

# Chance Constrained Optimization of Transient Groundwater Flow Considering Pump Characteristics

## Thesis

Submitted to the



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By

**Neha Dhapola**

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Pantnagar  
December, 2020

  
(**Neha Dhapola**)  
Authoress

# CERTIFICATE – I

This is to certify that the thesis entitled “**Chance Constrained Optimization of Transient Groundwater Flow considering Pump Characteristics**” submitted in partial fulfilment of the requirements for the degree of **Master of Technology in Civil Engineering** with major in **Hydraulic Engineering** of the College of Post Graduate Studies, G. B. Pant University of Agriculture & Technology, Pantnagar, is a record of *bona fide* research carried out by **Ms. Neha Dhapola, Id No. 54098** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

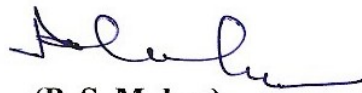
The assistance and help received during the course of this investigation have been acknowledged.

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December 2020

  
(P. S. Mahar)  
Chairman  
Advisory Committee

## CERTIFICATE – II

We, the undersigned, member of Advisory Committee of **Ms. Neha Dhapola, Id. No. 54098**, a candidate for the degree of **Master of Technology in Civil Engineering with Major in Hydraulic Engineering** agree that the thesis entitled **“Chance Constrained Optimization of Transient Groundwater Flow considering Pump Characteristics”** may be submitted in partial fulfilment of the requirements for the degree.



**(P. S. Mahar)**  
Chairman  
Advisory Committee



**(R. P. Singh)**  
Member



**(Sandeep Gupta)**  
Member

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## LIST OF NOMENCLATURES/SYMBOLS

$W [u(i,j,k) ]$	: Well function of $u$ at at $k^{\text{th}}$ period, $j^{\text{th}}$ control point and $i^{\text{th}}$ discharging well;
$r_{ij}$	: Distance between the $i^{\text{th}}$ pump well and $j^{\text{th}}$ control point;
$S$	: Storage coefficient;
$T$	: Aquifer transmissivity;
$t_k$	: Time instant at the end of $k^{\text{th}}$ period;
$v$	: Expressed as a number between 1.0 to 9.9 with an appropriate exponent;
$\beta(i,j,k)$	: The unit response function for the $k^{\text{th}}$ period relating drawdown at $j^{\text{th}}$ control point to unit pumpage at the $i^{\text{th}}$ discharging well;
$\psi(i,j,k)$	: Function used to calculate well function;
$CV$	: Coefficient of variation;
$\sigma$	: Standard deviation of the parameter;
$\mu$	: Mean of the parameter;
$\eta$	: Pump efficiency (%);
$Q$	: Pump discharge ( $\text{m}^3/\text{s}$ );
$a, b, c, d$	: Constants;
$Pr$	: Probability;
$Z$	: A standard normal deviate corresponding to the normal cumulative distributive function of $\alpha(j,n)$ ;
$s^*(j,n)$	: Drawdown at $j^{\text{th}}$ control point at the end of $n^{\text{th}}$ time period, (m);
$E[s(j,n)]$	: Stochastic unit response function of drawdown at the control point $j$ at the end of the $n^{\text{th}}$ time period, derived from the Cooper-Jacob equation;
$Q(i,n-k+1)$	: Pumpage at the $i^{\text{th}}$ discharging well during the $k^{\text{th}}$ period, where $k \leq n$ , ( $\text{m}^3/\text{s}$ );
$\sqrt{\text{var}[s(j,n)]}$	: Standard deviation of drawdown at $j^{\text{th}}$ control point at the end of $n^{\text{th}}$ time period;

$\alpha(j,n)$	: Reliability at $j^{\text{th}}$ control point at the end of $n^{\text{th}}$ time period;
$F^{-1}[\alpha(j,n)]$	: A standard normal deviate corresponding to the normal cumulative distribution function of reliability $\alpha(j,n)$ ;
$m$	: Higher order term used in first order analysis;
$\bar{A}(i,j,k), \bar{B}(i,j,k)$	: Coefficient of function of mean transmissivity and storage coefficient;
$Q(i,n)$	: Pumpage at $i^{\text{th}}$ discharging well during period $n$ , ( $\text{m}^3/\text{s}$ );
$M$	: Total number of wells;
$N$	: Total number of time period;
$\bar{s}^*(j,n)$	: Maximum allowable drawdown at $j^{\text{th}}$ control point at the end of $n^{\text{th}}$ time period, (m);
$E(i,j,k)$	: Stochastic unit response function derived from the Cooper-Jacob equation;
$Q_{i,n}$	: Pumpage from $i^{\text{th}}$ well at the end of $n^{\text{th}}$ time period, ( $\text{m}^3/\text{s}$ );
$\bar{Q}_{i,n}$	: Maximum allowable pumpage at $i^{\text{th}}$ well at the end of $n^{\text{th}}$ time period, ( $\text{m}^3/\text{s}$ );
$\bar{D}(n)$	: Water demand during $n^{\text{th}}$ period, ( $\text{m}^3/\text{s}$ );
$\sigma_T$	: Standard deviation of the transmissivity, (same unit as the original data)
$\sigma_S$	: Standard deviation of the storage coefficient, (same unit as the original data)
$\eta_l$	: Allowable minimum efficiency, (%);
$\eta_u$	: Allowable maximum efficiency, (%);



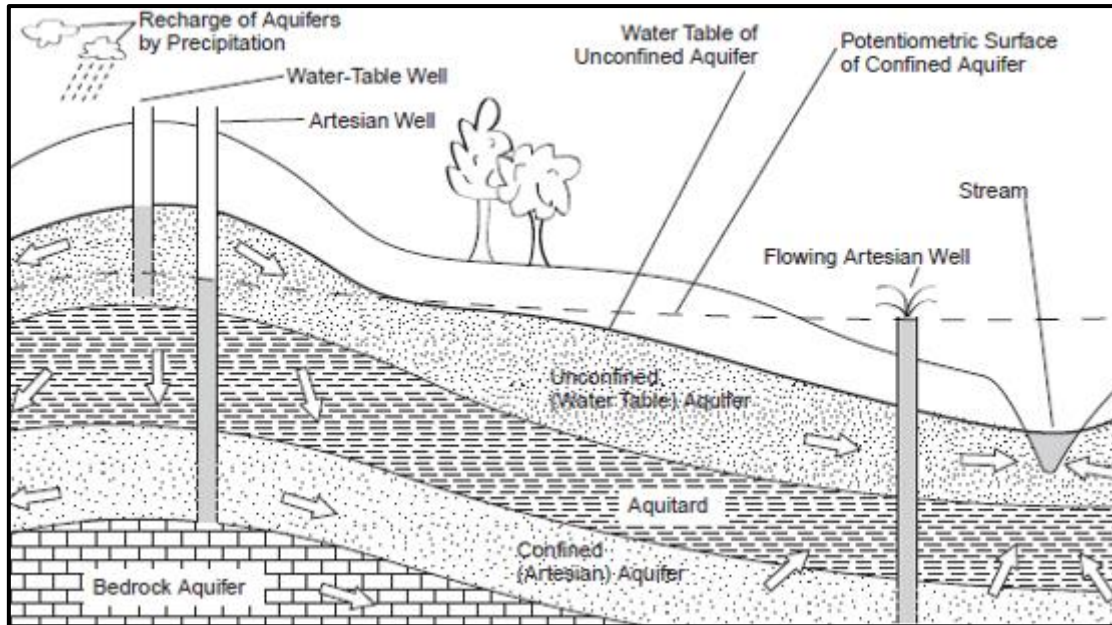
# *Introduction*



**1.1 General**

The term groundwater is typically used for water present beneath the earth's surface in soil pores and fully saturated geologic formations. Groundwater is an important feature of the natural environment. Groundwater is naturally replenishing from the surface through infiltration. When this recharge reaches the water table and is discharged naturally from the surface at springs and seeps, and can form oases and wetlands. The never-ending circulation of water between the ocean, atmosphere, and the land is called the hydrologic cycle. In a hydrological system, inflow occurs through the precipitation, in the form of rain or snowmelt, and in the form of streamflow, runoff, evaporation, and evapotranspiration, the outflow takes place. If we limit consideration to the utilizable freshwater resources (minus the icecaps and glaciers including South and the North Pole), global groundwater makes up about 30% of the world's fresh water supply, which is approximately 0.76% of the Earth's water, including oceans and permanent ice (**Gleick, 1993**). This makes it a major natural resource of fresh water that can safeguard against the paucity of surface water.

An aquifer is a geological formation that is capable of yielding economic quantities of water through wells. It must be porous, permeable, and saturated. The aquifers are classified as unconfined and confined aquifers. An aquifer is known as an unconfined aquifer when water flows directly between the surface and saturated aquifer zone. The water is under atmospheric pressure. It is also known as free aquifer, non-artesian aquifer because the water table serves as the upper zone of saturation in these aquifers. When an aquifer is confined between two very less permeable layers, often made of clay is called a confined aquifer. In these wells water level generally rises above the water table, hence it is considered that these wells exist under artesian flow conditions and known as artesian wells (**Freeze and Cherry, 1979**). The groundwater movement and types of the aquifer are depicted in figure 1.1.



**Figure 1.1 Groundwater movement and Aquifer types**

(Source: Groundwater Resource Availability, West Fork and White River Basin, 2002)

Groundwater is an abundant and quite valuable natural freshwater resource, but with the population explosion, its demand is increasing at an alarming rate. As the river water is getting polluted and being over-used, over-abstracted, it is not sufficient to meet the increasing demands. And the water table into the wells is also getting lower. For example, in the Punjab region of India, the groundwater level has been dropped 10 meters since 1979 with an accelerating depletion rate (**Lall, 2009**). As a consequence of extensive well pumping, there is a need for tapping well deeper to reach groundwater. It can cause harm to our ecosystems both aquatic and terrestrial and can lead to many calamities such as land subsidence, with associated infrastructure damage. In an engineering context, groundwater contributes to geotechnical problems such as land subsidence and slope stability.

To achieve optimal utilization of groundwater resources it becomes mandatory to prepare groundwater management models. As a source of water supply, the use of aquifers is increasing and as per the increasing usage, our knowledge of groundwater systems should also be increased. Groundwater development is a progressive process having three main stages. The first one is a survey for suitable aquifers. The second stage includes the measurement of hydrogeological parameters, the design and analysis

of wells, and the calculation of aquifer yields is called the evaluation stage. The last stage includes the development of optimal approaches for managing the resource is called the management stage. Being a quantitative science the groundwater hydrology has mathematics as its language or the very least it is quite impossible to overlook. So it requires the understanding of mathematics to enable the powerful tools of groundwater management. The classical studies of groundwater flow were based on the models developed for heat flow, electricity, and magnetism, as these models were related to the area of applied mathematics that can also be used for groundwater problems (**Freeze and Cherry, 1979**). There are two general approaches for solving mathematical models, both have their merits and demerits. The analytical approach is limited to homogeneous, isotropic, or very simple layered systems. And can't be used where there is a use of simple algebraic functions for water-table configurations. With the technical and computing advancement, the groundwater analysis also involved numerical analysis, which uses a different mathematical approach. And all the limitations can be removed by numerical solutions.

The determination of the maximum possible pumping rates, that are also compatible with the hydrogeological environment, is the primary objective of most groundwater studies. The compatibility requirements should be in terms of the balance between the groundwater pumping and adverse changes induced by pumping. Depression in water levels is the most ubiquitous change produced due to pumping. Hence while modeling a problem, groundwater yield can be expressed in terms of the maximum rate of pumping, keeping the water-level declines within acceptable limits. If our unit of study is aquifer then we can consider aquifer yield. Aquifer yield can be defined as the maximum rate of withdrawal that can be sustained by an aquifer without causing an acceptable decline in the hydraulic head in the aquifer. Aquifer yield is highly dependent on the number and spacing of the wells tapping the aquifer. If all the wells in a highly developed aquifer pump at the rate equal to their well yield, the aquifer yield will likely be exceeded.

The determination of hydraulic conductivity is very important for groundwater problems. **Camp (1977)** obtained hydraulic conductivity (K) values ranging from  $<0.001$  m/d to 0.12 m/d in the same type of soil from laboratory experiments. **Anderson and**

**Cassel (1986)** reported that the coefficient of variability of K-values determined from core samples in a Portsmouth sandy loam varied between 130 to 3300%. Pumping tests provide one of the most important procedures in practical groundwater investigations. Thus a pumping test is required to provide data from which the transmissivity and hydraulic conductivity of the tested aquifer can be calculated. And it can provide information about the performance and efficiency of the well being pumped.

## 1.2 Groundwater modeling

In analyzing many groundwater problems, one of the major tools is groundwater modeling. Groundwater models are very suitable for the study of a hypothetical aquifer in which various type of flow behavior is examined and are also helpful in reconnaissance studies, for interpretive studies, and predictive studies before actual study.

### 1.2.1 Types of groundwater model

Groundwater models are generally classified into four categories which are described as following:

**Groundwater flow model:** This model provides the solution of the problem concerned with water supply, which is normally represented by a single equation, usually in terms of hydraulic head.

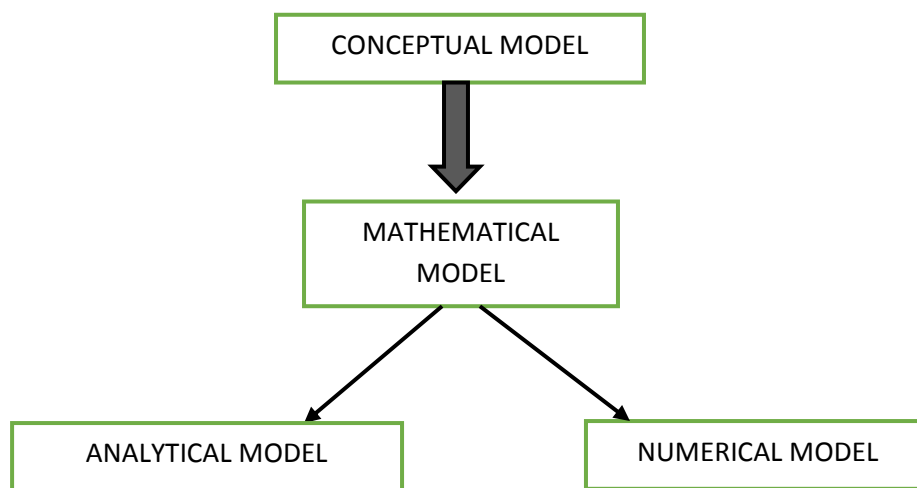
**Solute transport model:** This model involves the problems of water quality, so an additional equation (s) to the groundwaterflow equation must be solved for the concentration (s) of chemical specie (s).

**Heat transfer model:** In this model problems involving heat are considered, which requires the addition of an equation in terms of temperature, to the groundwater flow equation.

**Deformation model:** It comprises a model having a set of equations that describe aquifer deformation combined with a groundwater flow equation.

The simulation of groundwater systems generally discussed with the development and operation of the model, whose behavior is analogous to actual aquifer behavior. The model may be physical, electrical analog, or mathematical. Alternative

model divisions could also be found in **Karplus (1976)** and **Thomas (1973)**. A mathematical model is just a group of equations consisting of partial differential equations in conjunction with acceptable boundary and initial conditions that are subjected to definite assumptions, describes the physical process active in the aquifer. Mathematical models could also be statistical, deterministic, or the combination of the two. In Figure 1.2, the logic diagram for developing a mathematical model is presented. The first step is to understand how the system operates and a conceptual model is created and also the second step is to develop a mathematical model by forming governing equations. After formulating the mathematical model, the next step is to obtain a solution using any of two general approaches i.e., analytical or numerical.



**Figure 1.2 Logic diagram for developing a mathematical model**

(Source: Mercer and Faust, 1980)

The groundwater flow equation can be further simplified into general equations that are agreeable to an analytical solution. The equations and solutions to these subsections are denoted as analytical solutions. Alternatively, for problems where the simplified analytical models can't be used for solving problems, then the partial differential equations can be approximated numerically as finite-difference techniques or finite-element method. In this method, a computer program is formulated and equations are solved. Analytical methods such as type curve analysis require several simplified assumptions and are relatively easy to use than numerical models, which are more difficult to apply, are not limited by any assumptions necessary for the analytical

methods. In describing poorly understood systems and in classifying data, pure statistical methods are useful. Each approach has its advantages and disadvantages. Whichever approach is taken particularly based on the aquifer problem. The final step is to convert the mathematical results back to their physical meaning. And these results must be effective in answering the hydrologic questions and should be compatible with reality (**Mercer and Faust, 1980**).

The groundwater models are further subdivided into those describing porous media and fractured media. Some groundwater management models can be used to obtain optimal values and decision parameters like pumping rates and cost. These models combine flow models with linear programs. If we have problems concerning aquifers having irregular boundaries, heterogeneity, and high variable pumping and recharge rates, then the most appropriate model is a numerical model. To characterize uncertainty in model parameters and to determine aquifer parameters, these models can be combined with statistical techniques. Groundwater models associated with uncertainty are generally difficult to solve hence it requires a mathematical model which explicitly combines the uncertainty. Some researchers used variations in hydrologic factors such as aquifer parameters while some used variations in economic factors such as pumping costs and crop prices. **Maddock (1972, 1973, and 1974)** used the algebraic technological function to solve the model problems with uncertainties.

Uncertainty can also be incorporated into the groundwater management model using a chance-constrained method of optimization. Some researchers as **Tung (1986, 1987)**, **Wagner and Gorelick (1987)**, **Yeh and Wang (1987)**, **Hantush and Marino (1989)**, **Wagner et al. (1992)**, **Morgan et al. (1993)**, **Datta and Dhiman (1996)** and **Chang et al. (2007)** converted deterministic model into a stochastic model employing chance-constrained programming. When distributed parameter simulation models are employed stochastically, the effect of uncertainty and random nature of surface flow can be observed through sensitivity analysis (**Bredehoeft and Young, 1972; Maddock, 1974**). The stochastic groundwater management model formulated through chance-constrained programming is not mathematically operational, to solve the model we need to transform the chance-constrained equation into their deterministic equivalent. Generally, the aquifer parameters such as transmissivity are derived from

the pump well test, this test provides the averaged value for aquifer parameters as the aquifer volume and area is very large. Hence transmissivity can be treated as a random variable and not as a deterministic one. As per its consequences, the constraints also become random and their compliance at each control point can't be certain. It would be more realistic and appropriate to examine constraint performance probabilistically. In a stochastic approach, it would be operationally feasible to impose restrictions on allowable risk or reliability requirements on constraint performance. It is mandatory to assess all the statistical properties of variables at various control points **Tung (1987)**. **Wanakuleet al. (1986)** developed an optimization-based model to determine optimal pumping and recharge with groundwater flow equations as constraints. For groundwater management, pumping tests are performed by centrifugal pumps. Pump characteristics influence the flow conditions and discharge, so it would be reasonable to include them in the optimization model.

The groundwater management model with the inclusion of pump characteristics and uncertainties within the same model have not been considered hitherto. Therefore it is logical to consider these points while formulating the model. In this thesis, the Influence function or Unit response function approach is used to relate seasonal pumping from the wells to the drawdown at the wells. The objective function is to develop the optimal pumping of a largely confined aquifer. The Cooper-Jacob equation is used to develop the model to obtain the unit response function that can be related to drawdown at each control point. The final chance-constrained expression, which specifies the reliability requirements of system performance is linearized into its deterministic equivalent. Then for the cases below: (1) Constraints imposed on efficiency of pumping well, drawdown at control points and pumping rates, (2) Inclusion of uncertainties into aquifer parameters (transmissivity and storage coefficient), (3) In presence of different reliability requirements (90%,95% and97.5%).

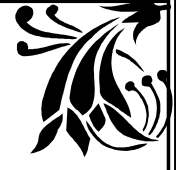
### 1.3 Objectives of the Thesis

The main objective of this thesis is to develop a chance-constrained programming model for optimal quantitative management of pumping from a confined aquifer. The specific objectives of this study are:

- ❖ To develop a chance-constrained optimization model for transient flow through a confined aquifer using pump characteristics to maximize total the pumping rate from all wells.
- ❖ To illustrate the applicability of the developed optimization model with example and study the effect of randomness in aquifer parameters, with different reliabilities.
- ❖ To investigate the consequences of restrictions imposed on pump characteristics, pump efficiency, pumping rates, and drawdown on optimal pumping rate.

## 1.4 Organization of the Thesis

The work in this thesis has been presented in five chapters comprising the current introductory chapter. In chapter 2, the pertinent works regarding the optimal groundwater modeling and management schemes and the literature concerning chance-constrained programming in the confined aquifer has been reviewed. The motivation for the present work has also been discussed in this chapter. Chapter 3 contains model description, and various governing equations of groundwater flow and chance-constrained model development. Chapter 4 presents the application of the developed model for confined aquifer with the help of an example and detailed discussion of results. Summary and conclusion drawn are presented in chapter 5.



*Review  
of  
Literature*



**2.1 General**

In arid and semi-dry regions, due to the high inconsistency of surface water, the groundwater becomes the vital natural resource. Many groundwater resources are contaminated and on the verge of depletion due to mismanagement hence groundwater management has become indispensable. With the advancement in knowledge on geophysical subsurface flow phenomena and computational capability, it becomes practical and feasible.

Groundwater management basically divided into two categories, simulation and direct optimization. The simulation model is tedious and necessitates trial and error. Direct optimization model is less time consuming as it involves some algorithms that automatically seek out optimal. This model is further classified as lumped parameter model and distributed parameter model. The model with temporal allocation of water is computationally simple and comes under lumped parameter models. The models which consider both the temporal and spatial allocation of water are classified as distributed parameter model. Distributed parameter models can be solved either by response matrix approach or embedding technique. Response matrix technique uses linear system theory and unit response function (influence function) and embedding technique directly includes flow equations as constraints.

Generally, the study of groundwater management is carried in the environment with uncertainties. There is always an uncertainty about aquifer parameters due to: (1) Geological variability, (2) Lack of aquifer data, (3) Lack of knowledge about flow characteristics, transport characteristics and aquifer systems, (4) Economic considerations. There are some other factors such as engineering designs, cost and operation of the system. Hence, it is required to incorporate uncertainty into management model for proper functioning of the model and predicting system behavior with certainty. An important area of groundwater management modeling research involves formulation and solution of management problems with uncertainties. In recent years, uncertainties in groundwater models are incorporated by means of

stochastic optimization techniques. The stochastic method has two approaches, in the first approach uncertainty is incorporated directly into management model through mean and covariance. The second approach uses Monte Carlo analysis which involves a series of realization of uncertain parameters. In stochastic groundwater management the system involves a number of random elements.

**Gorelick (1983)** divided groundwater management model into three categories: groundwater hydraulic management, ground quality management and groundwater policy evaluation and allocation.

The recent advancements in formulating and solving groundwater models is described by **Wagner (1995)**, he divides a groundwater management model in its basic form, which has following characteristics: (1) It is stochastic, with the uncertainty associated with the aquifer simulation model. (2) It contains both discrete and continuous decision variables. (3) It is nonlinear with respect to decision variables. (4) It requires solution of partial differential equations relating groundwater flow and transport. Recent studies in groundwater management involve stochastic management techniques to design groundwater hydraulic and quality management studies.

**Tung (1986), Wagner and Gorelick (1987)**, applied nonlinear chance constrained programming for groundwater management. There are two stages of solving chance constrained programming models, first one involves combination of two-dimensional steady state groundwater flow and transient contaminant transport simulation model with nonlinear least square regression and optimal estimates of uncertain model parameters are calculated. In second stage the nonlinear chance constrained program is solved. In this stage, probabilistic interpretation of model prediction is done which is defined as the function of pumping decisions and parameter uncertainty. It recognizes the need of overdesigning the model in high reliability requirement. In this case it was determined that to achieve a reliability of 90%, an overdesign up to 27% was needed.

## **2.2 Simulation Approach in Groundwater Management Model Analysis**

**Bredehoeft and Young (1970)** presented a groundwater management model using simulation approach. They stated that the simulation approach is better than dynamic and mathematical programming and assumes that drawdowns are uniformly

distributed through the basin, in response to withdrawals. The earlier efforts for providing solutions for temporal allocation of groundwater model were based upon simplified models, the author have used the digital model by replacing the past electrical analog model, which can be easily solved by computers. In this model nonhomogeneous parameters such as transmissivity and storage coefficient can also be incorporated for future extension of this work.

**Bear (1979)** explained the simulation model based on partial differential equations of groundwater flow and solute transport.

**Gorelick (1983)** incorporated the simulation model into groundwater management model by two methods that is embedding technique and response matrix approach. He used response matrix approach to develop unit responses from the external groundwater simulation model. Simulation model not directly provides optimal solution, but they offer an approximate method to produce the value of an objective function.

### 2.3 Direct Optimization Approach in Groundwater Management

**Maddock III (1972)** describes that it is difficult to couple management model explicitly with the distributed parameter groundwater simulation models, which seeks optimization based on economic objective. The author represented his 2-D linear partial differential equation model by producing an algebraic technological function (unit response function). This function can be related to well hydraulics and explicitly includes the behavior of ground water systems such as distances between wells, well radii, irregular shaped boundaries, or nonhomogeneous flow parameters in the management model.

**Maddock III (1973)** explains an approach in a primitive way, to determine the pertinent data with the help of a management or planning model. The author dealt with uncertainty by treating them as random variable and inferred that the uncertainty lies to the more variable term. And calculation of diffusivity coefficient could be critical in decision process.

**Maddock III (1974)** developed a deterministic model for developing operating rules, when supply sources and demands are stochastic in nature. He assumed a single

stream and single aquifer. These rules are helpful in conjunctive use of water. He incorporated algebraic technological function with well pumping, drawdowns, demands and stream flow interactions considering these factors as uncertainty.

**Wanakule *et al.* (1986)** developed a methodology that couples simulation model with optimization model for determining optimal pumping and recharge in artesian and non-artesian aquifers.

## 2.4 Linear Programming Approach in Aquifer Management

**Deninger (1970)** formulated a linear programming management model for minimizing the cost of water production and to increase the total discharge. For obtaining drawdown response matrix, the author used non equilibrium formula (**Theis, 1935**).

**Alley *et al.* (1976)** used approximation in two dimensional artesian aquifer where the flow was transient and steady-state, using finite differences. The authors formulated a linear programming model by combining linear differential equations with linear physical management constraint and linear objective function, and found the optimal solution and determined optimal well distribution and optimal pumping rates.

**Aguado *et al.* (1977)** explained a linear programming management model with the help of sensitivity analysis to determine the extent, to which the variations in input data and aquifer parameters could affect the optimal solution. This model uses finite element and finite difference method, in which groundwater flow equations are taken as constraints. The results showed that the optimal steady state solution is most sensitive to the hydraulic conductivity at or near the aquifer boundaries.

**Bathla *et al.* (1980)** formulated a generalized linear management model for analyzing several types of ground water problems. The authors developed a procedure to predict aquifer response behavior due to pumping. They used historical pumping rates, groundwater levels, stream stages and estimated storage coefficients as model inputs and the aquifer transmissivity and predicted groundwater levels as model outputs. They investigated the variations in storage coefficients and carried sensitivity analysis. They found that the technique was insensitive even to large errors estimated in storage coefficient and better than conventional numerical models of aquifer

system as the computational time required was very less comparatively. It computes the aquifer transmissivity and storage coefficient therefore those expensive pumping tests can be skipped which are used to compute those parameters.

**Heidari (1982)** did hydrogeological investigation of a valley in Kansas and represented by a linear system model with linear programming approach. The objective function of the model is to maximize the total pumpage, from the system with physical capabilities and institutional constraints. The author then compared the solutions with numerical and analytical solutions and provided a guide plan for groundwater management over the next ten years in that area.

## 2.5 Nonlinear Optimization in Groundwater Management Model

**Flores *et al.* (1978)** developed an optimal water management model in stream connected aquifers using conditional probability approach. In this paper the authors took linear reservoir model, and estimated the effect caused by variations in parameters. They used simple lumped parameter stochastic model, and a drawdown correction model is used, which proved very crucial. In the model chance constrained equation has been introduced and the expected values of discounted costs was minimized by linear decision rule, which is then solved by an iterative linear programming scheme. They concluded that the stochastic effects are very important in determining expected cost than arriving operating policy.

**Gorelick *et al.* (1984)** developed a planning model to determine the optimal design of reclamation schemes for contaminated groundwater. This model was applied in two stages first has steady state aquifer reclamation and second considers the transient flow and transport. This model combines the SUTRA (solute transport) model and distributed groundwater model with a MINOS (non- linear optimization model).

**Das and Datta (1999a, 1999b, and 2000)** demonstrated the application of nonlinear programming in the series of three series of papers. They formulated a highly nonlinear problem of sea water intrusions in coastal aquifers. They combined simulation model with optimization model.

## 2.6 Stochastic Conceptual Analysis of Groundwater Problem

**Freeze (1975)** represented non uniformity of a porous medium, stochastically through the concept of frequency distribution of three basic hydrogeological parameters. The author analyzed two one dimensional saturated flow problems: steady state flow and transient flow in a clay layer. He gave the term “equivalent” because each flow parameter can possibly have a single value for each flow parameter. But for the transient flow it is not possible to have “equivalent” value for each flow parameter, due to large uncertainties. He determined that as the level of uncertainty increases it will affect our predictions for the groundwater models.

**Bakr *et al.* (1978 b)** have taken a statistically homogeneous porous medium and derived a stochastic differential equation for one-dimensional flow. They illustrated the application of stochastic analysis with the example of two network design problems and concluded that the correlation length of hydraulic conductivity and measurement of errors has an important and vital role to play in the stochastic model studies. As the standard deviation of logarithm of conductivity is large, correlation length of hydraulic conductivity will be larger and head measurement errors cause a significant variance in discharge estimate.

**Smith and Freeze (1979)** conducted stochastic analysis for a steady state two dimensional groundwater model in a bounded region. They used the Monte Carlo techniques, and divided the flow domain into squares. Each discrete block represents the realistic spatial variation in hydraulic conductivity. The simulation results showed that the uncertainties in hydraulic head distribution output is influenced by spatial trend in mean hydraulic conductivity.

### 2.6.1 Spatial Variability in Stochastic Groundwater Management (Spectral Techniques)

**Bakr *et al.* (1978a)** represented the complex variations in aquifer transmissivity as spatial variability in stochastic conditions using covariance function. The stochastic differential equations describing flow through porous media were solved using theory of spectral analysis by assuming statistical homogeneity, and random variations in hydraulic conductivity. The results of their study showed that the stochastic approach for solving aquifer heterogeneity is effective and realistic.

**Gelhar *et al.* (1979)** illustrated the use of stochastic techniques in analyzing applied subsurface mass transfer model under field conditions. They showed the variations in hydraulic conductivity by using the term covariance, and represented it by spectral method. They took the conductivity as a stochastic process and developed the solutions for flow equations and demonstrated that for large time the longitudinal dispersivity approaches to a constant value, and depends upon stochastic properties of the medium. This approach proves to be good, where probabilistic approach should be applied on a large area.

**Dettinger and Wilson (1981)** developed numerical groundwater model, and analysed it by applying the first order and second order analysis. And estimated the mean, variance-covariance properties of piezometric head predictions. They used the compact matrix calculus notation and Taylor series expansion for calculation and concluded that the results of first order are alike to its deterministic model. The authors developed a probabilistic approach for solving the model to develop a situation where data availability is a limit and we have to predict aquifer behaviour. The authors also used variability in input data such as initial or boundary conditions, one-, two- or three-dimensional flow in bounded domains. But they did not cover the temporal probabilistic variations.

**Gelhar (1986)** showed how we can estimate the large scale parameters and reliability of three dimensional numerical simulations from stochastic analysis. He took the heterogeneity of aquifer into consideration and represented it in terms of randomly variable hydraulic parameters, which will then appear in partial differential equations representing flow process. The results were represented by probabilistic distribution or by statistical moments. He showed the importance of variance and spatial correlation scales of log hydraulic conductivity.

**Wagner and Gorelick (1989)** explained the importance of spatial variability of hydraulic conductivity in groundwater management models. They explicitly incorporated the uncertainty with optimization model for aquifer remediation strategies. They first solved the stochastic inverse model for aquifer remediation strategy and generated numerous maps of spatially variable hydraulic conductivity based on statistical characterization and conditional simulation. Then they solved the

optimization model. They presented two management model formulations. They provided relation between pumping costs and reliability. First one solved nonlinear simulation-optimization problem simultaneously for hydraulic conductivity realization which is termed as multiple realization management model, second one solved the nonlinear equations individually and is termed as Monte-Carlo management model. Each model was linked with stochastic inverse model.

**Massmann and Freeze (1987)** explained how resource allocation is sensitive to stochastic parameters of hydraulic conductivity and concluded that hydraulic conductivity proves to be the most valuable in reducing the uncertainties.

## 2.7 Chance Constrained Approach in Groundwater Management

**Tung (1986)** presented a management model that explicitly considers the random nature of transmissivity and storage coefficient, for determination of optimal pumping pattern in a well field subjected to a specified system performance reliability requirement. Application of the model was demonstrated using a hypothetical example through which factors affecting model results were investigated. The probability density function of random drawdown at each control point was taken as normally distributed. For easy application of chance constrained equation it is transformed into its deterministic equivalent to make it mathematically operable. As the reliability requirement and uncertainty level of aquifer properties decreases, the total maximum pumpage increases. The model results were found to be insensitive to the value of uncertainty of storage coefficient and quite sensitive to the uncertainty value of transmissivity and it could be concluded that the value of storage coefficient can be taken as deterministic.

**Tung (1987)** divided the groundwater management methodologies into two categories: simulation and optimization. He considers the randomness in aquifer parameters in a chance constrained framework, and further extended to multi-objective optimization framework. In that framework he showed the explicit relation between optimal pumpage and performance reliability of the system and then applied to a hypothetical steady, homogeneous, nonuniform, confined hypothetical aquifer. He concluded that at a given uncertainty level, as the reliability requirement decreases optimal pumping increases and at a given reliability level, as uncertainty increases total maximum allowable pumpage decreases.

**Wagner and Gorelick (1987)** gave a non-linear and statistical optimization for groundwater management, in which they incorporated parameter uncertainty into decision making process. The management model aims to best well selection and pumping-recharge rates, for meeting water quality assurance. Their whole work falls into two categories: parameter estimation and contaminant transport in simulation model. They used non-linear stochastic method for problem formulation, as deterministic model can't be used. This approach will be used to future groundwater management problems.

**Yeh and Wang (1987)** conducted their study on confined aquifer and estimated dispersivity by solving solute transport model and finite element method. They carried numerical experiments for parameter identification for three-dimensional solute transport model, which has unidirectional, steady and nonuniform flow field. Using a modified Gauss-Newton algorithm for least square minimization and sensitivity coefficients were calculated by solving sensitivity equations. For assessing the reliability of estimated parameters, statistical procedure was used.

**Tung (1987)** explained effects of parameter uncertainty on optimal risk based design of hydraulic structures. He also examined the effect of inherent hydrologic and parameter uncertainty.

**Hantush and Marino (1989)** developed a chance-constrained optimization model which explicitly considers the variations in aquifer transmissivity and specific yield of unconfined aquifer. They considered a hypothetical model and sensitivity analysis was carried by varying probability requirement for 30 days planning period. The optimal solution of the model was solved using the MINOS 5 package. They used the probability variations as 90%, 95% and 97.5%, and plotted a graph between log hydraulic conductivity and optimal pumpage in cubic meter per second and it was clearly visible from the graphs that the optimal pumping decreases in nonlinear fashion as the probability increases. The values of coefficient of variation for log hydraulic conductivity were taken from 0.017 to 0.056, and for log specific yield were taken as -0.31, -0.29, -0.27 and -0.23. They concluded that the results were highly sensitive to log hydraulic conductivity for coefficient of variation greater than 0.05 and higher probability levels, and insensitive to log specific yield.

**Wagner *et al.* (1992)** gave a model that explains how stochastic optimization is best suited for incorporating uncertainties into management model.

**Morgan *et al.* (1993)** developed a new chance constrained technique called MICCP (Mixed Integer Chance Constrained Programming) that does not require pre knowledge of distribution. This technique considers the uncertainties into all linear programming constraints. Their objective was to find the globally optimal trade off curve for maximum reliability versus minimum pumping objective. The results showed that the increase in standard deviation (0.6 to 0.8) leads to the decrease in the performance of MICCP. This technique is different from the conventional chance constrained programming. In previous chance constrained modeling chance constraints are converted into deterministic equivalents and then solved for the objective values. In MICCP all the uncertainties are taken into right hand side which automatically converts them into deterministic equivalent.

**Datta and Dhiman (1996)** developed a chance constrained model for optimization and solved using MICCP. They designed a groundwater quality monitoring network for pollutant transport. They considered tritium as the radioactive pollutant. In saturated zones, the uncertainties are incorporated in model for prediction of pollutant movement. Nonlinearities were accommodated through piecewise linearization scheme in management model. The cumulative distribution functions of the actual spatial concentrations were included in management model. The authors solved two mathematical models, one a simulation model and other an optimization model. Simulation model predicted the location of pollutant w.r.t. time and space and chance constrained optimization model determined optimal location of monitoring wells and maximum number of such wells.

**Chang *et al.* (2007)** developed a stochastic groundwater management model considering land subsidence. The authors firstly developed a deterministic model by using response matrix technique and one-dimensional consolidation equation. The unit response technique is used to quantify hydraulic conductivity. The deterministic model was converted into stochastic form by chance constrained programming. The first order variance method was adopted to analyze the uncertainties. The objective of the model was to determine optimal pumping subject to constraints that drawdown and land subsidence do not exceed the allowable values of specified reliabilities. The conclusion

of their research can be applied to regional groundwater resource management in conjunction with controlling land subsidence.

## 2.8 Analysis of Multiple Well Pumping Test Data

**Thiem (1906)** conducted a study on steady state flow conditions and in his model aquifer had been pumped long enough that equilibrium had been reached (drawdown does not change with the time). He concluded that the drawdown has been influenced by well losses screen and pump intake.

**Theis (1935)** had taken the transient flow conditions in compressible and completely elastic aquifer and showed that the early drawdown data may not closely represent theoretical drawdown data. He derived an equation for determining coefficient of transmissibility and storage coefficient from the observed drawdown, by superimposing the observed data on ‘type curve’.

**Hantush (1964)** performed pumping test with partial penetration where there is an anisotropy in horizontal plane and assumed that there is no vertical flow at the top and the bottom of the aquifer.

**Cooper and Jacob (1946)** assumed the aquifer to be compressible and completely elastic and considered the flow conditions to be transient. They determined the coefficient of transmissibility and storage coefficient by a straight line graphical method in which there is no requirement of ‘type curve’. This method can be applied to both the artesian and non-artesian aquifers under favorable conditions, for determining their capacities to transmit water under hydraulic gradient and to yield water under low artesian pressure or when water table is low. This test can be applied to both the single and multiple discharging wells.

## 2.9 Advancement in Groundwater Management

**Demirbas *et al.* (2008)** used the combined simulation-optimization approach for searching optimal decision of dewatering of an unconfined aquifer. The groundwater management problem converted into linear integer programming model by generating response matrix with the help of MODFLOW software. Their study was focused on a hypothetical dewatering area, in which nonlinear response due to pump characteristics has been examined. Genetic algorithm is used as an optimization tool to

compare the results. Results showed that the linear optimization in combination with MODFLOW software has proved an efficient method in groundwater management.

**Liu et al. (2013)** presented a scientific model of cloud computing for integrated stochastic analysis of uncertainty in groundwater. They took the first study area in Arizona where the effects of impact of groundwater pumping was evaluated and the second study area in Texas where sensitivity analysis was carried on regional groundwater availability model. The results of this study assisted in groundwater planning and sustainable management. This research used a public cloud, MODFLOW on Azure to develop a groundwater service.

## 2.10 Motivation for Present Work

In preceding sections the analysis of literature divulges that various groundwater management models have been developed for quantitative and qualitative management. The application of optimization models has significantly increased in recent years with the technological advancements. Several optimization methods such as linear programming, nonlinear programming, dynamic programming, stochastic programming, chance constrained programming, genetic algorithm, fuzzy logic and cloud computing have been used.

The literature reviewed shows the pump characteristics have not been considered in the chance constrained groundwater management models. Therefore, in this study a chance constrained groundwater management model with uncertainties using pump characteristics is developed. This model is then applied to a hypothetical study area to optimize total pumping. This study will assist in understanding the effect of uncertainties in aquifer parameters under different reliability conditions and pump efficiency in total discharge.

## 2.11 Summary

This chapter clearly tells about necessity of managing groundwater as it has become meagre due to technical advancements. There are different chance constrained optimization models that have been developed with different conditions. This literature review shows the need for development of chance constrained model under different reliabilities considering the pumping characteristics.



*Materials  
and  
Methods*



### **3.1 General**

This chapter provides details of the materials and methodology used in this study which is applied to a hypothetical study area of confined aquifer having four wells and six control points. This study explains about the chance-constrained methodology adopted to obtain optimal discharge when there is randomness in aquifer parameters. The random aquifer parameters taken are transmissivity and storage coefficient, under various reliability requirements. The objective of this study is to maximize the pumping rates while applying constraints on allowable drawdown at different control points, efficiencies and pump characteristics. Cooper-Jacob equation is used to demonstrate the development of a chance-constrained management model based on Theise equation (**Theis, 1935**), which are mainly used to solve the problems related to transient groundwater hydraulics.

### **3.2 Materials Used**

The study area chosen is a hypothetical confined, homogeneous and nonuniform aquifer. A simple transient nonlinear chance-constrained programming model is developed to maximize the total discharge of pumping. The random characteristics of transmissivity and storage coefficient are also considered in this model. The Unit response method relates the state variables of aquifer systems (drawdown) to the management decision variables (pumpage). The unit response function ( $\beta$ ) can be obtained by simulation model described by **Heidari (1982)**.

The formulated chance constrained programming model is implemented by writing a program in LINGO 10.0 software. LINGO software is a comprehensive and less time consuming tool, designed to build and solve the Linear, Nonlinear (convex and nonconvex/Global), quadratic, quadratically constrained, semi-definite, stochastic, and integer optimization models faster, easier and more efficiently. LINGO includes Algebraic Modeling Language, Convenient Data Options, Model Interactively information from databases and spreadsheets. LINGO is equipped with fast and inbuilt

solver to select the appropriate formulation. LINGO also includes examples for Excel, C/C++, FORTRAN, Java, Visual Basic, Delphi, VB.NET, C#.NET and ASP.NET.

Microsoft Excel 2010 spreadsheets were used to calculate time dependent drawdown response function, coefficient that are function of the mean transmissivity and storage coefficient and stochastic unit response function derived from the Cooper-Jacob equation. Microsoft Corporation for Windows, Android, iOS and macOS built and developed the Microsoft Excel software. This software enables the users in calculating organizing and manipulating the data by formulas with a system of spreadsheets having rows and columns. Since the launch of its 5<sup>th</sup> version in 1993, it has become a broadly used spreadsheet program. Generally it comes along with Microsoft Office package and also compatible with other Microsoft Corporation applications. An excel file stores numerous worksheets into a single workbook(**Wikipedia**).

### **3.3 Unsteady flow in confined aquifer and governing equations**

In a confined aquifer, when a well starts discharging, the water from the aquifer releases in the form of cone of depression of piezometric surface. The following assumptions are considered to develop the stochastic groundwater management model for confined aquifer:

1. The aquifer is homogeneous, nonuniform, and leaky having infinite horizontal extent.
2. The flow pattern considered is radial in the aquifer.
3. Wells are assumed to fully penetrate the entire aquifer thickness.
4. Flow assumed is unsteady.
5. Piezometric head is assumed to be uniform throughout the entire aquifer.
6. Water is released instantaneously from the storage, by ignoring the hydraulic head.
7. Very small diameter pumping wells are taken, to neglect the storage in the wells.

### 3.3.1 Unit Response Function

Under the aforementioned assumptions, the unit response function can be obtained from the well functions (**Maddock III, 1974 and Bathla et al., 1976**). A non-equilibrium formula was derived by (**Theis, 1935**), considering the analogy between the groundwater flow and the heat conduction under steady state conditions. In groundwater hydraulics, he introduced the concept of time. The well function can be written as:

$$W[u(i, j, k)] = \int_{u(i, j, k)}^{\infty} \frac{e^{-v}}{v} dv \dots\dots\dots (3.1)$$

Where,

W [u] = Well function of u;

The value of the function, u (i,j,k) (dimensionless) is given as:

$$u(i, j, k) = \frac{r_{ij}^2 S}{4 \pi T t_k} \dots\dots\dots (3.2)$$

Here, i, j and k denote k<sup>th</sup> period, j<sup>th</sup> control point and i<sup>th</sup> discharging well;

r<sub>ij</sub>= distance between the i<sup>th</sup> discharge well and j<sup>th</sup> control point, (m);

S = storage coefficient, (dimensionless);

T = aquifer transmissivity, (m<sup>2</sup>/day); and

t<sub>k</sub> = time instant at the end of k<sup>th</sup> period, (days).

The right hand side integral of Eq. 3.1 can be expressed as:

$$\int_{u(i, j, k)}^{\infty} \frac{e^{-v}}{v} dv = -0.57725665 - \ln v + v - \frac{v^2}{2 * 2!} + \frac{v^3}{3 * 3!} + \dots\dots\dots (3.3)$$

This exponential integral, cannot be integrated easily hence it is programmed using the infinite series. The value of v is indicated as:

$$v = \frac{1.87 r_{ij}^2 S}{T t_k} \dots\dots\dots (3.4)$$

To determine the value of W[ ] for a given value of v, it is necessary to express v as some number between 1.0 and 9.9, multiplied by 10 with an appropriate exponent **Wenzel (1942)**.

The Cooper-Jacob equation can be written as:

$$W [u(i, j, k)] = \ln \left( \frac{2.25 T t_k}{r_{ij}^2 S} \right) \dots\dots\dots (3.5)$$

Unit response function is also derived from Cooper-Jacob equation.

$$\beta(i, j, k) = \left\{ \begin{array}{l} \psi(i, j, k); k = 1 \\ \psi(i, j, k) - \psi(i, j, k - 1); k \geq 2 \end{array} \right\} \dots\dots\dots (3.6)$$

Where;

$\beta(i,j,k)$ = the unit response function for the  $k^{th}$  period relating drawdown at  $j^{th}$  control point to unit pumpage at the  $i^{th}$  discharging well; and

$\psi(i,j,k)$  is a function used in place of  $(1/4\pi T)W[u(i,j,k)]$  in equation (3.6).

### 3.3.2 Boundary conditions

In groundwater management models, generally two types of boundary conditions are used:

1. Dirichlet's boundary condition
  2. Neuman boundary condition
- **Dirichlet's boundary condition:** Generally the hydraulic head is a time dependent variable or it may have a constant value. These boundary conditions specify the value of hydraulic head with aquifer boundaries. For this study, as per the Dirichlet's boundary condition the value of head is taken as constant.
  - **Neuman boundary condition:** These boundary conditions represent flux across the boundary. For impervious boundary, the boundary condition as no flux or zero flux boundary.

### 3.3.3 Reliability requirements

The transient groundwater flow problem is solved with many levels of uncertainty in aquifer parameters and with different reliability ranges. The reliability requirement and uncertainty analysis is carried to observe the effect on optimal solutions. The coefficients of variation of transmissivity(CV of T) and storage

coefficient(CV of S)are used to define the degree of aquifer uncertainty. The range of coefficients of variation are taken as 0.2, 0.4, 0.6 and 0.8 for both transmissivity and storage coefficient.

The value of coefficient of variation (CV) can be calculated by the formula:

$$CV = \frac{\sigma}{\mu} \dots\dots\dots (3.7)$$

Where,

$\sigma$  = standard deviation of the parameter, (same unit as the original data);

$\mu$  = mean of the parameter, (same unit as the original data).

For chance-constrained formulation statistical properties of aquifer parameters are used in model. To meet the stated operation constraints in normal distribution, the mean values of the aquifer parameters are used which roughly correspond to 50% performance reliability of the model(Tung, 1987). The reliability used in the study is 90%, 95% and 97.5%.

### 3.3.4 Pump Characteristic Curves

When we represent pump behavior and its performance under different operating conditions through graphs it is termed as pump characteristic curve.Pump characteristic curves can be categorized into four types:

**Head-Discharge Curve:** This curve relates Head produced by the pump and Discharge (Flow rate) of the pump. Usually for the increasing discharge, head produced by the pump falls steadily.The head generated at zero discharge (i.e., when the pump is operating against a closed valve) is known as Shut-off head, used to determine the priming losses of pump.

**Energy Curve:**This curve is a graph of Brake Horse Power vs. Flow rate. The brake horse power also rises linearly, with the increase in discharge. Even at null flow the pump consumes minimum power and this power is used to develop the shut off head of pump.

**Efficiency Curve:** It is the plot of Efficiency vs. Flow rate. This curve starts from no flow conditions, and increases till a peak point is reached then starts falling back. The

ultimate point is called Best Efficiency Point (BEP), and corresponds to the flow rate having maximum efficiency.

**Net Positive Suction Head vs. Flow Curve:** This curve is usually flat till discharge reaches to corresponding BEP, then start rising sharply. To avoid cavitation, required NPSH should always be greater than NPSH at any time.

Figure 3.3, shows typical characteristic curves for centrifugal pump. The relation between efficiency and discharge is a third order polynomial and is represented as:

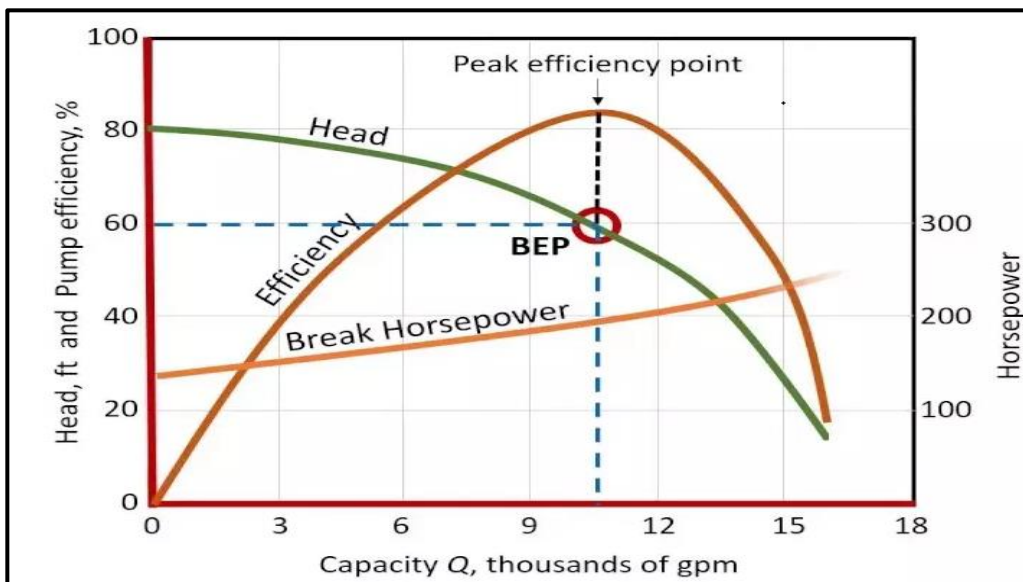
$$\eta = a + bQ + cQ^2 + dQ^3 \dots\dots\dots(3.8)$$

Where,

$\eta$  = pump efficiency, (%) ,

$Q$  = pump discharge, (m<sup>3</sup>/sec) and

$a, b, c$  and  $d$  = constants.



**Figure 3.1 Typical pump characteristic curves**

(Source: americanwatercollege.org)

### 3.4 Probabilistic consideration of model elements with uncertainty

In this study, a nonlinear chance constrained optimization model is developed because it seemed more realistic and correct to examine performance of constraints probabilistically. It is possible to impose restrictions on allowable risk or required

reliability of constraint performance in stochastic model. As we won't be able to solve the probabilistic expression mathematically, so in chance-constrained model, it is essential to calculate the statistical properties of random variables.

### 3.4.1 Derivation of stochastic groundwater management model

After considering the uncertainties in aquifer parameters the management model requires stochastic approach for finding optimal solution. This requires replacement of constraints from deterministic model by some stochastic ones. The use of linear programming technique is inappropriate as the deterministic equivalent of chance-constrained equation is nonlinear. Linearization procedure linearizes the nonlinear terms, this procedure is called quasi-linearization. This process requires the initial estimate of pumping rates from all wells during all periods, which are used to calculate the stochastic influence coefficient. As a result optimal solution is obtained. The objective is to maximize the total amount of groundwater that can be pumped from the system.

The study problem contains a hypothetical aquifer basin with four wells and six control points. Total drawdown at any control point is equal to the sum of drawdowns from individual pump wells. It is assumed that at each control point drawdown is normally distributed. And the statistical properties of random aquifer parameters are calculated using central limit theorem (CLT).

The actual chance-chance constrained model, under normality assumption can be represented as (Tung, 1986):

$$\Pr \left\{ Z \leq \frac{s^*(j, n) - E[s^*(j, n)]}{\sqrt{\text{var}[s^*(j, n)]}} \right\} \geq \alpha(j, n) \dots \dots \dots (3.9)$$

Where,

Pr = Probability;

Z = a standard normal deviate corresponding to the normal cumulative distributive function of  $\alpha(j, n)$ ;

$s^*(j, n)$  = drawdown at  $j^{\text{th}}$  control point at the end of  $n^{\text{th}}$  time period (m).

The value of  $E[s^*(j,n)]$  can be approximately written as:

$$E[s^*(j,n)] \approx \sum_{i=1}^M \sum_{k=1}^n \beta(i,j,k)Q(i,n-k+1) \dots\dots\dots (3.10)$$

$E[s^*(j,n)]$  = stochastic unit response function of drawdown at the control point  $j$  at the end of the  $n^{\text{th}}$  time period, derived from the Cooper-Jacob equation;

$Q(i,n-k+1)$  = pumpage at the  $i^{\text{th}}$  discharging well during the  $k^{\text{th}}$  period, where  $k \leq n$ , ( $\text{m}^3/\text{s}$ );

$\sqrt{\text{var}[s^*(j,n)]}$  = standard deviation of drawdown at the control point  $j$  at the end of the  $n^{\text{th}}$  time period (same unit as that of original data); and

$[\alpha(j,n)]$  = reliability at the  $j^{\text{th}}$  control point at the end of  $n^{\text{th}}$  time period.

After substituting Eq. (3.10) into Eq. (3.9), an equivalent equation to Eq. (3.9) can be written as:

$$\sqrt{\text{var}[s^*(j,n)]} F^{-1}[\alpha(j,n)] + \sum_{i=1}^M \sum_{k=1}^n \beta(i,j,k)Q(i,n-k+1) \leq s^*(j,n); \text{ for all } j \text{ and } n$$

..... (3.11)

Where,

$F^{-1}[\alpha(j,n)]$  = the value of standard normal deviate, it is calculated using the standard normal distribution table, which is the function of  $[\alpha(j,n)]$ ;

To linearize the nonlinear terms, the first order analysis (**Cornell, 1972**) is applied, which involves the expansion of equation (3.11) in Taylor series about any arbitrary pumping rate, say  $Q^0(i,n-k+1)$  for all  $i=1 \dots M$ ;  $k \leq n$  and  $n = 1 \dots N$ , as

$$f(Q) = \{\text{var}[s^*(j,n)]\}^{1/2} = f[Q^0] + \sum_{i=1}^M \sum_{k=1}^n \left. \frac{df(Q)}{dQ(i,n-k+1)} \right|_{Q^0} [Q(i,n-k+1) - Q^0(i,n-k+1)] + m$$

..... (3.12)

Where,

$f(Q)$  = standard deviation of drawdown at the control point  $j$  at the end of the  $n^{\text{th}}$  time period, in terms of unknown pumpage,  $Q$  more explicitly;

$f(Q^0)$  = the value of function  $f(Q)$  calculated by using arbitrary assumed pumpage,  $Q^0$ ;

$m$  = higher order term.

After neglecting the higher order terms and manipulation, the first order linear approximation can be expressed as:

$$f(Q) = \{\text{var} [s^*(j, n)]\}^{1/2} \approx \sum_{i=1}^M \sum_{k=1}^n \bar{D}(i, j, k) Q(i, n - k + 1); \text{ For all } j \text{ and } n \dots\dots\dots (3.13)$$

Where,

$$\bar{D}(i, j, k) = \frac{1}{f(Q^0)} \left\{ \left[ \sum_{i=1}^M \sum_{k=1}^n \bar{A}(i, j, k) \sigma_T Q^0(i, n - k + 1) \right] \bar{A}(i, j, k) \sigma_T + \left[ \sum_{i=1}^M \sum_{k=1}^n \bar{B}(i, j, k) \sigma_S Q^0(i, n - k + 1) \right] \bar{B}(i, j, k) \sigma_S \right\} \dots\dots\dots (3.14)$$

Here,

$\sigma_T$ = standard deviation of the transmissivity, (same unit as the original data);

$\sigma_S$ = standard deviation of the storage coefficient, (same unit as the original data);

and,  $\bar{A}(i,j,k)$  and  $\bar{B}(i,j,k)$  are coefficients that are function of the mean transmissivity and storage coefficient(**Tung, 1986**).

### 3.5 Optimization Model

In this study, a chance constrained nonlinear optimization model is developed for maximizing the total pumpage from a confined aquifer having transient flow with multiple number of pumping wells. Constraints are imposed on the maximum allowable drawdown at different control points. The pump characteristics are used as binding constraints. Different uncertainty levels of aquifer parameters and different reliability levels are used. The limitations on the desired efficiency are also imposed in the developed model.

#### 3.5.1 Objective function

The objective of the model is to maximize the sum of pumping rates from all the pumps over all durations of pumping. It can be written as:

$$\text{Maximize } Z = \sum_{i=1}^M \sum_{n=1}^N Q(i, n) \dots\dots\dots(3.15)$$

Where,

$Q(i,n)$  = pumpage at  $i^{\text{th}}$  discharging well during  $n^{\text{th}}$  period, ( $\text{m}^3/\text{s}$ );

$M$  = total number of wells; and

$N$  = total number of pumping time periods.

### 3.5.2 Constraints

The constraints incorporated in developed groundwater management model are maximum allowable drawdown at different control points, minimum desired efficiency of the pumps, maximum allowable pumping rate of different pumps and minimum amount of total discharge from all the pumps.

The allowable drawdown is computed using Cooper-Jacob formula for unsteady flow of groundwater through confined aquifer and the efficiency of the pump is calculated using the pump efficiency-discharge characteristic equation. These constraints can be written as:

1. Drawdown at each control point should not exceed the maximum allowable drawdown at each control point with a reliability

$$0 \leq s^*(j, n) \leq \bar{s}^*(j, n) \dots\dots\dots (3.16)$$

Where,

$\bar{s}^*(j, n)$  = maximum allowable drawdown at  $j^{\text{th}}$  control point at the end of  $n^{\text{th}}$  time period, (m).

The values of  $s^*(j, n)$  can be calculated as:

$$\sum_{i=1}^M \sum_{k=1}^n E(i, j, k) Q(i, n - k + 1) = s^*(j, n) \dots\dots\dots (3.17)$$

Where,

$$E(i, j, k) = \beta(i, j, k) + F^{-1}[\alpha(j, n)] D(i, j, k) \dots\dots\dots (3.18)$$

It is the linear approximation of deterministic equivalent of original chance-constraint.

2. Pump efficiency-discharge characteristic equation 3.8 should be satisfied at each pumping well for each period of pumping.
3. For efficient utilization of energy, the pump efficiency must be within the certain limit.

$$\eta_u \geq \eta \geq \eta_l \dots\dots\dots (3.19)$$

Where,

$\eta_l$ = allowable minimum efficiency of pump, (%); and

$\eta_u$ = allowable maximum efficiency of pump, (%).

4. The pumping rate from a well should be within specified range.

$$0 \leq Q_{i,n} \leq \bar{Q}_{i,n} \dots\dots\dots (3.20)$$

Where,

$Q_{i,n}$ = pumpage from  $i^{\text{th}}$  well at the end of  $n^{\text{th}}$  time period, ( $\text{m}^3/\text{s}$ ); and

$\bar{Q}_{i,n}$ = maximum allowable pumpage at  $i^{\text{th}}$  well at the end of  $n^{\text{th}}$  time period, ( $\text{m}^3/\text{s}$ ).

5. The total amount of pumpage from all the wells together should be more than demand.

$$\sum_{i=1}^M Q(i, n) \geq \bar{D}(n); \text{ for all } n \dots\dots\dots (3.21)$$

Where,

$\bar{D}(n)$ = water demand during  $n^{\text{th}}$  time period or minimum discharge of pumpage in the time period  $n$ , ( $\text{m}^3/\text{s}$ ).

6. Discharge at each pumping well should not exceed the upper and lower range of discharge.

$$Q_u \leq Q_{i,n} \leq Q_l \dots\dots\dots (3.22)$$

Where,

$Q_u$ = allowable maximum discharge at each pumping well, ( $\text{m}^3/\text{s}$ ); and

$Q_l$ = allowable minimum discharge at each pumping well, ( $\text{m}^3/\text{s}$ ).

7. Limiting the number of pumping wells, keeping the number of control points the same.

### 3.5.3 Resulting stochastic groundwater model constraints

The unit response equations have been derived in detail for different control points for deterministic model. In chance-constrained modeling, it is mandatory to estimate the statistical properties of random drawdown. The first order analysis of uncertainty is required and simplification of Taylor's expansion of drawdown about the mean values of  $T$  and  $S$  is also required for calculating constraint equations. It is needed to specify limits on allowable risk or required reliability of constraint performance in stochastic approach. To calculate drawdown values at different control points, the following expression is used.

$$s^*(j, n) = \sum_{i=1}^M \sum_{k=1}^n E(i, j, k) Q(i, n - k + 1) \dots \dots \dots (3.23)$$

#### 1. Control point 1

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i, j, k) + F^{-1}[\alpha(j, n)] \bar{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.24)$$

$$\begin{aligned} s^*(1,1) &= (\beta(1,1,1) + F^{-1}[\alpha(1,1)] \bar{D}(1,1,1)) Q(1,1) + (\beta(2,1,1) + F^{-1}[\alpha(1,1)] \bar{D}(2,1,1)) Q(2,1) \\ &+ (\beta(3,1,1) + F^{-1}[\alpha(1,1)] \bar{D}(3,1,1)) Q(3,1) + (\beta(4,1,1) + F^{-1}[\alpha(1,1)] \bar{D}(4,1,1)) Q(4,1) \\ &\dots \dots \dots (3.25) \end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i, j, k) + F^{-1}[\alpha(j, n)] \bar{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.26)$$

$$\begin{aligned} s^*(1,2) &= (\beta(1,1,1) + F^{-1}[\alpha(1,2)] \bar{D}(1,1,1)) Q(1,2) + (\beta(1,1,2) + F^{-1}[\alpha(1,2)] \bar{D}(1,1,2)) Q(1,1) \\ &+ (\beta(2,1,1) + F^{-1}[\alpha(1,2)] \bar{D}(2,1,1)) Q(2,2) + (\beta(2,1,2) + F^{-1}[\alpha(1,2)] \bar{D}(2,1,2)) Q(2,1) \\ &+ (\beta(3,1,1) + F^{-1}[\alpha(1,2)] \bar{D}(3,1,1)) Q(3,2) + (\beta(3,1,2) + F^{-1}[\alpha(1,2)] \bar{D}(3,1,2)) Q(3,1) \\ &+ (\beta(4,1,1) + F^{-1}[\alpha(1,2)] \bar{D}(4,1,1)) Q(4,2) + (\beta(1,1,2) + F^{-1}[\alpha(1,2)] \bar{D}(4,1,2)) Q(4,1) \\ &\dots \dots \dots (3.27) \end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i, j, k) + F^{-1}[\alpha(j, n)] \bar{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.28)$$

$$\begin{aligned}
s^*(1,3) &= (\beta(1,1,1) + F^{-1}[\alpha(1,3)]\overline{D}(1,1,1))\mathcal{Q}(1,3) + (\beta(1,1,2) + F^{-1}[\alpha(1,3)]\overline{D}(1,1,2))\mathcal{Q}(1,2) \\
&+ (\beta(1,1,3) + F^{-1}[\alpha(1,3)]\overline{D}(1,1,3))\mathcal{Q}(1,1) + (\beta(2,1,1) + F^{-1}[\alpha(1,3)]\overline{D}(2,1,1))\mathcal{Q}(2,3) \\
&+ (\beta(2,1,2) + F^{-1}[\alpha(1,3)]\overline{D}(2,1,2))\mathcal{Q}(2,2) + (\beta(2,1,3) + F^{-1}[\alpha(1,3)]\overline{D}(2,1,3))\mathcal{Q}(2,1) \\
&+ (\beta(3,1,1) + F^{-1}[\alpha(1,3)]\overline{D}(3,1,1))\mathcal{Q}(3,3) + (\beta(3,1,2) + F^{-1}[\alpha(1,3)]\overline{D}(3,1,2))\mathcal{Q}(3,2) \\
&+ (\beta(3,1,3) + F^{-1}[\alpha(1,3)]\overline{D}(3,1,3))\mathcal{Q}(3,1) + (\beta(4,1,1) + F^{-1}[\alpha(1,3)]\overline{D}(4,1,1))\mathcal{Q}(4,3) \\
&+ (\beta(4,1,2) + F^{-1}[\alpha(1,3)]\overline{D}(4,1,2))\mathcal{Q}(4,2) + (\beta(4,1,3) + F^{-1}[\alpha(1,3)]\overline{D}(4,1,3))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.29)
\end{aligned}$$

## 2. Control point 2

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.30)$$

$$\begin{aligned}
s^*(2,1) &= (\beta(1,2,1) + F^{-1}[\alpha(2,1)]\overline{D}(1,2,1))\mathcal{Q}(1,1) + (\beta(2,2,1) + F^{-1}[\alpha(2,1)]\overline{D}(2,2,1))\mathcal{Q}(2,1) \\
&+ (\beta(3,2,1) + F^{-1}[\alpha(2,1)]\overline{D}(3,2,1))\mathcal{Q}(3,1) + (\beta(4,2,1) + F^{-1}[\alpha(2,1)]\overline{D}(4,2,1))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.31)
\end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.32)$$

$$\begin{aligned}
s^*(2,2) &= (\beta(1,2,1) + F^{-1}[\alpha(2,2)]\overline{D}(1,2,1))\mathcal{Q}(1,2) + (\beta(1,2,2) + F^{-1}[\alpha(2,2)]\overline{D}(1,2,2))\mathcal{Q}(1,1) \\
&+ (\beta(2,2,1) + F^{-1}[\alpha(2,2)]\overline{D}(2,2,1))\mathcal{Q}(2,2) + (\beta(2,2,2) + F^{-1}[\alpha(2,2)]\overline{D}(2,2,2))\mathcal{Q}(2,1) \\
&+ (\beta(3,2,1) + F^{-1}[\alpha(2,2)]\overline{D}(3,2,1))\mathcal{Q}(3,2) + (\beta(3,2,2) + F^{-1}[\alpha(2,2)]\overline{D}(3,2,2))\mathcal{Q}(3,1) \\
&+ (\beta(4,2,1) + F^{-1}[\alpha(2,2)]\overline{D}(4,2,1))\mathcal{Q}(4,2) + (\beta(4,2,2) + F^{-1}[\alpha(2,2)]\overline{D}(4,2,2))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.33)
\end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.34)$$

$$\begin{aligned}
s^*(2,3) &= (\beta(1,2,1) + F^{-1}[\alpha(2,3)]\overline{D}(1,2,1))\mathcal{Q}(1,3) + (\beta(1,2,2) + F^{-1}[\alpha(1,3)]\overline{D}(1,2,2))\mathcal{Q}(1,2) \\
&+ (\beta(1,2,3) + F^{-1}[\alpha(2,3)]\overline{D}(1,2,3))\mathcal{Q}(1,1) + (\beta(2,2,1) + F^{-1}[\alpha(2,3)]\overline{D}(2,2,1))\mathcal{Q}(2,3) \\
&+ (\beta(2,2,2) + F^{-1}[\alpha(2,3)]\overline{D}(2,2,2))\mathcal{Q}(2,2) + (\beta(2,2,3) + F^{-1}[\alpha(1,3)]\overline{D}(2,2,3))\mathcal{Q}(2,1) \\
&+ (\beta(3,2,1) + F^{-1}[\alpha(2,3)]\overline{D}(3,2,1))\mathcal{Q}(3,3) + (\beta(3,2,2) + F^{-1}[\alpha(1,3)]\overline{D}(3,2,2))\mathcal{Q}(3,2) \\
&+ (\beta(3,2,3) + F^{-1}[\alpha(2,3)]\overline{D}(3,2,3))\mathcal{Q}(3,1) + (\beta(4,2,1) + F^{-1}[\alpha(1,3)]\overline{D}(4,2,1))\mathcal{Q}(4,3) \\
&+ (\beta(4,2,2) + F^{-1}[\alpha(2,3)]\overline{D}(4,2,2))\mathcal{Q}(4,2) + (\beta(4,2,3) + F^{-1}[\alpha(1,3)]\overline{D}(4,2,3))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.35)
\end{aligned}$$

### 3. Control point 3

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.36)$$

$$s^*(3, 1) = (\beta(1, 3, 1) + F^{-1}[\alpha(3, 1)]\overline{D}(1, 3, 1))Q(1, 1) + (\beta(2, 3, 1) + F^{-1}[\alpha(3, 1)]\overline{D}(2, 3, 1))Q(2, 1) \\ + (\beta(3, 3, 1) + F^{-1}[\alpha(3, 1)]\overline{D}(3, 3, 1))Q(3, 1) + (\beta(4, 3, 1) + F^{-1}[\alpha(3, 1)]\overline{D}(4, 3, 1))Q(4, 1) \\ \dots \dots \dots (3.37)$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.38)$$

$$s^*(3, 2) = (\beta(1, 3, 1) + F^{-1}[\alpha(3, 2)]\overline{D}(1, 3, 1))Q(1, 2) + (\beta(1, 3, 2) + F^{-1}[\alpha(3, 2)]\overline{D}(1, 3, 2))Q(1, 1) \\ + (\beta(2, 3, 1) + F^{-1}[\alpha(3, 2)]\overline{D}(2, 3, 1))Q(2, 2) + (\beta(2, 3, 2) + F^{-1}[\alpha(3, 2)]\overline{D}(2, 3, 2))Q(2, 1) \\ + (\beta(3, 3, 1) + F^{-1}[\alpha(3, 2)]\overline{D}(3, 3, 1))Q(3, 2) + (\beta(3, 3, 2) + F^{-1}[\alpha(3, 2)]\overline{D}(3, 3, 2))Q(3, 1) \\ + (\beta(4, 3, 1) + F^{-1}[\alpha(3, 2)]\overline{D}(4, 3, 1))Q(4, 2) + (\beta(4, 3, 2) + F^{-1}[\alpha(3, 2)]\overline{D}(4, 3, 2))Q(4, 1) \\ \dots \dots \dots (3.39)$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.40)$$

$$s^*(3, 3) = (\beta(1, 3, 1) + F^{-1}[\alpha(3, 3)]\overline{D}(1, 3, 1))Q(1, 3) + (\beta(1, 3, 2) + F^{-1}[\alpha(3, 3)]\overline{D}(1, 3, 2))Q(1, 2) \\ + (\beta(1, 3, 3) + F^{-1}[\alpha(3, 3)]\overline{D}(1, 3, 3))Q(1, 1) + (\beta(2, 3, 1) + F^{-1}[\alpha(3, 3)]\overline{D}(2, 3, 1))Q(2, 3) \\ + (\beta(2, 3, 2) + F^{-1}[\alpha(3, 3)]\overline{D}(2, 3, 2))Q(2, 2) + (\beta(2, 3, 3) + F^{-1}[\alpha(3, 3)]\overline{D}(2, 3, 3))Q(2, 1) \\ + (\beta(3, 3, 1) + F^{-1}[\alpha(3, 3)]\overline{D}(3, 3, 1))Q(3, 3) + (\beta(3, 3, 2) + F^{-1}[\alpha(3, 3)]\overline{D}(3, 3, 2))Q(3, 2) \\ + (\beta(3, 3, 3) + F^{-1}[\alpha(3, 3)]\overline{D}(3, 3, 3))Q(3, 1) + (\beta(4, 3, 1) + F^{-1}[\alpha(3, 3)]\overline{D}(4, 3, 1))Q(4, 3) \\ + (\beta(4, 3, 2) + F^{-1}[\alpha(3, 3)]\overline{D}(4, 3, 2))Q(4, 2) + (\beta(4, 3, 3) + F^{-1}[\alpha(3, 3)]\overline{D}(4, 3, 3))Q(4, 1) \\ \dots \dots \dots (3.41)$$

### 4. Control point 4

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.42)$$

$$s^*(4, 1) = (\beta(1, 4, 1) + F^{-1}[\alpha(4, 1)]\overline{D}(1, 4, 1))Q(1, 1) + (\beta(2, 4, 1) + F^{-1}[\alpha(4, 1)]\overline{D}(2, 4, 1))Q(2, 1) \\ + (\beta(3, 4, 1) + F^{-1}[\alpha(4, 1)]\overline{D}(3, 4, 1))Q(3, 1) + (\beta(4, 4, 1) + F^{-1}[\alpha(4, 1)]\overline{D}(4, 4, 1))Q(4, 1) \\ \dots \dots \dots (3.43)$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times Q(i, n - k + 1) \dots \dots \dots (3.44)$$

$$\begin{aligned}
s^*(4,2) &= (\beta(1,4,1) + F^{-1}[\alpha(4,2)]\overline{D}(1,4,1))\mathcal{Q}(1,2) + (\beta(1,4,2) + F^{-1}[\alpha(4,2)]\overline{D}(1,4,2))\mathcal{Q}(1,1) \\
&+ (\beta(2,4,1) + F^{-1}[\alpha(4,2)]\overline{D}(2,4,1))\mathcal{Q}(2,2) + (\beta(2,4,2) + F^{-1}[\alpha(4,2)]\overline{D}(2,4,2))\mathcal{Q}(2,1) \\
&+ (\beta(3,4,1) + F^{-1}[\alpha(4,2)]\overline{D}(3,3,1))\mathcal{Q}(3,2) + (\beta(3,4,2) + F^{-1}[\alpha(3,2)]\overline{D}(3,4,2))\mathcal{Q}(3,1) \\
&+ (\beta(4,4,1) + F^{-1}[\alpha(4,2)]\overline{D}(4,3,1))\mathcal{Q}(4,2) + (\beta(4,4,2) + F^{-1}[\alpha(3,2)]\overline{D}(4,4,2))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.45)
\end{aligned}$$

$$s^*(j,n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i,j,k) + F^{-1}[\alpha(j,n)]\overline{D}(i,j,k)] \times \mathcal{Q}(i,n-k+1) \dots\dots\dots (3.46)$$

$$\begin{aligned}
s^*(4,3) &= (\beta(1,4,1) + F^{-1}[\alpha(4,3)]\overline{D}(1,4,1))\mathcal{Q}(1,3) + (\beta(1,4,2) + F^{-1}[\alpha(4,3)]\overline{D}(1,4,2))\mathcal{Q}(1,2) \\
&+ (\beta(1,4,3) + F^{-1}[\alpha(4,3)]\overline{D}(1,4,3))\mathcal{Q}(1,1) + (\beta(2,4,1) + F^{-1}[\alpha(4,3)]\overline{D}(2,4,1))\mathcal{Q}(2,3) \\
&+ (\beta(2,4,2) + F^{-1}[\alpha(4,3)]\overline{D}(2,4,2))\mathcal{Q}(2,2) + (\beta(2,4,3) + F^{-1}[\alpha(4,3)]\overline{D}(2,4,3))\mathcal{Q}(2,1) \\
&+ (\beta(3,4,1) + F^{-1}[\alpha(4,3)]\overline{D}(3,4,1))\mathcal{Q}(3,3) + (\beta(3,4,2) + F^{-1}[\alpha(4,3)]\overline{D}(3,4,2))\mathcal{Q}(3,2) \\
&+ (\beta(3,4,3) + F^{-1}[\alpha(4,3)]\overline{D}(3,4,3))\mathcal{Q}(3,1) + (\beta(4,4,1) + F^{-1}[\alpha(4,3)]\overline{D}(4,4,1))\mathcal{Q}(4,3) \\
&+ (\beta(4,4,2) + F^{-1}[\alpha(4,3)]\overline{D}(4,4,2))\mathcal{Q}(4,2) + (\beta(4,4,3) + F^{-1}[\alpha(4,3)]\overline{D}(4,4,3))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.47)
\end{aligned}$$

## 5. Control point 5

$$s^*(j,n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i,j,k) + F^{-1}[\alpha(j,n)]\overline{D}(i,j,k)] \times \mathcal{Q}(i,n-k+1) \dots\dots\dots (3.48)$$

$$\begin{aligned}
s^*(5,1) &= (\beta(1,5,1) + F^{-1}[\alpha(5,1)]\overline{D}(1,5,1))\mathcal{Q}(1,1) + (\beta(2,5,1) + F^{-1}[\alpha(5,1)]\overline{D}(2,5,1))\mathcal{Q}(2,1) \\
&+ (\beta(3,5,1) + F^{-1}[\alpha(5,1)]\overline{D}(3,5,1))\mathcal{Q}(3,1) + (\beta(4,5,1) + F^{-1}[\alpha(5,1)]\overline{D}(4,5,1))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.49)
\end{aligned}$$

$$s^*(j,n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i,j,k) + F^{-1}[\alpha(j,n)]\overline{D}(i,j,k)] \times \mathcal{Q}(i,n-k+1) \dots\dots\dots (3.50)$$

$$\begin{aligned}
s^*(5,2) &= (\beta(1,5,1) + F^{-1}[\alpha(5,2)]\overline{D}(1,5,1))\mathcal{Q}(1,2) + (\beta(1,5,2) + F^{-1}[\alpha(5,2)]\overline{D}(1,5,2))\mathcal{Q}(1,1) \\
&+ (\beta(2,5,1) + F^{-1}[\alpha(5,2)]\overline{D}(2,5,1))\mathcal{Q}(2,2) + (\beta(2,5,2) + F^{-1}[\alpha(5,2)]\overline{D}(2,5,2))\mathcal{Q}(2,1) \\
&+ (\beta(3,5,1) + F^{-1}[\alpha(5,2)]\overline{D}(3,5,1))\mathcal{Q}(3,2) + (\beta(3,5,2) + F^{-1}[\alpha(5,2)]\overline{D}(3,5,2))\mathcal{Q}(3,1) \\
&+ (\beta(4,5,1) + F^{-1}[\alpha(5,2)]\overline{D}(4,5,1))\mathcal{Q}(4,2) + (\beta(4,5,2) + F^{-1}[\alpha(5,2)]\overline{D}(4,5,2))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.51)
\end{aligned}$$

$$s^*(j,n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i,j,k) + F^{-1}[\alpha(j,n)]\overline{D}(i,j,k)] \times \mathcal{Q}(i,n-k+1) \dots\dots\dots (3.52)$$

$$\begin{aligned}
s^*(5,3) &= (\beta(1,5,1) + F^{-1}[\alpha(5,3)]\overline{D}(1,5,1))\mathcal{Q}(1,3) + (\beta(1,5,2) + F^{-1}[\alpha(5,3)]\overline{D}(1,5,2))\mathcal{Q}(1,2) \\
&+ (\beta(1,5,3) + F^{-1}[\alpha(5,3)]\overline{D}(1,5,3))\mathcal{Q}(1,1) + (\beta(2,5,1) + F^{-1}[\alpha(5,3)]\overline{D}(2,5,1))\mathcal{Q}(2,3) \\
&+ (\beta(2,5,2) + F^{-1}[\alpha(5,3)]\overline{D}(2,5,2))\mathcal{Q}(2,2) + (\beta(2,5,3) + F^{-1}[\alpha(5,3)]\overline{D}(2,5,3))\mathcal{Q}(2,1) \\
&+ (\beta(3,5,1) + F^{-1}[\alpha(5,3)]\overline{D}(3,5,1))\mathcal{Q}(3,3) + (\beta(3,5,2) + F^{-1}[\alpha(5,3)]\overline{D}(3,5,2))\mathcal{Q}(3,2) \\
&+ (\beta(3,5,3) + F^{-1}[\alpha(5,3)]\overline{D}(3,5,3))\mathcal{Q}(3,1) + (\beta(4,5,1) + F^{-1}[\alpha(5,3)]\overline{D}(4,5,1))\mathcal{Q}(4,3) \\
&+ (\beta(4,5,2) + F^{-1}[\alpha(5,3)]\overline{D}(4,5,2))\mathcal{Q}(4,2) + (\beta(4,5,3) + F^{-1}[\alpha(5,3)]\overline{D}(4,5,3))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.53)
\end{aligned}$$

## 6. Control point 6

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-1} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.54)$$

$$\begin{aligned}
s^*(6,1) &= (\beta(1,6,1) + F^{-1}[\alpha(6,1)]\overline{D}(1,6,1))\mathcal{Q}(1,1) + (\beta(2,6,1) + F^{-1}[\alpha(6,1)]\overline{D}(2,6,1))\mathcal{Q}(2,1) \\
&+ (\beta(3,6,1) + F^{-1}[\alpha(6,1)]\overline{D}(3,6,1))\mathcal{Q}(3,1) + (\beta(4,6,1) + F^{-1}[\alpha(6,1)]\overline{D}(4,6,1))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.55)
\end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-2} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.56)$$

$$\begin{aligned}
s^*(6,2) &= (\beta(1,6,1) + F^{-1}[\alpha(6,2)]\overline{D}(1,6,1))\mathcal{Q}(1,2) + (\beta(1,6,2) + F^{-1}[\alpha(6,2)]\overline{D}(1,6,2))\mathcal{Q}(1,1) \\
&+ (\beta(2,6,1) + F^{-1}[\alpha(6,2)]\overline{D}(2,6,1))\mathcal{Q}(2,2) + (\beta(2,6,2) + F^{-1}[\alpha(6,2)]\overline{D}(2,6,2))\mathcal{Q}(2,1) \\
&+ (\beta(3,6,1) + F^{-1}[\alpha(6,2)]\overline{D}(3,6,1))\mathcal{Q}(3,2) + (\beta(3,6,2) + F^{-1}[\alpha(6,2)]\overline{D}(3,6,2))\mathcal{Q}(3,1) \\
&+ (\beta(4,6,1) + F^{-1}[\alpha(6,2)]\overline{D}(4,6,1))\mathcal{Q}(4,2) + (\beta(4,6,2) + F^{-1}[\alpha(6,2)]\overline{D}(4,6,2))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.57)
\end{aligned}$$

$$s^*(j, n) = \sum_{i=1}^4 \sum_{k=1}^{n-3} [\beta(i, j, k) + F^{-1}[\alpha(j, n)]\overline{D}(i, j, k)] \times \mathcal{Q}(i, n - k + 1) \dots\dots\dots (3.58)$$

$$\begin{aligned}
s^*(6,3) &= (\beta(1,6,1) + F^{-1}[\alpha(6,3)]\overline{D}(1,6,1))\mathcal{Q}(1,3) + (\beta(1,6,2) + F^{-1}[\alpha(6,3)]\overline{D}(1,6,2))\mathcal{Q}(1,2) \\
&+ (\beta(1,6,3) + F^{-1}[\alpha(6,3)]\overline{D}(1,6,3))\mathcal{Q}(1,1) + (\beta(2,6,1) + F^{-1}[\alpha(6,3)]\overline{D}(2,6,1))\mathcal{Q}(2,3) \\
&+ (\beta(2,6,2) + F^{-1}[\alpha(6,3)]\overline{D}(2,6,2))\mathcal{Q}(2,2) + (\beta(2,6,3) + F^{-1}[\alpha(6,3)]\overline{D}(2,6,3))\mathcal{Q}(2,1) \\
&+ (\beta(3,6,1) + F^{-1}[\alpha(6,3)]\overline{D}(3,6,1))\mathcal{Q}(3,3) + (\beta(3,6,2) + F^{-1}[\alpha(6,3)]\overline{D}(3,6,2))\mathcal{Q}(3,2) \\
&+ (\beta(3,6,3) + F^{-1}[\alpha(6,3)]\overline{D}(3,6,3))\mathcal{Q}(3,1) + (\beta(4,6,1) + F^{-1}[\alpha(6,3)]\overline{D}(4,6,1))\mathcal{Q}(4,3) \\
&+ (\beta(4,6,2) + F^{-1}[\alpha(6,3)]\overline{D}(4,6,2))\mathcal{Q}(4,2) + (\beta(4,6,3) + F^{-1}[\alpha(6,3)]\overline{D}(4,6,3))\mathcal{Q}(4,1) \\
&\dots\dots\dots (3.59)
\end{aligned}$$

### 3.6 Summary

In this chapter, materials and methods to be used in this study are described. The governing groundwater flow equations for homogeneous, non-uniform flow through confined aquifer are explained and unit response function approach for drawdown calculations is discussed in detail. The use of well functions, Cooper-Jacob equation formula and use of pump characteristics equations are also discussed. The generation of stochastic management model, stochastic influence function has also been elaborated within the chapter. At last, the development of chance-constrained optimization model is presented and written to enhance the understanding of model and its execution.



*Results  
and  
Discussion*



**4.1 General**

In chapter 3, groundwater flow equations, uncertainties in aquifer parameters and generation of stochastic influence function using well functions and Cooper-Jacob equation under different reliability requirement and pump characteristics were elaborated. Then the nonlinear chance-constrained model was formulated to determine the optimal pumpage for managing drawdown in confined aquifers, using unit response approach. In the current chapter, the applicability of developed chance-constrained management model is illustrated through a hypothetical study problem. The uncertainty is considered in terms of coefficient of variability of transmissivity and storage coefficient, to maximize the total pumpage subjected to drawdown constraints at specified reliability. The effects of uncertainty in aquifer parameters and reliability levels on the optimal pumpage are examined and solved in this chapter.

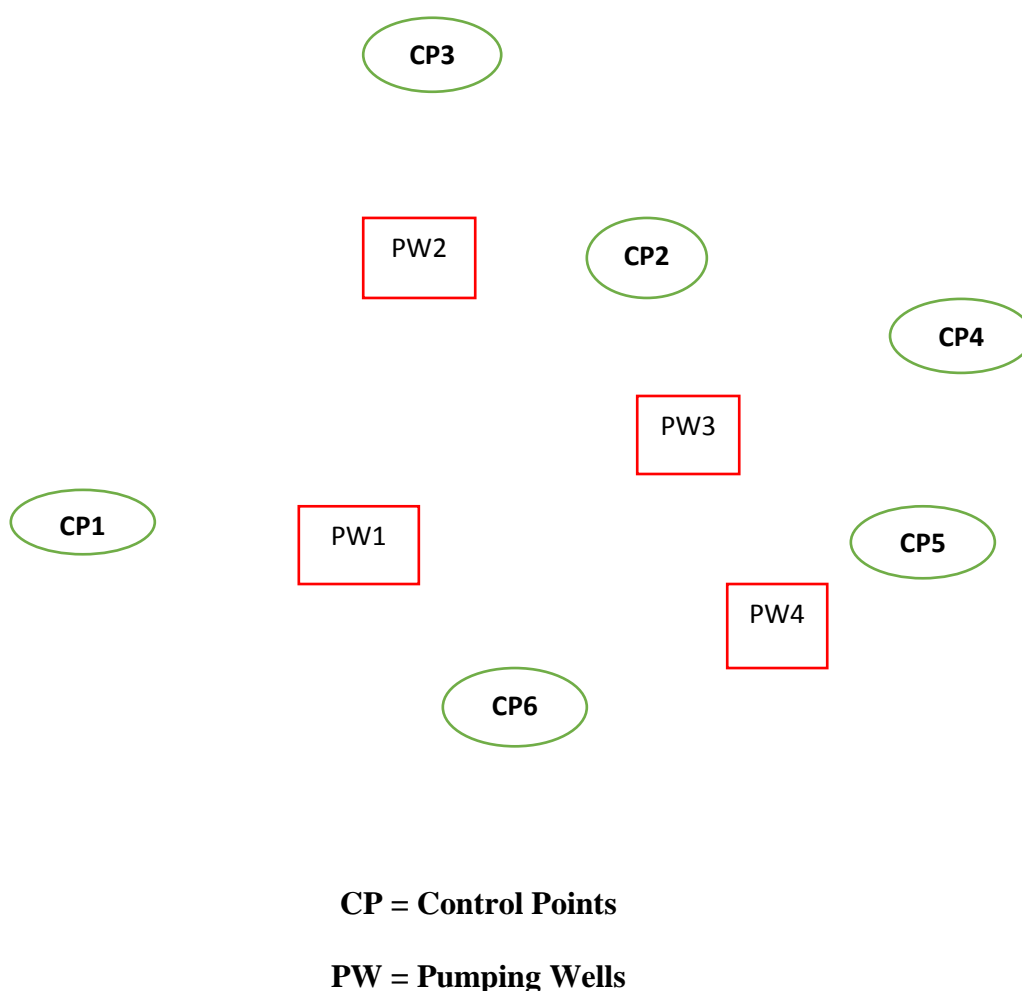
**4.2 Optimal Aquifer Management System**

The proposed nonlinear stochastic chance-constrained programming model presented in previous chapter, is applied to a hypothetical study area with an objective of maximizing the total pumpage (discharge) from a group of four pumping wells with known location and the drawdown constraints of specified reliability at selected control points (or observation wells). Cooper-Jacob equation is used to calculate the drawdown and other constraints related to maximum allowable efficiency and imposed pump characteristics.

**4.3 Model Example**

A hypothetical confined, homogeneous aquifer is used for illustrating the applicability of the proposed nonlinear chance constrained optimization model. There are 4 pumping wells and 6 control points to observe drawdown as shown in Figure 4.1. The assumed mean transmissivity is 5000 ft<sup>2</sup>/day (465 m<sup>2</sup>/day) and mean storage coefficient assumed over the basin is 0.002. The problem is to determine the optimal

pumping rates for pumping wells over three time periods of 50 days each, such that the resulting drawdown at each control point should not exceed a maximum allowable value with a specified reliability. The reliability values considered are 90%, 95% and 97.5%. The maximum allowable drawdown values at each control point for the three time periods are 3.5, 4 and 4.5 m respectively. The cells denoted as CP1, CP2,...,CP6 represent the control points, and the cells denoted as PW1, PW2,...,PW4 represent pumping well location. The distances between the potential production wells and control points are given in Table 4.1.



**Figure 4.1 Study area of a confined aquifer**

**Table 4.1 Distance (meter), between Potential Pumping Wells and Control Points**

Pumping Well	Control Point					
	1	2	3	4	5	6
1	76.15	141.42	150	156.2	150	70.71
2	117	72.8	76.15	151.32	197	152.64
3	154	87.32	105.5	76.32	112.36	110.12
4	165	210.24	166.44	95	60.83	72.11

The uncertainty in aquifer parameters is measured in terms of coefficient of variation. In present study, the extent of variability in data is shown by coefficient of variation of Transmissivity and Storage Coefficient. For calculation purposes the coefficient of variation values are referred from the Table 4.4 (**Tung, 1986**). Typically, the value of Storage Coefficient for confined aquifers lie in the range  $5 \times 10^{-5}$  to  $5 \times 10^{-3}$  (**Subramanya, 2010**).

**Table 4.2 Coefficient of Variation**

Transmissivity	0.2	0.4	0.6	0.8
Storage Coefficient	0.2	0.4	0.6	0.8

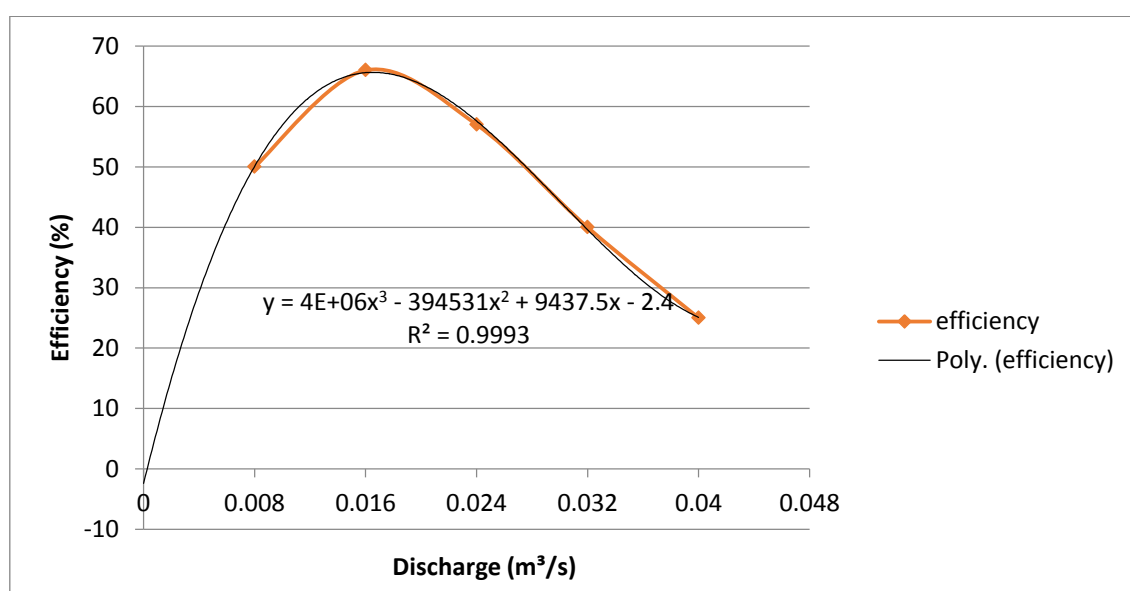
#### 4.4 Preliminary Calculations

The procedure of obtaining the pump characteristic equations and the graph representing pump characteristics, is presented in chapter 3 and Figure 3.2 respectively. Two pumps of different characteristics are used for computing results in this study. I have represented these pumps as pump I and pump II. The data related to these pumps is given in Table 4.3 and Table 4.4. This data is taken from **Punmia et al. (2009)**.

**Table 4.3 Characteristic Data for Pump I**

Discharge (L/s)	0	8	16	24	32	40
Efficiency (%)	-	50	66	57	40	25

From the above characteristic data of pump I, discharges are converted into m<sup>3</sup>/s and then equation are developed in Microsoft Excel and the graph is plotted as shown in Figure 4.2 between discharge v/s efficiency for pump I. The values of coefficients in the equation 3.6 are obtained as: a = -2.4, b = 9437, c = -39453 and d = 4\*10<sup>6</sup>.

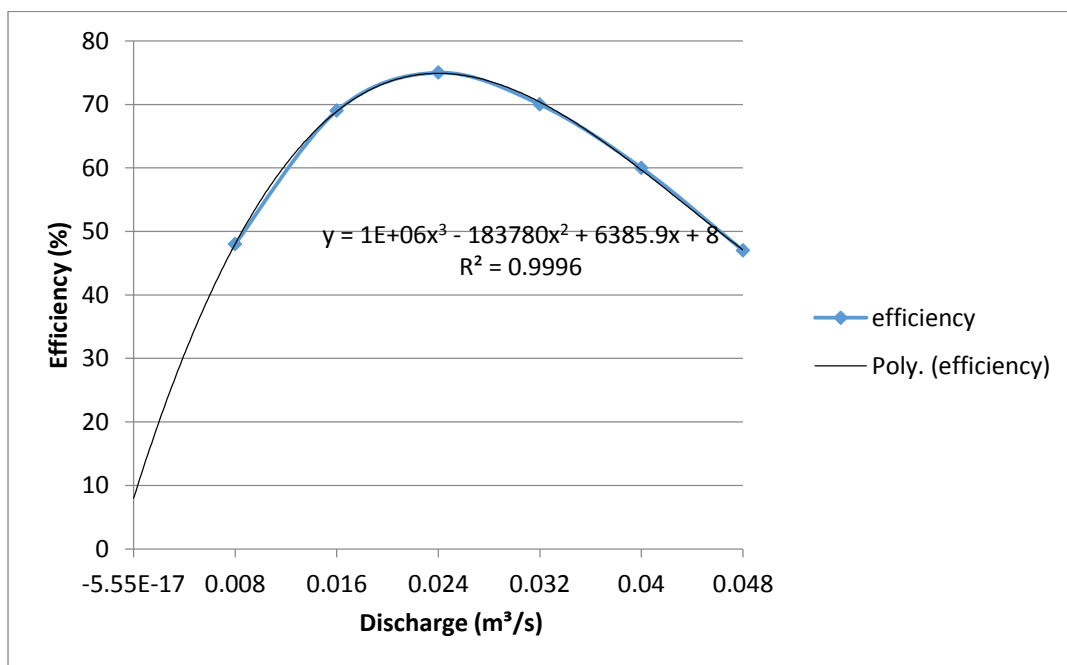


**Figure 4.2 Characteristic curve for pump I**

**Table 4.4 Characteristic Data for Pump II**

Discharge (L/s)	0	8	16	24	32	40	48
Efficiency (%)	-	48	69	75	70	60	47

Similarly, from the characteristic data of pump II, discharges are converted into m<sup>3</sup>/s then equations are developed in Microsoft Excel and graph is plotted between discharge v/s efficiency as shown in Figure 4.3. The values of coefficients in the equation 3.8 are obtained as: a = 8, b = 6385, c = -18378 and d = 1\*10<sup>6</sup>.



**Figure 4.3 Characteristic curve for pump II**

#### 4.5 Optimal Results

The developed chance-constrained model is solved by LINGO 10.0, using the related data and the pump characteristics given in study problem. For pump I, the maximum efficiency is taken as 66%, as per the Table 4.3 and then results are calculated for each value of coefficient of variation ranging from 0.2 to 0.8. It is assumed that all the 4 pumping wells are working for all three periods. The values of discharges are converted to L/s for ease. With the restriction imposed on the maximum efficiency, the value of optimal (discharge) got affected. The optimal results are obtained for different uncertainty levels in the values of transmissivity and storage coefficient with a desired reliability requirement.

The optimal results are first calculated for 90% reliability using different values of transmissivity and storage coefficient to understand the effect on optimal pumpage due to uncertainties. At 90% reliability, the results are calculated for 16 different combinations of coefficient of variation in transmissivity (CV of T) and coefficient of variation in storage coefficient (CV of S). All these combinations can be applied to both the pumps. One sample of the formulated program for the case CV of T 0.2 and

CV of S 0.2 at 90% reliability using pump I is shown in Appendix 1. All results using pump I characteristics are tabulated below in Tables 4.5 to 4.20.

#### 4.5.1 Optimal pumping results for different values of CV of T with CV of S equal to 0.2

The model was solved using CV of S equal to 0.2 and values of CV of T equal to 0.2, 0.4, 0.6 and 0.8, separately. The results of these four combinations are presented in Tables 4.5 to 4.8. These tables contain the values of pumping rate and efficiency at each pumping well during three time periods each of 50 days. The upper limit of efficiency is fixed for both the pumps i.e., 66% for pump I and 75% for Pump II, but lower limit of efficiency is considered according to the minimum value below which the optimal solution becomes infeasible. It differs and varies according to the coefficient of variation and reliability, shown in tables below.

**Table 4.5 CV of Transmissivity 0.2 and CV of Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.95	58	6.51	63.27	5.95	58	5.95	58
<b>50-100</b>	5.95	58	6.03	58.73	5.95	58	5.95	58
<b>100-150</b>	5.95	58	6.8	66	6.3	61.32	5.95	58

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.95 L/s and 58%. The values for discharge and efficiency are different for well 2 and for well 3 at different time intervals. Total discharge from all the wells during all periods of pumping is 73.24 L/s.

**Table 4.6 CV of Transmissivity 0.4 and CV of Storage Coefficient 0.2**

<b>Time Period (Days)</b>	<b>Pumping Well 1</b>		<b>Pumping Well 2</b>		<b>Pumping Well 3</b>		<b>Pumping Well 4</b>	
	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>
<b>0-50</b>	4.99	49	5.67	55.32	4.99	49	4.99	49
<b>50-100</b>	4.99	49	5.42	53.04	4.99	49	4.99	49
<b>100-150</b>	4.99	49	6.44	62.63	4.99	49	4.99	49

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.99 L/s and 49%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 62.43 L/s.

**Table 4.7 CV of Transmissivity 0.6 and CV of Storage Coefficient 0.2**

<b>Time Period (Days)</b>	<b>Pumping Well 1</b>		<b>Pumping Well 2</b>		<b>Pumping Well 3</b>		<b>Pumping Well 4</b>	
	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>
<b>0-50</b>	4.35	43	4.83	47.47	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.72	46.51	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.64	55.11	4.35	43	4.35	43

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 54.31 L/s.

**Table 4.8 CV of Transmissivity 0.8 and CV of Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	3.96	39.38	3.92	39	3.92	39
<b>50-100</b>	3.92	39	3.94	39.23	3.92	39	3.92	39
<b>100-150</b>	3.92	39	4.78	47.01	3.92	39	3.92	39

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 47.93 L/s.

All these results presented above show the variation in efficiency ranges and decrease in total optimal pumpage as well as in pumping rates at each pumping well when CV of transmissivity increases from 0.2 to 0.8. The observed differences in efficiency and total discharge values are large as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 58% for CV of Transmissivity 0.2 and 39% for CV of Transmissivity 0.8. Hence there is a decrease of 19% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

#### **4.5.2 Optimal pumping results for different values of CV of T with CV of S equal to 0.4**

The model was solved using CV of S equal to 0.4 and values of CV of T equal to 0.2, 0.4, 0.6 and 0.8, separately. The results of all four combinations are presented below from Table 4.9 to Table 4.12. The results contain the values of discharges and efficiency at each pumping well for every time interval.

**Table 4.9 CV of Transmissivity 0.2 and CV of Storage Coefficient 0.4**

<b>Time Period (Days)</b>	<b>Pumping Well 1</b>		<b>Pumping Well 2</b>		<b>Pumping Well 3</b>		<b>Pumping Well 4</b>	
	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>
<b>0-50</b>	5.84	57	6.74	65.39	5.84	57	5.84	57
<b>50-100</b>	5.84	57	6.28	61.12	5.84	57	5.84	57
<b>100-150</b>	5.84	57	6.8	66	6.44	62.6	5.84	57

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.84 L/s and 57%. The values for discharge and efficiency are different in all periods for wells 2 and 3. Total discharge from all the wells during all periods of pumping is 73.01 L/s.

**Table 4.10 CV of Transmissivity 0.4 and CV of Storage Coefficient 0.4**

<b>Time Period (Days)</b>	<b>Pumping Well 1</b>		<b>Pumping Well 2</b>		<b>Pumping Well 3</b>		<b>Pumping Well 4</b>	
	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>	<b>Discharge (L/s)</b>	<b>Efficiency (%)</b>
<b>0-50</b>	4.99	49	5.62	54.89	4.99	49	4.99	49
<b>50-100</b>	4.99	49	5.39	52.73	4.99	49	4.99	49
<b>100-150</b>	4.99	49	6.41	62.37	4.99	49	4.99	49

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.99 L/s and 49%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 62.33 L/s.

**Table 4.11 CV of Transmissivity 0.6 and CV of Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.8	47.25	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.7	46.34	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.62	54.93	4.35	43	4.35	43

It is seen from this table that the discharges and efficiency from well 1, well 2 and well 3 are equal at each time interval i.e., 4.35 L/s and 43%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 54.25 L/s.

**Table 4.12 CV of Transmissivity 0.8 and CV of Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	3.95	39.31	3.92	39	3.92	39
<b>50-100</b>	3.92	39	3.94	39.22	3.92	39	3.92	39
<b>100-150</b>	3.92	39	4.78	47.01	3.92	39	3.92	39

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 47.92 L/s.

All these results presented above show the variation in efficiency ranges and decrease in total optimal pumpage as well as in pumping rates at each pumping well when CV of Transmissivity increases from 0.2 to 0.8. The observed difference in efficiencies and total discharge values are large as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 57% for CV of Transmissivity 0.2 and 39% for CV of Transmissivity 0.8. Hence there is a decrease of 18% in the minimum allowable efficiency value when we increase the value of CV of Transmissivity from 0.2 to 0.8.

#### 4.5.3 Optimal pumping results for different values of CV of T with CV of S equal to 0.6

The model was solved using CV of S equal to 0.6 and values of CV of T equal to 0.2, 0.4, 0.6 and 0.8, separately. The results of all four combinations are presented below from Table 4.13 to Table 4.16. The results contain the values of discharges and efficiency at each pumping well for every time period of pumping.

**Table 4.13 CV of Transmissivity 0.2 and CV of Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.84	57	6.49	63.27	5.84	57	5.84	57
<b>50-100</b>	5.84	57	6.1	58.73	5.84	57	5.84	57
<b>100-150</b>	5.84	57	6.8	66	6.29	61.17	5.84	57

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.84 L/s and 57%. The values for discharge and efficiency are different in all periods for wells 2 and 3. Total discharge from all the wells during all periods of pumping is 72.43 L/s.

**Table 4.14 CV of Transmissivity 0.4 and CV of Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.99	49	5.52	53.99	4.99	49	4.99	49
<b>50-100</b>	4.99	49	5.31	52.03	4.99	49	4.99	49
<b>100-150</b>	4.99	49	6.34	61.7	4.99	49	4.99	49

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.99 L/s and 49%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 62.08 L/s.

**Table 4.15 CV of Transmissivity 0.6 and CV of Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.67	46.84	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.67	46.03	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.59	54.65	4.35	43	4.35	43

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 54.14 L/s.

**Table 4.16 CV of Transmissivity 0.8 and CV of Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.81	38	4.3	42.61	3.81	38	3.81	38
<b>50-100</b>	3.81	38	4.29	42.5	3.81	38	3.81	38
<b>100-150</b>	3.81	38	5.13	50.29	3.81	38	3.81	38

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.81 L/s and 38%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 48.01 L/s.

All these results presented above show the variation in efficiency ranges and decrease in total optimal pumpage as well as in pumping rates at each pumping well when CV of Transmissivity increases from 0.2 to 0.8. The value of minimum allowable efficiency is 57% for CV of Transmissivity 0.2 and 39% for CV of Transmissivity 0.8. Hence there is a decrease of 18% in the minimum allowable efficiency value when we increase the value of CV of Transmissivity from 0.2 to 0.8.

#### **4.5.4 Optimal pumping results for different values of CV of T with CV of S equal to 0.8**

The model was solved using CV of S equal to 0.8 and values of CV of T equal to 0.2, 0.4, 0.6 and 0.8, separately. The results of all four combinations are presented below from Table 4.17 to Table 4.20. The results contain the values of discharges and efficiency at each pumping well for every time interval of 50 days.

**Table 4.17 CV of Transmissivity 0.2 and CV of Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.84	57	6.18	60.13	5.84	57	5.84	57
<b>50-100</b>	5.84	57	5.86	57.16	5.84	57	5.84	57
<b>100-150</b>	5.84	57	6.8	66	6.08	59.26	5.84	57

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.84 L/s and 57%. The values for discharge and efficiency are different in all periods for wells 2 and 3. Total discharge from all the wells during all periods of pumping is 71.67 L/s.

**Table 4.18 CV of Transmissivity 0.4 and CV of Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.99	49	5.4	52.86	4.99	49	4.99	49
<b>50-100</b>	4.99	49	5.22	51.18	4.99	49	4.99	49
<b>100-150</b>	4.99	49	6.26	60.96	4.99	49	4.99	49

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.99 L/s and 49%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 61.79 L/s.

**Table 4.19 CV of Transmissivity 0.6 and CV of Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.7	46.25	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.63	45.64	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.55	54.26	4.35	43	4.35	43

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 53.99 L/s.

**Table 4.20 CV of Transmissivity 0.8 and CV of Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.81	38	4.27	42.26	3.81	38	3.81	38
<b>50-100</b>	3.81	38	4.26	42.24	3.81	38	3.81	38
<b>100-150</b>	3.81	38	5.1	50.05	3.81	38	3.81	38

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.81 L/s and 38%. The values for discharge and efficiency are different in all periods for well 2. Total discharge from all the wells during all periods of pumping is 47.92 L/s.

All these results presented above show the variation in efficiency ranges and decrease in total optimal pumpage as well as in pumping rates at each pumping well

when CV of Transmissivity increases from 0.2 to 0.8. The value of minimum allowable efficiency is 57% for CV Transmissivity 0.2 and 38% for CV Transmissivity 0.8. Hence, there is a decrease of 19% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

#### 4.5.5 Variation of optimal pumpage

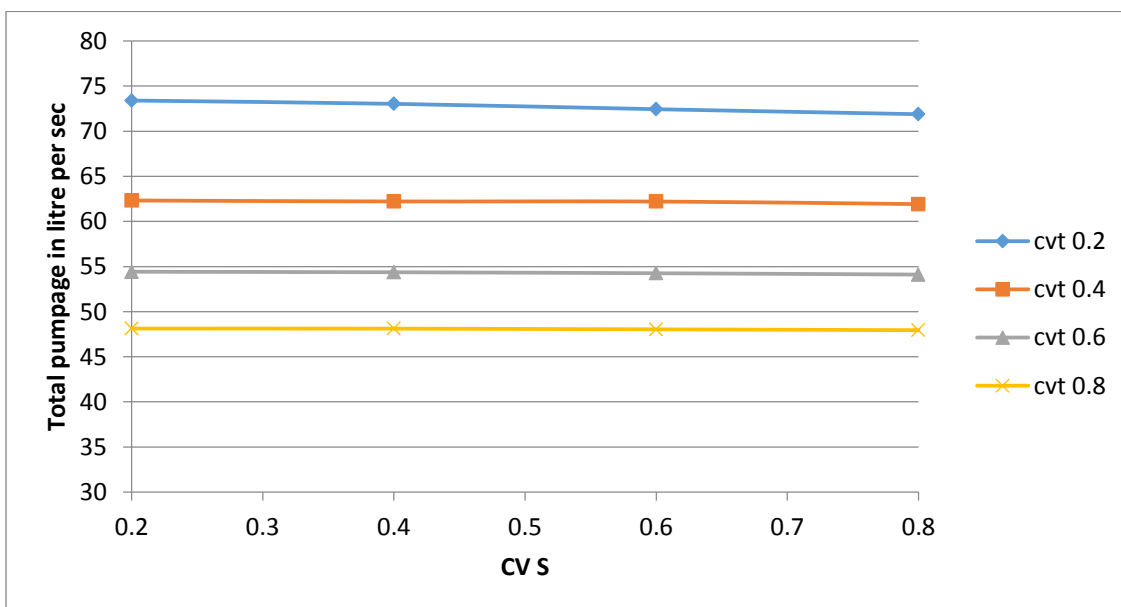
The values of total optimal pumping rate from all the wells corresponding to different values of transmissivity and storage coefficient for 90% reliability are summarized in Table 4.21 and in this Table the results are calculated using pump I and all the values of optimal discharges for all the 16 combinations for variation in CV of Transmissivity and CV of Storage Coefficient.

**Table 4.21 Total optimal pumpages for all combinations of CV of S and CV of T with 90% reliability using pump I**

Discharge (L/s)					
CV of S	CV of T				Total decrease in discharge (%)
	0.2	0.4	0.6	0.8	
0.2	73.24	62.43	54.31	47.93	34.56
0.4	73.01	62.33	54.25	47.92	34.36
0.6	72.43	62.09	54.14	48.01	33.71
0.8	71.67	61.79	53.99	47.92	33.14
<b>Total decrease in discharge (%)</b>	2.15	1.02	0.58	0.19	

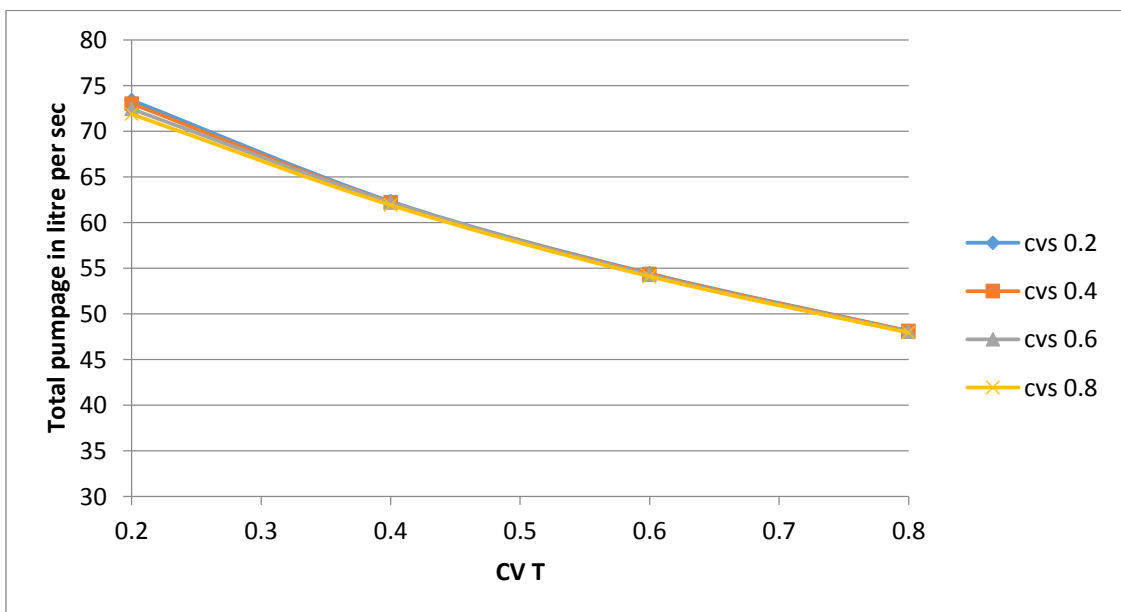
It is observed from the above results that the value of optimal pumpage decreases with the increase in the variation. Although the decrement is gradual with the increase in the CV of Storage Coefficient in which the maximum decrease is around 2%. But the optimal results are very sensitive towards the variations in CV of Transmissivity, because the maximum decrease is around 35%.

These results are also shown in Figures 4.4 and 4.5 as variation of the total optimal discharge against coefficient of variation of storage coefficient and coefficient of variation of transmissivity, respectively.



**Figure 4.4 Variation of Total Pumpage with CV of S for 90% Reliability using Pump I**

It is seen from Figure 4.4 that if we vary CV of Storage Coefficient, keeping CV of Transmissivity constant the variation is very close to a horizontal straight line. Hence the variation in optimal discharge seems to be insensitive to variation in CV of Storage Coefficient. This can also be seen from Table4.21 that the maximum decrease in optimal pumpages is around 2% only.



**Figure4.5 Variation of Total Pumpage with CV of T for 90% Reliability using Pump I**

It is understood from Figure 4.5 that if we vary CV of Transmissivity, keeping CV of Storage Coefficient as constant, there is a drastic decrement in the optimal discharge. The variation of optimal discharge is highly sensitive to CV of Transmissivity. Hence care must be taken to consider the variation in CV of Transmissivity. The decrement and sensitivity in the optimal pumpages can also be seen from Table 4.21 as the decrease is around 35%. Further, from Figure 4.5 it can be seen that the plots corresponding to different constant values of CV of S, total pumpage from the aquifer is almost independent of CV of S.

#### 4.6 Effect of reliability on optimal solution

To study the effect of reliability on the optimal solution, the formulated chance constrained optimization model for transient groundwater flow is solved for two other values of reliabilities i.e., 95% and 97.5%. Using pump I characteristics, for each 16 combinations of CV of Transmissivity and Storage Coefficient are considered and their results in brief are presented in Tables 4.22 and 4.23, respectively for 95% and 97.5% reliability, while the detailed tabulated results are presented in Appendix 2 (Tables A2.1 to 2.16 and A2.17 to 2.32, respectively for 95% and 97.5% reliability). Further, the variation in optimal pumpage is demonstrated through graphs in Figures 4.6 and 4.7 for 95% and Figures 4.8 and 4.9 for 97.5% reliability.

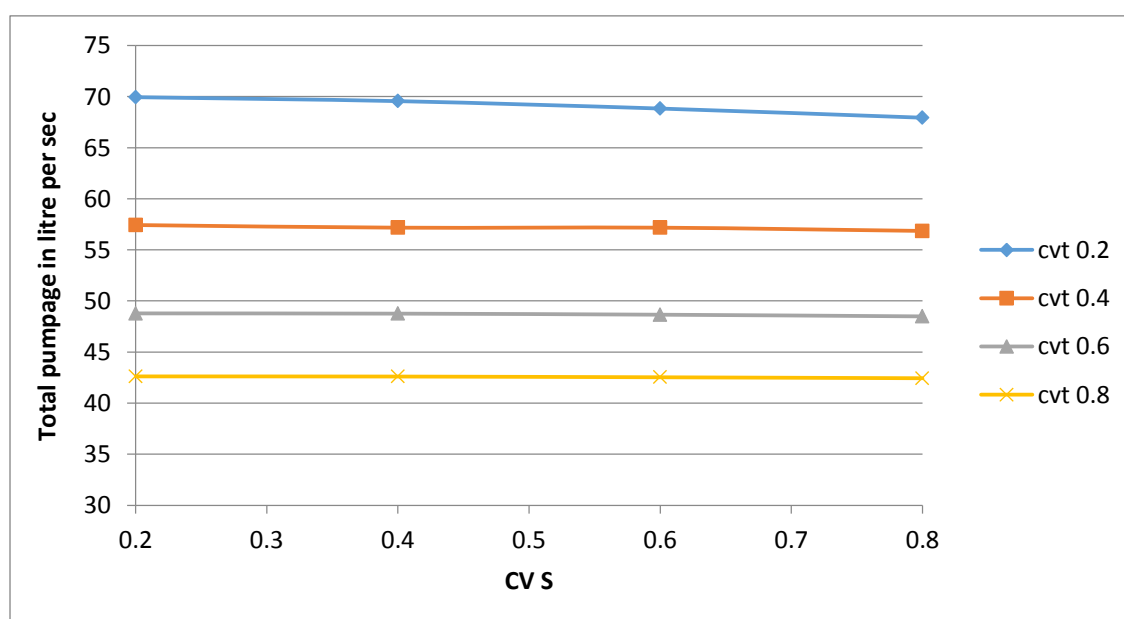
**Table 4.22 Total optimal pumpages for all combinations of CV of S and CV of T with 95% reliability using pump I**

Discharge (L/s)					
CV of S	CV of T				Total decrease in discharge (%)
	0.2	0.4	0.6	0.8	
0.2	69.80	57.43	48.87	42.51	39.10
0.4	69.42	57.31	48.81	42.50	38.78
0.6	68.84	57.21	48.71	42.43	38.37
0.8	67.96	56.89	48.54	42.33	37.71
<b>Total decrease in discharge (%)</b>	2.63	0.94	0.67	0.42	

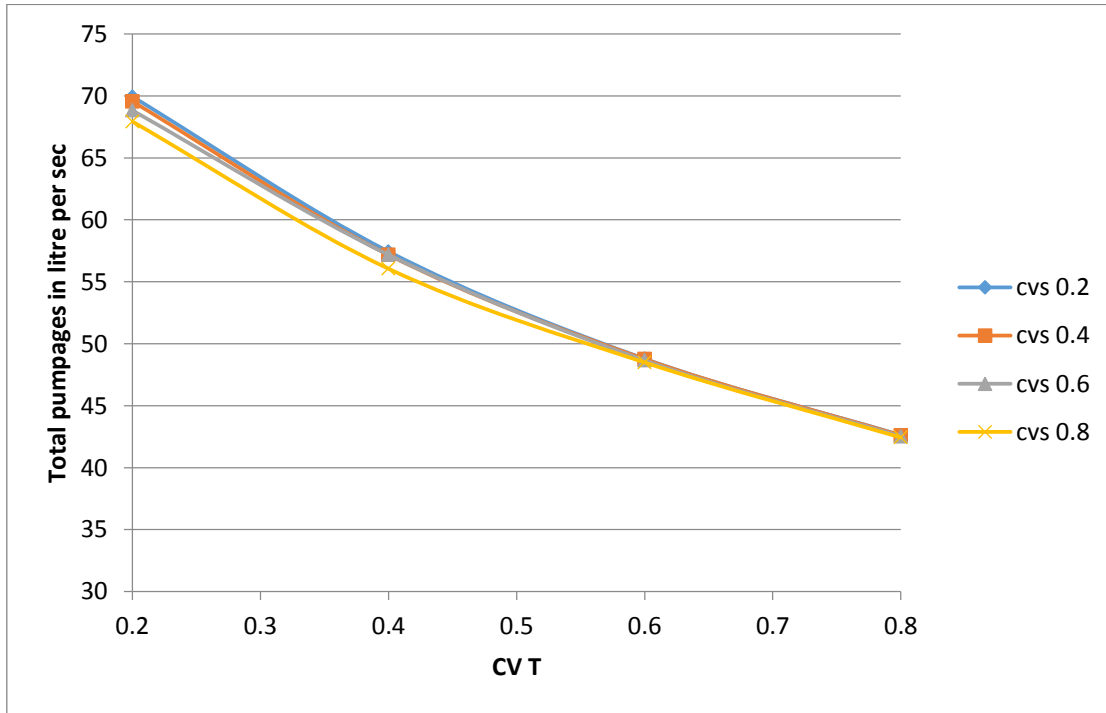
**Table 4.23 Total optimal pumpages for all combinations of CV of S and CV of T for 97.5% reliability using pump I**

Discharge (L/s)					
CV of S	CV of T				Total decrease in discharge (%)
	0.2	0.4	0.6	0.8	
0.2	67.02	53.80	44.99	38.65	42.32
0.4	66.55	53.68	44.93	38.64	41.94
0.6	65.93	53.41	44.82	38.57	41.12
0.8	64.70	53.23	44.65	38.48	40.53
<b>Total decrease in discharge (%)</b>	3.45	1.06	0.74	0.46	

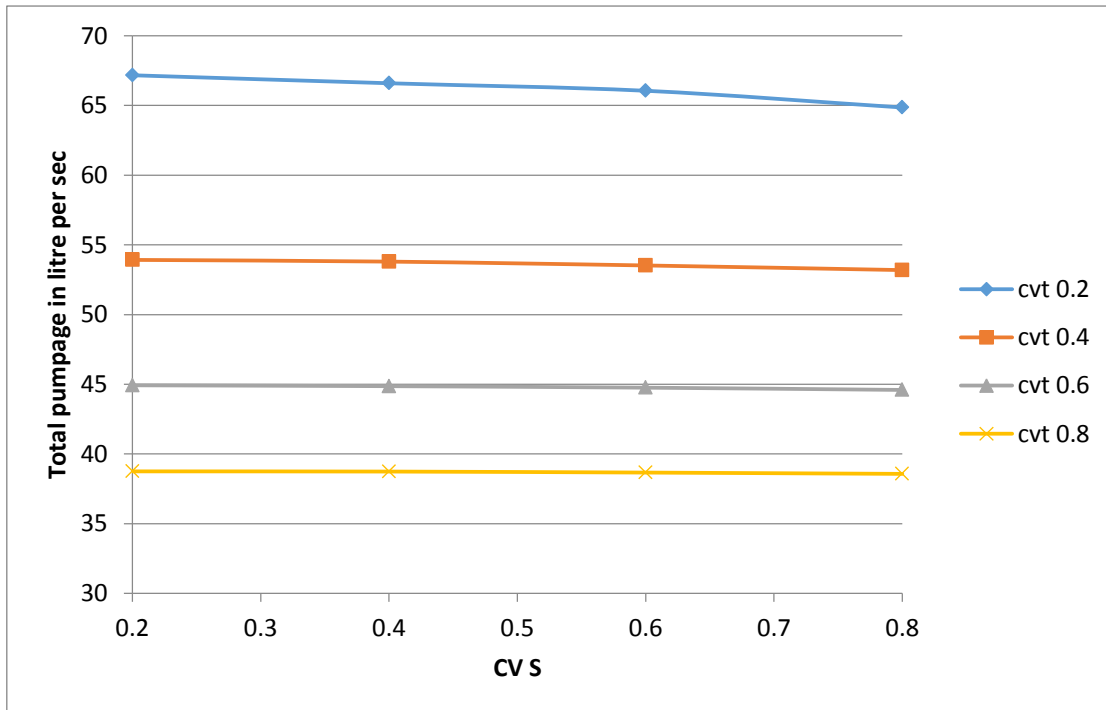
If CV of T taken for comparison is 0.2, the range of optimal pumpage for all four combinations with CV of S can be averaged to 66 L/s for 97.5% reliability, 69 L/s for 95% reliability (Table 4.22) and 73 L/s for 90% reliability (Table 4.21). This pattern is similar for all the other 12 combinations of CV of T and CV of S. Hence, on comparing the results of Table 4.21 (for reliability 90%) and Table 4.22 (for reliability 95%), it can be inferred that for each combination of CV, with the increase in the model reliability the optimal pumpage decreases.



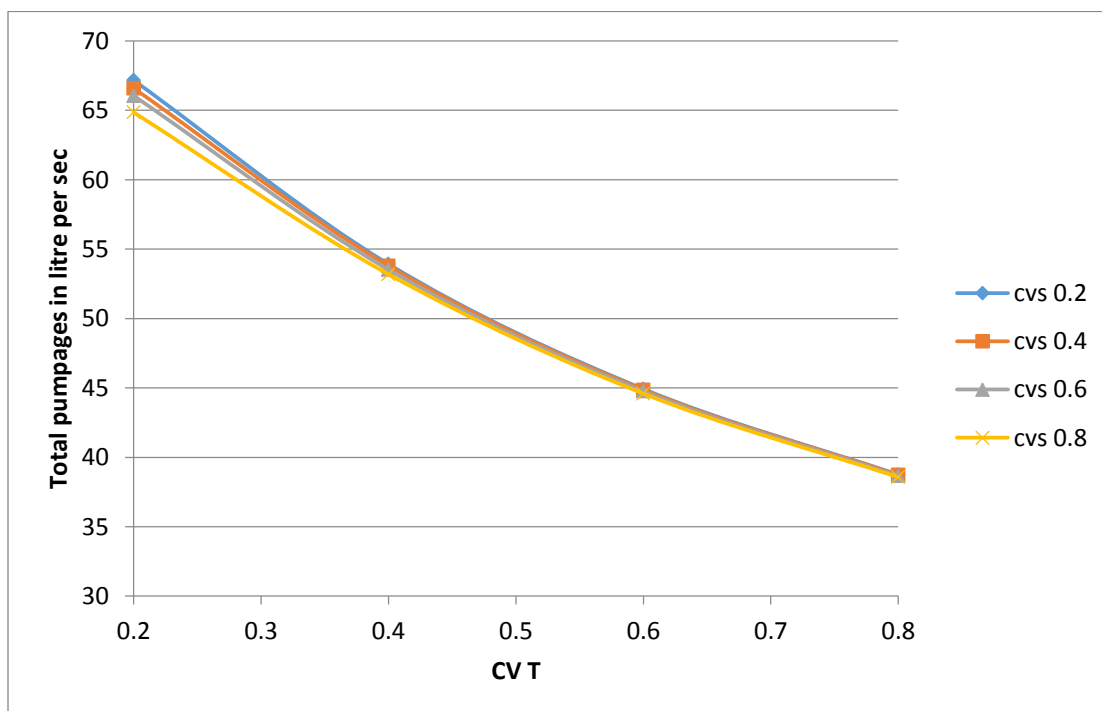
**Figure 4.6 Variation of Total Pumpage with CV of S with 95% Reliability using Pump I**



**Figure 4.7 Variation of Total Pumpage with CV of T with 95% Reliability using Pump I**



**Figure 4.8 Variation of Total Pumpage with CV of Swith97.5% Reliability using Pump I**



**Figure 4.9 Variation of Total Pumpage with CV of T with 97.5% Reliability using Pump I**

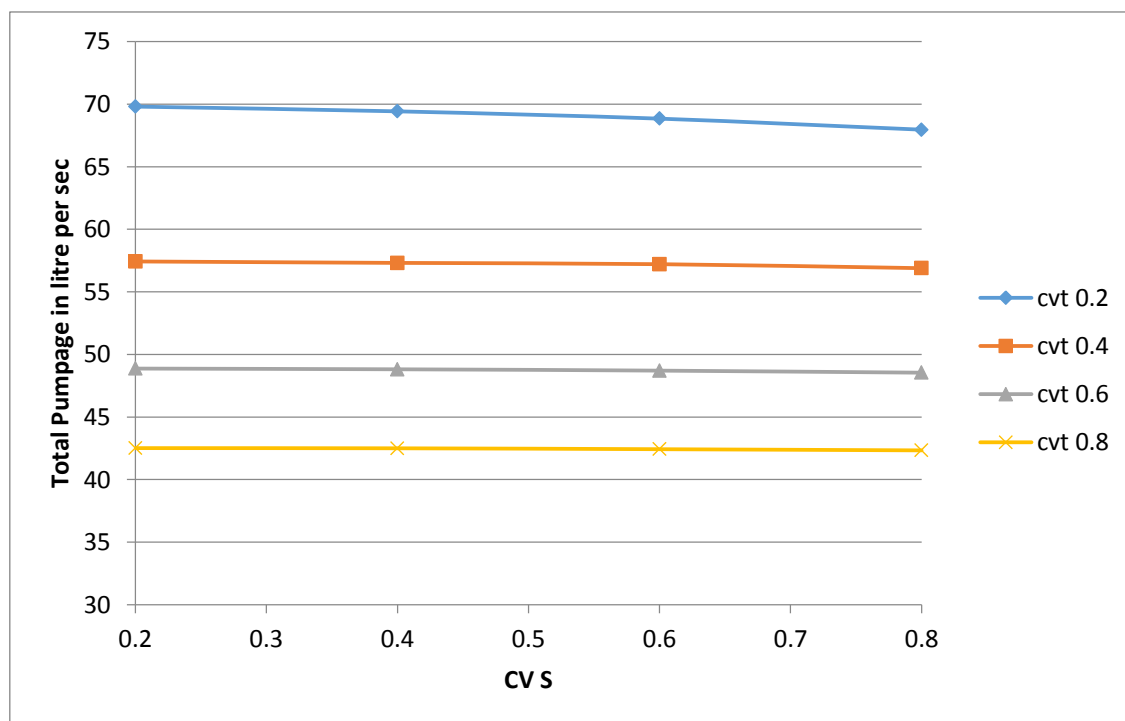
It is seen from the Figures 4.6 and 4.8 that with the increment in reliability requirement from 90% to 95% and 97.5%, the total optimal pumpage decreases but the pattern of variation remains similar to the case of 90% reliability under same condition. Further, from Figures 4.7 and 4.9, it can be seen that there is some effect of CV of S on total pumpage from the aquifer especially for values of CV of T up to 0.6 as is clear from the plots for CV of S equal to 0.2 and 0.8. Thus, it is observed that the percentage drop in optimal value of pumpage is more sensitive to CV of T than CV of S.

#### **4.7 Effect of using pumps with different characteristics**

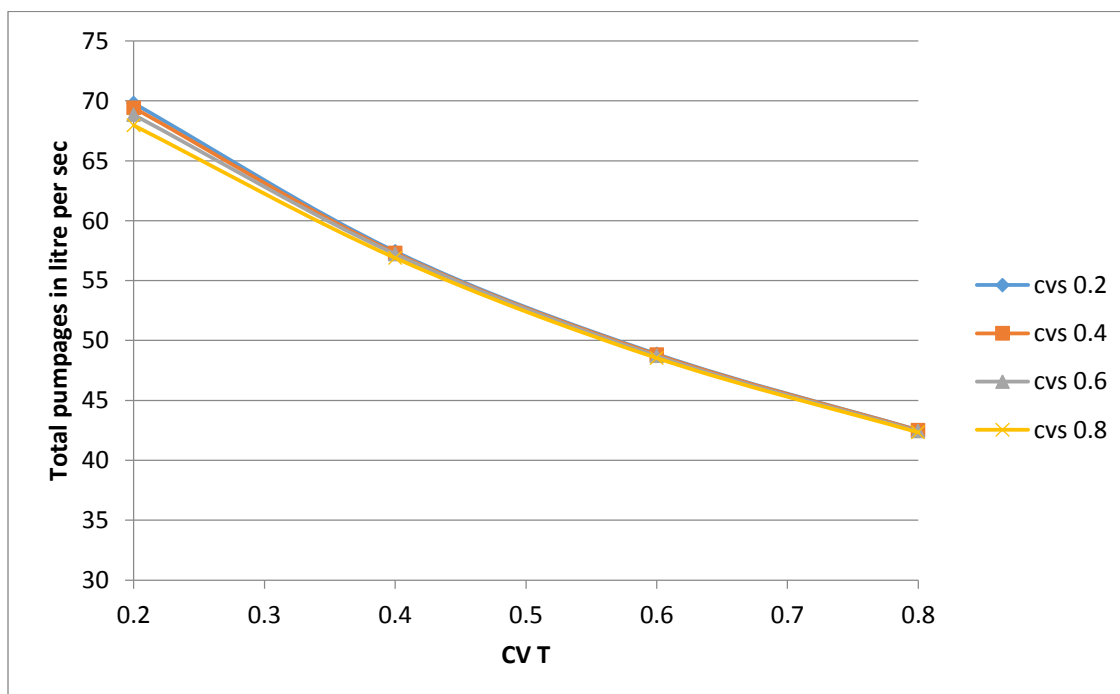
To investigate the effect of using pumps with different characteristics on pumping plan, the developed chance constrained optimization model was also solved using characteristic data of pump II. The optimal results obtained for 95% reliability using pump II are given in Table 4.24 and also presented in Figures 4.10 and 4.11.

**Table 4.24 Total optimal pumpages for all combinations of CV of S and CV of T for 95% reliability using pump II**

Discharge (L/s)					
CV of S	CV of T				Total decrease in discharge (%)
	0.2	0.4	0.6	0.8	
0.2	69.94	57.43	48.78	42.62	39.10
0.4	69.55	57.17	48.76	42.60	38.75
0.6	68.83	57.21	48.65	42.53	38.21
0.8	67.93	56.85	48.49	42.43	37.53
<b>Total decrease in discharge (%)</b>	2.87	1.02	0.60	0.43	



**Figure 4.10 Variation of Total Pumpage with CV of S for 95% Reliability using Pump II**



**Figure 4.11 Variation of Total Pumpage with CV of T with 95% Reliability using Pump II**

Comparing the results for pump I from Table 4.22 and for pump II from Table 4.24 it can be observed that the optimal pumpage for both the pumps at this reliability are almost similar, but the range of efficiency for pump I is 34%-66% and for pump II it is 29%-75%. These values of maximum and minimum efficiency are observed for all values of coefficient of variation. Hence the range of efficiency can be used to determine which pump one should use.

#### 4.8 Effect of limiting the range of efficiency

For further analysis of the results, I have applied restrictions on the range of minimum and maximum efficiency i.e., 40% to 66% for pump I and 40% to 75% for pump II. The results for all reliabilities are summarized in Table 4.25. From this table it can be concluded that within a specified range of efficiency the less is the variation and reliability required, the more promising are the results. Although there is a little difference in the discharge values for both the pumps when compared for CV of T value of 0.2, but as the uncertainty increases the solutions for both the pumps become infeasible shown as Inf in the table. Even so, the results for pump I are better than pump II.

**Table 4.25 Total Optimal Pumpages for All Combinations of CV of S and CV of T for Efficiency Range 40%-66%**

Reliability	CV of S	CV of T							
		0.2		0.4		0.6		0.8	
		Pump I	Pump II	Pump I	Pump II	Pump I	Pump II	Pump I	Pump II
		Q(L/s)	Q(L/s)	Q(L/s)	Q(L/s)	Q(L/s)	Q(L/s)	Q(L/s)	Q(L/s)
90%	0.2	73.64	74.53	63.29	62.32	54.80	Inf	Inf	Inf
	0.4	73.33	74.15	63.18	62.21	54.74	Inf	Inf	Inf
	0.6	72.82	73.54	62.96	Inf	54.63	Inf	Inf	Inf
	0.8	72.17	72.74	62.68	Inf	54.48	Inf	Inf	Inf
95%	0.2	70.37	70.63	58.15	Inf	Inf	Inf	Inf	Inf
	0.4	70.04	70.23	58.15	Inf	Inf	Inf	Inf	Inf
	0.6	69.42	69.50	57.90	Inf	Inf	Inf	Inf	Inf
	0.8	68.64	68.58	57.60	Inf	Inf	Inf	Inf	Inf
97.5%	0.2	67.77	67.61	54.30	Inf	Inf	Inf	Inf	Inf
	0.4	67.73	67.05	54.17	Inf	Inf	Inf	Inf	Inf
	0.6	66.71	66.28	53.89	Inf	Inf	Inf	Inf	Inf
	0.8	65.37	64.80	53.55	Inf	Inf	Inf	Inf	Inf

\*Inf = Infeasible Solution

#### 4.9 Effect of limiting the pumping rate

To see the effect of limiting the minimum allowable pumping at each pumping well optimization model was solved for CV of T equal to 0.4 using pump I for 95% reliability by taking the minimum allowable pumping rate of 4.6 L/s for pump I and pump II separately. The efficiency range used is 40% to 66%, for pump I and 40% to

75% for pump II. The results are given in Table 4.26. From Table 4.26, it can be easily seen that the values of total optimal discharge are nearly the same as the values obtained without imposing restriction on minimum allowable pumping rate (Table 4.22). Thus, the limit imposed on minimum allowable pumping rate does not significantly affect the pumping rate at each pumping well.

The model is also tested for minimum allowable pumping rate of 5 L/s. But it is observed from the calculations that the model results are infeasible for minimum pumping rate greater than 4.6 L/s for pump I.

**Table 4.26 Total optimal pumpage at minimum discharge restriction, for pump I**

CV of T	CV of S	Reliability	Pumping rate at each well	Total pumpage
0.4	0.2	95%	4.6 L/sec	57.42
0.4	0.4	95%	4.6 L/sec	57.42
0.4	0.6	95%	4.6 L/sec	57.15
0.4	0.8	95%	4.6 L/sec	56.83

However, for pump II, at reliability of 95%, when CV of transmissivity is 0.4 with different value of CV of storage coefficient, the result of total optimal pumpage value has infeasible solution. So there will be no results for applying restrictions on minimum allowable pumping rate.

#### **4.10 Effect of demand on management model**

To see the effect of demand on the performance of proposed model can be observed by imposing a constraint on the demand into the model. It helps in determining the minimum value of demand that a model can satisfy. For calculating results, the model is tested under the demands from 70L/s to 50L/s, for every combination of CV of T and CV of S at reliability 95%. The results are given in Table 4.27 calculated at efficiencies 40-66% for pump I and 40-75% for pump II.

**Table 4.27 Optimal pumping for minimum water demand with 95% Reliability**

Reliability	CV of S	CV of T							
		0.2		0.4		0.6		0.8	
		Pump I	Pump II	Pump I	Pump II	Pump I	Pump II	Pump I	Pump II
		Demand	Demand	Demand	Demand	Demand	Demand	Demand	Demand
95%	0.2	70L/s	70L/s	58L/s	Inf	Inf	Inf	Inf	Inf
	0.4	70L/s	70L/s	58L/s	Inf	Inf	Inf	Inf	Inf
	0.6	69L/s	69L/s	57L/s	Inf	Inf	Inf	Inf	Inf
	0.8	68L/s	68L/s	57L/s	Inf	Inf	Inf	Inf	Inf

It is clear from the above table that at 95% reliability, for pump I at CV of T value higher than 0.4, the model is unable to satisfy any demand condition under 40-66% efficiency and for pump II, the model is incapable to meet any demands for CV of T higher than 0.2 at 40-75% efficiency. However, on lowering the allowable minimum efficiency to 34% for pump I and 29% for pump II, for all the combinations of CV of T and CV of S, the optimal solution is obtained. But it is evident that the solutions obtained under these conditions are not efficient, hence the minimum demand value then satisfied will not be beneficial too. It is finally concluded that the model results are highly sensitive to uncertainty in transmissivity.

#### 4.11 Effect of number of pumping wells

The effect of number of pumping wells on pumping plan can be investigated by changing the number of pumping wells to three, keeping the number of control points same as before. Different combination of pumping wells considered for the reliability value of 95%, taking CV of transmissivity fixed at 0.4 as the results are more sensitive to CV of transmissivity. The drawdown values considered are 3.5, 4 and 4.5m using both pump I and pump II. The results obtained for the efficiency range of 40%-66% for pump I and 40%-75% for pump II are given in Table 4.28

**Table 4.28 Total discharge from Pump I and Pump II**

CV of T	CV of S	Without PW1		Without PW2		Without PW3		Without PW4	
		Pump I	Pump II	Pump I	Pump II	Pump I	Pump II	Pump I	Pump II
		Q(L/sec)	Q(L/sec)	Q(L/sec)	Q(L/sec)	Q(L/sec)	Q(L/sec)	Q(L/sec)	Q(L/sec)
<b>0.4</b>	<b>0.2</b>	58.28	58.73	55.33	55.81	56.84	57.97	57.33	57.80
<b>0.4</b>	<b>0.4</b>	58.28	58.73	55.33	55.81	56.83	57.97	57.33	57.80
<b>0.4</b>	<b>0.6</b>	58.04	58.49	55.13	55.58	56.62	57.71	57.33	57.80
<b>0.4</b>	<b>0.8</b>	57.75	58.16	54.89	55.31	56.36	57.40	56.89	57.28

From the results in Table 4.28, it is inferred that the results for pump I are more close to optimal value of discharge for the same efficiency range (Table 4.22), when we exclude PW1 from the combination. While all the other combinations of excluding PW2, PW3 and PW4 gives results less than the optimal value of discharge.

From Table 4.28, it is seen that the results for pump II within this efficiency range has infeasible solution. But it is better to exclude any of the pumping wells (PW1, PW2, PW3 and PW4) from the combination to get feasible solution. When we only exclude PW1 from the combination it gives best results.

#### **4.12 Effect of pump characteristics on optimal pumpage**

To see the effect of pump characteristics, the chance constrained optimization model was also solved without pump characteristics for values of both CV of T and CV of S as 0.4 at 95% reliability. The results are given in Table 4.29. The value of optimal total discharge is 58.83 L/s.

To calculate the efficiency corresponding to optimal discharges in Table 4.29, the discharge values were put in the characteristics equation of pump I. The values of efficiency are obtained more than 100% for pumping well 2. Thus these results are

practically meaningless. Therefore, it can be concluded that the chance constrained optimization model should be solved using pump characteristics to obtain practically meaningful results.

**Table 4.29 Optimal solution for CV of T 0.4 and CV of S 0.4 at 95% reliability without pump characteristics**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	0	0	10.62	102.96	0	0	8.84	85.50
<b>50-100</b>	0	0	10.40	100.78	0	0	8.86	85.70
<b>100-150</b>	0	0	10.76	104.36	0	0	9.34	90.36
<b>Total Discharge</b>	58.83 L/s							

However, when the model was solved by imposing limits on the maximum and minimum allowable discharge values at each well without pump characteristics, practically meaningful results were obtained as given in Table 4.30. The minimum and maximum allowable pumping rates of 4 and 8 L/s, respectively were used.

**Table 4.30 CV of T 0.4 and CV of S 0.4 at 95% reliability without pump characteristics with limitations on discharge**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4	39.77	7.24	70.17	4	39.77	4	39.77
<b>50-100</b>	4	39.77	7.06	68.47	4	39.77	4	39.77
<b>100-150</b>	4	39.77	7.89	76.37	4	39.77	4	39.77
<b>Total Discharge</b>	58.30 L/s							

The model was also solved with pump characteristics for pump I with limits on the maximum and minimum allowable discharge values of 8 and 4 L/s, respectively at each well. The results are presented in Table 4.31.

**Table 4.31 CV of T 0.4 and CV of S 0.4, at 95% reliability with pump characteristics for pump I and limitations on discharge**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.67	46	4.86	47.88	4.67	46	4.67	46
<b>50-100</b>	4.67	46	4.73	46.59	4.67	46	4.67	46
<b>100-150</b>	4.67	46	5.7	55.64	4.67	46	4.67	46

Efficiency = 46-66%; Total Discharge = 57.32 L/s.

On comparing Tables 4.30 and 4.31 it is seen that the discharge and efficiency at pumping wells 1, 3 and 4 is almost similar. But the variations in the discharges and efficiencies at pumping well 2 is noticeable. The pumps without using characteristics are running at a low efficiency.

### 4.13 Summary

In this chapter, the applicability of the developed chance constrained nonlinear optimization model for transient flow through confined aquifer using pump characteristics is demonstrated. The developed model is applied to a hypothetical aquifer having 4 pumping wells and 6 observation wells and solved using LINGO 10.0. It was subjected to constraint of maximum allowable drawdown of 3.5, 4 and 4.5 m for three time periods. The pump characteristics were included as binding constraints in the model. Mainly, this study is based on the inclusion of uncertainties in the form of aquifer parameters as these parameters are not considered deterministic because of the geologic formations of aquifer parameters and hydraulic characteristics of water keep on varying place to place. The effect of uncertainties in aquifer parameters especially

the transmissivity and storage coefficient were considered to see its effect on optimal pumping plan for the aquifer. The effect of reliability was also considered while evaluating the applicability of the model. The limitation is also imposed on the maximum pumping rates for each pump during different periods. The effect of limiting the number of pumping wells is also studied. It is inferred from this study that the total pumpage increases as the reliability requirement decreases. If we have to increase the optimal pumpage we need to decrease the reliability requirement. Hence it is concluded that the actual reliability is less than required reliability. The values of pumpage at higher uncertainty levels decrease. Hence it is concluded that the pumpage at lower uncertainty values are acceptable, because at higher uncertainty the values for pump efficiency decreases very much leading to unsatisfactory results. It is also concluded that the model results are insensitive for variations in storage coefficient and highly sensitive to the variations in transmissivity values. Further, for the large values of transmissivity, log normal distribution should be considered. The optimal pumping plan is affected by pump characteristics.



*Summary  
and  
Conclusions*



**5.1 Summary**

In the groundwater management, groundwater model is a primary quantitative tool to achieve its best use, so that the longevity of the resource can be assured. As the aquifer dimensions and characteristics cannot be fixed hence the data related to aquifer properties cannot be deterministic. The purpose of this study is to observe the effects of uncertainty in aquifer parameters on optimal pumpage.

In this study a hypothetical homogeneous, non-uniform, confined aquifer for non-linear stochastic multi-period groundwater management is developed. The variability is incorporated in two aquifer parameters i.e., Transmissivity and Storage Coefficient and developed a Chance-Constrained optimization model which is then solved using the software LINGO 10.0. And calculated the optimum discharge from the wells, subjected to constraints such as drawdown and efficiency of pumps. The study also shows, how uncertainty affects our actual planning and management decisions. The process of developing the model has one of the important step that is generating the unit response function (**Maddock, 1973**). The unit response function is derived using Cooper-Jacob equation. Generally all the models in their primitive stages are developed assuming ideal conditions as it is much easier to do mathematical calculations in the ideal models, for more accurate and realistic results it is required to focus more on uncertainty and realistic parameters.

The model is solved for seven different cases. First, the model is solved for a specified range of efficiencies using different pumps. Then, the values for coefficient of variability for storage coefficient are varied, keeping the coefficient of variation values for transmissivity constant or vice-versa. After that, the model is tested for three reliability requirements. In addition to this the effect of pump characteristics on model results is observed. Furthermore, the results are calculated after limiting the pumping rate in the model. Moreover, the model is tested for various demand constraints. Finally the effect of limiting the number of pumping wells is observed.

## 5.2 Conclusions

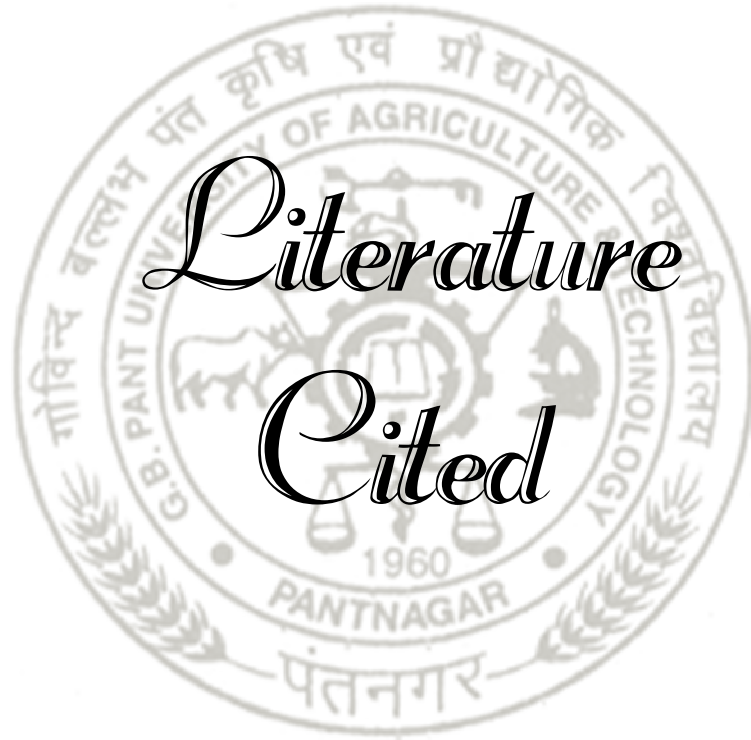
The following conclusions can be made from this study.

1. The developed nonlinear multi-period chance-constrained model can be used to maximize the pumping rate from different wells driven through a confined aquifer having transient flow when limitations are imposed on the maximum values of the drawdown at a number of control points considering pump characteristics and uncertainties in the aquifer parameters.
2. The model results are found to be very sensitive to the uncertainty levels of transmissivity whereas the effect of uncertainty level of storage coefficient on the optimal results is negligible. Therefore, efforts should be made to accurately evaluate the aquifer transmissivity including its variability and the storage coefficient can be treated as deterministic.
3. The optimal value of total pumpage from the aquifer is found to decrease as the reliability requirement and uncertainty level of aquifer properties especially transmissivity increases.
4. The effect of pump characteristics on optimal results is not significant except the difference in efficiency range suggesting to select the pump which results in larger range of efficiency.
5. The model can also be used in deciding the optimum number of pumping wells without compromising with the total amount of pumpage to be achieved in a situation.
6. When results are drawn after fixing the lower and upper range of efficiency to 40% and 66% for pump I and 40% to 75% for pump II, the results obtained are infeasible for higher reliability and high variation in aquifer transmissivity.

## 5.3 Scope for Future Research

Scope for future research of this study are highlighted as below:

1. For more accuracy HYDRUS-2D/3D can be used.
2. Coupling of two models like HYDRUS and MODFLOW can be considered for more accurate modelling.
3. Further studies can be made including artificial recharge.



*Literature  
Cited*



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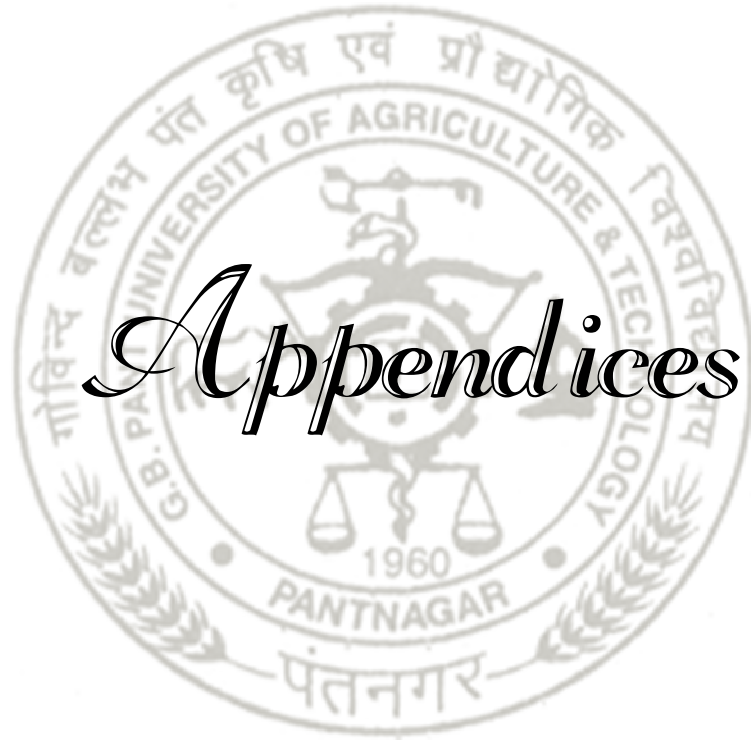
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# *Appendices*



## APPENDIX 1

Considering the pump characteristics with the maximum allowable drawdown (3.5, 4 and 4.5 m) at 90% reliability for CV of T 0.2 and CV of S 0.2, for all the four pumping wells are:

!Objective function;

$$\max=q11+q12+q13+q21+q22+q23+q31+q32+q33+q41+q42+q43;$$

! constraints;

$$\begin{aligned} 152.66*q11+136.75*q21+126.57*q31+124.02*q41 &\leq 3.5; \\ 7.66*(q11+q21+q31+q41)+168.74*q12+152.49*q22+142.09*q32+139.48*q42 &\leq 4; \\ 4.49*(q11+q21+q31+q41)+7.5*(q12+q22+q32+q42)+178.41*q13+161.95*q23+151.42*q33+148.78*q43 &\leq 4.5; \end{aligned}$$

$$\begin{aligned} 129.74*q11+154.35*q21+147.61*q31+115.06*q41 &\leq 3.5; \\ 7.66*(q11+q21+q31+q41)+145.30*q12+170.42*q22+163.55*q32+130.30*q42 &\leq 4; \\ 4.49*(q11+q21+q31+q41)+7.5*(q12+q22+q32+q42)+154.65*q13+180.09*q23+173.12*q33+139.47*q43 &\leq 4.5; \end{aligned}$$

$$\begin{aligned} 127.61*q11+152.74*q21+140.66*q31+123.76*q41 &\leq 3.5; \\ 7.66*(q11+q21+q31+q41)+143.14*q12+168.8*q22+156.46*q32+139.2*q42 &\leq 4; \\ 4.49*(q11+q21+q31+q41)+7.5*(q12+q22+q32+q42)+152.48*q13+178.46*q23+165.97*q33+148.5*q43 &\leq 4.5; \end{aligned}$$

$$\begin{aligned} 126.08*q11+127.26*q21+152.62*q31+144.51*q41 &\leq 3.5; \\ 7.66*(q11+q21+q31+q41)+141.55*q12+142.75*q22+168.65*q32+160.36*q42 &\leq 4; \\ 4.49*(q11+q21+q31+q41)+7.5*(q12+q22+q32+q42)+150.85*q13+152.06*q23+178.27*q33+169.89*q43 &\leq 4.5; \end{aligned}$$

$$\begin{aligned} 127.6*q11+117.5*q21+138.3*q31+161.04*q41 &\leq 3.5; \\ 7.66*(q11+q21+q31+q41)+143.13*q12+132.81*q22+154.06*q32+177.29*q42 &\leq 4; \\ 4.49*(q11+q21+q31+q41)+7.5*(q12+q22+q32+q42)+152.46*q13+142.02*q23+163.54*q33+187.05*q43 &\leq 4.5; \end{aligned}$$

$$\begin{aligned} 155.48*q11+126.96*q21+139.05*q31+154.75*q41 &\leq 3.5; \\ 7.65*(q11+q21+q31+q41)+171.41*q12+142.31*q22+154.66*q32+170.67*q42 &\leq 4; \\ 4.48*(q11+q21+q31+q41)+7.49*(q12+q22+q32+q42)+180.82*q13+151.51*q23+163.99*q33+180.19*q43 &\leq 4.5; \end{aligned}$$

```

eta11= -b0+(b1*q11)-(b2*q11^2)+(b3*q11^3);
eta12= -b0+(b1*q12)-(b2*q12^2)+(b3*q12^3);
eta13= -b0+(b1*q13)-(b2*q13^2)+(b3*q13^3);
eta21= -b0+(b1*q21)-(b2*q21^2)+(b3*q21^3);
eta22= -b0+(b1*q22)-(b2*q22^2)+(b3*q22^3);
eta23= -b0+(b1*q23)-(b2*q23^2)+(b3*q23^3);
eta31= -b0+(b1*q31)-(b2*q31^2)+(b3*q31^3);
eta32= -b0+(b1*q32)-(b2*q32^2)+(b3*q32^3);
eta33= -b0+(b1*q33)-(b2*q33^2)+(b3*q33^3);
eta41= -b0+(b1*q41)-(b2*q41^2)+(b3*q41^3);
eta42= -b0+(b1*q42)-(b2*q42^2)+(b3*q42^3);
eta43= -b0+(b1*q43)-(b2*q43^2)+(b3*q43^3);

```

```

etad=40;

```

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etah=66;

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eta11>=etad;

```

```

eta12>=etad;

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eta13>=etad;

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eta21>=etad;

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eta22>=etad;

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eta23>=etad;

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eta31>=etad;

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eta32>=etad;

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eta33>=etad;

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eta41>=etad;

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eta42>=etad;

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eta43>=etad;

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eta11<=etah;

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eta12<=etah;

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eta13<=etah;

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eta21<=etah;

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eta22<=etah;

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eta23<=etah;

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eta31<=etah;

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eta32<=etah;

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eta33<=etah;

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eta41<=etah;

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eta42<=etah;

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eta43<=etah;

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b0=2.4;

```

```

b1=9437;

```

```

b2=39453;

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```

b3=4000000;

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end

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## APPENDIX 2

### Optimal Results for pump I at Reliability 95%

At 95% reliability, the results are calculated for 16 different combinations of coefficient of variation in transmissivity and coefficient of variance in storage coefficient. All these combinations can be applied to both pumps. Here the results for pump I are tabulated from Table A2.1 to Table A2.16.

#### Case 1: Fixing coefficient of variation of storage coefficient equals to 0.4, and varying coefficient of variation of transmissivity

The results of these four combinations are presented from Table A2.1 to Table A2.4. These tables contain the value of pumping rates and efficiency at each pumping well for three time periods each of 50 days. The range of efficiency is fixed for maximum allowable efficiency, but minimum allowable efficiency is considered according to the minimum value below which the optimal solution becomes infeasible.

**Table A2.1 CV Transmissivity 0.2 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.63	55	6.27	61.03	5.63	55	5.63	55
<b>50-100</b>	5.63	55	5.88	57.31	5.63	55	5.63	55
<b>100-150</b>	5.63	55	6.8	66	5.81	55.65	5.63	55

Efficiency = 55-66%; Total Discharge = 69.80124 L/s.

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.63 L/s and 55%. But the values for discharge and efficiency is different in all periods for well 2 and well 3.

**Table A2.2 CV Transmissivity 0.4 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.67	46	4.92	48.32	4.67	46	4.67	46
<b>50-100</b>	4.67	46	4.77	46.93	4.67	46	4.67	46
<b>100-150</b>	4.67	46	5.76	55.93	4.67	46	4.67	46

Efficiency = 46-66%; Total Discharge = 57.4284 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.67 L/s and 46%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.3 CV Transmissivity 0.6 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	4.28	42.35	3.92	39	3.92	39
50-100	3.92	39	4.25	42.08	3.92	39	3.92	39
100-150	3.92	39	5.09	49.96	3.92	39	3.92	39

Efficiency = 39-66%; Total Discharge = 48.86914 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A4.4 CV Transmissivity 0.8 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.38	34	3.75	37.49	3.38	34	3.38	34
<b>50-100</b>	3.38	34	3.79	37.85	3.38	34	3.38	34
<b>100-150</b>	3.38	34	4.55	44.86	3.38	34	3.38	34

Efficiency = 34-66%; Total Discharge = 42.51294 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.38 L/s and 34%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 55% for CV Transmissivity 0.2 and 34% for CV Transmissivity 0.8. Hence there is a decrease of 21% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

### **Case 2: Fixing coefficient of variation of storage coefficient equals to 0.4, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.5 to Table A2.8. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days. The range of efficiency is fixed for maximum allowable efficiency, but minimum allowable efficiency is considered according to the minimum value above which the optimal solution becomes infeasible.

**Table A2.5 CV Transmissivity 0.2 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.63	55	6.1	59.37	5.63	55	5.63	55
<b>50-100</b>	5.63	55	5.76	56.23	5.63	55	5.63	55
<b>100-150</b>	5.63	55	6.8	66	5.72	55.83	5.63	55

Efficiency = 55-66%; Total Discharge = 69.42228 L/s.

It is seen from this table that the discharges and efficiency from well 1 and well 4 are equal at each time interval i.e., 5.63 L/s and 55%. But the values for discharge and efficiency is different in all periods for well 2 and well 3.

**Table A2.6 CV Transmissivity 0.4 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.67	46	4.86	47.88	4.67	46	4.67	46
<b>50-100</b>	4.67	46	4.73	46.59	4.67	46	4.67	46
<b>100-150</b>	4.67	46	5.7	55.64	4.67	46	4.67	46

Efficiency = 46-66%; Total Discharge = 57.3129 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.67 L/s and 46%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.7 CV Transmissivity 0.6 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	4.25	42.12	3.92	39	3.92	39
<b>50-100</b>	3.92	39	4.23	41.92	3.92	39	3.92	39
<b>100-150</b>	3.92	39	5.08	49.81	3.92	39	3.92	39

Efficiency = 39-66%; Total Discharge = 48.81152 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.8 CV Transmissivity 0.8 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.38	34	3.75	37.42	3.38	34	3.38	34
<b>50-100</b>	3.38	34	3.79	37.81	3.38	34	3.38	34
<b>100-150</b>	3.38	34	4.54	44.83	3.38	34	3.38	34

Efficiency = 34-66%; Total Discharge = 42.4988 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.38 L/s and 34%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 55% for CV Transmissivity 0.2 and 34% for CV Transmissivity 0.8. Hence there is a decrease of 21% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

**Case 3: Fixing coefficient of variation of storage coefficient equals to 0.6, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.9 to Table A2.12. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days.

**Table A2.9 CV Transmissivity 0.2 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.52	54	6.19	60.28	5.52	54	5.63	55
<b>50-100</b>	5.52	54	5.92	57.69	5.52	54	5.63	55
<b>100-150</b>	5.52	54	6.8	66	5.74	56	5.63	55

Efficiency = 54-66%; Total Discharge = 68.8388 L/s.

It is seen from this table that the discharges and efficiency from well 1 at each time interval is same i.e., 5.52 L/s and 54% and the values for discharge and efficiency is 5.63L/s and 55 % in all periods for well 4. But the values of discharge and efficiency is different for all periods for well 2 and well 3.

**Table A2.10 CV Transmissivity 0.4 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.56	45	5.14	50.44	4.56	45	4.56	45
<b>50-100</b>	4.56	45	5.02	49.32	4.56	45	4.56	45
<b>100-150</b>	4.56	45	5.99	58.41	4.56	45	4.56	45

Efficiency = 45-66%; Total Discharge = 57.20961 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.56 L/s and 45%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.11 CV Transmissivity 0.6 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	4.21	41.7	3.92	39	3.92	39
<b>50-100</b>	3.92	39	4.2	41.64	3.92	39	3.92	39
<b>100-150</b>	3.92	39	5.05	49.52	3.92	39	3.92	39

Efficiency = 39-66%; Total Discharge = 48.70535 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.12 CV Transmissivity 0.8 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.38	34	3.72	37.15	3.38	34	3.38	34
<b>50-100</b>	3.38	34	3.77	37.59	3.38	34	3.38	34
<b>100-150</b>	3.38	34	4.52	44.63	3.38	34	3.38	34

Efficiency = 34-66%; Total Discharge = 42.42522 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 2 and well 3 are equal at each time interval i.e., 3.38 L/s and 34%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 54% for CV Transmissivity 0.2 and 34% for CV Transmissivity 0.8. Hence there is a decrease of 20% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

#### **Case 4: Fixing coefficient of variation of storage coefficient equals to 0.8, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.13 to Table A2.16. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days.

**Table A2.13 CV Transmissivity 0.2 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.52	54	5.83	56.88	5.52	54	5.52	54
<b>50-100</b>	5.52	54	5.62	54.93	5.52	54	5.52	54
<b>100-150</b>	5.52	54	6.79	65.89	5.52	54	5.52	54

Efficiency = 54-66%; Total Discharge = 67.9575 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 5.52 L/s and 54%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.14 CV Transmissivity 0.4 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.56	45	5.01	49.22	4.56	45	4.56	45
<b>50-100</b>	4.56	45	4.93	48.4	4.56	45	4.56	45
<b>100-150</b>	4.56	45	5.9	57.56	4.56	45	4.56	45

Efficiency = 45-66%; Total Discharge = 56.8903 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.56 L/s and 45%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.15 CV Transmissivity 0.6 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.92	39	4.14	41.09	3.92	39	3.92	39
<b>50-100</b>	3.92	39	4.14	41.12	3.92	39	3.92	39
<b>100-150</b>	3.92	39	4.14	49.01	3.92	39	3.92	39

Efficiency = 39-66%; Total Discharge = 48.5402 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.92 L/s and 39%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.16 CV Transmissivity 0.8 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.38	34	3.68	36.8	3.38	34	3.38	34
<b>50-100</b>	3.38	34	3.74	37.32	3.38	34	3.38	34
<b>100-150</b>	3.38	34	4.49	44.38	3.38	34	3.38	34

Efficiency = 34-66%; Total Discharge = 42.3318 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.38 L/s and 34%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 54% for CV Transmissivity 0.2 and 34% for CV Transmissivity 0.8. Hence there is a decrease of 20% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

### **Optimal Results for pump I at Reliability 97.5%**

At 97.5% reliability, the results are calculated for 16 different combinations of coefficient of variation in transmissivity and coefficient of variance in storage coefficient. All these combinations can be applied to both pumps. Here the results for pump I are tabulated from Table A2.17 to Table A2.32.

#### **Case 1: Fixing coefficient of variation of storage coefficient equals to 0.4, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.17 to Table A2.20. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days. The range of efficiency is fixed for maximum allowable efficiency, but minimum allowable efficiency is considered according to the minimum value above which the optimal solution becomes infeasible.

**Table A2.17 CV Transmissivity 0.2 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.42	53	5.95	57.96	5.42	53	5.42	53
<b>50-100</b>	5.42	53	5.62	54.87	5.42	53	5.42	53
<b>100-150</b>	5.42	53	6.7	65.05	5.42	53	5.42	53

Efficiency = 53-66%; Total Discharge = 67.01576 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 5.42 L/s and 53%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.18 CV Transmissivity 0.4 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.65	45.85	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.56	44.99	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.47	53.53	4.35	43	4.35	43

Efficiency = 43-66%; Total Discharge = 53.8025 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.19 CV Transmissivity 0.6 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.59	36	3.94	39.19	3.59	36	3.59	36
<b>50-100</b>	3.59	36	3.95	39.33	3.59	36	3.59	36
<b>100-150</b>	3.59	36	4.74	48.68	3.59	36	3.59	36

Efficiency = 36-66%; Total Discharge = 44.98511 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.59 L/s and 36%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.20 CV Transmissivity 0.8 and CV Storage Coefficient 0.2**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.06	31	3.43	34.51	3.06	31	3.06	31
<b>50-100</b>	3.06	31	3.5	35.15	3.06	31	3.06	31
<b>100-150</b>	3.06	31	4.2	41.98	3.06	31	3.06	31

Efficiency = 31-66%; Total Discharge = 38.6536 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.06 L/s and 31%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 53% for CV Transmissivity 0.2 and 31% for CV Transmissivity 0.8. Hence there is a decrease of 22% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

**Case 2: Fixing coefficient of variation of storage coefficient equals to 0.4, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.21 to Table A2.24. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days.

**Table A2.21 CV Transmissivity 0.2 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.42	53	5.75	56.15	5.42	53	5.42	53
<b>50-100</b>	5.42	53	5.47	53.51	5.42	53	5.42	53
<b>100-150</b>	5.42	53	6.57	63.82	5.42	53	5.42	53

Efficiency = 53-66%; Total Discharge = 66.54765 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 5.42 L/s and 53%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.22 CV Transmissivity 0.4 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.6	45.36	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.52	44.65	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.44	53.23	4.35	43	4.35	43

Efficiency = 43-66%; Total Discharge = 53.6817 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.23 CV Transmissivity 0.6 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.59	36	3.91	38.97	3.59	36	3.59	36
<b>50-100</b>	3.59	36	3.93	39.18	3.59	36	3.59	36
<b>100-150</b>	3.59	36	4.72	46.54	3.59	36	3.59	36

Efficiency = 36-66%; Total Discharge = 44.9229 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.59 L/s and 36%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.24 CV Transmissivity 0.8 and CV Storage Coefficient 0.4**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.06	31	3.43	34.44	3.06	31	3.06	31
<b>50-100</b>	3.06	31	3.5	35.11	3.06	31	3.06	31
<b>100-150</b>	3.06	31	4.2	41.59	3.06	31	3.06	31

Efficiency = 31-66%; Total Discharge = 38.64052 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.06 L/s and 31%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 53% for CV Transmissivity 0.2 and 31% for CV Transmissivity 0.8. Hence there is a decrease of 22% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

**Case 2: Fixing coefficient of variation of storage coefficient equals to 0.4, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.25 to Table A2.28. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days.

**Table A2.25 CV Transmissivity 0.2 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	5.31	52	5.82	56.81	5.31	52	5.31	52
<b>50-100</b>	5.31	52	5.6	54.71	5.31	52	5.31	52
<b>100-150</b>	5.31	52	6.72	65.23	5.31	52	5.31	52

Efficiency = 52-66%; Total Discharge = 65.9346 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 5.31 L/s and 52%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.26 CV Transmissivity 0.4 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.35	43	4.49	44.36	4.35	43	4.35	43
<b>50-100</b>	4.35	43	4.44	43.83	4.35	43	4.35	43
<b>100-150</b>	4.35	43	5.36	52.47	4.35	43	4.35	43

Efficiency = 43-66%; Total Discharge = 53.406 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.35 L/s and 43%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.27 CV Transmissivity 0.6 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.59	36	3.87	38.55	3.59	36	3.59	36
<b>50-100</b>	3.59	36	3.9	38.84	3.59	36	3.59	36
<b>100-150</b>	3.59	36	4.69	46.24	3.59	36	3.59	36

Efficiency = 36-66%; Total Discharge = 44.81554 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.59 L/s and 36%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.28 CV Transmissivity 0.8 and CV Storage Coefficient 0.6**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.06	31	3.4	34.18	3.06	31	3.06	31
<b>50-100</b>	3.06	31	3.48	34.9	3.06	31	3.06	31
<b>100-150</b>	3.06	31	4.17	41.39	3.06	31	3.06	31

Efficiency = 31-66%; Total Discharge = 38.56813 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.06 L/s and 31%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 52% for CV Transmissivity 0.2 and 31% for CV Transmissivity 0.8. Hence there is a decrease of 21% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

**Case 4: Fixing coefficient of variation of storage coefficient equals to 0.8, and varying coefficient of variation of transmissivity**

The results of these combinations are presented from Table A2.29 to Table A2.32. These tables contain the value of pumping rates and efficiency at each pumping well at three time periods each of 50 days.

**Table A2.29 CV Transmissivity 0.2 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
0-50	5.20	51	5.81	56.65	5.20	51	5.2	51
50-100	5.20	51	5.20	51	5.24	51.38	5.2	51
100-150	5.20	51	6.80	66	5.22	51.19	5.2	51

Efficiency = 51-66%; Total Discharge = 64.70078 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 5.2 L/s and 51%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.30 CV Transmissivity 0.4 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	4.24	42	4.74	46.64	4.24	42	4.24	42
<b>50-100</b>	4.24	42	4.71	46.36	4.24	42	4.24	42
<b>100-150</b>	4.24	42	5.64	55.07	4.24	42	4.24	42

Efficiency = 42-66%; Total Discharge = 53.2345 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 4.24 L/s and 42%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.31 CV Transmissivity 0.6 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.59	36	3.80	37.94	3.59	36	3.59	36
<b>50-100</b>	3.59	36	3.85	38.36	3.59	36	3.59	36
<b>100-150</b>	3.59	36	4.65	45.81	3.59	36	3.59	36

Efficiency = 36-66%; Total Discharge = 44.6523 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 2 and well 3 are equal at each time interval i.e., 3.59 L/s and 36%. But the values for discharge and efficiency is different in all periods for well 2.

**Table A2.32 CV Transmissivity 0.8 and CV Storage Coefficient 0.8**

Time Period (Days)	Pumping Well 1		Pumping Well 2		Pumping Well 3		Pumping Well 4	
	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)	Discharge (L/s)	Efficiency (%)
<b>0-50</b>	3.06	31	3.36	33.84	3.06	31	3.06	31
<b>50-100</b>	3.06	31	3.45	34.63	3.06	31	3.06	31
<b>100-150</b>	3.06	31	4.15	41.14	3.06	31	3.06	31

Efficiency = 31-66%; Total Discharge = 38.4754 L/s.

It is seen from this table that the discharges and efficiency from well 1, well 3 and well 4 are equal at each time interval i.e., 3.06 L/s and 31%. But the values for discharge and efficiency is different in all periods for well 2.

The observed difference in efficiencies and total discharge value is huge as we vary the value of coefficient of transmissivity. The value of minimum allowable efficiency is 51% for CV Transmissivity 0.2 and 31% for CV Transmissivity 0.8. Hence there is a decrease of 20% in the minimum allowable efficiency value when we increase the value of CV from 0.2 to 0.8.

*The authoress, **Neha Dhapola** was born in Bageshwar, Uttarakhand on 1<sup>st</sup> June 1995. She passed her High School Examination in 2010 from Jawahar Navodaya Vidyalaya, Bageshwar and Higher Secondary Examination in 2012 from Jawahar Navodaya Vidyalaya, Nainital. She did her B.Tech Degree in Civil Engineering from Bipin Tripathi Kumaon Institute of Technology, Dwarahat (affiliated to Uttarakhand Technical University) in 2017 with Honor. She took admission in Gobind Ballabh Pant University of Agriculture and Technology, Pantnagar in August 2018 for M.Tech in Civil Engineering with major in Hydraulic Engineering.*

## ***Address:***


*Village and Post Office Chaura  
Distt. Bageshwar,  
Pin: 263619  
Uttarakhand  
Contact number: 8755215250  
Email id. nehadhapola720@gmail.com*



## सारांश

नाम	: नेहा धपोला	आईडी नंबर	: 54098
सेमेस्टर और प्रवेश का वर्ष	: 1 <sup>st</sup> , 2018-2019	डिग्री	: मास्टर ऑफ टेक्नोलॉजी (सिविल इंजीनियरिंग)
प्रमुख	: हाइड्रोलिक इंजीनियरिंग	विभाग	: सिविल इंजीनियरिंग
मामूली	: शून्य		
थीसिस शीर्षक	: पंप विशेषताओं को ध्यान में रखते हुए क्षणिक भूजल प्रवाह का मौका विवश अनुकूलन		
सलाहकार	: डॉ पी एस महर		

भूजल मॉडलिंग में विकास के साथ, भूजल मापदंडों से जुड़ी अनिश्चितता पर विचार करना अपरिहार्य हो गया है। निर्धारक दृष्टिकोण के साथ पारंपरिक मॉडल वांछनीय परिणाम ड्राइंग के लिए अनुपयुक्त साबित होते हैं और नतीजतन यह भूजल मॉडल के बेहतर अध्ययन के लिए एक मौका विवश दृष्टिकोण की मांग करता है। इस अध्ययन में, क्षणिक भूजल प्रवाह का एक मौका विवश अनुकूलन मॉडल अधिकतम स्वीकार्य ड्राडाउन, पंप विशेषताओं, विभिन्न विश्वसनीयताओं पर जलभृत मापदंडों में भिन्नता के साथ आवश्यक मांग जैसी बाधाओं के अधीन कई पंपिंग कुओं से पंपिंग दर को अधिकतम करने के लिए विकसित किया गया है। मॉडल प्रयोज्यता चार पंपिंग कुओं और छह अवलोकन कुओं को शामिल एक काल्पनिक अध्ययन क्षेत्र का उपयोग कर सचित्र किया गया है। प्रत्येक 50 दिनों की तीन अवधियों के लिए पंपिंग दरों को अधिकतम किया जाता है। मॉडल के भीतर बाधा समीकरणों को बीजीय तकनीकी कार्यों का उपयोग करके विकसित किया जाता है जो कूपर-जैकब समीकरणों का उपयोग करके उत्पन्न होते हैं, मॉडल को लिंगो 10.0 सॉफ्टवेयर का उपयोग करके हल किया जाता है। अध्ययन से निकाले गए प्रमुख निष्कर्ष से पता चलता है कि ट्रांसमिसिजिटी और स्टोरेज गुणांक की भिन्नता में वृद्धि के साथ, इष्टतम निर्वहन कम हो जाता है। इष्टतम निर्वहन मान भंडारण गुणांक की तुलना में संक्रामकता में भिन्नता के प्रति अधिक संवेदनशील होते हैं। अध्ययन यह भी दर्शाता है कि मौका विवश मॉडल में बाधाओं के रूप में पंप विशेषताओं का उपयोग करके, व्यावहारिक रूप से सार्थक समाधान प्राप्त किया जाता है। यह भी अनुमानित है कि उच्च विश्वसनीयता आवश्यकता के लिए इष्टतम पंपेज कम हो जाता है।

  
(पी एस महर)  
सलाहकार

  
(नेहा धपोला)  
लेखिका