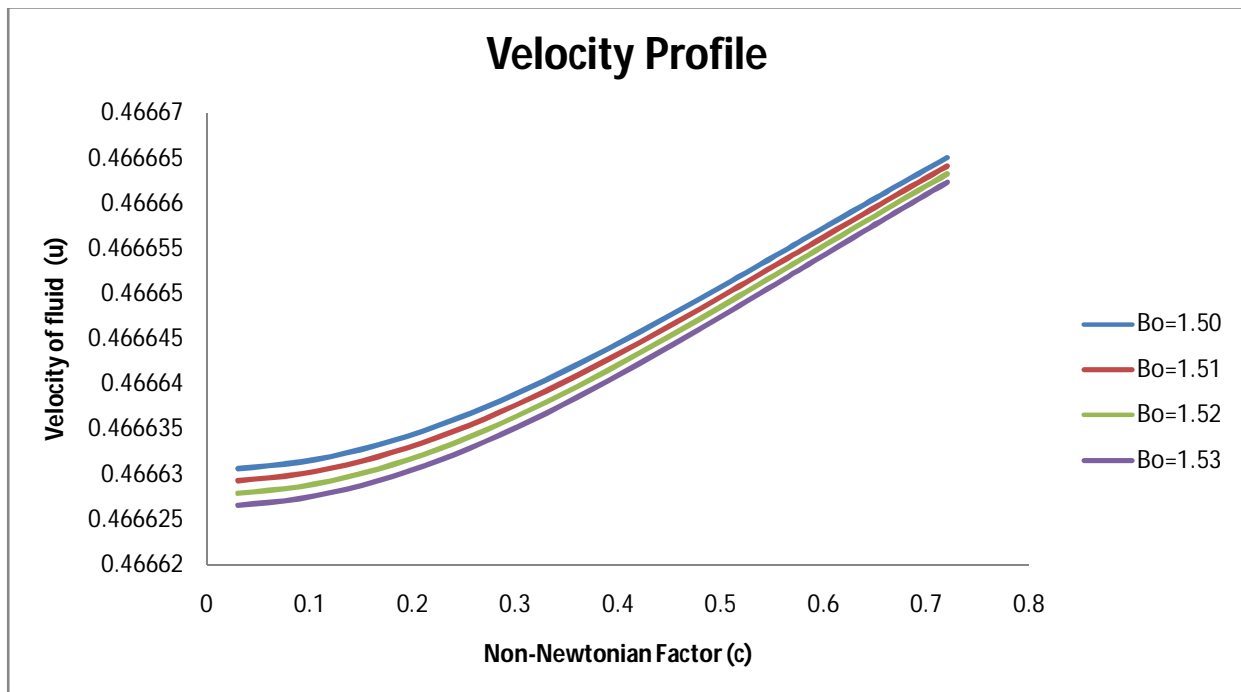
Considering the derived expression of the velocity pattern in chapter 5(B), suitable values are chosen and the graph has been plotted for different parameters.

Table (5.4) : Velocity Profile and Non-Newtonian factor at different values of Magnetic Field

$(\mu = 0.01, \rho = 1.03, G_r = 2, G_c = 2, y = 0.2)$

Sr. No.	Non-Newtonian Factor (c)	Velocity of fluid			
		Magnetic Field ($B_0=1.50$)	Magnetic Field ($B_0=1.51$)	Magnetic Field ($B_0=1.52$)	Magnetic Field ($B_0=1.53$)
1.	0.03	0.466631	0.466629	0.466628	0.466627
2.	0.09	0.466631	0.466630	0.466629	0.466627
3.	0.15	0.466633	0.466631	0.466630	0.466629
4.	0.21	0.466635	0.466634	0.466632	0.466631
5.	0.27	0.466637	0.466636	0.466635	0.466634
6.	0.33	0.466640	0.466639	0.466638	0.466637
7.	0.39	0.466644	0.466643	0.466642	0.466640
8.	0.45	0.466648	0.466646	0.466645	0.466644
9.	0.51	0.466651	0.466650	0.466649	0.466648
10.	0.57	0.466655	0.466654	0.466653	0.466652
11.	0.66	0.466661	0.466660	0.466659	0.466658
12.	0.72	0.466665	0.466664	0.466663	0.466662

Fig:- (5.4): Velocity Profile with respect to Non-Newtonian Factor (c).



Results and Discussion

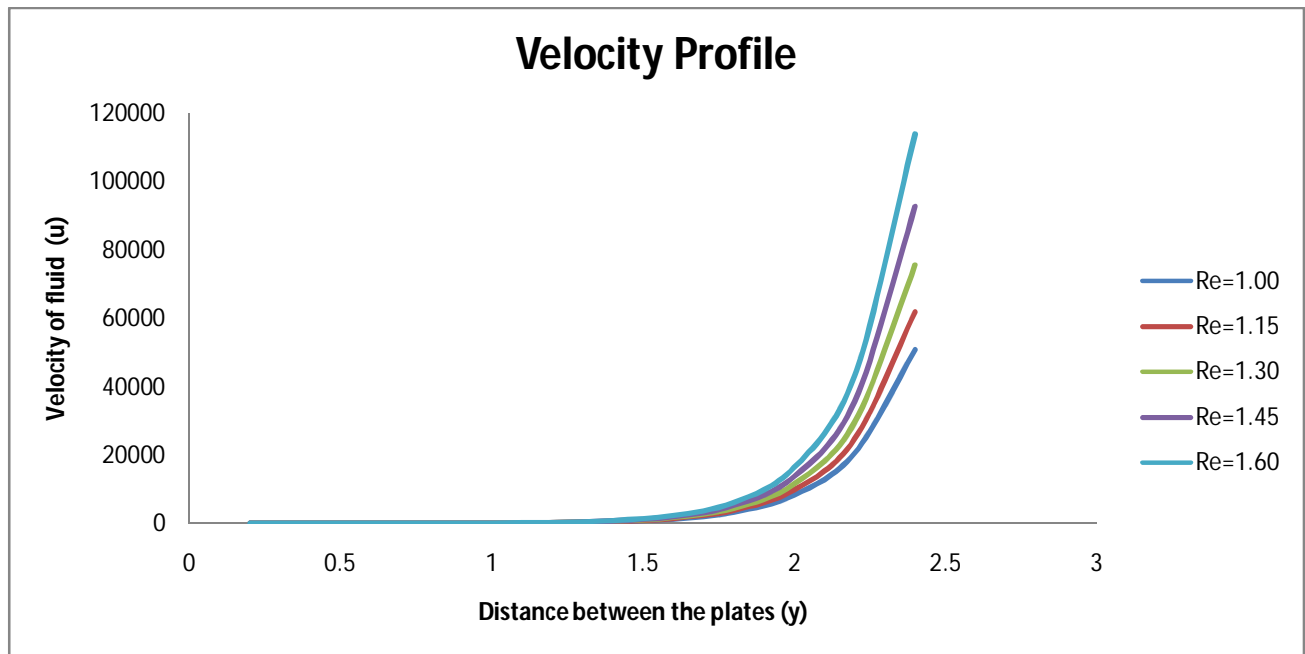
The above graph shows the velocity profile of a non-Newtonian fluid between two infinite porous plates which are placed in an inclined magnetic field. It appears from the graph that the velocity increases with the increase in non-Newtonian factor (c) for all values of Magnetic field and on further increasing the Non-Newtonian factor the graph adopts hyperbolic tendency due to the presence of exponential terms in the derived relation. However the presence of magnetic field and other parameters on the velocity profile cannot be ignored. Further the velocity profile starts showing a diverging pattern at Non-Newtonian factor ($c = 0.002$) and goes on increasing and then converges for Non-Newtonian factor ($c = 0.72$), the present work can be extended in this field and the results can be applied in the electric power generator, extrusion of plastic in the manufacture of rayon and nylon etc.

Table (5.5) : Velocity of fluid and Distance from the plate at different values of Reynolds number

$(\mu = 0.01, \rho = 1.03, G_r = 2, G_c = 2, c = 0.001)$

Sr. No.	Distance from the base plate (y)	Velocity of fluid				
		Reynolds number (Re = 1.00)	Reynolds number (Re = 1.15)	Reynolds number (Re = 1.30)	Reynolds number (Re = 1.45)	Reynolds number (Re = 1.60)
1.	0.2	0.46672039	0.507630	0.549875	0.593494	0.638528
2.	0.4	4.08548709	4.288994	4.502656	4.727012	4.962636
3.	0.6	13.0124589	13.77177	14.58229	15.44783	16.3725
4.	0.8	35.0343098	37.55265	40.28578	43.25399	46.47963
5.	1.0	89.3599293	97.19046	105.8309	115.3739	125.9232
6.	1.2	223.375666	246.7505	272.9743	302.4289	335.5510
7.	1.4	553.978795	621.8182	699.1981	787.5873	888.6958
8.	1.6	1369.54295	1562.415	1786.090	2045.927	2348.278
9.	1.8	3381.45643	3921.251	4557.719	5309.642	6199.678
10.	2.0	8344.64152	9836.753	11625.51	13774.63	16362.36
11.	2.2	20588.3123	24671.69	29648.74	35730.00	43178.64
12.	2.4	50792.1978	61874.83	75608.85	92674.94	113938.7

Fig:- (5.5): Velocity Profile with respect to distance between the plates (y).



Results and Discussion

In the above graph the velocity of fluid remains constant in the beginning as the distance of the plate increases ($y = 0.2$) upto a certain value but then the velocity gradually increases as the distance between the plates is further increased ($y = 2.4$) giving a curvy pattern at different values of Reynolds Number. Further the velocity profile takes a converging pattern in the beginning and as the distance from the base plate increases, the pattern of the profile diverges, also the curviness of the graph is due to the presence of exponential terms present in the velocity profile equation.

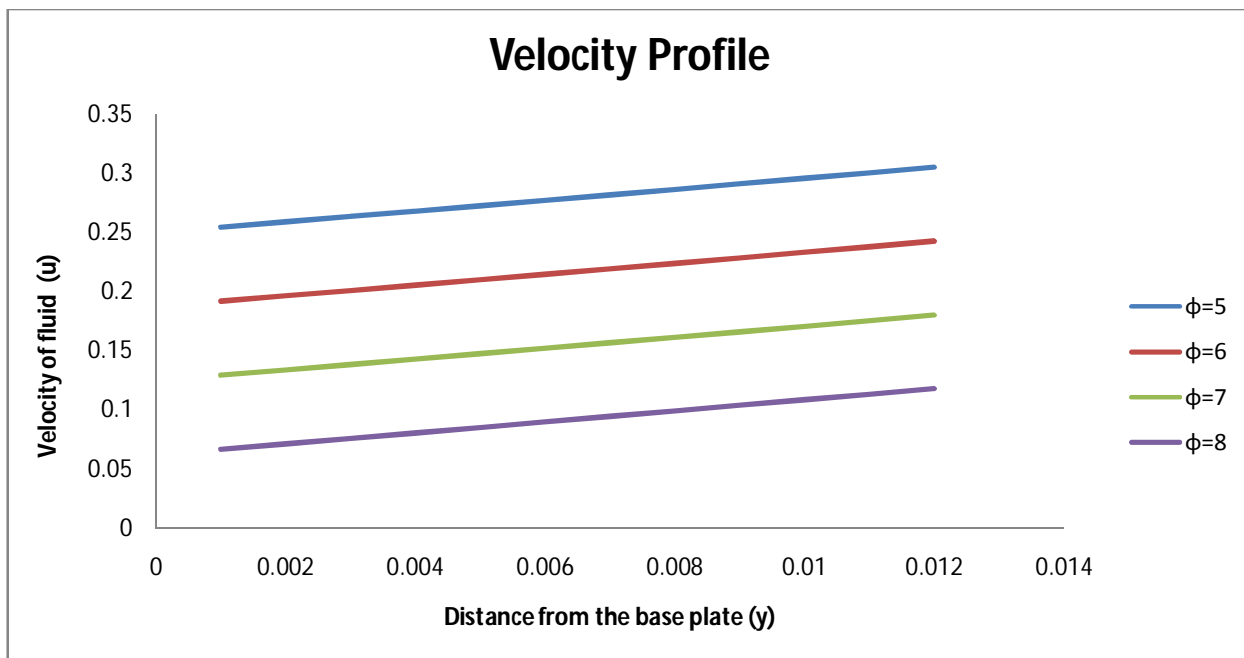
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Table (5.6): Velocity of fluid and Distance from the base plate at different values of angle of inclinations of the magnetic field.

$$(\mu = 0.01, \rho = 1.03, G_r = 2, G_c = 2, B_0 = 0.5)$$

Sr. No.	Distance from the base plate (y)	Velocity of fluid			
		Angle of inclination ($\phi=5$)	Angle of inclination ($\phi=6$)	Angle of inclination ($\phi=7$)	Angle of inclination ($\phi=8$)
1.	0.001	0.254192	0.191699	0.129208	0.066709
2.	0.002	0.258739	0.196246	0.133755	0.071255
3.	0.003	0.263306	0.200813	0.138322	0.075822
4.	0.004	0.267894	0.205401	0.142910	0.080410
5.	0.005	0.272502	0.210010	0.147518	0.085019
6.	0.006	0.277132	0.214639	0.152148	0.089648
7.	0.007	0.281782	0.219289	0.156798	0.094299
8.	0.008	0.286454	0.223961	0.161469	0.098970
9.	0.009	0.291146	0.228653	0.166162	0.103662
10.	0.010	0.295860	0.233367	0.170876	0.108376
11.	0.011	0.300595	0.238102	0.175611	0.113111
12.	0.012	0.305351	0.242859	0.180367	0.117868

Fig:- (5.6): Velocity Profile with respect to Distance between the plates (y) at different angle of inclinations of magnetic field (ϕ).



Results and Discussion

The graph shows that the velocity of the fluid acquires a constant motion which is continuously increasing in nature as the distance from the base plate is increased at different angles of inclinations of the magnetic field on the parallel plates. The graph starts from a fixed value of $y = 0.001$ and gradually increases up to distance $y = 0.012$. This pattern of the velocity profile is highly governed by the magnetic sensitive particles but the presence of other factors cannot be ignored.

CHAPTER 6

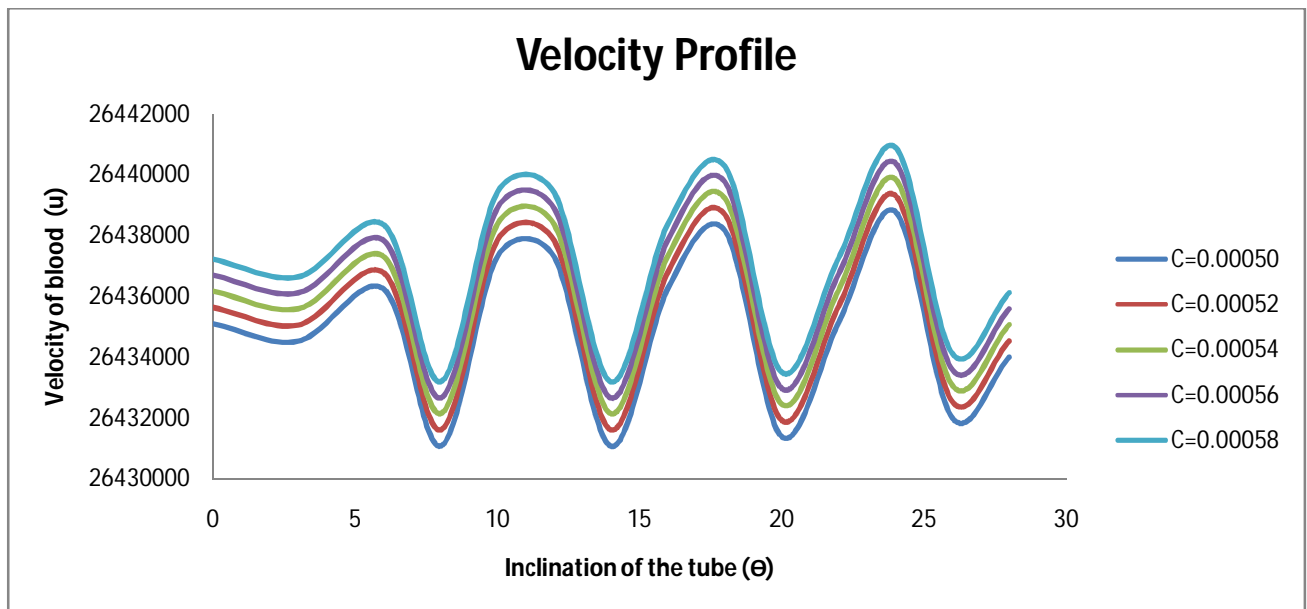
Considering the derived expression of the velocity pattern in chapter 6, suitable values are chosen and the graph has been plotted for different parameters.

Table (6.1) : Angle of Inclination of the tube and Velocity Profile

($R_0 = 0.01, B_0 = 1.5, y = 0.2, \rho = 1.06$)

Sr. No.	Angle of inclination of the tube (θ)	Velocity of blood				
		Non-Newtonian factor (C)=0.00050	Non-Newtonian factor (C)=0.00050	Non-Newtonian factor (C)=0.00050	Non-Newtonian factor (C)=0.00050	Non-Newtonian factor (C)=0.00050
1.	0	264351092.9	264356377.3	264361661.7	264366946.1	264372230.5
2.	3	264345351.8	264350636.1	264355920.3	264361204.6	264366488.9
3.	6	264362460.3	264367744.9	264373029.5	264378314.1	264383598.7
4.	8	264310843.2	264316126.8	264321410.4	264326693.9	264331977.5
5.	10	264373225.2	264378510.0	264383794.8	264389079.6	264394364.4
6.	12	264372922.1	264378207.0	264383491.8	264388776.6	264394061.4
7.	14	264310792.4	264316076.0	264321359.5	264326643.1	264331926.7
8.	16	264362805.6	264368090.2	264373374.8	264378659.4	264383944.1
9.	18	264381645.1	264386930.1	264392215.1	264397500.1	264402785.1
10.	20	264313951.9	264319235.5	264324519.2	264329802.8	264335086.4
11.	22	264351453.0	264356737.4	264362021.8	264367306.2	264372590.6
12.	24	264387934.3	264393219.4	264398504.5	264403789.6	264409074.7
13.	26	264320070.0	264325353.8	264330637.6	264335921.3	264341205.1
14.	28	264340071.8	264345355.9	264350640.1	264355924.2	264361208.4

Fig:- (6.1): Velocity Profile with respect to Non-Newtonian factor (C).



Results and Discussion

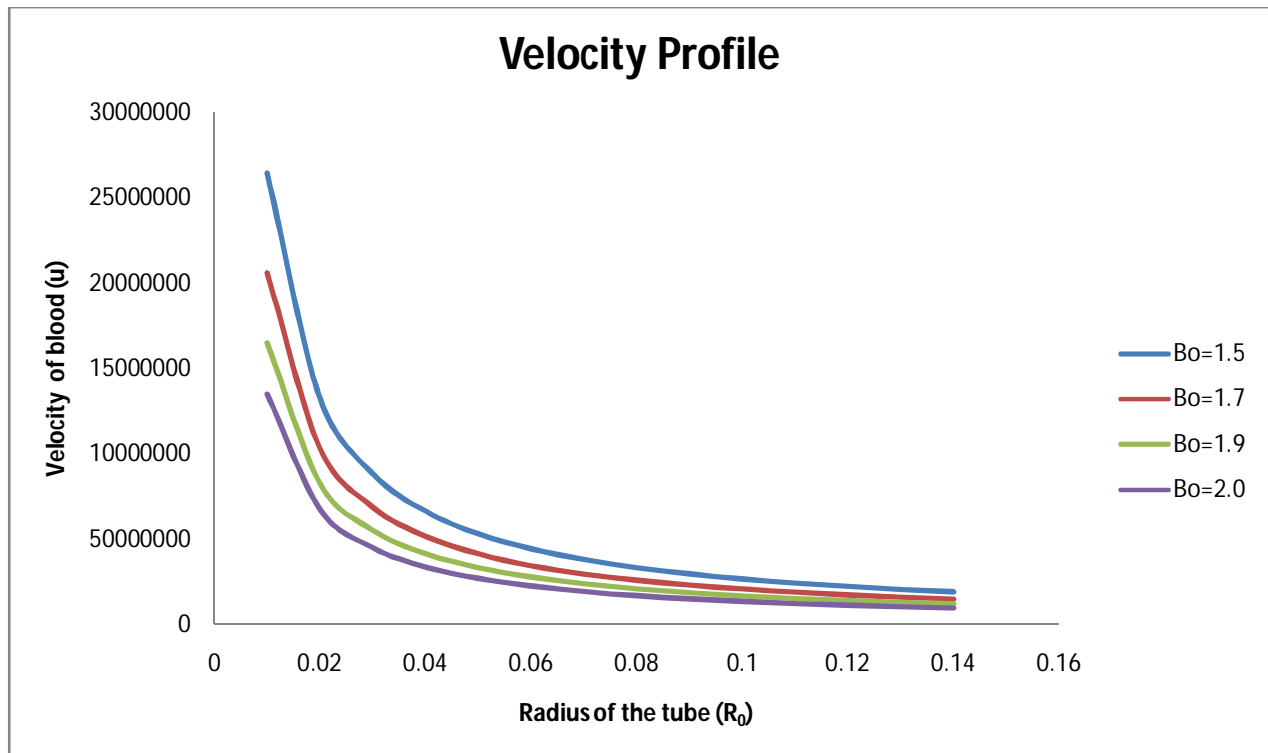
The above graph is obtained for different values of non-Newtonian (C) and the angle of inclination (θ) for which the velocity of fluid shows similar feature in each case except the variations in the maximum/minimum values of amplitudes of velocity which appears to have a wavy nature about a mean value. This pulsating pattern is due to the sine function present in the velocity profile and hence it takes the form of a sinusoidal wave therefore when the angle of inclination of the tube increases, the amplitude increases on both sides, in each case. The experimental and theoretical studies of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human physiology and for other practical applications.

Table (6.2) : Radius of the tube and Velocity Profile at different values of magnetic field.

($C = 0.0005, y = 0.2, \rho = 1.06$)

Sr. No.	Radius of the tube (R_0)	Velocity of blood			
		Magnetic field ($B_0 = 1.5$)	Magnetic field ($B_0 = 1.7$)	Magnetic field ($B_0 = 1.9$)	Magnetic field ($B_0 = 2.1$)
1.	0.01	2.64E+08	2.06E+08	1.65E+08	1.35E+08
2.	0.02	1.32E+08	1.03E+08	82418232	67466643
3.	0.03	88155949	68632725	54943485	44975758
4.	0.04	66114858	51472440	41205508	33729713
5.	0.05	52889721	41175786	32962240	26981602
6.	0.06	44072561	34310947	27466323	22482457
7.	0.07	37774243	29407144	23540322	19268434
8.	0.08	33050202	25728987	20595515	16857610
9.	0.09	29375677	22867927	18304837	14982251
10.	0.10	26435812	20578834	16472048	13481716
11.	0.11	24030246	18705715	14972268	12253777
12.	0.12	22025402	17144576	13722243	11230285
13.	0.13	20328805	15823420	12664335	10364058
14.	0.14	18874402	14690821	11757376	9621394

Fig:- (6.2): Velocity Profile with respect to Radius of the tube at different values of magnetic field. (B_0).



Results and Discussion

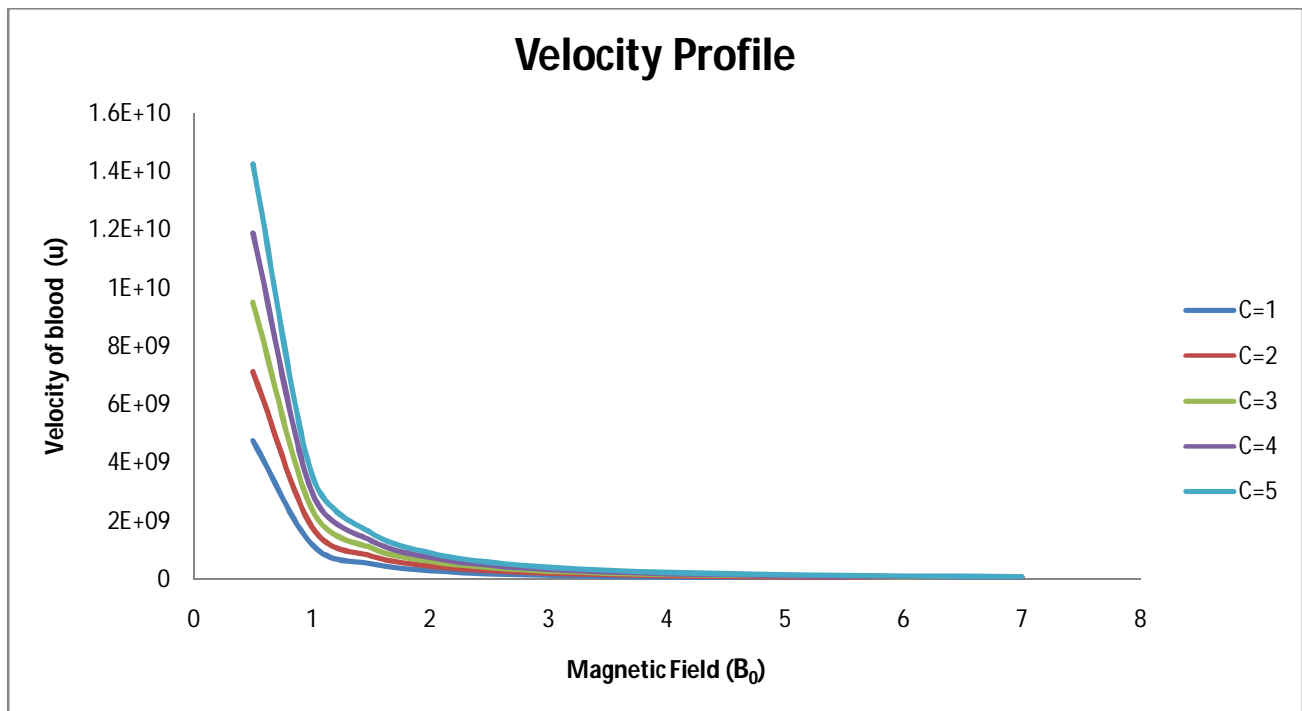
The above figure shows that the velocity profile of the blood appears to be in a decreasing manner as the radius of the tube increases for different values of magnetic field, hence as blood acts as a non-Newtonian fluid, therefore, the effect of magnetic field can clearly be observed. Further it can be clearly seen that the velocity pattern starts forming a diverging pattern for the fixed radius of the tube $R_0 = 0.01$ and then decreases gradually showing a converging pattern for $R_0 = 0.14$ but the magnetic field plays a very important role in the behavior of the pattern of the velocity profile but the presence of other terms cannot be ignored.

Table (6.3) : Magnetic Field and Velocity Profile

$(R_0 = 0.01, \gamma = 0.2, \rho = 1.06)$

Sr. No.	Magnetic Field (B_0)	Velocity of blood				
		Non-Newtonian factor (C=1.0)	Non-Newtonian factor (C=2.0)	Non-Newtonian factor (C=3.0)	Non-Newtonian factor (C=4.0)	Non-Newtonian factor (C=5.0)
1.	0.5	4755857614	7133786420	9511715227	11889644034	14267572841
2.	1.0	1188962603	1783443905	2377925207	2972406508	3566887810
3.	1.5	528426490.4	792639735.5	1056852981	1321066226	1585279471
4.	2.0	297238850.7	445858276.1	594477701.5	743097126.9	891716552.2
5.	2.5	190232000.4	285348000.5	380464000.7	475580000.9	570696001.1
6.	3.0	132104822.3	198157233.5	264209644.6	330262055.8	396314466.9
7.	3.5	97055967.22	145583950.8	194111934.4	242639918	291167901.7
8.	4.0	74307912.17	111461868.3	148615824.3	185769780.4	222923736.5
9.	4.5	58711920.45	88067880.68	117423840.9	146779801.1	176135761.4
10.	5.0	47556199.26	71334298.89	95112398.52	118890498.1	142668597.8
11.	5.5	39302227.14	58953340.71	78604454.28	98255567.85	117906681.4
12.	6.0	33024404.36	49536606.55	66048808.73	82561010.91	99073213.09
13.	6.5	28138781.99	42208172.99	56277563.99	70346954.99	84416345.98
14.	7.0	24262190.14	36393285.21	48524380.28	60655475.34	72786570.41

Fig:- (6.3): Velocity Profile with respect to Magnetic Field (B_0).



Results and Discussion

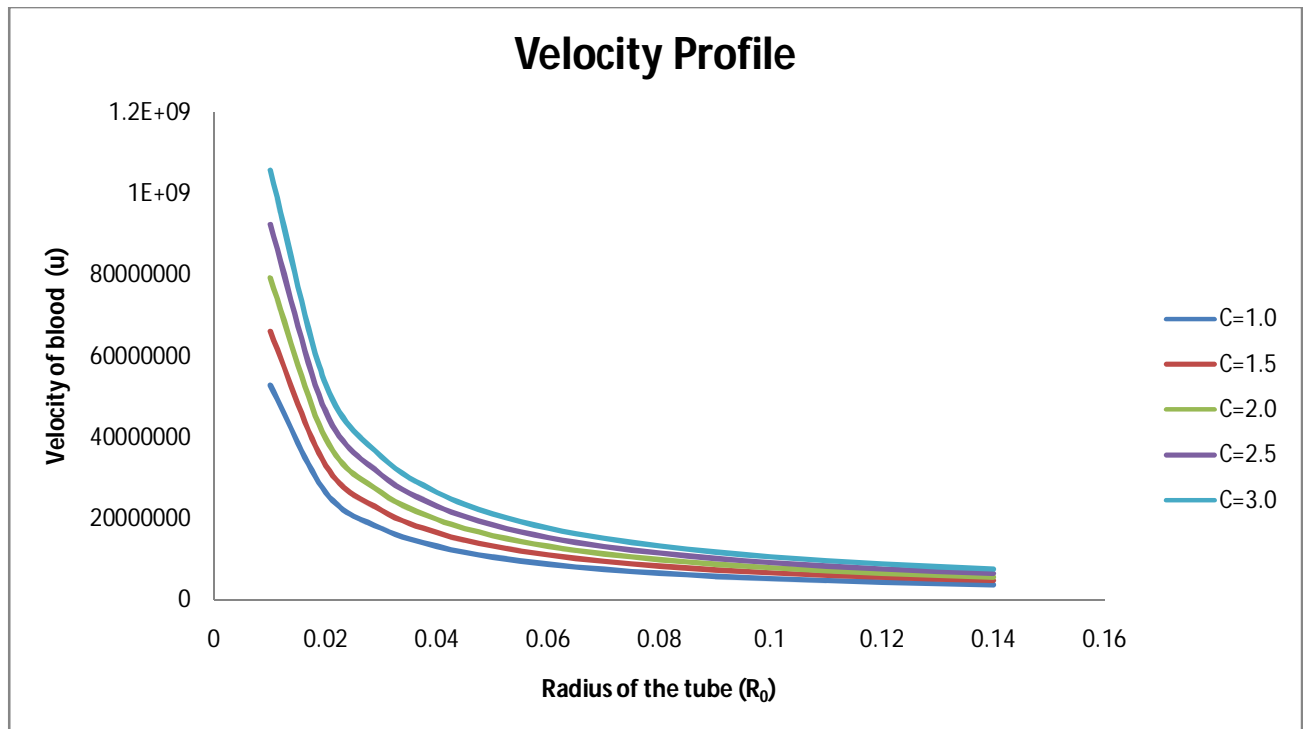
In the above graph, it is seen that the velocity profile of the blood decreases sharply on increasing the magnetic field and after a certain interval of time; the motion of the blood flow becomes constant for different values of non-Newtonian factor. Further it can be seen that the velocity profile diverges for lower values of magnetic field ($B_0 = 0.5$) and then decreases as the magnetic is increased ($B_0 = 1$) and then attains a constant profile at $B_0 = 3$ which is having a converging pattern in nature. Hence it can figured out that the blood which is bearing magnetic sensitive particles when comes in contact with magnetic field, its velocity decreases.

Table (6.4) : Radius of the tube and Velocity Profile

($B_0 = 1.5, y = 0.2, \rho = 1.06$)

Sr. No.	Radius of the narrow tube (R_0)	Velocity of blood				
		Non-Newtonian factor (C=1.0)	Non-Newtonian factor (C=1.5)	Non-Newtonian factor (C=2.0)	Non-Newtonian factor (C=2.5)	Non-Newtonian factor (C=3.0)
1.	0.01	528426490.4	660533112.9	792639735.5	924746358.1	1056852981
2.	0.02	264209644.6	330262055.8	396314466.9	462366878.1	528419289.3
3.	0.03	176135761.4	220169701.7	264203642.0	308237582.4	352271522.7
4.	0.04	132097617.5	165122021.8	198146426.2	231170830.6	264195234.9
5.	0.05	105673767.9	132092209.9	158510651.8	184929093.8	211347535.8
6.	0.06	88057064.00	110071330.0	132085596.0	154099862.0	176114128.0
7.	0.07	75473013.19	94341266.49	113209519.8	132077773.1	150946026.4
8.	0.08	66034368.94	82542961.18	99051553.41	115560145.6	132068737.9
9.	0.09	58692660.73	73365825.91	88038991.09	102712156.3	117385321.5
10.	0.10	52818806.03	66023507.54	79228209.05	92432910.55	105637612.1
11.	0.11	48012479.44	60015599.3	72018719.16	84021839.02	96024958.88
12.	0.12	44006797.17	55008496.47	66010195.76	77011895.06	88013594.35
13.	0.13	40616993.39	50771241.74	60925490.08	71079738.43	81233986.78
14.	0.14	37711092.34	47138865.42	56566638.51	65994411.59	75422184.68

Fig:- (6.4): Velocity Profile with respect to Radius of the narrow tube (R_0).



Results and Discussion

The graph shows that the velocity of the blood flow gradually decreases with the increase in the radius of the tube for every increasing values of non-Newtonian factors. Further it can be seen that the velocity profile diverges for lower values of Radius of tube ($R_0 = 0.01$) and then decreases as the radius is increased ($R_0 = 0.02$) and then attains a constant profile at $R_0 = 0.1$ which is showing that the flow pattern is converging in nature. Hence the radius of the tube shares a proportional relationship with the velocity profile in the velocity profile equation but the presence of other factors cannot be ignored which are responsible for the steady decrease of the blood flow through the tube.

CHAPTER 8

Summary and Conclusion

SUMMARY

The study of visco-elastic, incompressible and non-Newtonian fluid through various channels placed in magnetic field is of great importance in the field of engineering and technology and also in industries. New mathematical models have been developed (IV, V(A), V(B), VI) which govern the velocity of the fluid under the given boundary conditions, hence relations of velocity profile of the fluid under consideration have been derived with respect to various parameters for which graphical and analytical studies and its physical interpretation has been made to justify the nature of the problem.

In Chapter I, the introductory part comprises of the subject along with its applications and importance and briefly described theory and equations which have been used in this work.

Chapter II consists of a brief review of literature and the developments made by the researchers in this field of science, also some important contributions are mentioned with critical analysis in chronological order.

The Methodology of the work is described in chapter III briefly along with the parameters taken into consideration.

In chapter IV, heat and mass transfer of an oscillatory flow of fluid between two inclined porous plates in magnetic field have been studied and the expression for velocity profile of the fluid has been derived with respect to various parameters and corresponding effects have also been observed with the help of graphs.

Chapter V (A and B), a study on the effect of flow of a Newtonian and non-Newtonian fluids through parallel plates having porous medium has been examined and the relation of the velocity profile of the fluid and the combined effect of magnetic field and other parameters are taken in consideration and graphical study has been conducted between the velocity profile and other parameters respectively.

Chapter VI, a study on the influence of magnetic field on the blood flow through an inclined narrow tube has been made and the velocity profile of blood flow has been derived which has been examined under the effect of non-Newtonian factor and other factors are also taken into consideration and the results have been discussed graphically.

Chapter VII consists of pictorial presentations of graphs and their numerical data in tabular form. After this, the results are discussed for each graph and the physical consistency of the problem under assumed parameters with the results so obtained has been justified.

At the end, recommendations and limitations of this study along with bibliography has been well mentioned.

CONCLUSION

In Chapters (4-6), mathematical models have been developed for the velocity profile of the fluid with respect to various parameters such as magnetic field, non-Newtonian factor, Reynolds number. After solving the equations, the relations for velocity and these variables have been observed and further used to plot the graphs to verify the results by altering the values of the parameters and boundary conditions which were found to be consistent with the physical nature of the problem.

CHAPTER 4

In this chapter, the study on heat and mass transfer of an oscillatory flow of fluid between two inclined porous plates in magnetic field has been done taking into account that the fluid is non-Newtonian in nature and the conclusion of the studies are given as:

- In all the cases, the role of magnetic field is very important as it controls the velocity of the fluid. As the fluid is considered to be non-Newtonian, therefore, magnetic sensitive particles are attracted when magnetic field comes into play.
- The velocity profile with respect to magnetic field for different non-Newtonian factors, the higher value of non-Newtonian factor brings more bending around maximum value of velocity and the velocity profile shows a resonance character which is due to the presence of trigonometric function bearing non-Newtonian terms and it puts the motion in hyperbolic nature and for a certain value of magnetic field the resonance occurs in each time. Further the profile has a steady increase in a linear way for different values of Grashof number.
- The velocity of the fluid exponentially increases as the boundary layer distance increases taking a resonance character.
- The increasing value of chemical reaction parameter decreases the velocity of the fluid.
- The velocity profile decreases as the frequency parameter of the particles increases.

- The velocity profile follows a linear path and after a definite time the velocity sharply increases as the Schmidt number (Sc) increases and after attaining a suitable amplitude, the velocity sharply decreases on further increasing the Schmidt number for different values of frequency of oscillation of fluid particles (ω).

CHAPTER 5

The effect of heat and mass transfer on the flow of Newtonian and non-Newtonian flow through parallel plates having porous medium in the presence of magnetic field have been observed and the following conclusions have been derived:

- The velocity of the fluid decreases with increase in magnetic field for all values of Reynolds number. The graph adopts a hyperbolic tendency which is due to the trigonometric terms present in the velocity profile equation.
- As the boundary layer distance increases along the positive x-axis, the velocity of the fluid increases showing a curvy pattern which is due to the presence of magnetic field in each case.
- The velocity pattern adopts a linear path through the motion as the Grashoff number increases.
- Graph concludes that the velocity profile increases with the increase in non-Newtonian factor (c) for all values of Magnetic field and on further increasing the Non-Newtonian factor the graph adopts hyperbolic tendency due to the presence of exponential terms in the derived relation.
- The velocity of fluid adopts a constant motion in the beginning as the boundary layer distance increases ($y = 0.2$) upto a certain value but then the velocity gradually increases as the boundary layer distance is further increased ($y = 2.4$) giving a curvy pattern at different the values of Reynolds Number.
- The velocity linearly increases with the increasing boundary layer distance for different values of angle of inclination of magnetic field on the parallel plates which highly effects the pattern of the flow.

CHAPTER 6

In this chapter, the study of blood flow has been carried out which behaves as a non-Newtonian fluid therefore a non-Newtonian factor 'C' is introduced which influences the velocity profile of blood under the effect of magnetic field. The conclusions drawn are as follows:

- The graphs obtained for different values of non-Newtonian (C) and the angle of inclination (θ) for which the velocity of fluid shows similar feature in each case except the variations in the maximum/minimum values of amplitudes of velocity which appear to have a wavy nature about a mean value. This pulsating pattern is due to the trigonometric terms present in the derived equation of the velocity profile, hence as the magnetic field increases the amplitude increases on both the sides.
- The velocity of the blood appears to be in a decreasing manner as the radius of the tube increases for different values of magnetic field, hence as blood acts as a non-Newtonian fluid, therefore, the effect of magnetic field can clearly be observed. The curve with lower value of magnetic field lies at upper position and has more slopes.
- Graph concludes that the velocity of the blood decreases sharply on increasing the magnetic field and after a certain interval of time, the motion of the blood flow becomes linear in nature for different values of non-Newtonian factor.
- The velocity of the blood flow gradually decreases with the increase in the radius of the tube for every increasing values of non-Newtonian factor.

The results obtained numerically are in consistency with the physical nature of the problem as expected. Thus the study appears to have opened a new dimension for multiphase MHD flow which will be fruitful for industrial sectors.

RECOMMENDATIONS

The results in this study came out to be very useful in various areas of science, industries and technologies where such type of flow occur in different channels placed in magnetic field under different physical conditions.

- The position of the channel can be changed under different conditions.
- It can be used to study flows occurring in three dimensional spaces.
- This study can be extended to other channels and parameters.
- This study can further be extended where the flows occur under the influence of electrical and magnetic field applied simultaneously.

LIMITATIONS:

- The study is limited because of the assumed parameters are few in number.
- The lengthy work of calculations seems to reduce the accuracy of the result derived.

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