METAHEURISTIC APPROACHES FOR ADAPTIVE ARRAY SIGNAL PROCESSING IN SMART ANTENNA

Thesis

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By

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The tenure of human in this world is supported by many others. Acknowledgement for a few might be just a trifle thing written on a piece of paper. Nevertheless, in the true essence, it gives us an opportunity to remember and express our feelings to those, whom we love, revere and share our secrets. Here I get a great chance to express my token of thanks to people who in a way helped and supported me to complete this record.

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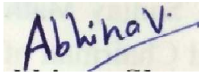

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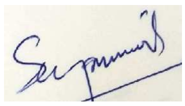
CERTIFICATE-I

This is to certify that the thesis entitled **"Metaheuristic Approaches for Adaptive Array Signal Processing in Smart Antenna"** submitted in partial fulfilment of the requirements for the degree of **Doctor of Philosophy** with major in **Electronics and Communication Engineering** and minor in **Computer Science** of the College of Post-Graduate Studies, G. B. Pant University of Agriculture and Technology, Pantnagar, is a record of *bona fide* research carried out by **Mr. Abhinav Sharma**, Id. No. **39344** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been acknowledged.



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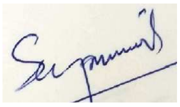
We, the undersigned, members of Advisory Committee of **Mr. Abhinav Sharma**, Id. No. **39344**, a candidate for the degree of **Doctor of Philosophy** with major in **Electronics and Communication Engineering** and minor in **Computer Science**, agree that the thesis entitled " **Metaheuristic Approaches for Adaptive Array Signal Processing in Smart Antenna** ", may be submitted in partial fulfilment of the requirements for the degree.



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ACRONYMS AND ABBREVIATIONS

ACO	Ant colony optimization
A/D	Analog to digital
ADIWO	Adaptive dispersion invasive weed optimization
AI	Artificial intelligence
AMBPSO	Adaptive mutated boolean particle swarm optimization
AWGN	Additive white Gaussian noise
BER	Bit error rate
B-ESPRIT	Beamspace ESPRIT
BFO	Bacteria foraging algorithm
CDMA	Code division multiple access
CFO	Central force optimization
CM	Constant modulus
CRLB	Crammer rao lower bound
dB	decibel
DMI	Direct matrix inversion
DML	Deterministic maximum likelihood
DOA	Direction of arrival estimation
EA	Evolutionary algorithms

ESMI	Enhanced sample matrix inversion
ESPRIT	Estimation of signal parameters via rotation invariance technique
FCC	Federal communication commission
FDMA	Frequency division multiple access
FM	Frequency modulation
FPGA	Field programmable gate array's
FSK	Frequency shift keying
GA	Genetic algorithm
GSA	Gravitational search algorithm
LCMV	Linear constraint minimum variance
LMS	Least mean square
MC-CDMA	Multi carrier code division multiple access
MIMO	Multiple input multiple output
ML	Maximum likelihood
MN-MUST	Modified neural multiple source tracking
MP	Matrix Pencil
MUSIC	Multiple signal classification
MVDR	Minimum variance distortionless response
N-MUST	Neural multiple source tracking
NN	Neural network

NSMI	Normalized sample matrix inversion
PAM	Pulse amplitude modulation
PCS	Personal communications services
PDF	Probability density function
PSK	Phase shift keying
PSO	Particle swarm optimization
QAM	Quadrature amplitude modulation
RBFNN	Radial basis function neural network
RF	Radio frequency
RLS	Recursive least square
SA	Simulated annealing
SD	Standard deviation
SDMA	Space division multiple access
SINR	Signal to interference and noise ratio
SLC	Side lobe canceller
SLL	Side lobe level
SML	Stochastic maximum likelihood
SMI	Sample matrix inversion
SMW	Sherman Morrison-woodbury
SNOI	Signal not of interest

SNR	Signal to noise ratio
SOI	Signal of interest
SVD	Singular value decomposition
TDMA	Time division multiple access
TLS	Total least square
TLS-MP	Total least square matrix pencil
ULA	Uniform linear array
UMP	Unitary matrix pencil
VLSI	Very large scale integration

Introduction

CHAPTER 1 INTRODUCTION

The field of wireless communication has shown a rapid growth in recent past, which requires higher channel capacities and data rates to satisfy increasing demand of smart phones, tablets, ipads and other electronic gadgets. The increasing demand of these services without a corresponding increase in radio frequency (RF) spectrum allocation motivates the need for new techniques to improve spectrum utilization. There exists numerous tools and techniques such as advanced coding and compression technique, higher order modulation technique, adaptive interference management, dynamic spectrum management and smart antennas which enables the efficient utilization of spectrum. *Joseph Mitola* first proposed dynamic spectrum management technique i.e. cognitive radios in 1998 that autonomously coordinate the usage of spectrum. They identify radio spectrum when it is unused by the incumbent (primary, license holding) users and use this spectrum in an intelligent way based on spectrum observation.

The smart antenna approach which has shown promise in terms of capacity improvement is the use of adaptive array antenna also known as smart antenna. Smart essentially means computer control of the antenna performance. This antenna has the capability of spatial filtering by steering its beam towards the direction of interest and placing nulls in the direction of interfering signals. This outcome is achieved by proper weighting of each received signal according to an adaptive algorithm. This concept is called space division multiple access (SDMA). It is used in conjunction with time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) technologies in order to provide the latter ones with the additional ability of identifying a single user with its unique *spatial signature*. Adaptive antennas were first introduced in 1960s and initially deployed in military communication systems, where narrow beams are used to avoid interference from noise and jamming signals.

The researchers extended the concept of adaptive antennas in wireless communication which offers many advantages like improvement in bit error rate (BER) and network throughput, reduction in fading due to multipath propagation but it could be best explained by words of Andrew Viterbi, a pioneer in the global spread of wireless communication, "*Spatial processing remains as the most promising, if not the last frontier,* *in the evolution of multiple access systems*". Signal processing stage allows the antenna beam pattern to adapt to the changing RF environment. Signal processing combined with antenna array produces a directive beam that can be scanned electronically by varying the excitation of the individual elements. The objectives of the signal processing stage are estimation of direction of arrival (DOA) of all the impinging signals and the calculation of appropriate weights to steer the maximum radiation pattern towards the signal of interest (SOI) and place nulls towards signal not of interest (SNOI).

DOA estimation of narrow band plane waves in real time is an integral part of many practical signal processing problems. With the advent of third and fourth generation wireless technology federal communication commission (FCC) requires that mobile communication system should locate and track mobile phones for efficient spectrum utilization. It has widespread applications in the field of radar, sonar, seismic exploration, electronic surveillance, satellite and mobile communication. DOA estimation techniques can be categorized on the basis of the data analysis into three categories, spectral estimation methods, Eigen structure methods and maximum likelihood (ML) techniques. Spectral estimation methods search for peaks in all possible direction while Eigen structure based methods exploit the structure of the received data, in particular array correlation matrix. Multiple signal classification (MUSIC), estimation of signal parameters via rotation invariance technique (ESPRIT) are some of the high resolution Eigen structure methods. In this thesis, the performance of spectral and Eigen structure based DOA estimation algorithms have been compared.

ML is a standard technique in statistical estimation theory. This method gives superior statistical performance compared to other methods in multipath channel environment and in the environment where signal to noise ratio (SNR) and the number of snapshots are small. The ML estimate is computed by maximizing the likelihood function or minimizing the negative likelihood function with respect to all unknown parameters, which may include the source DOA's, the signal covariance and the noise parameters. Due to large computational burden direct maximization of multimodal nonlinear likelihood function over the large parameter space is unrealistic. Thus, different optimization techniques have been considered to jointly estimate source DOA's and other parameters over a high dimensional space. Many researchers have proposed different optimization techniques to optimize ML function. Alternating projection, simulated annealing and data supported grid search algorithm optimize ML function but they cannot guarantee global convergence. Metaheuristic approaches have evolved as one of the leading techniques for solving optimization problems in engineering applications. These approaches find applications in the field of adaptive array signal processing problems in smart antennas. Genetic algorithm (GA), differential evolution (DE) algorithm, particle swarm optimization (PSO), ant colony optimization (ACO), artificial bees colony algorithm, gravitational search algorithm (GSA), etc. have widely been used to obtain ML solution by optimizing complex non linear multimodal function. On the basis of nature of signals, there exists two types of ML models viz. deterministic model (signals are assumed to be deterministic) and stochastic ML model (signals are assumed to be random) **(Stoica and Nehorai, 1990)**. **Magdy, *et al.* (2013)** addressed GSA for estimating direction of deterministic signals in circular arrays by optimizing deterministic ML function.

In this thesis, GSA has been used for estimating direction of closely spaced narrow band uncorrelated deterministic signals by optimizing deterministic ML function in linear arrays. The performance of the algorithm is judged on the basis of root mean square error (RMSE) and resolution probability and the results are compared with PSO and conventional algorithms.

Coherent signals are common in practical scenario of wireless communication. Multipath propagation is a common example of such scenario in which signals originate from a single source and reach the destination by successive reflections from various objects. The spectral estimation and Eigen structure based methods fail to estimate the direction in such scenarios. **Minghui and Yilong (2007)** proposed a modified and refined genetic algorithm to find exact solution of stochastic ML function for linear and circular arrays. **Jiankui, *et al.* (2006)** has estimated direction of uncorrelated signals by optimizing stochastic ML function using PSO algorithm. **Minghui and Yilong (2008**) have used PSO algorithm for estimating the direction in uncorrelated and colored noise environment.

In this research work, PSO algorithm has been explored for optimizing stochastic ML function for estimating direction of uncorrelated, partially correlated and coherent signals in linear arrays. PSO algorithm gives best statistical performance than conventional approaches in estimating direction of incoming signals in all environments. The direction of partially correlated signals with different correlation coefficient are estimated, in order to analyze the efficiency of the proposed algorithm in multipath channel environment.

Adaptive beamforming (ABF) also known as optimum combining finds its roots with the invention of sidelobe canceller (SLC) in 1959 by Howells. In 1965 Applebaum developed fully adaptive antenna array which is based on maximization of signal to noise ratio (SNR) at the array output. Since then many ABF techniques have been proposed which dynamically shape the beam pattern of antenna array by applying appropriate excitation weights that vary with time. There are two types of beamforming approaches, first in which arrival angle is fixed and doesn't change with time and second in which arrival angle is continuously changing with time.

Minimum mean square error (MMSE), Maximum Likelihood (ML) and Minimum variance distortionless response (MVDR) are fixed beamforming approaches. MVDR is an efficient ABF technique that aims to maximize the signal to interference and noise ratio (SINR) in order to increase the efficiency of adaptive antennas. This algorithm can maximize the SINR but simultaneously can't minimize the side lobe level (SLL) which is an essential requirement to avoid unreasonable spread of radiated power. The dual problem of maximizing SINR with reduced SLL can't be solved by conventional beamforming techniques. This multi-objective problem can be modeled as an optimization problem. Since classical gradient based optimization methods have a tendency to stuck in local minima and have a slow rate of convergence, thus these techniques fail to provide a optimum beamforming solution. In recent past, researchers have explored metaheuristic approaches in ABF and pattern synthesis problems.

Goswami and Mandal (2012) have explored GA for controlling nulls and SLL in linear antenna arrays. **Zaharis *et al.*** (**2011)** proposed adaptive mutated boolean particle swarm optimization (AMBPSO) for ABF in linear antenna arrays at different power levels. **Zaharis *et al.* (2012, a, b, c)** proposed adaptive dispersion invasive weed optimization (ADIWO) technique for ABF with reduced SLL at different number of interfering signals and power levels.

In this research work GSA has been explored for estimating weights of linear arrays for ABF. The results are analyzed at different power levels and for different number of interfering signals. The linear and polar radiation pattern obtained using GSA are compared with MVDR and the previous reported results.

The ABF algorithm in which arrival angle is continuously changing with time can be categorized based on whether the algorithm uses a reference signal, direction of the desired signal or system properties of the signal to adaptively form its beam. Reference signal based algorithms minimize the mean square error (cost function) between the reference signal and array output. If the cost function is minimized, then the quality of signal is maximized. The most common adaptive algorithms are Least Mean Square (LMS), Sample Matrix Inversion (SMI) and Recursive Least Square (RLS). Constant Modulus (CM) is a blind beamforming algorithm which takes advantage of the constant amplitude of phase modulated signals. In this research work, the performance of all ABF algorithms has been compared.

1.1 Objectives

The objective of the present study is to explore metaheuristic approaches viz. PSO and GSA in the field of array signal processing. Towards this end, the outline of the work to be carried out in this dissertation is as follows:

1. Simulation of DOA estimation algorithms and their statistical analysis based on RMSE and probability of resolution.

2. Simulation of blind and non-blind ABF algorithms and their performance analysis based on the beamwidth, maximum null depth, maximum SLL and rate of convergence.

3. Simulation of GSA and PSO algorithms for DOA estimation using ML criteria for uncorrelated deterministic signals and their comparison with conventional techniques.

4. Simulation of PSO algorithm for DOA estimation using ML criteria for random signals in different channel environment and their comparison with conventional techniques. The different channel environment includes sources which are:

    i. Uncorrelated sources

    ii. Partially correlated sources

    iii. Coherent sources

5. Simulation of GSA for optimizing single objective fitness function for ABF and its comparison with MVDR technique.

6. Simulation of GSA for optimizing multi-objective fitness function for ABF with reduced SLL and its comparison with MVDR and previous reported results.

1.2 Organization of thesis

The thesis is organized in five chapters and the first chapter presents the introduction and objectives of the thesis. The outline of rest of the four chapters are as follows:

- **Chapter 2** presents the literature review which covers the review of DOA estimation and ABF algorithms. The metaheuristic approaches addressed in the proposed research work and their applications in the field of array signal processing are briefly reviewed.

- **Chapter 3** presents the data model and step-wise description of all the DOA estimation and ABF algorithms. The optimization techniques and the MATLAB tool used in the simulation work are briefly discussed. The chapter also presents the methodology of the proposed research work.

- **Chapter 4** presents the simulation results of DOA estimation techniques and the ABF algorithms. The simulation results of ML techniques for DOA estimation using metaheuristic approaches are presented and discussed. The simulation results of ABF using GSA are also presented and discussed and compared with previously reported results.

- **Chapter 5** presents conclusion of this study, contributions to knowledge and scope of extension of this research work.

Review of Literature

CHAPTER 2 REVIEW OF LITERATURE

In this chapter, the work reported in the field of array signal processing which includes the DOA estimation and ABF is reviewed. In the recent years, there has been a tremendous growth in the field of array signal processing with the introduction of new research ideas to a concrete system. The limitations of basic approaches have urged the researchers to explore new techniques for array signal processing. Soft-computing approaches have gained momentum in recent years and have been explored by many researchers for DOA estimation and ABF. The contribution of researchers in the field of array signal processing using conventional techniques and soft-computing approaches are reviewed in two sections. In section one, the DOA estimation techniques are reviewed and in section two the ABF techniques are briefly reviewed.

2.1. Review related to direction of arrival estimation

Capon, J. (1969) proposed an algorithm known as Capon method which is a high resolution method of estimation of parameters of incident wavefronts. The output of antenna array elements is considered to be a random field. The frequency-wavenumber spectral representation of this field gives the information of the power spectral density which is basically a mean-square amplitude of these waves. The conventional method of frequency-wavenumber spectral density has a fixed wave number window, thus it has limited resolution. The author introduced a new method of estimation which uses a wavenumber window whose shape changes and is a function of the wavenumber at which an estimate is to be obtained. Thus, the method presents better resolution and finds application in the fields such as radar, sonar, radio-astronomy, etc.

Barabell, A. (1983) proposed two methods for DOA estimation in low SNR scenario with good resolution probability. The first method finds the direction of incoming signals by finding the roots of the polynomial and is applicable to uniform linear array (ULA). The second method finds the direction of incoming signals with improved resolution properties by defining a (pole-zero) spectrum function which is formulated using signal subspace eigen vectors.

Shan, T. J., *et al.* (1985) addressed the problem of DOA estimation for coherent signals using spatial smoothing preprocessing technique. The technique is used in conjunction with eigen structure technique, Capon and linear prediction method, etc. The results verifies the effectiveness of this spatial smoothing approach.

Schmidt, R. O. (1986) proposed high resolution MUSIC algorithm for processing the signals received on an antenna array elements in a noise/ interference environment. The algorithm is a Eigen structure method since it divides the signal subspace into signal and noise subspace and DOA estimation is performed from one of the subspaces, assuming that noise in each channel is highly uncorrelated. The algorithm gives unbiased estimates of number of incident signals, elevation and azimuth angles, range, polarization for multiple incident wavefronts, cross-correlation among the incident wavefronts, noise/ interference strength. The author has compared the simulation results with the Maximum Likelihood (ML), maximum entropy (ME) and conventional beamforming criteria.

Hua, Y. and Sarkar, T. K. (1988) proposed a novel matrix pencil method for estimation of frequencies of exponentially damped or undamped sinusoidal sequences. This method presents different ways of solving generalized Eigen value problem and possesses different fundamental perturbation properties. The simulation and theoretical result shows that the method gives good performance than the high performance polynomial method.

Ziskind, I. and Wax, M. (1988) proposed alternating projection (AP) method for ML function optimization in order to estimate the direction of signals impinge on the sensor array. The algorithm estimate the direction of coherent signals and in the scenario of single snapshot. The AP is an iterative approach and the global optimum solution is reached through successive approximations. The global convergence of the algorithm depends on the initialization scheme.

Sharman, K. (1988) proposed simulated annealing algorithm for ML function optimization in estimating direction of incoming signals which impinge on the sensor array. Simulated annealing is a stochastic optimization algorithm which solves the combinatorial optimization problems and finds wide range of applications in the field of science and technology. The simulation results show that the algorithm give accurate results as compared to existing methods, i.e. MUSIC under severe condition such as irregular sampling and short data length.

Roy, R. and Kailath, T. (1989) proposed ESPRIT algorithm for signal parameters estimation which is a significant problem in many signal processing applications. It includes DOA estimation, system identification and time series analysis. The technique exploits the rotation invariance in the signal subspace induced on arrays with a translational invariance structure. The method assumes multiple identical subarrays called doublets instead of single element array which are displaced by fixed displacement vector. The simulation results show that the algorithm presents significant performance and computational advantage over other algorithms such as MUSIC, Capon, ML and ME method.

Rao, B. D. and Hari, K. V. S. (1989) analyzed the performance of the Root-MUSIC algorithm. The algorithm is basically the variation of the MUSIC algorithm in which the direction of plane waves is determined by finding the roots of the polynomial equation. The performance is analyzed by noticing the perturbation in the roots of the polynomial found in the algorithm. The simulation results show that the algorithm performs well in terms of root-mean square error as compared to MUSIC and ESPRIT algorithm.

Stoica, P. and Nehorai, A. (1989) derived the Crammer-Rao bound (CRB) for the MUSIC and the ML algorithms and analyzed their statistical performance. The authors investigated the relationship between the two algorithms. A numerical study of statistical efficiency of the MUSIC estimator is made in which the direction of two plane waves for the uniform linear array is estimated.

Hua, Y. and Sarkar, T. K. (1990) proposed matrix pencil method for estimating frequencies and damping factors of exponentially damped or undamped sinusoids in noise. The matrix prediction approach has two variants viz. polynomial method and matrix pencil method. The theoretical analysis and simulation results shows that pencil method is found to be computationally less complex and is less sensitive to noise than polynomial method.

Stoica, P. and Sharman, K. C. (1990) addressed five different methods of DOA estimation based on the ML principle. The ML method is derived by applying the ML principle to the statistics of observed raw data. MUSIC-1 is obtained by applying the brute-force approximation to the ML method while MUSIC-2 is obtained by applying the ML principle to the statistics of the certain linear combination of the sample noise space eigen vectors. Method of direction estimation (MODE)-1 method is derived as the large sample realization of the ML method and its performance lies between the ML method and the MUSIC algorithm. MODE-2 is obtained by applying the ML principle on the statistics of the certain linear combination of the sample eigen vectors. The author presented the numerical comparison in terms of RMSE of five different DOA estimation methods.

Stoica, P. and Nehorai, A. (1990) presented the numerical and analytical study of conditional and unconditional ML DOA estimation for ULA. The authors derived the expression for Crammer-Rao Lower bound for conditional and unconditional ML estimator. The author showed that many DOA estimation methods have same statistical properties for conditional and unconditional models.

Long, L. and Da, L. Y. (1993) proposed a neural network approach for computing the noise subspace in real time for the MUSIC algorithm. The simulation results show that the proposed neural network approach finds solution close to the conventional approach in a very short time. The advantage of the proposed approach is its asynchronous parallel processing and high speed computational compatibility.

Xu, G., *et al.* (1994) proposed Beamspace ESPRIT algorithm for restoring the displacement invariance structure in ESPRIT algorithm. The Eigen structure based ESPRIT algorithm is difficult to implement in a parallel manner and requires $O(M^3)$ multiplications for an $M X M$ matrix, corresponding to M sensors. Thus, ULA lose their displacement invariance structure which increases the computation complexity of the algorithm and it becomes difficult to implement the ESPRIT algorithm. The simulation results show that beamspace ESPRIT algorithm reduce the computational complexity of the algorithm and improves the estimation accuracy.

Haardt, M., *et al.* (1994) proposed unitary ESPRIT algorithm which utilizes the fact that the phase delays between the two subarrays are unitary. This approach estimates more accurate results than the ESPRIT algorithm especially when the standard ESPRIT algorithm finds difficulty in estimating direction of closely spaced sources. Thus, the algorithm is found to be computationally less complex and separates and reconstructs superimposed signals arriving from different directions.

Bachl, R. (1995) proposed a hybrid algorithm which is a combination of ESPRIT algorithm and forward-backward averaging technique. ESPRIT algorithm is applied to two identical subarrays and forward-backward averaging is applied to arrays in which the elements are symmetrically placed. The forward-backward averaging technique provides efficient solution to decorrelating coherent or highly correlated signals. The combination of two techniques implies a uniformly spaced linear arrays and shows substantial reduction in the computational complexity of the ESPRIT algorithm.

Kennedy, J. (1995) proposed particle swarm optimization, a new stochastic optimization method inspired by social behaviour of bird flocking or fish schooling. The heuristic approach is judged for non-linear function optimization and neural network training. The author discussed the relationship between the proposed approach and other evolutionary techniques.

Sarkar, T. K. and Pereira, O. (1995) discussed the linear techniques for parameter estimation. Matrix pencil method which is more robust to noise is briefly described. The method is computationally more efficient and has lower variance of the estimates of the parameter of interest as compare to polynomial type method. The author implemented the band-pass version of matrix pencil method in hardware using AT&T DSP32C chip for real time implementation.

Southall, H. L., *et al.* (1995) discussed adaptive radial basis function neural network (ARBFNN) for finding the direction of sources from an eight-element X-band array having multiple failures and degradations. The performance of two ARBFNN viz. one whose internal weights are computed using a modified gradient descent algorithm and second whose weights are calculated using linear algebra are compared.

El Zooghby, A. H., *et al.* (1997) addressed the problem of DOA estimation for linear antenna arrays using three layer RBFNN. The neural network trained with input-output pairs presents reduced computational complexity as compared to superresolution algorithms. The proposed approach shows high performance as compared to MUSIC algorithm for detecting directions of uncorrelated and correlated signals.

Godara, L. C. (1997) discussed the advantage of using an antenna array in mobile communications systems. The author gave an overview of mobile communication system and briefly described how an array can be beneficial for different communication system which includes land-mobile, indoor-mobile and satellite based system. The author highlighted the advantages and feasibility of antenna array in mobile communications applications.

Godara, L. C. (1997) discussed all aspects of array signal processing. The author provides a comprehensive and detailed analysis of different DOA estimation and ABF algorithms. The performance of different algorithms are analyzed and the effect of error on the performance of antenna array system is briefly acknowledged.

Roy, R. H. (1998) outlined the principles and architecture of SDMA wireless communication system. The author addressed that spatial processing would be the key for next generation wireless communication system as it provides significant reduction in cost and improvement in coverage, capacity of the network. The advantages of switched beam and adaptive antenna system employed at base station were discussed. Experimental results built with this new architecture shows the efficiency of the SDMA technology.

Stoica, P. and Gershman, A. B. (1999) proposed a novel and computationally efficient approach of maximizing the likelihood function over a set of points derived from the data. The author used the ESPRIT algorithm to obtain the grid points. The data supported grid search algorithm shows similar performance as genetic algorithm at a lower computational cost.

El Zooghby, A. H., *et al.* (2000) discussed the problem of DOA estimation using radial basis function network for wireless terrestrial and satellite mobile communications. The algorithm works in two stages viz. detection and estimation. The field of view of antenna array is divided into spatial angular sectors and these sectors are assigned to different pairs of neural network. Initially, when the first pair of network detects the signal then the second stage of the network is activated to estimate the direction of incoming signals. The performance of the algorithm is investigated for various angular separation with sources of random relative SNR.

Pesavento, M., *et al.* (2000) proposed unitary root-MUSIC algorithm for DOA estimation of plane waves. The algorithm reduced the computation complexity since it exploits the eigen values of real-valued covariance matrix. The performance of the algorithm is compared with conventional root-MUSIC algorithm and the results show that unitary root-MUSIC give approximately same performance as root-MUSIC algorithm for detecting direction of uncorrelated sources while give superior performance in finding direction of partially correlated and correlated sources. The algorithm shows improved threshold performance when the directions of sources are found in real time environment.

Thus, the authors recommend unitary root-MUSIC in comparison to conventional root-MUSIC algorithm in finding direction of narrow band sources.

Karamalis, P. M., *et al.* (2001) addressed the problem of DOA estimation of several signals impinging the ULA using genetic algorithm. In the first part of the paper, authors briefly described genetic algorithm and its parameters. In the second part of the paper simulation results are presented where the changes in the receiving antenna structure and the influence of the GA are examined. The statistical analysis is done to conclude the algorithm reliability and accuracy.

Du, K. L., *et al.* (2002) reviewed the application of neural networks in the field of array signal processing. Neural network is gaining momentum in the field of array signal processing because of its real time application and hardware implementation. The authors discussed different models of neural networks viz. multi layer perceptron (MLP), radial basis function neural network (RBFNN), Hopfield neural network for DOA estimation and ABF.

Jiankui, Z., *et al.* (2006) proposed PSO for optimizing ML function in order to estimate direction of incoming signals towards the sensor array. The proposed method finds the global solution and overcomes the drawback of previous optimization techniques which get trapped in local optimum solution. The PSO-ML estimator possess modest computational complexity and shows improved performance in terms of RMSE as compared to alternating projection (AP) ML solution.

Caylar, S., *et al.* (2006) proposed modified neural multiple-source tracking algorithm (MN-MUST) for real time target tracking problem. The proposed approach inserts a spatial filtering stage and reduces the input size of the neural network without any degradation in the accuracy of the system for detecting uncorrelated sources.

Mestre, X. (2006) proposed weighted MUSIC algorithm for the scenarios where the sample size is small. The author initially analyzes the performance of traditional MUSIC algorithm in the situation where the number of samples and number of array elements increases without bound. The results show that MUSIC is inconsistent as the part of the energy for the noise subspace eigen vectors spills into signal subspace. Then, the performance of the weighted MUSIC algorithm is analyzed and the results show that it provides consistent performance even when the observation dimension increases without bound. Finally the performance of the conventional and the proposed weighted MUSIC algorithm is compared.

Li, M. and Lu, Y. (2007) proposed modified and refined GA to find the exact solution of the ML function. The proposed evolutionary technique has some new features viz. intelligent initialization, emperor-selective mating scheme, carefully selected crossover and mutation operators. The heuristic technique achieves fast global convergence because of fine-tuned parameters such as population size and probability of crossover and mutation. The proposed GA based ML solution presents improved and faster performance as compared to conventional DOA estimation techniques and other ML-based DOA estimation techniques in different scenarios.

Wasylkiwskyj, W., *et al.* (2007) proposed modified root-MUSIC algorithm with a simple algebraic structure. The performance of the algorithm is evaluated and the results are as accurate as conventional root-MUSIC algorithm. The authors demonstrated the performance of the algorithm in a hardware setup comprising of decoupled array and phase shifters. The experimental results show a good agreement with the simulation results.

Li, M. and Lu, Y. (2008) proposed particle swarm optimization for DOA estimation by optimizing complex multimodal nonlinear ML function in unknown colored noise fields. The direction of incoming signals are computed with other noise and signal parameters and the performance of the proposed approach is compared to approximate maximum likelihood (AML) and MUSIC algorithm in different noise scenarios. The PSO-(exact maximum likelihood) EML solution estimate the direction of signals at less computational costs as compared to other algorithms.

Zhang, Y. and Ye, Z. (2008) proposed an efficient method of DOA estimation when both uncorrelated and coherent signals impinge on an ULA. The technique first estimate the direction of uncorrelated signals and the information is eliminated from the signal subspace. After elimination of information of uncorrelated signals, the matrix contains the information of coherent signals and is called as C-matrix. The direction of coherent signals are obtained from the remaining C-matrix. The simulation results show the effectiveness of the proposed approach.

Rashedi, E., *et al.* (2009) proposed GSA, a new heuristic optimization method based on the gravity and mass interactions. The optimization method contains a collection of masses as search agents and they interact with other based on the Newtonian gravity and the laws of motion. The performance of the proposed approach is compared with other well-known heuristic optimization methods by optimizing standard benchmark functions and the results show the high performance of the proposed method in solving different non-linear functions.

Dhope, T. S. (2010) estimated the performance of MUSIC, Root-MUSIC and ESPRIT DOA estimation algorithms in the presence of Gaussian noise on ULA. The simulation results show that MUSIC algorithm gives minimum RMSE as compared to other two algorithms. The performance of all three DOA estimation algorithms improves with the increase of SNR, number of array elements and number of snapshots.

Hanumantharao, A. D., *et al.* (2011) proposed bacteria foraging algorithm (BFO) for DOA estimation by optimizing complex multimodal nonlinear function over a high dimensional space. The proposed approach gives better performance over other conventional techniques and PSO based solution in low SNR scenarios at less computational costs.

Guimei, Z., *et al.* (2012) proposed a new unitary ESPRIT algorithm for estimating direction of arrival and direction of departure of signals in bistatic MIMO radar. The author utilized the properties of centro-Hermitian matrices to transform complex valued data matrix into a real-valued data matrix. The real valued rotation invariance equations for signal subspace are evaluated to estimate the direction of arrival and direction of departure of signals in radar. The proposed algorithm presents reduced computational complexity and simulation results verify the effectiveness of proposed approach in terms of estimation accuracy.

Liu, F., *et al.* (2012) addressed the problem of DOA estimation in the scenario where uncorrelated and coherent sources impinge upon the ULA. The proposed method firstly estimate the direction of uncorrelated sources using conventional subspace method and then using spatial differencing technique their effect is being eliminated. Finally, the coherent signals are estimated using spatial differencing matrix. The simulation results show that the proposed method improves the estimation accuracy and increases the maximal number of detectable signals.

He, J., *et al.* (2012) proposed an efficient MUSIC algorithm which detected and classified the sources in the scenario where both far-field and near-field sources coexist. The proposed algorithm neither requires the multi-dimensional search nor the higher order statistics. The stochastic Crammer-Rao bound (CRB) was also determined for the problem under consideration. The performance of the proposed approach is compared with the existing method and the CRB.

Zaman, F., *et al.* (2012) proposed a method based on PSO for single snapshot DOA estimation. MSE is used as a fitness function for the optimization problem. Multiple sources are assumed to impinge on the linear array and the performance is analyzed in two separate cases viz. when separation between sources is more and separation between sources is less. The proposed approach shows effective results and can be applied for DOA estimation in real time environment.

Zhou, Z., *et al.* (2012) proposed an iterative approach for fundamental frequency estimation for real harmonic sinusoids. An optimally weighed harmonic multiple signal classification (OW-HMUSIC) estimator is devised using the subspace technique and Markov based eigen analysis. The simulation results show the superior performance of the proposed approach as compared to least square and harmonic multiple signal classification (HMUSIC) algorithms and also with the CRB.

Zwick, T., *et al.* (2012) proposed a novel high resolution two-dimensional (2D) DOA estimation algorithm using neural networks. The field of view of antenna array is divided into spatial elevation and azimuth sectors. The DOA estimation is implemented in two stages viz. detection and estimation. Multilayer perceptron (MLP) is used for detecting the signals and RBFNN is used for estimating the signals. The trained neural network estimates the directions of incoming signals in a fraction of seconds, thus is suitable for real time implementation. The proposed approach shows good accuracy as compared to 2D MUSIC algorithm.

Basha, T. S. G., *et al.* (2013) proposed an improved MUSIC algorithm which incorporates a tuned correlation matrix after solving the objective model for the matrix. The tuned correlation matrix is obtained using GA. The simulation shows that estimated DOA is more precise compared to the conventional MUSIC DOA estimation technique. The ABF is derived on the basis of the results of estimated DOA and the experimental results show the improved results of proposed beamforming techniques as compared to conventional beamforming technique.

Dhope, T. S., *et al.* (2013) compared the performance of MUSIC, Root-MUSIC and Capon DOA estimation algorithms in the presence of Gaussian noise on ULA. The performance of all the algorithms are compared on the basis of number of array elements, SNR and number of snapshots. The simulation results show that MUSIC gives best statistical performance among Capon and Root-MUSIC algorithms.

Errasti-Alcala, B. and Fernandez-Recio, R. (2013) addressed the problem of single snapshot DOA estimation by metaheuristic approaches. The authors investigate the performance of five different optimization methods viz. PSO, ACO for continuous domains, DE, simulated annealing (SA), GA for optimizing function for DOA estimation. The performance is compared in terms of rate of convergence, accuracy, resolution and computational cost. The simulations are carried out for uniform and non-uniform arrays and for different noise scenarios and the results show that ACO for continuous domains gives best results in comparison to other optimization techniques.

Magdy, A., *et al.* (2013) proposed GSA, a optimization method for DOA estimation based on unconditional ML solution. The proposed heuristic method is used to optimize complex ML function for a uniform circular array of twelve elements and the results are compared with PSO and conventional techniques. The performance of GSA-ML algorithm is superior in terms of RMSE as compared to other techniques.

Panigrahi, T., *et al.* (2013) proposed PSO for DOA estimation by optimizing complex ML function in wireless sensor networks. The proposed approach presents a centralized method in which every node participates in bearing estimation in order to achieve accurate solution with minimum computational cost. The simulation results show the advantage of the proposed approach as compared to conventional approach.

Yan, F. G., *et al.* (2013) proposed a compressed MUSIC (C-MUSIC) algorithm which involves search over a small sector and reduces the computational complexity. The technique shows advantage over alternating projection and singular value decomposition method. The simulation results show that the RMSE and probability of resolution of proposed method has certain trade-off as compared to MUSIC algorithm.

Wen, F., *et al.* (2014) addressed the problem of DOA estimation with multiple noncoherent subarrays. The proposed approach finds the overall spatial spectrum by combining the weighted MUSIC spectrum of the subarrays. The ML approach is utilized to find the weighted MUSIC algorithm for such arrays. The simulation results show that the weighted algorithm gives improved result as compared to conventional MUSIC algorithm for noncoherent subarrays.

Yan F. G., *et al.* (2014) proposed real valued MUSIC algorithm which reduces the computational complexity by 75 %. The array output covariance matrix is complex and thus requires complex computations. The null space of real and imaginary part of array output covariance is same and coincides with the noise subspace. Thus using this fact, the proposed technique is used with arbitrary array geometries. The simulation results show that the accuracy of the real valued MUSIC is very close to standard MUSIC.

Shi, H., *et al.* (2015) proposed a novel method of estimating direction of narrowband sources in which the sources are uncorrelated and coherent. The proposed method firstly estimate the uncorrelated sources by utilizing the property of moduli of eigen values of the DOA matrix. Thereafter, the contribution of uncorrelated sources and noise interference are eliminated using the improved spatial differencing technique and lastly the direction of coherent sources are estimated. The simulation results show that the proposed method resolves more sources than the array elements and distinguish the uncorrelated and coherent sources from the same direction.

2.2. Review related to adaptive beamforming

Widrow, B., *et al.* (1967) addressed the LMS algorithm for beamforming in adaptive antennas for wide range of frequencies. Adaptive antenna comprising of antenna arrays and adaptive processor can process the signals in both time and frequency domain and thus direct the narrow beam towards desired user and nulls towards interfering signals. The author proposed LMS algorithm which requires a pilot signal and minimize the mean square error for adjusting the weights of antenna array for ABF. The simulation results are demonstrated which shows the effectiveness of the proposed approach. Experimental results checked the rate of convergence and misadjustments of predicted theoretical results.

Godard, D. N. (1980) proposed a novel scheme to solve the adaptive channel equalization problem with minimum distortion and without restoring to the known training sequence.

Conventional channel equalization algorithm require initial training period during which the known data sequence is synchronized at the receiver. The criteria for equalizer adaptation is the minimization of non convex cost function. The author mathematically and through simulations analyzed the convergence properties of proposed self recovering algorithms. This concept is used in CMA for estimating the weights of antenna array for ABF.

Agee, B. G. (1986) presented, analyzed and demonstrated a novel least-squares CMA (LSCMA) for two element antenna array. The performance of static and dynamic LSCMA have been analyzed in the presence of severe noise and co-channel interference. The simulation results show that the proposed algorithm is stable and has fast rate of convergence than the conventional CMA.

Fabre, P. and Gueguen, C. (1986) proposed an improved form of least squares algorithm. The author analyzed the performance of least-squares algorithm and particularly with their normalization. A comparison of different algorithms was made in terms of their rate of convergence, complexity and initialization procedures. The normalized algorithms shows improved performance in terms of their good numerical behaviour and simple structure.

Godara, L. C. and Gray, D. A. (1989) proposed an alternative scheme for estimating the gradient for calculating the weights for ABF. The LMS algorithm find the gradient by multiplying the array output with the array receiver outputs. The proposed scheme used the structured estimate of the array correlation matrix for estimating the gradient. The results show that the estimated gradient is unbiased and the scheme has good convergence rate means array weights quickly converges to optimal weights. The weights estimated by structured gradient algorithm are less noisy than the standard LMS algorithm.

Slock, D. T. M. (1993) proposed a very simple model for the input signal vectors that simplifies the convergence behaviour of LMS and normalized least mean square (NLMS) algorithm. The author quantitatively analyzed the performance of the model and concludes that convergence rate of NLMS algorithm can be speeded up by using time-varying step-size. The step size for the white noise input signal with arbitrary distribution can be specified before calculating the weights for the algorithm.

Barrett, M. and Arnott, R. (1994) developed a practical adaptive antenna signal processing structures. Adaptive antennas increases the system capacity, extend the range of each cell and improve immunity to cochannel and multichannel interference. The author presented the results obtained from the adaptive antenna test bed by series of analysis and field trials activities.

Shynk, J. and Gooch, R. (1996) investigated the steady state properties of constant modulus (CM) array and the signal canceller. The CM array is a blind adaptive beamformer which detects the narrowband signal among cochannel signals which impinge on the antenna array without having the knowledge of the pilot signal. The CM array and signal canceller are integrated and a multi-stage system is constructed to recover cochannel sources. The simulation results verified the proposed approach and illustrated the transient behaviour of the system.

Florens, C., *et al.* (1998) proposed GA for ABF in linear antenna arrays. The author considered two different systems viz. narrow band and broad band system for beamforming in adaptive antenna arrays. The simulation results show that the algorithm is easily implemented but possesses slow rate of convergence than LMS algorithm. The good convergence is dependent on good initial conditions and size of the search space. The GA is less sensitive to complex environment and detect the direction of correlated signals. The complexity of real-life systems make the algorithm unsuitable for real-time implementation. But, with the advancement of digital circuit technology the algorithm can be thought of as the beamforming algorithm of near future.

El Zooghby, A. H., *et al.* (1998) proposed a neural network based beamformer for modern cellular satellite mobile communication system and in global positioning system (GPS). The author considered a 1-D and two 2-D antenna arrays where desired signal and interfering signals are assumed to impinge from the array broadside in the presence of Gaussian noise. The three layer RBFNN is trained for random cases by the optimum weights estimated by MVDR algorithm. The network is generalized for random DOA of desired and interfering signals. The simulation results show good agreement with the Weiner solution.

El Zooghby, A. H., *et al.* (1999) proposed neural network beamforming approach for linear antenna arrays. The author trained a three layer RBFNN with input-output pairs of correlation matrix and weights of antenna arrays. The network generalizes for random angle of desired and interfering signals and calculates the array weight in real time environment. The simulation results are in good agreement with Wiener solution and the algorithm constantly track the desired user and place nulls in the direction of interfering signals.

Chen, Y. (2004) proposed RLS optimization technique for finding approximate solution of CM algorithm. The author presented a RLS constant modulus algorithm (RLS-CMA) in which the power of array output can take arbitrary values. The simulations are demonstrated to analyzed the performance of proposed approach. The results show that the proposed approach has fast rate of convergence and better tracking ability.

Karaboga, D., *et al.* (2004) proposed an efficient modified touring ant colony optimization (MATCO) technique for steering nulls in the direction of interference by controlling both the amplitude and phase of the linear antenna array. The MATCO technique is an modified version of touring ant colony optimization (TACO) algorithm which is based on the behaviour of ants searching for food. In the proposed pattern synthesis problem the parameters which are considered are null depth level, maximum side lobe level, and the dynamic range ratio. The simulation results show the effectiveness of the proposed approach with single, multiple and broad nulls imposed for the Chebyshev pattern.

Zhang, L., *et al.* (2004) addressed two different novel adaptive multiple beamformers for efficient reception of coherent signals and uncorrelated interferences. The first approach is a two step approach, in the first step amplitude of all the coherent signals are estimated and in the second step linear constraints minimum variance beamformer is being constructed. The second approach developed a minimum variance beamformer that can achieve efficient signal utilization. The simulation results are presented and the performance of the proposed algorithm is compared with minimum mean square error approach.

Mouhamadou, M. and Vaudon, P. (2006) proposed an efficient Sequential Programming (SQP) algorithm for the pattern synthesis of linear antenna arrays. The algorithm solves the multi-objective optimization problem and achieves multi-lobe pattern and adaptive nulling by controlling the phase of each array element. The objective of multi-objective optimization problem is to optimize vector of objective functions and find the optimum solution. The simulations results show that multi-beam patterns are demonstrated with main lobe is directed in the direction of desired signal are nulls are placed in the direction of interfering signals.

Song, X., ***et al.*** **(2008)** proposed a neural network approach for robust ABF in linear antenna arrays. The proposed approach modeled the uncertainty in the desired signal array response and RBFNN, which belongs to the diagonal loading approaches. The problem of calculation of weights of antenna array is modeled as an mapping problem, which can be modeled using a RBFNN trained with input/output pairs. The algorithm presents robust performance against signal steering vector mismatches and makes the output SINR of linear antenna array close to the optimal one. The simulation results show the performance improvement as compared to other ABF algorithms.

Mathur, S. and Gangwar, R. P. S. (2010) addressed the problem of DOA estimation and ABF using multi-layer perceptron neural network. The authors proposed a decision directed approach for blind adaptation of smart antenna system using a complex neural estimation of parameters for ABF. The novel approach is simulated for multi-quadrature amplitude modulated (M-QAM) signal environment with 4, 16, 64 constellations and the results indicate the effectiveness of the proposed approach.

Yasin, M. and Pervez, A. (2010) proposed enhanced sample matrix inversion (ESMI) algorithm for beamforming in adaptive antennas. The adaptive antennas direct narrow beam towards desired user and nulls toward interfering signals and thus enhances the signal quality and capacity of the network. The simulation results are presented and compared with SMI and normalized sample matrix inversion (NSMI) to analyze the performance of ESMI algorithm. The results confirm that the proposed algorithm shows enhanced capacity and service quality as compared to SMI and NSMI algorithms.

Basha, T. S. G., ***et al.*** **(2011)** proposed GA for beamforming in smart antenna. In the proposed method, direction of desired and interfering signals are given as input and the beamformer presents maximum gain in the antenna beampattern with corresponding position and phase angle of the desired user. The beamforming method considered different factors viz. length of the beam, number of patterns and interference and phase angle. The simulation results show that the proposed GA based beamformer increases the gain and reduces the interference of the system.

Blom, K. C. H., ***et al.*** **(2011)** proposed an angular CMA, a blind beamforming algorithm which calculated steering angle updates instead of weight vector updates to constantly track the desired user. The simulation results show that the proposed algorithm has reduced computational complexity and possesses fast rate of convergence as compared to other conventional approaches.

Ji, C. P., *et al.* (2011) proposed an improved method for low side lobe beampattern optimization for environment in which jamming signals are present. Conventional beamforming algorithms fail to give robust anti-jamming performance. Since these algorithms require the knowledge of the direction of interfering signals and the SLL must be too high. The simulation results show the effectiveness of the proposed approach in such environment.

Zaharis, Z. D. and Yioultsis, T. V. (2011) proposed a novel optimization technique for ABF in linear antenna arrays. The proposed heuristic approach is a variant of PSO called as AMBPSO. The optimization technique possess update formula which are implemented in boolean form by using an adaptive mutation process. The desired signal and several interference signals are assumed to impinge the ULA in the presence of Gaussian noise. The optimization technique calculates the excitation weights by minimizing certain chosen fitness function and doesn't takes into account the knowledge of interference correlation matrix. The simulations are performed for different spacing between adjacent elements and different noise power levels and the results are compared with MVDR technique.

Chang, Y. J. and Ho, C. L. (2011) proposed an efficient reduced symmetric self-constructing fuzzy neural network (RS-SCFNN) beamformer for multi-antenna assisted system. The authors proposed a training algorithm which is based on adaptive minimum bit error rate method and clustering of array input vectors. The training procedure of RS-SCFNN exploits inherent symmetric property of the array input signals and requires less amount of fuzzy rules thus found to be more efficient than SCFNN. The simulation results demonstrate that RS-SCFNN beamformer improves the BER, reduces the computational complexity and training data size as compared to classical and other non-linear approaches.

Wang, Y., *et al.* (2011) proposed complex-valued GA optimization algorithm for pattern synthesis in linear antenna arrays. The conventional GA represents the variable with binary numbers but in complex-valued GA the population (excitation weights) are represented by complex numbers. The proposed algorithm for beamforming shows enhanced searching efficiency and avoids premature convergence. The simulation results show the advantage of the proposed technique over conventional pattern synthesis method.

Basha, T. S. G., *et al.* (2012) proposed a hybrid soft-computing algorithm for beamforming in smart antenna. The proposed method used GA for obtaining the training data for neural network and fuzzy inference system. The parameters considered in the optimization are length of the beam, interference, phase angle, number of patterns and spatial diversity. The trained network give corresponding beam, position and phase angle as output if angle is given as input. The simulation results show that the proposed method efficiently determine the smart antenna parameters and presents low SLL than LMS algorithm.

Chang, J. C. (2012) proposed diagonal variable loading recursive least square (VLRLS) algorithm for antenna array beamforming applications. The proposed algorithm is based on a generalized side lobe canceller. The author analyzed the performance of the proposed approach through simulations and the results show the effectiveness of the proposed approach.

Imtiaj, S. K., *et al.* (2012) compared the performance of LMS and SMI ABF algorithms on the basis of their null depth, beamwidth and SLL. The performance of the algorithms is analyzed by varying number of antenna elements and spacing between the elements with multiple incident wavefronts impinging on the ULA. The simulation results show that SMI algorithm gives better null depth when the spacing is varied as compared to LMS algorithm.

Omar, O. K., *et al.* (2012) proposed a hybrid honey bee and tabu search algorithms for beamforming in ULA arrays. The proposed approach obtained the weights of antenna array such that the main lobe is steered towards desired signal and nulls are placed towards several interference signals with lowest possible SLL. The algorithm controls the phase of each array element and the simulation results are compared with LMS and GA. The simulation results proved the effectiveness of the proposed approach.

Salem, B., *et al.* (2012) addressed the significance of the beamforming technique employed for the next generation wireless communication system. The authors described the significance of the beamforming technique which includes improvement in data rates, null steering and coverage of cellular system. The performance of two different algorithms viz. LCMV and MVDR algorithm are analyzed for a four element array operating at 2.4 GHz, noise power of 0.5 dB with half a wavelength of spacing between elements. The simulation results show that MVDR algorithm shows improved performance than LCMV algorithm.

Singh, H. and Jha, R. M. (2012) addressed various facts of array signal processing and ABF. The authors gave a detailed comparison of different ABF algorithms and DOA estimation algorithms. A detailed study on various types of errors which effect the performance of antenna arrays and their remedial measures are made. The author concluded the article with the brief discussion on the future aspects of array signal processing across various disciplines.

Zaharis, Z. D., *et al.* (2012, a) proposed a novel invasive weed optimization (IWO) variant called as ADIWO algorithm for ABF in linear antenna arrays. The proposed optimization algorithm and IWO are applied on well known test functions and the results show that the proposed algorithm have faster rate of convergence. The ADIWO algorithm is applied on multi-objective fitness function so as to steer the main lobe of the beamformer towards desired signal and nulls towards interfering signals with reduced SLL. The simulations are performed for different number of interfering signals and different power levels of Gaussian noise. The results show that proposed algorithm has faster rate of convergence and achieves better SLL reduction as compared to PSO and MVDR algorithms.

Zaharis, Z. D., *et al.* (2012, b) presented a comparative study of two adaptive beamformer based on neural network trained by MBPSO and MVDR algorithm. The ULA receives a desired signal and several interference signals in the presence of Gaussian noise and the MBPSO and MVDR algorithm extract weights of beamformer such that ULA directs the main lobe towards desired user and nulls towards interfering signals with reduced SLL. The neural network is trained by the weights extracted by the two algorithms. The two trained neural network are generalized for random sets of DOA of desired user and interfering signals. The simulation results show superiority of the beamformer trained by MBPSO as compare to beamformer trained by MVDR algorithm.

Zaharis, Z. D., *et al.* (2012, c) proposed a neural network based beamformer trained by novel MBPSO algorithm. The ULA receives a desired signal and several interference at respective DOA in the presence of Gaussian noise. The MBPSO algorithm extract the excitation weights of beamformer by optimizing multi-objective fitness function so as to direct main lobe of beamformer towards desired signal and nulls toward interfering signals. The trained network is generalized for new random cases and the beampattern obtained by MBPSO based neural network is compared with MBPSO and MVDR algorithms. The proposed beamformer combines the advantages of MBPSO optimization technique and the speed of neural network, thus is suitable for real time beamforming applications.

Goswami, B. and Mandal, D. (2013) proposed a real-coded genetic algorithm (RGA) for pattern synthesis in linear antenna arrays. The objective of optimization algorithm is to improve the null depth, reduce the SLL and fix the first null beamwidth (FNBW). The RGA is used to obtain the weights of the antenna array and optimum inter-element spacing in order to achieve the desired objectives. The simulations are demonstrated for three different cases and the optimization goals in each cases are easily achieved.

Nwalozie, G. C., *et al.* (2013) compared the performance of ABF algorithms. The author analyzed the performance of LMS, SMI, RLS, CMA and LS-CMA algorithms. The simulation results show that RLS algorithm requires less iterations to converge and thus have best rate of convergence as compared to other four algorithms.

Darzi, S., *et al.* (2014) proposed a novel approach in which PSO, dynamic mutated artificial immune system (DM-AIS), and GSA are incorporated with LCMV technique so as to improve the weights obtained by LCMV ABF technique. The LCMV beamforming technique is one of the most commonly used beamforming technique but the weights computed by LCMV technique are not good enough to precisely direct the main lobe and nulls towards desired and interfering signals. Conventional empirical approach can't obtain precise and accurate solution to improve and optimize LCMV beamforming technique. The artificial intelligence (AI) is explored to enhance the LCMV beamforming solution. The simulation results show that the integration of PSO, DM-AIS and GSA with LCMV beamforming technique improves the SINR and GSA provides the best results among the other two algorithms.

Rao, A. P. and Sarma, N. V. S. N. (2014) addressed a novel DE algorithm based beamforming for smart antenna system. The proposed algorithm calculates the weights of antenna array such that main lobe is directed towards desired user and deep nulls are directed towards several interfering signals with controlled side lobe levels. Three different scenarios are considered for beamforming and the result shows that DE based beamformer is better than conventional beamforming algorithms.

Ahmed, S. K. (2015) proposed an efficient DE algorithm for pattern synthesis problem of linear antenna array. The optimization algorithm estimates the weights of antenna array such that the main lobe is steered in the direction of desired user and nulls are steered in the direction of interfering signals. The optimization problem is a maximization problem and fitness function is the inverse of the difference between actual pattern and the desired pattern. In order to analyze the performance of the algorithm a number of simulations are performed for different number of array elements. The simulation results show the effectiveness of the proposed optimization approach.

Magdy, A., *et al.* (2015) proposed a novel hybrid particle swarm optimization gravitational search algorithm (PSOGSA) for ABF in *N* element uniform circular arrays (UCA). The UCA receives several signals which includes desired and interfering signals and the complex excitation phases are optimized using POSGSA algorithm. The simulation results show that the PSOGSA algorithm shows improved performance in terms of rate of convergence and accuracy than GSA.

From the review of literature it is found that researchers have contributed to analyze different aspects of DOA estimation and ABF in the field of array signal processing. The field of array signal processing was first explored in adaptive antennas which was initially deployed in military radar, navy sonar and satellite tracking systems. Later the adaptive antennas became the most promising technology in the field of mobile communication, especially with the development of digital signal processing and innovative algorithms. Although there is a great progress in the research concerning the commercial use of adaptive antennas but their deployment is limited due to different reasons. **Kaiser (2005) and Rayal (2005)** addressed the problems such as cost barrier, mismatching between the forward and reverse link in beamforming, bunch of wires used in smart antenna for connecting antenna elements and main body of base station, etc. that causes limited use of smart antenna in present wireless communication.

Capon (1969) first estimated the direction of incident narrowband plane waves but if had limited resolution. **Barabell (1983)** proposed a technique which estimates the direction by finding roots of the polynomial but the approach was applicable to ULA.

Schmidt (1986) proposed a high resolution MUSIC algorithm that finds unbiased estimates of number of incident signal parameters in a noisy environment. **Roy and Kailath (1989)** exploits the rotation invariance in the signal subspace induced on arrays with a translational invariance structure for DOA estimation. **Hua and Sarkar (1990)** proposed matrix pencil method for signal parameters estimation and founds to be less sensitive to noise. **Southall, *et al.* (1995) and El Zooghby, *et al.* (1997)** proposed RBFNN for DOA estimation in linear arrays. **Godara (1997)** gave a review and theoretical comparison of all discussed DOA estimation techniques. Since then many researchers **Wu, *et al.* (2010) and Dhope (2013)** have simulated and compared different DOA estimation techniques. But no one has made a detailed theoretical and statistical comparison of all the DOA estimation techniques. Thus, in this research work a detailed performance analysis of all conventional and neural network based DOA estimation techniques is presented.

Widrow, *et al.* (1967) proposed LMS algorithm for beamforming in adaptive antennas and through simulation and experimental setup analyzed the performance of the algorithm. SMI, a block adaptive approach introduced by **Reed, *et al.* (1974)** obtained the weights of antenna array for beamforming and have a fast rate of convergence than LMS algorithm. RLS an adaptive filtering algorithm, approximates the Weiner solution using method of least squares. **Godard (1980)** introduced the concept of CMA for adaptive channel equalization. **Shynk and Gooch (1996)** investigated the steady state properties of constant modulus (CM) array and the signal canceller. **El Zooghby, *et al.* (1998, 1999)** proposed RBFNN approach for beamforming in linear antenna arrays. **Godara (1997)** gave a review and theoretical comparison of different beamforming algorithms. **Imtiaj, *et al.* (2012)** have simulated and compared LMS and SMI algorithms on the basis of null depth, maximum SLL and rate of convergence. **Nwalozie, *et al.* (2013)** have simulated LMS, SMI, CMA and RLS algorithms and analyzed the convergence of LMS algorithm.

Researchers have analyzed different algorithms but no one has compared all the algorithms and have considered all the parameters which affect the performance of the algorithms. Thus, in this research work a performance analysis of all the ABF algorithms is presented by considering their beamwidth, maximum null depth, maximum SLL and their rate of convergence.

AI has gained momentum in the field of science and technology because of their strong numerical approximation capabilities. The AI approaches such as neural networks **(Haykins, 1999)**, fuzzy logic **(Yager and Zadeh, 2012)** and metaheuristic techniques **(Haupt, 2004)** were reported being used in the field of array signal processing. Although, researchers proposed different techniques and methods for DOA estimation and ABF but certain research areas are still unidentified and have not been addressed. The DOA estimation for deterministic signals were obtained in circular arrays using GSA **(Magdy, *et al.*, 2013)** but was not explored for linear arrays thus DOA estimation in linear arrays is addressed in this research work. **Ziskind, *et al.* (1988)** addressed ML based DOA estimation for random signals using alternating projection (AP) technique. **Sharman (1988)** used simulated annealing algorithm for estimating direction of narrow band random signals using ML criteria. **Jiankui, *et al.* (2006)** first introduced PSO algorithm for estimating direction using unconditional ML criteria in uncorrelated channel environment. The refined genetic algorithm has been used to estimate direction in partially correlated environment (**Minghui and Yilong, 2007). Zhang and Ye (2008)** proposed an efficient method of DOA estimation when both the uncorrelated and coherent signals impinge on an ULA. **Minghui and Yilong (2008**) have used PSO algorithm for estimating the direction in uncorrelated and colored noise environment. **Hanumantharao, *et al.* (2011)** have used bacteria foraging optimization algorithm for exact ML DOA estimation of uncorrelated random signals. **Panigrahi, *et al.* (2013)** used PSO for optimizing ML function to estimate direction in wireless sensor networks. **Shi, *et al.* (2014)** proposed a method of estimating direction of narrowband sources in which the sources are uncorrelated and coherent.

It is found that no work has been reported to use any algorithm which may be used for all possible channel environments viz. uncorrelated, partially correlated and coherent channel environment. Thus, PSO algorithm is explored to estimate direction in uncorrelated, partially correlated and coherent channel environment. The direction of signals has not been previously identified for narrow angular separation thus we explored the efficiency of the algorithms by identifying signals for narrow angular separation for uncorrelated deterministic signals and random signals in uncorrelated, partially correlated and coherent channel environment.

Soft computing techniques were explored by researchers in the field of ABF or pattern synthesis problems for linear and circular antenna arrays. Researchers have explored different optimization techniques to control either phase or both amplitude and phase so as to place main lobe in the desired user and nulls in the direction of interfering signals. **Karaboga, *et al.* (2004)** proposed MTACO algorithm for null steering of ULA by controlling both amplitude and phase of array elements. **Mouhamadou and Vaudon (2006)** proposed SQP for controlling phase of each array element by modeling the beamforming problem as a multi-objective optimization problem. **Wang, *et al.* (2011)** proposed complex valued GA for optimization of beamforming in ULA. **Basha, *et al.* (2012)** integrates the NN and fuzzy logic approach with GA to obtain smart antenna parameters for beamforming. **Goswami and Mandal (2012)** used GA for placing deeper nulls in the direction of interfering signals under the constraints of reduced SLL and fixed FNBW.

Zaharis, *et al.* (2011) proposed AMBPSO for ABF in linear antenna arrays at different power levels. **Zaharis, *et al.* (2012, a)** proposed ADIWO technique for ABF with reduced SLL at different number of interfering signals and power levels. **Zaharis, *et al.*, (2012, c)** integrates NN with MBPSO for ABF in linear antenna arrays. The NN is trained by the data obtained using MBPSO algorithm and the results are compared with MVDR algorithm.

In this research work, GSA is explored for ABF in linear antenna arrays. This algorithm has never been used for optimizing the function which is being considered in our research problem. The problem of ABF is first modeled as a single objective problem in which the optimization function is inverse of SINR. Secondly, the problem is formulated as a multi-objective optimization problem where the SINR is increased and SLL are reduced at certain angular region. The beamforming results in terms of rate of convergence of the optimization algorithm, main lobe deviation, null deviation and SLL are compared with the previous reported results.

Materials and Methods

In this chapter signal processing techniques in smart antennas that are DOA estimation and ABF have been discussed. The chapter presents some basic concepts of optimization and the classification of optimization techniques and briefly introduces the MATLAB tool used in the simulation of this work. The methodology of DOA estimation and ABF using optimization techniques has been discussed.

Smart antennas have the capability of estimating the direction of desired and interfering signals in multipath channel environment and accordingly optimizing its radiation pattern in response to changing signal environment. Thus, signal processing in smart antenna deals with two key objectives:

- Direction of arrival estimation
- Adaptive beamforming

3.1 Direction of arrival estimation

In many practical signal processing problems, the objective is to estimate from a collection of noise contaminated measurements a set of constant parameters upon which the underlying true signal depends. The problem of direction finding in real time is an integral part of SDMA scheme for terrestrial and satellite communication systems. Direction finding has various diverse applications such as target tracking, radar, sonar, electronic surveillance and telemetry. The performance of adaptive array antenna improves with respect to recovering the signal of interest and suppressing the interfering signals by accurate estimation of DOA of all highly correlated signals. Many researchers investigated the performance bounds of an arbitrary array geometry because DOA and beamforming algorithms exploit properties of array geometries and could not provide conclusive comparative results.

In DOA estimation, Cramer-Rao lower bound (CRLB) measures the goodness of an estimator. CRLB expresses a lower bound on the variance of estimator of a deterministic parameter. In order to understand the basics of different DOA estimation techniques, initially we present the data model of wireless channel environment.

Data model

Let us consider a ULA composed of *M* isotropic antenna elements (sensors), as shown in figure 3.1. The elements are placed with inter-element spacing of *d* respectively. Suppose ULA receives *D* narrow band signals, centered at frequency f_o and it also includes additive zero mean Gaussian noise.

The direction of the incoming signal of the *i-th* user is represented by the array steering vector of dimension *M X 1* and is given as:

$$\bar{a}(\theta_i) = \left[e^{j(m-1)\beta d sin\theta_i}\right]^T, \qquad m = 1, 2, \dots\dots\dots.M, \tag{3.1}$$

where, $\beta = {}^{2\pi}/_{\lambda}$ is the wave number, θ_i is the angle from the array broadside, $\lambda = {}^{c}/_{f_o}$ is the signal wavelength, c being the velocity of light. Time is represented by k-th time sample.

The received signal by the array elements can be written as:

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ . \\ . \\ . \\ x_M(k) \end{bmatrix} = [\bar{a}(\theta_1)\bar{a}(\theta_2)\dots..\bar{a}(\theta_D)] * \begin{bmatrix} s_1(k) \\ s_2(k) \\ . \\ . \\ . \\ s_D(k) \end{bmatrix} + \bar{n}(k),$$

$$\bar{x}(\mathrm{k}) = \bar{A} * \bar{s}(k) + \bar{n}(k). \tag{3.2}$$

where,

$\bar{s}(k) =$ vector of incident complex monochromatic signals at sample time *k*,

$\bar{n}(k) =$ noise vector of dimension *M X 1* with i. i. d. elements having a mean zero and variance σ_n^2,

$\bar{a}(\theta_i) =$ *M*-element array steering vector for the θ_i direction of arrival,

$\bar{A} = [\bar{a}(\theta_1)\bar{a}(\theta_2)\bar{a}(\theta_3)\dots..\bar{a}(\theta_D)]$ *M* X *D* matrix of steering vectors $\bar{a}(\theta_i)$.

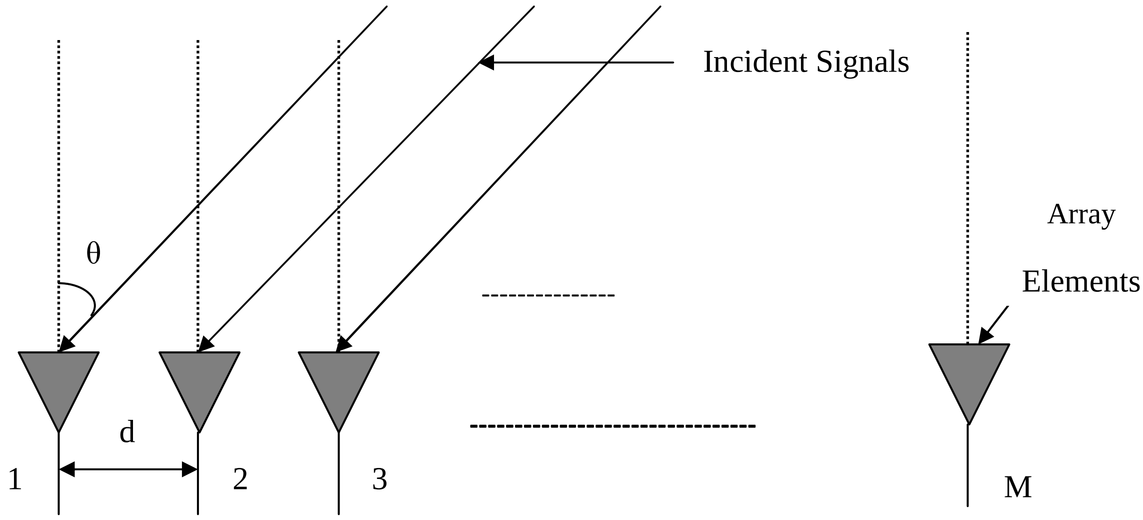


Fig. 3.1 Uniform linear array.

In array signal processing, correlation matrices are used instead of the actual array output $\bar{x}(k)$ for determining the direction of arrival of incoming signal since it contains sufficient information about the received signal. The M X M array correlation matrix $\bar{R}_{xx}$ of the received signal in case the signal statistics are not changing with time is defined as:

$$\bar{R}_{xx} = E[\bar{x} * \bar{x}^H] = E[(\bar{A}\bar{s} + \bar{n})(\bar{s}^H \bar{A}^H + \bar{n}^H)],$$

$$= \bar{A}E[\bar{s} * \bar{s}^H]\bar{A}^H + E[\bar{n} * \bar{n}^H],$$

$$= \bar{A}\bar{R}_{ss}\bar{A}^H + \bar{R}_{nn}. \tag{3.3}$$

where,

$\bar{R}_{ss}$ represents the *D X D* source correlation matrix,

$\bar{R}_{nn}$ represents the *M X M* noise correlation matrix,

E[] and $()^H$ denote the expectation and Hermitian operation respectively.

The DOA estimation techniques can be classified on the basis of the data analysis into spectral estimation methods, eigen structure methods and ML techniques. The goal of the spectral and some of the eigen structure DOA estimation technique is to define a function called as pseudospectrum P(θ). This function represents the output power of the beamformer and the true DOA is the angle θ that corresponds to the peak value of this output power spectrum. The ML technique computes the source DOA by optimizing a complex multimodal nonlinear likelihood function over a high dimensional space. Based on the computation complexity, resolution and convergence speed different DOA estimation techniques are reviewed in this section.

3.1.1 Spectral estimation methods

These methods are also known as conventional methods. These methods estimate the direction of incoming signals by computing the pseudospectrum and then determine the local maxima of this spectrum. The methods differ by how the weights are estimated to obtain the spectrum.

Bartlett estimate

It is one of the earliest and simplest DOA estimation technique also referred to as delay and sum method. In this method the array is uniformly weighted and the pseudospectrum is given as:

$$P_B(\theta) = \bar{a}^H(\theta)\bar{R}_{xx}\bar{a}(\theta) \tag{3.4}$$

This method is also referred to as Fourier method since it is a natural extension of the classical Fourier based spectral analysis with different window functions. The pseudospectrum has a resolution threshold. The technique fails to resolve the closely spaced signals. Thus, the poor resolution is a significant weakness of this method.

Capon estimate

The capon DOA estimate is a ML method of spectral estimation and is known as a minimum variance distortion response (MVDR). This method overcome the poor resolution problem associated with the bartlett method and it results a significant improvement.

The goal is to minimize the power contributed by noise and signals coming from undesired direction.

$$\min_{\bar{w}} \bar{w}^H \bar{R}_{xx}^{-1}\bar{w} \qquad \text{subject to } |\bar{w}^H a(\theta)| = 1$$

The maximized SIR is obtained with a set of array weights which is given by :

$$\bar{w} = \frac{\bar{R}_{xx}^{-1}\bar{a}(\theta)}{\bar{a}^H(\theta)\bar{R}_{xx}^{-1}\bar{a}(\theta)} \tag{3.5}$$

and the pseudospectrum is given by:

$$P_C(\theta) = \frac{1}{\bar{a}^H(\theta)\bar{R}_{xx}^{-1}\bar{a}(\theta)} \tag{3.6}$$

The advantages of spectral estimation techniques are that these give nonparametric solutions and thus one does not need prior knowledge of the specific statistical properties. But, these techniques fail in multipath channel environment where highly correlated signals are present.

3.1.2 Eigen structure methods

Eigen structure methods are high resolution methods and have achieved high success during the last two decades due to their low complexity and reasonable performance. These methods are also known as subspace method as it decomposes the correlation matrix into orthogonal spaces i.e. signal subspace and noise subspace. The noise subspace is spanned by the Eigen vectors associated with smaller Eigen values and the signal subspace is spanned by the Eigen vectors associated with larger Eigen values.

Pisarenko method was the first subspace method to utilize the data structure model and has shown better resolution than spectral estimation methods. These methods are used for DOA estimation under different environment conditions, e.g. when the noise is not white and has known covariance or an unknown covariance.

Pisarenko harmonic decomposition

This algorithm is named after the Russian mathematician who devised this minimum mean square approach **(Gross, 2005)**. The basic principle of this algorithm is to minimize the mean squared error of the array output under the constraint that the norm of the weight vector be equal to unity. The Eigen vector that minimizes the mean squared error corresponds to the smallest Eigen value of the correlation matrix.

The corresponding pseudospectrum of this algorithm is given by:

$$P_{PHD} = \frac{1}{\left|\bar{a}^H(\theta)\bar{e}_1\right|^2} \tag{3.7}$$

where, $\bar{e}_1$ is the eigen vector associated with the smallest eigen value.

Minimum-norm estimate

This DOA technique was proposed by Reddi, Kumaresan and Tufts. The algorithm was also briefly explained by Ermolaev and Gershman and is applicable only for ULA. The algorithm optimizes the array weights by solving the optimization problem which is stated as:

$$\min_{\bar{w}} \bar{w}^H\bar{w} \qquad \bar{E}_S^H\bar{w} = 0 \qquad \bar{w}^H\bar{u}_1 = 1 \tag{3.8}$$

where, $\bar{w}$ = array weights,

$\bar{E}_S$ = subspace of D signal eigen vectors,

$\bar{u}_1$ = Cartesian basis vector (first column of the M X M identity matrix)

$= [1 \quad 0 \quad 0 \quad \ldots\ldots \quad 0]^T$

The min-norm pseudospectrum obtained by solving the optimization problem mentioned above is given by:

$$P_{MN}(\theta) = \frac{(\bar{u}_1^T\bar{E}_N\bar{E}_N^H\bar{u}_1)^2}{\left|\bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{u}_1\right|^2} \tag{3.9}$$

where, $\bar{E}_N$ = subspace of M - D noise eigen vectors,

In eq. (3.9), since the numerator is a constant, the pseudospectrum can be normalized and the resultant pseudospectrum is given by:

$$P_{MN}(\theta) = \frac{1}{\left|\bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{u}_1\right|^2} \tag{3.10}$$

Since, the min-norm method combines all noise eigen vectors whereas Pisarenko harmonic decomposition method uses only first noise eigen vector, thus the pseudospectrum of both the algorithms are almost identical. As this approach is applicable to ULA, the optimization problem to solve for the array weights may be transformed to a polynomial rooting problem, leading to a root-min-norm method similar to root-MUSIC.

Researchers are exploring various ways to speed up the min-norm DOA estimation technique and reduce its computational complexity.

MUSIC

MUSIC is an acronym which stands for Multiple Signal Classification and was proposed by Schmidt in 1986. It is relatively simple, efficient and widely examined DOA estimation technique. This is a high resolution Eigen structure technique which estimates number of signals and multiple parameters (e.g., azimuth and elevation angle, range, polarization) for multiple incident wavefronts. The algorithm is applicable to arrays of arbitrary but known configuration and response and is a suitable candidate for real time hardware implementation. DOA estimation is performed from one of either signal or noise subspaces, assuming that noise in each channel is highly uncorrelated.

Algorithm

Step 1. Collect data and estimate the correlation matrix $\bar{R}_{xx}$.

Step 2. Compute the Eigen decomposition of $\bar{R}_{xx}$. The Eigen vectors corresponding to noise subspace of dimension M X M-D is expressed as:

$$\bar{E}_N = [\bar{e}_1 \bar{e}_2 \ldots . \bar{e}_{M-D}]. \tag{3.11}$$

Step 3. Estimate the DOA by looking for steering vectors that are orthogonal to the noise subspace and because of this orthogonality condition at every arrival angle the Euclidean distance is zero,

$$d^2 = \bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{a}(\theta) = 0. \tag{3.12}$$

Step 4. Compute the expression for MUSIC pseudospectrum by placing the Euclidean distance expression in the denominator. Thus, the MUSIC pseudospectrum is given as:

$$P_{MU}(\theta) = \frac{1}{|\bar{a}^H(\theta)\bar{E}_N\bar{E}_N^H\bar{a}(\theta)|}. \tag{3.13}$$

Step 5. Find the peaks in the pseudospectrum, as it indicates the DOA of the incoming signal.

The MUSIC algorithm fails to detect the incoming signal under high signal correlation, so to overcome this weakness a spatial averaging technique is developed to suppress signal correlation in multipath environments.

Root-MUSIC

Root-MUSIC is the polynomial version of MUSIC algorithm in which array steering vector in MUSIC pseudospectrum is expressed in polynomial form by evaluating at z=$e^{j\beta d sin\theta}$. The MUSIC pseudospectrum becomes equivalent to the polynomial on the unit circle and peaks in the MUSIC spectrum exists as roots of polynomial that lie close to the unit circle **(Barabell, 1983)**. The algorithm requires that the elements must be uniformly placed and must be physically identical with no mutual coupling effects.

<u>Algorithm</u>

Step 1. Collect data from the ULA and estimate the correlation matrix $\bar{R}_{xx}$.

Step 2. Compute the expression for the MUSIC pseudospectrum as discussed above.

Step 3. Compute the Root-MUSIC expression by simplifying the MUSIC expression,

$$P\ (\theta) = \frac{1}{|\bar{a}^H(\theta)\bar{C}\bar{a}(\theta)|}, \tag{3.14}$$

where $\bar{C} = \bar{E}_N\bar{E}_N^H$.

Step 4. Find the sum of the (2M-1) diagonal elements of $\bar{C}$.

Step 5. Estimate 2(M-1) roots of the polynomial.

Step 6. Plot all the roots in the complex plane and identify the roots that are closer to the unit circle.

Step 7. Express all the roots closer to the unit circle in polar form:

$$z_i = |z_i|e^{jarg(z_i)}, \qquad i=1, 2, ...2(M-1). \tag{3.15}$$

where arg (z_i) is the phase of z_i.

Step 8. Estimate the DOA of incoming signal by comparing $e^{jarg(z_i)}$ to $e^{jkdsin\theta_i}$ to get:

$$\theta_i = sin^{-1}(\frac{1}{\beta d} \arg{(z_i)}). \tag{3.16}$$

Thus, the DOA estimation is obtained by solving a polynomial rooting problem in contrast to the identification and localization of spectral peaks using spectral MUSIC algorithm.

ESPRIT

ESPRIT, proposed by **Roy and Kailath (1989)** stands for Estimation of Signal Parameters via Rotation Invariance Technique. This technique exploits the rotational invariance in the signal subspace induced on arrays with a translational invariance structure. The method assumes multiple identical subarrays called doublets instead of single element array which are displaced by fixed displacement vector. Let the signal induced on each of the arrays be $\bar{x}_1(k)$ and $\bar{x}_2(k)$ then from equation (3.2) these are given by:

$$\bar{x}_1(k) = \bar{A}_1 * \bar{s}(k) + \bar{n}_1(k), \tag{3.17}$$

$$\bar{x}_2(k) = \bar{A}_1 * \bar{\Phi} * \bar{s}(k) + \bar{n}_1(k), \tag{3.18}$$

where $\bar{\Phi}$ is an D X D diagonal unitary matrix with phase shifts between the elements for every direction of arrival, with its D diagonal elements are given by:

$$\bar{\Phi} = \{e^{jkdsin\theta_1}, e^{jkdsin\theta_2}, \dots \dots, e^{jkdsin\theta_D}\}. \tag{3.19}$$

The correlation matrices corresponding to two subarrays are given as:

$$\bar{R}_{11} = E[\bar{x}_1 * \bar{x}^H{}_1] = \bar{A}\bar{R}_{ss}\bar{A}^H + \bar{R}_{nn}, \tag{3.20}$$

$$\bar{R}_{22} = E[\bar{x}_2 * \bar{x}^H{}_2] = \bar{A}\bar{\Phi}\bar{R}_{ss}\bar{\Phi}^{\mathrm{H}}\bar{A}^H + \bar{R}_{nn}. \tag{3.21}$$

Let $\bar{E}_1$ and $\bar{E}_2$ represents the Eigen vectors of dimension M X D corresponding to signal subspace for the two subarrays. Since the array is translationally related, the subspaces of signal Eigen vectors are related by a unique non-singular transformation matrix $\bar{\Psi}$ such that:

$$\bar{E}_1\bar{\Psi} = \bar{E}_2, \tag{3.22}$$

where, $\bar{\Psi}$ is a rotation operator that maps the signal subspace $\bar{E}_1$ into the signal subspace $\bar{E}_2$.

There must also exist a unique non-singular transformation matrix $\bar{T}$ such that:

$$\bar{E}_1 = \bar{A}\bar{T}, \tag{3.23}$$

$$\bar{E}_2 = \bar{A}\bar{\Phi}\bar{T}. \tag{3.24}$$

Using the relations, obtained in equation (3.23, 3.24) in equation (3.22) we obtain the relationship

$$\bar{T}\bar{\Psi}\bar{T}^{-1} = \bar{\Phi}. \tag{3.25}$$

The Eigen values of $\bar{\Psi}$ corresponds to the diagonal elements of $\bar{\Phi}$. Thus, the direction of the incoming signal can be evaluated by finding the Eigen values of $\bar{\Psi}$. The rotation operator can be estimated using the total least square (TLS) criteria outlined in the algorithm.

<u>Algorithm</u>

Step 1. Collect data from the ULA and estimate the correlation matrix $\bar{R}_{xx}$.

Step 2. Compute the Eigen decomposition of $\bar{R}_{xx}$. The Eigen vectors corresponding to signal subspace of dimension M X D is expressed as:

$$\bar{E}_s = [\bar{e}_1\bar{e}_2 \dots . \bar{e}_D]. \tag{3.26}$$

Step 3. Compute the signal subspace $\{\bar{E}_1, \bar{E}_2\}$ for the two subarrays by selecting the first and last (M-1) rows from the signal subspace $\bar{E}_s$.

Step 4. Calculate the matrix $\bar{C}$ of dimension 2D X 2D using the signal subspaces $\{\bar{E}_1, \bar{E}_2\}$.

$$\bar{C} = \begin{bmatrix} \bar{E}_1^H \\ E_2^H \end{bmatrix} [\bar{E}_1 \quad \bar{E}_2]. \tag{3.27}$$

Step 5. Compute the Eigen decomposition of matrix $\bar{C}$. The Eigen vectors $\bar{E}_C$ are partitioned into four D X D submatrices such that,

$$\bar{E}_C = \begin{bmatrix} \bar{E}_{11} & \bar{E}_{12} \\ \bar{E}_{21} & \bar{E}_{22} \end{bmatrix}. \tag{3.28}$$

Step 6. Calculate the rotation operator $\bar{\Psi}$ by:

$$\bar{\Psi} = -\bar{E}_{12}\bar{E}_{22}^{-1}. \tag{3.29}$$

Step 7. Estimate the Eigen values of $\bar{\Psi}$, $\lambda_1, \lambda_2, \dots \dots . \lambda_D$.

Step 8. Express the Eigen values in polar form and find the DOA of incoming signal using equation (3.16).

Thus, this algorithm is more efficient without the need of searching peaks throughout the entire range of angles. The algorithm finds its application in multiple inputs and multiple outputs (MIMO) radar but because of the large size of the array complexity of the algorithm increases. In order to reduce the complexity beamspace ESPRIT (B-ESPRIT) algorithm which focuses in a particular directional sector of interest is developed. However, the algorithm possesses certain drawbacks. A popular variant of ESPRIT algorithm is Unitary ESPRIT method in which the computational burden is reduced (to about one-fourth) by converting the complex correlation matrix into a real matrix by using a unitary matrix transformation. Multicarrier code-division multiple access (MC-CDMA) have received attention in fourth generation communication system because of its capabilities. The problem of DOA estimation for a MC-CDMA system using multi invariance ESPRIT algorithm with improved performance has been developed.

Matrix pencil method

The Matrix pencil (MP) method was proposed by **Hua and Sarkar (1988, 1990)**. This method is a variation of the idea of Pencil-of-Function method and was first used to extract the parameters from damped exponentials transient electromagnetic signals and later for estimating frequency and DOA estimation. It is a direct data domain method and doesn't require a search procedure to estimate the DOA of incoming signal. Therefore, it is a less complicated approach and eliminates problems encountered in search procedure with regard to memory storage and system calibration. The algorithm can handle a non-stationary environment relatively easily since it estimates the required parameters on snapshot by snapshot basis.

Algorithm

Step 1. Collect data from the ULA and choose a parameter L, known as pencil parameter that lies between M/3 to M/2.

Step 2. Estimate the output y(k) in the form of Henkel matrix from the set of inputs data samples and each column of the matrix is the part of the original data vector $\{x_1 x_2 .... x_M\}$.

$$Y = \begin{bmatrix} x_1 & \cdots & x_L \\ \vdots & \ddots & \vdots \\ x_{M+1-L} & \cdots & x_M \end{bmatrix}_{(M-L+1)x\,L} \tag{3.30}$$

Step 3. Compute Y_1 & Y_2 matrices by deleting the last and the first row of Y matrix. It is given by:

$$Y_1 = \begin{bmatrix} x_1 & \cdots & x_L \\ \vdots & \ddots & \vdots \\ x_{M-L} & \cdots & x_{M-1} \end{bmatrix}_{(M-L)x\,L} \tag{3.31}$$

$$Y_2 = \begin{bmatrix} x_2 & \cdots & x_{L+1} \\ \vdots & \ddots & \vdots \\ x_{M+1-L} & \cdots & x_M \end{bmatrix}_{(M-L)x\,L} \tag{3.32}$$

Step 4. Compute Moore-Penrose pseudo inverse of Y_1 which is defined as:

$$[Y_1]^+ = \{[Y_1]^H[Y_1]\}^{-1}[Y_1]^H. \tag{3.33}$$

Step 5. Find the Eigen values of the expression:

$$\{Y_1^+Y_2 - \lambda I\}. \tag{3.34}$$

Step 6. Express the Eigen values in polar form and find the DOA of incoming signal using equation (3.16).

The MP method does not give accurate results in the presence of noise. To mitigate the effect of noise total least squares (TLS) approach is applied by taking singular value decomposition (SVD) of Y. This method is known as total least squares matrix pencil (TLS-MP) method. The rapid estimation of parameters in real time requires very fast processing of MP algorithm. Therefore, it is highly desirable to minimize the computational burden of the algorithm with no or negligible loss of accuracy. The unitary matrix pencil (UMP) algorithm has been developed to reduce the computational complexity of the algorithm. Many researchers have used the beamspace approach where prior information of the parameters of incoming signal is utilized to minimize the computational burden of one and two dimensional MP algorithms.

3.1.3 Maximum likelihood techniques

The ML method is a standard technique in statistical estimation theory. In this technique, initially a parametric data model of the observed data has to be specified and a likelihood function is formulated from the sampled data. The ML estimate is computed by maximizing the likelihood function or minimizing the negative likelihood function with respect to all unknown parameters, which may include the source DOA's, the signal covariance and the noise parameters.

In the field of array signal processing, the ML method for DOA estimation can be categorized into two main methods, depending on the assumption on the signal model. When the emitter signals are modeled as deterministic quantities, the resulting ML estimator is referred to as the deterministic maximum likelihood (DML) estimator and when the emitter signals are modeled as Gaussian random processes, the resulting estimator is referred to as a stochastic maximum likelihood (SML) estimator.

Deterministic maximum likelihood estimator

In the field of radar and radio communications, the signal waveforms are often far from being Gaussian random variables as stated by central limit theorem. Thus, the model is assumed as the deterministic model, since it makes no assumptions at all on the signals. In microwave communications, the estimation of signal (*$s(t_i)$, $i = 1,.....,k$*) is of more interest than estimation of direction of incoming signals. The ML estimator for this model is termed as DML method. The observation process $x(t_i)$, is often Gaussian distributed given the unknown quantities. The first and second order moments are given by:

$$E\{x(t_i\} = A(\theta)s(t_i) \qquad (3.35)$$

$$E\left\{(x(t_i) - E\{x(t_i)\})(x(t_j) - E\{x(t_j)\}^H\right\} = \sigma^2 I \delta_{ij} \qquad (3.36)$$

$$E\left\{(x(t_i) - E\{x(t_i)\})(x(t_j) - E\{x(t_j)\}^T\right\} = 0. \qquad (3.37)$$

The unknown parameters in this case are direction of arrival i.e. θ, input signals *$\{s(t_i), i = 1,........, k,\}$* and variance σ^2. δ_{ij} represents the kronecker delta which is equal to 1 if i=j and 0 otherwise. The parameters of the deterministic model, i.e. the signal parameters, the noise variance and the waveforms are conditioned to formed the joint probability distribution of the observations data **(Ottersten, *et al.*, 1993)**. The conditional density for the independent snapshots are given by :

$$p(x(t_1) \ldots x(t_k)|\theta, \sigma^2, s_k) = \prod_{i=1}^{k} \frac{1}{|\pi\sigma^2 I|} e^{-\sigma^{-2}(x(t_i) - A(\theta)s(t_i))^H (x(t_i) - A(\theta)s(t_i))} \qquad (3.38)$$

and the negative log likelihood function has the following form:

$$-\log\big(p(\theta,\sigma^2,s_k)\big) = Nmlog(\pi\sigma^2) + \sigma^{-2}Tr\{(x_k - A(\theta)s_k)^H(x_k - A(\theta)s_k)\}$$

$$= Nmlog(\pi\sigma^2) + \sigma^{-2}\|x_k - A(\theta)s_k\|_F^2, \quad (3.39)$$

where, $\|.\|_F$ is the Frobenius norm of a matrix, and x_k and s_k are input and signal vector as defined in eq. (3.2). The DML estimates are obtained by minimizing arguments of eq. (3.39). For fixed angle θ and s_k, the minimum with respect to σ^2 is derived as:

$$\hat{\sigma}^2 = \frac{1}{m}Tr\{P_A^{\perp}(\theta)\hat{R}\}. \quad (3.40)$$

The estimated angle and the signal parameters are obtained using linear least square problem by substituting eq.(3.40) in eq.(3.39)

$$[\hat{\theta},\hat{S}_k] = arg\min_{\theta,S_k}\|x_k - A(\theta)s_k\|_F^2 \quad (3.41)$$

As the above criterion function is quadratic in the signal waveform parameters, it becomes easy to minimize with respect to s_k. This results in the following estimates:

$$\hat{s}_k = A^{\dagger}(\theta)x_k \quad (3.42)$$

$$\hat{\theta} = arg\min_{\theta} V_{DML}(\theta) \quad (3.43)$$

$$V_{DML} = Tr\{(I - A(A^HA)^{-1}A^H\ )\hat{R}\} \quad (3.44)$$

The DML function obtained in eq. (3.44) is nonlinear and often possesses a large number of local minima, thus direct optimization of this function is unrealistic. Thus, different optimization techniques have been considered to jointly estimate source DOA's and other parameters over a high dimensional space.

Stochastic maximum likelihood estimator

In the field of wireless communication, the signal is often modeled as stationary stochastic processes, having a certain probability distribution. Gaussian distribution is by far the most commonly advocated stochastic distribution because of its mathematical convenience. The detection and estimation schemes derived from the stochastic model is found to yield superior performance, regardless of the actual emitter signals.

The received signal waveforms are assumed to be narrowband and the signal $s(t_i)$ and noise $n(t_i)$ for {i=1,...., k}are independent zero mean complex Gaussian random processes with second order moments given by:

$$E\{x(t_i)x^H(t_j)\} = R\delta_{ij} = (A(\theta)SA^H(\theta) + \sigma^2 I)\delta_{ij} \quad (3.45)$$

$$E\{x(t_i)x^T(t_j)\} = 0. \quad (3.46)$$

The apriori information on the signal covariance matrix is not available in most of the wireless communication applications. The signal covariance matrix is a Hermitian matrix, thus it can be uniquely parameterized by d^2 real parameters, which are the real diagonal elements and the real and imaginary parts of the lower or upper off-diagonal elements. d and p represents the number of point sources and array elements. The other possible assumptions are for uncorrelated signals in which covariance matrix is completely known or unknown but contains diagonal elements. The direction of signals, covariance matrix and the variance are all considered to be completely unknown which results in a total of $d^2 + pd + 1$ unknown parameters **(Ottersten, *et al.*, 1993)**.
The likelihood function of a single observation, $x(t_i)$, is

$$p_i(x) = \frac{1}{\pi^m|R|} e^{-x^H R^{-1} x}, \quad (3.47)$$

where, $|R|$ denotes the determinant of R. This is the complex *m-variate* Gaussian distribution and the likelihood of the complex data set $x(t_1), \ldots, x(t_k)$ is given by:

$$p(x(t_1), \ldots, x(t_k)|\theta, S, \sigma^2) = \prod_{i=1}^{k} \frac{1}{\pi^m|R|} e^{-x^H(t_i)R^{-1}x(t_i)}, \quad (3.48)$$

Maximizing $p(\theta, \sigma^2, S)$ is thus equivalent to minimizing the negative *log-likelihood function*,

$$-\log\big(p(\theta, \sigma^2, S)\big) = -\sum_{i=1}^{k} log\left[\frac{1}{\pi^m|R|} e^{-x^H(t_i)R^{-1}x(t_i)}\right]$$

$$= mklog\pi + klog|R| + \sum_{i=1}^{k} x^H(t_i)R^{-1}x(t_i). \quad (3.49)$$

The constant terms are being ignored and are normalized by k, SML estimate is obtained by finding the solution of the following optimization problem,

$$[\hat{\theta}, \hat{S}, \hat{\sigma}^2] = arg\, \min_{\theta, S, \sigma^2} l(\theta, S, \sigma^2), \quad (3.50)$$

where the criterion function is defined as:

$$l(\theta, S, \sigma^2) = log|R| + \frac{1}{k}\sum_{i=1}^{k} x^H(t_i)R^{-1}x(t_i). \qquad (3.51)$$

The normalized negative log-likelihood function can be expressed using the well-known properties of the trace operator as:

$$l(\theta, S, \sigma^2) = \log|R| + Tr\{R^{-1}\hat{R}\}, \qquad (3.52)$$

where, $\hat{R}$ is the sample covariance which is given by:

$$\hat{R} = \frac{1}{k}\sum_{i=1}^{k} x(t_i)x^H(t_i) \qquad (3.53)$$

The ML function can be concentrated with respect to S and σ^2 with some mathematical calculations and thus the dimension of the required numerical optimization is reduced to *pd*. The SML estimates of the noise power and the signal covariance matrix are obtained by inserting the SML estimates of θ in the following expressions:

$$\hat{S}(\theta) = A^{\dagger}(\theta)\left(\hat{R} - \hat{\sigma}^2(\theta)I\right)A^{\dagger H}(\theta) \qquad (3.54)$$

$$\hat{\sigma}^2(\theta) = \frac{1}{m-d} Tr\{P_A^{\perp}(\theta)\hat{R}\}, \qquad (3.55)$$

where $A^{\dagger}$is the pseudo-inverse of A and $P_A^{\perp}$is the orthogonal projector onto the null space of A^H, i.e.

$$A^{\dagger} = (A^H A)^{-1}A^H \qquad (3.56)$$

$$P_A = AA^{\dagger} \qquad (3.57)$$

$$P_A^{\perp} = I - P_A. \qquad (3.58)$$

The expression for SML criterion is obtained by substituting eq. (3.54, 3.55) into eq. (3.52). The signal parameter estimates are obtained by finding the solution of the following optimization problem,

$$\hat{\theta} = arg \min_{\theta} V_{SML}(\theta) \qquad (3.59)$$

$$V_{SML}(\theta) = \log\left|A(\theta)\hat{S}(\theta)A^H(\theta) + \hat{\sigma}^2(\theta)I\right|. \qquad (3.60)$$

Although the dimension of the parameter space is reduced substantially, the form of the function obtained in eq. (3.60) is complicated and thus it is difficult to obtain the analytical solution for the angle θ. Many researchers have proposed different optimization techniques to optimize SML function. Alternating projection, simulated annealing and data supported grid search algorithm optimize ML function but they cannot guarantee global convergence. Heuristic approaches have found a great success in finding global solution of complex multimodal function over a high dimensional space. The approaches like GA, PSO, bacteria foraging algorithm and GSA have been successfully used in the field of array signal processing to find direction of incoming signals using ML function.

3.1.4 Neural network approach for DOA estimation

Neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experimental knowledge and making it available for use **(Haykins, 1999)**. NN forms arbitrary nonlinear design boundaries for complex classification task and are extremely useful in problems where the relationship between inputs and outputs is not easily modeled. In array signal processing, antenna array can be thought of as performing mapping from the space of the DOA's to the space of array output. A NN is used to perform the inverse mapping and the network generalize for the inputs that are not included in the training phase. MLP, RBFNN and Hopfield NN are popular neural DOA estimation techniques.

Radial basis function neural network

Radial Basis Functions emerged as a variant of artificial neural network in late 1980s. The RBFNN in its most basic form is a three layered feedforward network and is considered as a curve-fitting problem in a high dimension space. It has universal approximations, optimization, and regularization capabilities. It has a faster learning speed and requires less iterations as compared to MLP with the BP rule using the sigmoid activation function. Basic principle of the RBFNN method is detailed in the remarkable literature of Haykins **(Haykins, 1999)**.

In **(El Zooghby, *et al.*, 1997)**, a RBFNN has been used to track the locations of mobile users. However, in this approach a different network had to be used for different number of users with some fixed angular separation. In (**El Zooghby, *et al.*, 2000)** authors proposed a new approach which is the generalization of the approach in **(El Zooghby, *et***

***al.*, 1997)** such that algorithm can track an arbitrary number of sources with any angular separation. This new neural multiple source tracking algorithm (N-MUST) divides the field of view of antenna array into angular spatial sectors and performs detection and DOA estimation in two stages. In the first stage of the algorithm network is trained to detect signals emanating from sources in a particular sector. Based on the results of first stage second stage is activated to estimate the exact location of the sources. The main advantage of this approach is a dramatic reduction in the size of the training set required to train each smaller neural network.

Algorithm

Detection stage

In the detection stage the entire angular spectrum is divided into *P* sectors. The p-th (1≤p≤P) RBFNN is trained to determine if one or more signals exist within a sector. The RBFNN have *M (M+1)* input and hidden nodes and one output node.

Training phase

Step1. Collect data from the ULA and estimate the correlation matrices $\{\bar{R}_{xx}^{n}; n = 1, 2, 3....,N\}$ for all DOA's ranging from $[-90^{o}, 90^{o}]$ and arrange it's upper triangular part in the form of a vector-b.

Step2. The vector-b is normalized by its norm to obtain the normalized z^{n}vector,

$$z^{n} = \frac{b}{||b||}. \tag{3.61}$$

Step3. Generate input-output pairs $\{z^{n}, 1\}$ for sources located in the sector, and $\{z^{n}, 0\}$ for sources located outside the sector where n=1, 2, 3, ….N.

Step4. Employ an appropriate RBFNN to learn in the detection stage.

Generalization phase

Step1. Collect data from the array and evaluate sample correlation matrix.

Step2. Compute the normalized $\bar{z}$ vector using equation (3.30).

Step3. Present input vector $\bar{z}$ to the RBFNN of the detection stage and obtain an output $\{0, 1\}$.

DOA estimation stage

This stage performs the exact DOA estimation. The *P* networks of the DOA estimation stage are assigned to the same spatial sectors as in the detection stage. When the output of one or more networks from the first stage is 1, the corresponding second stage networks are activated. The number of hidden nodes is same as number of input nodes given as *M (M+1)*. The number of output nodes is given by:

$$J = \begin{bmatrix} \theta_W \\ \Delta\theta_{min} \end{bmatrix}. \tag{3.62}$$

where, θ_W and $\Delta\theta_{min}$ are the sector width and minimum angular resolution. Same procedure as discussed in detection stage for training and generalization is implemented in DOA estimation stage. The outputs are post processed to estimate the exact direction of incoming signal.

A modified neural multiple source tracking (MN-MUST) **(Caylar, *et al.*, 2006)** algorithm inserts the spatial filtering stage and reduces the network size and training data as compared to previous approach (**El Zooghby, *et al.*, 2000)**.

Table 3.1 summarizes all the discussed DOA estimation techniques in terms of their advantages and disadvantages.

Table 3.1 Summary of DOA estimation algorithms.

Algorithm	**Advantages**	**Disadvantages**
Bartlett Estimate	• Array weights are uniformly weighted, therefore a simple and less complicated approach.	• Does not have good resolution and accuracy.
Capon Estimate	• A non-parametric solution and does not require prior knowledge of specific statistical properties.	• Does not have best resolution.

MUSIC	• High Resolution method. • Estimate multiple parameters per source.	• Failed to resolve highly correlated sources. • Performance degrades in low SNR conditions and closely spaced sources.
Root-Music	• Better resolution than MUSIC specially where the signals are closely related and have low SNR. • Requires no array calibration.	• Applicable only for uniform linear array.
ESPRIT	• Reduction in computational complexity. • Less sensitive to array imperfections and noise.	• Cannot handle correlated sources.
Matrix Pencil Method	• Estimation can be done in a single snapshot. • Efficiently determine DOA in presence of multipath coherent signals. • Easily handle non stationary environment.	• Sensitive to perturbation and measurement errors.
Maximum Likelihood techniques	• Gives superior statistical performance in low SNR environment and when the snapshots are small. • Efficiently estimate DOA in multipath channel environment.	• ML function is complicated and is difficult to solve analytically. • The ML function contains large number of local minima.
Radial Basis Function Neural Network	• Provides faster and more accurate results. • Reduces the computational complexity. • Locate Sources greater than the number of array elements.	• A large amount of diverse data is required in the training phase.

3.2 Adaptive beamforming

In the field of wireless communication, in recent past traditionally phase shifters were integrated with antenna arrays to steer the main beam towards signal of interest, and are called as phased arrays, beamsteered arrays and scanned arrays. This approach of phase shifting where the phase of the current is changed at each antenna element is referred to as electronic beamsteering.

In the present modern scenario, beam is steered by an antenna array based on certain optimum criteria and are known as adaptive array antenna or smart antenna. The term *adaptive* signifies the use of adaptive algorithm for beamforming and the term *smart* signifies the use of signal processing in order to shape the beam pattern according to certain conditions. Algorithms can be implemented electronically through analog devices but generally these algorithms can be implemented more easily using advanced digital signal processing technology. Therefore, each of the receiving antenna array must possess the necessary electronics i.e. analog to digital (A/D) converter as shown in figure 3.2 for digitizing the array output. Since an antenna beampattern is formed through digital signal processing, thus this process is often referred to as digital beamforming.

ABF is a subcategory under the more general subject of digital beamforming in which the algorithms are adaptive algorithms. ABF is the application of adaptive filters to spatial signal processing. An adaptive beamformer is a device that is able to separate signals collocated in the frequency band but separated in the spatial domain **(Litva and Lo, 1996)**. In beamforming array processor optimizes the beam pattern by adjusting the elemental control weights with respect to a prescribed criterion (scheme), so that the output contains the minimum contribution from noise and interference. The means by which optimization is achieved is specified by an algorithm. ABF also known as optimum combining has been primarily used in radar and sonar system. The term adaptive array was first coined by Van Atta in 1959 which describe a self-phased array or retrodirective array as it reflect all incident signals back in the direction of arrival by using clever phasing schemes.

ABF finds its roots with the invention of sidelobe canceller (SLC) in 1959 by Howells. In 1965 Applebaum developed fully adaptive antenna array which is based on maximization of SNR at the array output. This algorithm became known as Howells-Applebaum algorithm. The algorithm based on different beamforming criteria like maximizing the signal to interference ratio, minimum mean square error, maximum likelihood **(Van Trees, 1968)** and minimum variance **(Godara, 2004 and Haykins, *et al.*, 1985)** are fixed beamforming algorithms in which arrival angles are fixed and don't change with time.

In time varying signal propagation environment, arrival angle continuously changes with time and it is necessary to devise an optimization scheme that recursively updates the array weights. The adaptive algorithm solves the time varying situation and allows for the continuously updated array weights.

Based on different optimization criteria, speed of convergence and hardware complexity to implement the algorithm different techniques are reviewed in this section. ABF founds its application in various fields such as mobile communication especially in CDMA system and personal communications services (PCS).

3.2.1 Algorithm based on fixed weight beamforming criteria

In the field of array signal processing, the term beamforming relates to the function performed by a device or apparatus in which energy radiated by an aperture antenna is focussed along a specific direction in space. The network that controls the phase and amplitude of the excitation current is usually called the beamforming network. The fixed weight beamforming criteria were assumed to apply to fixed arrival angle emitters i.e. when the arrival angle of the emitters are fixed and don't change with time. There are several fixed beamforming approaches which includes maximum SINR, ML, and the minimum variance techniques which are briefly reviewed in the following section.

Data model

Let us consider a ULA having M isotropic antenna elements which are symmetrically placed about the origin as shown in figure 3.2. Suppose ULA receives monochromatic signals and having an inter element spacing of d respectively. The incoming signals consists of desired signal *s(k)* coming from angle $\theta_0$and N interference signals $i_n(k)$ coming from angle θ_n (n=1,....., N). The received signals also includes additive zero mean Gaussian noise *n(k)* having a variance σ^2. The signal at the input of the mth array element can be modeled as:

$$\bar{x}(k) = \bar{a}(\theta_0) * s(k) + [\bar{a}(\theta_1) \quad \bar{a}(\theta_2) \quad .. \quad \bar{a}(\theta_N)] * \bar{\imath}(k) + \bar{n}(\mathrm{k}),$$

$$\bar{x}(k) = \bar{a}(\theta_0) * s(k) + \bar{A} * \bar{\imath}(k) + \bar{n}(k), \tag{3.63}$$

$$\bar{x}(k) = \bar{d}(k) + \bar{u}(k). \tag{3.64}$$

where,

$\bar{a}(\theta_n)$ represents the array steering vector which contains information of incoming signals as discussed in section 3.1.

$\bar{d}(k) = \bar{a}(\theta_0) * s(k)$ represents the desired input signal vector,

$\bar{u}(k) = \bar{A} * \bar{\imath}(k) + \bar{n}(k)$ represents the interference plus noise signal vector.

The array output is given by the following expression:

$$y(k) = \bar{w}^H * \bar{x}(k)$$

$$y(k) = \bar{w}^H * \bar{d}(k) + \bar{w}^H * \bar{u}(k) \tag{3.65}$$

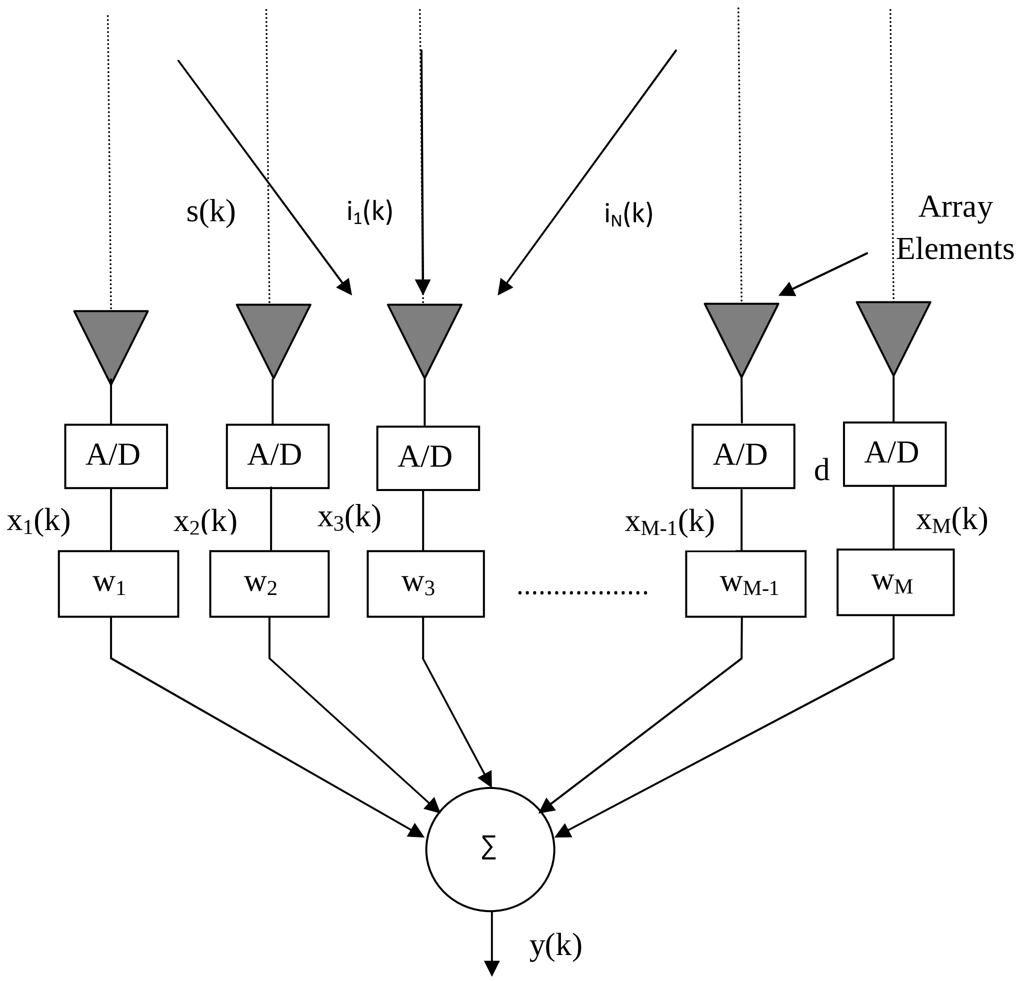


Fig. 3.2 Uniform linear antenna array for digital beamforming.

Maximum signal to interference and noise ratio

Maximum signal to interference and noise ratio beamformer is an extension of classical beamformer. This approach maximizes the output signal to interference and noise power ratio which enhance the received signal and minimizes the power corresponding to interfering signal and noise.

The mean array output power for the desired signal is given by:

$$\sigma_d^2 = E\left[\left|\bar{w}^H \bar{d}(k)\right|^2\right] = E[|\bar{w}^H \bar{a}(\theta_0) s(k)|^2] = S\bar{w}^H \bar{a}(\theta_0) \bar{a}^H(\theta_0) \bar{w} \qquad (3.66)$$

where, $S = E[|s(k)|^2]$ is the mean power of the signal of interest and is considered as unity thus the output power of desired signal is given by:

$$\sigma_d^2 = \bar{w}^H \bar{a}(\theta_0) \bar{a}^H(\theta_0) \bar{w} \qquad (3.67)$$

The mean array output power of the undesired signal is given by:

$$\bar{\sigma}_u^2 = E[|\bar{w}^H \bar{u}(k)|^2] = E[|\bar{w}^H [\bar{A} * \bar{\imath}(k) + \bar{n}(k)]|^2],$$

$$= \bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \bar{w}^H \bar{R}_{nn} \bar{w} \qquad (3.68)$$

where,

$\bar{R}_{ii} = E[\bar{\imath}(k) \bar{\imath}^H(k)]$ is the interference correlation matrix,

$\bar{R}_{nn} = E[\bar{n}(k) \bar{n}^H(k)] = \sigma^2 * I$ is the noise correlation matrix.

Thus, using the above expressions of noise correlation matrix in eq. (3.68) we obtain the following expression:

$$\sigma_u^2 = \bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w} \qquad (3.69)$$

Using the expressions of mean array output power for desired and undesired signal as obtained in eq. (3.67, 3.69) we obtain the following expression for SINR:

$$SINR = \frac{\sigma_d^2}{\sigma_u^2} = \frac{\bar{w}^H \bar{a}(\theta_0) \bar{a}^H(\theta_0) \bar{w}}{\bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w}} \qquad (3.70)$$

The SINR can be maximized by using an optimization technique. Since classical gradient based optimization methods have a tendency to stuck in local minima and have a slow rate of convergence, thus these techniques fail to provide a optimum beamforming solution. Heuristic approaches have evolved as one of the leading techniques for solving optimization problems in engineering applications. These approaches can provide optimum weights for maximizing the SINR and can efficiently direct the main lobe towards desired user and form pattern nulls in the direction of interfering signals.

Maximum signal to interference and noise ratio with reduced side lobe level

The approach discussed in the previous section can maximize SINR but simultaneously can't minimize the SLL, which is an essential requirement to avoid unreasonable spread of radiated power. The dual problem of maximizing SINR with reduced SLL can't be solved by conventional beamforming techniques. This multi-objective problem can be modeled as an optimization problem and can be solved using heuristic approaches. The optimization problem can be considered as an minimization problem and the fitness function used in the optimization problem is the combination of inverse of SINR and SLL.

$$F = \gamma_1 \frac{\bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w}}{\bar{w}^H \bar{a}(\theta_0) \bar{a}^H(\theta_0) \bar{w}} + \gamma_2 SLL \qquad (3.71)$$

where,

$\gamma_1$and $\gamma_2$are constants which balance the minimization of the two terms.

SLL is defined as the ratio of pattern value of the side lobe peak relative to the maximum pattern value in the direction of the main lobe. It can be mathematically expressed in decibel (dB) as:

$$SLL = 20\,\mathrm{LOG}_{10}\left(\frac{AF(SL)}{\max\,(AF)}\right) \qquad (3.72)$$

where, AF(SL) is the value of the array factor at the side lobe location.

Maximum likelihood

The ML approach is based on the assumption that we have an unknown desired signal and that the unwanted signal has a zero mean Gaussian distribution. The goal of this approach is to define a likelihood function which can give us an estimate on the desired signal. The input signal vector is given by:

$$\bar{x} = \bar{a}(\theta_0) * s + \bar{n} = \bar{x}_s + \bar{n} \qquad (3.73)$$

The overall distribution is assumed to be Gaussian but the mean is controlled by the desired signal $\bar{x}_s$. The probability density function can be described as the joint probability density $p(\bar{x}|\bar{x}_s)$. The density function can be viewed as the likelihood function that can be used to estimate the parameter $\bar{x}_s$. The probability density function (PDF) can be described as:

$$p(\bar{x}|\bar{x}_s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-((\bar{x}-\bar{a}(\theta_0)s)^H \bar{R}_{nn}^{-1}(\bar{x}-\bar{a}(\theta_0)s))} \quad (3.74)$$

The optimum array weights are obtained by maximizing the log likelihood function which is given by:

$$L[\bar{x}] = -\ln[p(\bar{x}|\bar{x}_s)] = C(\bar{x} - \bar{a}(\theta_0)s)^H \bar{R}_{nn}^{-1}(\bar{x} - \bar{a}(\theta_0)s) \quad (3.75)$$

where C is the constant. The optimum array weights based on maximizing the ML function is expressed as:

$$\bar{w}_{ML} = \frac{\bar{R}_{nn}^{-1}\bar{a}(\theta_0)}{\bar{a}^H(\theta_0)\bar{R}_{nn}^{-1}\bar{a}(\theta_0)} \quad (3.76)$$

Minimum variance

The minimum variance solution is also called the minimum variance distortionless response (MVDR) or the minimum noise variance performance measure. The term distortionless signifies that the received signal is undistorted after the application of the array weights. It is a robust beamforming technique and its solution is similar to the maximum likelihood solution.

The goal of this technique is to minimize the mean array output power of the undesired signal while maintaining the output power corresponding to the desired signal. Thus, the optimum array weights are derived by minimizing the quantity:

$$\min\{\bar{w}^H \bar{R}_{uu} \bar{w}\}, \qquad \text{while} \quad \bar{w}^H \bar{a}_0 = 1 \quad (3.77)$$

The variance can be minimized by using the method of Lagrange. The optimum array weights based on minimum variance criteria is expressed as:

$$\bar{w}_{mvdr} = \frac{\bar{R}_{uu}^{-1}\bar{a}(\theta_0)}{\bar{a}^H(\theta_0)\bar{R}_{uu}^{-1}\bar{a}(\theta_0)} \quad (3.78)$$

where, $\bar{R}_{uu} = E[\bar{u}(k)\bar{u}^H(k)]$ is the correlation matrix of $\bar{u}(k)$.

The minimum variance solution is identical in form to the maximum likelihood solution. The only difference between the two approaches is that the ML approach requires that all unwanted signals are zero mean and have a Gaussian distribution while the minimum variance approach can include interferers arriving at unwanted angles as well as the noise. This concludes that the minimum variance approach is more general in its application.

Minimum mean square error

This approach of digital beamforming is somewhat different from the previous discussed approaches as it requires the knowledge of the desired signal. In this approach the optimal weights array are obtained by minimizing the mean square error between the reference signal i.e. the desired signal and the outputs of the M dimensional antenna array. The error signal is given by:

$$\epsilon(k) = d(k) - \bar{w}^H \bar{x}(k) \tag{3.79}$$

The minimum mean square error criteria is mathematically expressed as:

$$\min_w E\{|e(k)|^2\} \tag{3.80}$$

The function $|e(k)|^2$ called the cost function or objective function forms the quadratic surface in M-dimensional space as shown in figure 3.3 and the optimum array weights are obtained by minimizing the cost function with respect to the weight vector $\bar{w}$.

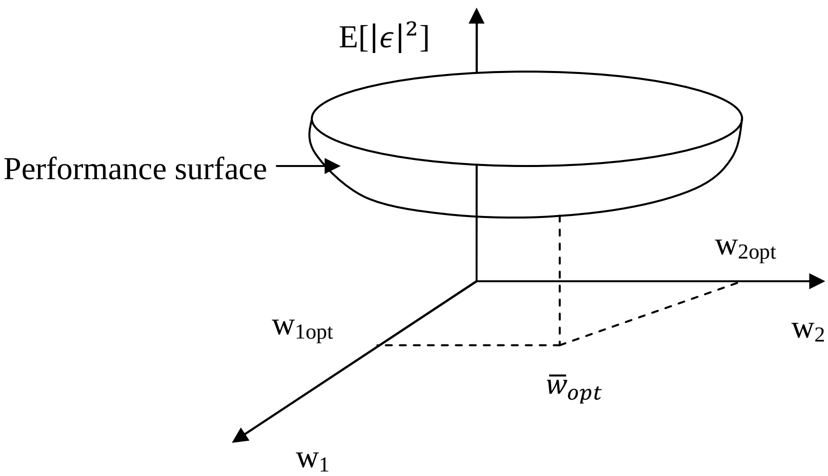


Fig. 3.3 Quadratic surface for mean square error.

The optimal minimum mean square error weight solution which is also referred to as Wiener-Hopf solution is given by:

$$\bar{w}_{MSE} = \bar{R}_{xx}^{-1} * \bar{r} \tag{3.81}$$

where, $\bar{R}_{xx} = E[\bar{x} * \bar{x}^H]$ is the array correlation matrix and $\bar{r} = E[\bar{x} * \bar{d}^*]$ is the signal correlation vector.

3.2.2 Adaptive beamforming algorithm

In the field of wireless cellular communication system, the signal statistics changes with time as the desired user and the interfering user moves around the cell. The receiver signal processing algorithms then must allow for the continuous adaptation to the ever changing signal environment which results in optimal transmission and reception of the desired signal. The ABF algorithms based on certain optimization criteria takes the fixed beamforming approaches one step further and recursively updates the array weights as shown in figure 3.4 to adapt to the changing signal environment. In the following section we will discuss different ABF algorithms.

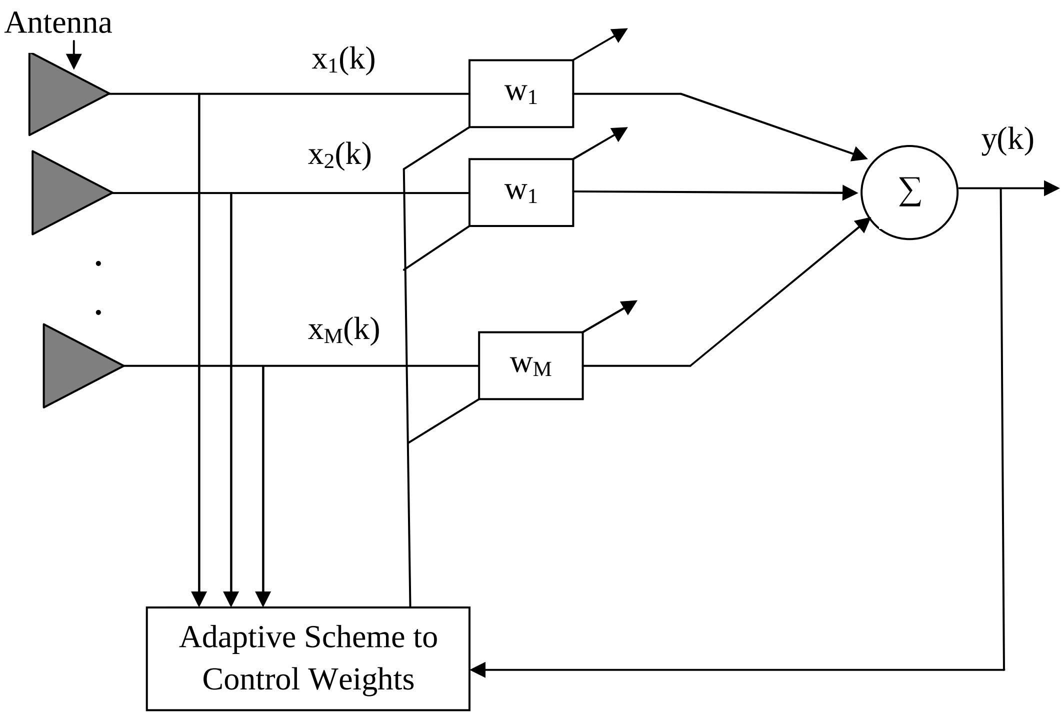


Fig. 3.4 Adaptive beamforming network.

Least mean square

The Least mean square algorithm, introduced by Widrow and Hoff **(Widrow, *et al.*, 1967)** in 1959 is a member of a family of stochastic gradient algorithm. This is the most widely used adaptive filtering algorithm because of its low complexity and proven robustness. The algorithm is based on the method of steepest descent and defines cost function as mean square error. It employs an iterative technique to make successive corrections to the weight vector in the direction of the negative of the gradient of cost function. Let us suppose $\bar{x}$(k) is the input vector, $\overline{w}$(k) is the weight vector and d(k), ε(k) are the desired signal and the error signal for the k-th time sample.

<u>Algorithm</u>

Step1. Collect data from the ULA and initialize the array weights $\overline{w}(0)$.

Step2. Estimate the error between desired and output signal as obtained in eq. (3.79),

$$e(k) = d(k) - \overline{w}^H(k)\bar{x}(k). \tag{3.82}$$

Step3. Estimate the gradient of the cost function,

$$\nabla_{\overline{w}}\big(J(\overline{w}(k))\big) = 2\hat{R}_{xx}(k)\,\overline{w}(k) - 2\hat{r}(\text{k}), \tag{3.83}$$

where, $$\hat{R}_{xx}(k) \approx \bar{x}(k)\bar{x}^H(k), \tag{3.84}$$

$$\hat{r} \approx d^*(k)\bar{x}(k) \tag{3.85}$$

are the instantaneous estimates of the array correlation matrix and signal correlation vector.

Step4. The steepest descent iteration approximation is given as:

$$\overline{w}(k+1) = \overline{w}(k) - \frac{1}{2}\mu\nabla_{\overline{w}}\Big(J\big(\overline{w}(k)\big)\Big),$$

$$= \overline{w}(k) - \mu[\bar{R}_{xx}\overline{w}(\text{k}) - \bar{r}(\text{k})],$$

$$= \overline{w}(k) + \mu\varepsilon^*(k)\bar{x}(k). \tag{3.86}$$

where μ is the step size which controls the rate of convergence and stability of the algorithm.

Step5. Repeat steps 2 to 4 for k number of samples.

In order to guarantee stability in the mean-squared sense, the step size is restricted in the interval:

$$0 \leq \mu \leq \frac{1}{2trace\,[\hat{R}_{xx}]}. \tag{3.87}$$

The algorithm shows limited performance in case of fast fading environment because it is difficult to have knowledge of the desired signal. A normalized version of the LMS algorithm, the NLMS algorithm is used to overcome the dynamic range problem and it substantially reduces the convergence time. The convergence rate and the required step size are sensitive to the signal power in the look direction. Godara and Gray proposed an structured gradient LMS algorithm which reduces the signal sensitivity by utilizing the Toeplitz structure of the correlation matrix. Many other variants of LMS algorithm have been proposed for its real time implementation in adaptive signal processing task.

Sample matrix inversion

This algorithm was developed by Reed, Mallet and Brennen in 1974 and is also known as direct matrix inversion (DMI) **(Van Trees, 1968)**. In LMS algorithm the weights are updated with the knowledge of the desired signal but since the signal environment undergoes frequent changes the adaptive algorithm may not allow tracking of the desired signal in a satisfactory manner. SMI algorithm overcomes the slow convergence problems of feedback loop adaptive arrays and is an attractive alternative to the LMS algorithm. It is a block adaptive approach which does not require the prior information of desired signal and estimates the statistics using temporal block of the array data.

Algorithm

Step1. Collect data from the ULA and estimate the array correlation matrix $\hat{R}_{xx}$ and signal correlation vector $\hat{r}$ in a finite interval of length K as:

$$\hat{R}_{xx} = \frac{1}{K}\sum_{k=1}^{K} \bar{x}(k)\bar{x}^{H}(k), \tag{3.88}$$

$$\hat{r} = \frac{1}{K}\sum_{k=1}^{K} d^{*}(k)\bar{x}(k). \tag{3.89}$$

Step2. Estimate the array weight using the optimum wiener solution,

$$\hat{w} = \hat{R}_{xx}^{-1}\hat{r}\ . \tag{3.90}$$

Apart from the high convergence rate of SMI algorithm, it has several drawbacks. The correlation matrix may be ill conditioned resulting in errors when inverted and there is a challenge of inverting large matrices.

An enhanced sample matrix inversion (ESMI) and normalized sample matrix inversion (NSMI) are developed and the results verified that ESMI algorithm is more efficient in mobile communication environment for CDMA system.

Recursive least square

Recursive Least Square algorithm is an adaptive filtering algorithm which approximates the Wiener solution using the method of least squares **(Liberti and Rappaport, 1999)**. The algorithm recursively updates the weight vector to minimize the cost function that consists of the weighted sum of error squares over a time window and is given as:

$$\mathrm{J(w)} = \sum_{i=1}^{k} \alpha^{k-i}|e(i)|^2, \tag{3.91}$$

where α lies between $0 \leq \alpha \leq 1$ and is a exponential scaling factor which determines how quickly the previous data are de-emphasized and is referred to as the forgetting factor. Differentiating the cost function and solving for the minimum yields:

$$\left[\sum_{i=1}^{k} \alpha^{k-i}x(i)x^H(i)\right]w(k) = \sum_{i=1}^{k} \alpha^{k-i}x(i)d^*(i). \tag{3.92}$$

Equation (3.92) approximates the wiener solution and defines the array correlation matrix and signal correlation vector as:

$$\hat{R}_{xx}(k) = \sum_{i=1}^{k} \alpha^{k-i}\bar{x}(i)\bar{x}^H(i), \tag{3.93}$$

$$\hat{r}(k) = \sum_{i=1}^{k} \alpha^{k-i}d^*(i)\bar{x}(i). \tag{3.94}$$

Let us break up the summation in the last two equations into two terms i.e. the summation for values up to i = k – 1 and last term for i = k.

$$\hat{R}_{xx}(k) = \alpha \sum_{i=1}^{k-1} \alpha^{k-1-i}\bar{x}(i)\bar{x}^H(i) + \bar{x}(k)\bar{x}^H(k),$$

$$= \alpha\hat{R}_{xx}(k-1) + \bar{x}(k)\bar{x}^H(k). \qquad (3.95)$$

$$\hat{r}(k) = \alpha \sum_{i=1}^{k-1} \alpha^{k-1-i} d^*(i)\bar{x}(i) + d^*(k)\bar{x}(k),$$

$$= \alpha\hat{r}(k-1) + d^*(k)\bar{x}(k). \qquad (3.96)$$

To find the optimum wiener solution the knowledge of the inverse of the array correlation matrix is required. The Sherman Morrison-Woodbury (SMW) theorem finds the inverse of the array correlation matrix obtained in equation (3.95) as:

$$\hat{R}_{xx}^{-1}(k) = \alpha^{-1}\hat{R}_{xx}^{-1}(k-1) - \alpha^{-1}\bar{g}(k)\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1), \qquad (3.97)$$

where $\bar{g}(k)$ is the gain vector and defined as:

$$\bar{g}(k) = \frac{\alpha^{-1}\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)}{1+\alpha^{-1}\bar{x}^H(k)\hat{R}_{xx}^{-1}(k-1)\bar{x}(k)}. \qquad (3.98)$$

Algorithm

Step1. Initialize the array weights $\bar{w}(0)$ and take initial estimate of the inverse of the array correlation matrix,

$$R^{-1}(0) = \delta^{-1}I.$$

where δ is the small positive constant and I is the identity matrix.

Step2. Estimate the inverse of the array correlation matrix using equation (3.97).

Step3. Estimate the error vector using equation (3.82).

Step4. Update the array weights,

$$\bar{w}(k+1) = \bar{w}(k) + e^*(k)\bar{g}(k). \qquad (3.99)$$

Step5. Repeat steps 2 to 4 for k number of samples.

The weight update equation for RLS is similar to LMS in which the gradient step size μ is replaced by the gain vector $\bar{g}(k)$ and hence the convergence speed of the algorithm becomes independent of the Eigen value spread. However, the algorithm is more complex. Many researchers proposed different forms of RLS algorithm to overcome this limitation. The performance of the adaptive array deteriorates when pointing and calibration errors occur, many techniques are developed to alleviate the effect of pointing errors.

Constant modulus algorithm

The Constant Modulus algorithm was first introduced by Godard for blind equalization of quadrature amplitude modulation (QAM) signals in 1980 and for equalization of pulse amplitude modulation (PAM), frequency modulation (FM) signals by Treichler in 1983. In wireless communication signals are generally frequency and phase modulated like FM, phase shift keying (PSK), frequency shift keying (FSK), and QAM and has a constant envelope and this property is known as constant modulus signal property. However, due to multipath effects, a signal travelling in frequency selective channel loses its constant modulus property. The constant modulus algorithm uses this property to adjust the weights of adaptive array so as to minimize the variation of the desired signal at the array. The CM algorithm minimizes the cost function of the form:

$$J(k) = E[(|y(k)|^p - 1)^q], \qquad (3.100)$$

where p and q are the positive integers and defines the convergence of the algorithm. A particular choice of p and q yields a specific cost function called (p, q) CM cost function.

Algorithm

Step1. Collect data from the ULA and initialize the array weights $\bar{w}(0)$ and choose a particular value of (p, q) in CM cost function. Let's take (1, 2) cost function.

Step2. Find the output vector y(k),

$$y(k) = \bar{w}(k) * \bar{x}(k). \qquad (3.101)$$

Step3. Find the instantaneous estimate of the gradient of the cost function,

$$\nabla_{\bar{w}}\left(J_{1,2}(\bar{w}(k))\right) = \left[y(k) - \frac{y(k)}{|y(k)|}\right]^* \bar{x}(k). \qquad (3.102)$$

Step4. Update the array weights using the method of steepest descent which is given by the following recursive equation,

$$\bar{w}(k+1) = \bar{w}(k) - \mu\nabla_{\bar{w}}(J_{1,2}(\bar{w}(k)),$$

$$\bar{w}(k+1) = \bar{w}(k) - \mu \left[y(k) - \frac{y(k)}{|y(k)|} \right]^* \bar{x}(k),$$

$$\bar{w}(k+1) = \bar{w}(k) + \mu e^*(k)\bar{x}(k). \quad (3.103)$$

Step5. Repeat step 2 to step 4 for k number of samples.

The CM and LMS algorithm are similar to each other, the only difference is that LMS require reference signal while CM algorithm doesn't require reference signal to generate the error signal. The algorithm has a slow convergence time thus not suitable for dynamic environment where the signal conditions are rapidly changing. LS-CMA algorithm based on method of non-linear least square converges 100 times faster than the conventional CM algorithm.

3.2.3 Adaptive beamforming based on radial basis function neural network

The performance of ABF algorithm discussed in previous section becomes worse in practical problems because of violation of underlying assumptions on the environment, sources and array and this may cause a mismatch between the assumed array response and true array response. Many approaches have been developed such as linear constrained minimum variance (LCMV) and diagonal loading to improve the robustness of the ABF algorithms. But these approaches fail to provide sufficient improvements. Neural network approach overcomes this problem by computing the weights of the adaptive array as a mapping problem. The approximation capability of the RBFNN by learning the optimum wiener beamforming solution and using the trained network to replace wiener beamformer for real time tracking have been developed. The algorithm is well described by El Zooghby **(El Zooghby, *et al.*, 1998)**.

The array output vector $\bar{x}(k)$ as obtained in equation (3.2) is given as:

$$\bar{x}(\mathrm{k}) = \bar{A} * \bar{s}(k) + \bar{n}(k).$$

The array correlation matrix derived from the induced signals on each array element is given as:

$$\bar{R}_{xx} = E\{\bar{x}(k)\bar{x}^H(k)\}.$$

The optimum weight vector is obtained by minimizing the array output so that the desired signals are received with specific gain, while the contribution due to noise and interference are minimized.

This yield,

$$\bar{w} = \bar{R}_{xx}{}^{-1} A [A^H \bar{R}_{xx}{}^{-1} A]^{-1} * r. \tag{3.104}$$

where A is the array steering matrix pointing to the desired signals. The term r in above equation is the characteristic parameter that determines if the signals are interfering or desired signals. If there are two desired signals, r = [1 1]. If there is a single desired and a single interfering signal, r = [1 0].

Algorithm

The algorithm is divided into two phases:

Training phase

Step1. Collect data from the ULA and estimate the correlation matrices $\{\bar{R}^n_{xx}; n = 1, 2, 3\ldots, N\}$ for all DOA's ranging from $[-90^o, 90^o]$ and arrange it's upper triangular part in the form of a vector-b.

Step2. The vector-b is normalized by its norm to obtain the normalized z^n vector, as obtained in equation (3.61).

Step3. Estimate the weights of the linear array elements $\{\bar{w}^n; n=1, 2,\ldots,N\}$ corresponding to all DOA's ranging from $[-90^o, 90^o]$ based on equation (3.104).

Step4. Generate input-output pairs of $\{(z^n, \bar{w}^n); n = 1, 2, \ldots\ldots, N\}$ for sources located in angular range of $[-90^o, 90^o]$.

Step5. Employ an appropriate RBFNN to learn all sets of input-output pairs.

Generalization phase

Step1. Collect data from the array and evaluate sample correlation matrix.

Step2. Compute the normalized $\bar{z}$ vector using equation (3.61).

Step3. Present input vector $\bar{z}$ to the trained RBFNN which will estimate the optimum weights for the array output.

In this approach the training is done offline and weights are estimated from the trained network in real time. Thus, it is a faster approach than LMS, SMI, RLS, CM algorithms where the optimization is carried out whenever the direction of desired and interfering signal changes. Table 3.2 summarizes all the discussed ABF techniques in terms of their advantages and disadvantages.

Table 3.2 Summary of ABF algorithms.

Algorithm	Advantages	Disadvantages
Least Mean Square	• Always converges and a simple algorithm.	• Requires reference signal. • Slow rate of convergence for large Eigen value spread.
Sample Matrix Inversion	• Always converges and faster than LMS. • Performance is independent of Eigen value spread.	• Increased computational complexity. • Numerical instability results due to inverting a large correlation matrix.
Recursive Least Square	• Always converges and has the fast rate of convergence.	• Requires initial estimate of $\bar{R}_{xx}$. • Forgetting factor is dependent on the fading rate of the channel.
Constant Modulus	• No reference signal is required. • Efficiently eliminate correlated signals.	• Theoretically may not converge. • Not suitable for CDMA system.
Radial Basis Function Neural Network	• Faster and less complex technique.	• A large amount of diverse data is required in training phase.

3.3 Optimization

Optimization is an act, process, or methodology of making something better. Researchers, scientist, engineers gave several new ideas and optimization deals with variation on the initial concept or idea and based on the information gained it tries to improvise the idea. In other words, it is a mathematical procedure of adjusting the inputs or characteristics of a device, process, or experiment, to find the optimal (i.e. maximum or minimum) solution subject to certain constraints. The inputs consists of variables, the process or function is known as fitness function, cost function or objective function and the output is the fitness or cost. Optimization has widespread applications in the field of Engineering, Science, Mathematics, Economics, Commerce and Management, etc. An optimization problem thus comprises of three ingredients:

- **Variables:** These represents components of the optimization model that can be changed to obtain optimum solution. The design variables can be continuous, discrete and Boolean.
- **Constraints:** These represents limitations on the variables and objective function.
- **Objective function:** It defines the process or the relationship between input and output.

The optimization problem is mathematically defined as:

Given $f: D^N \to R$,

Find $\vec{x}^* = [x_1^*, x_2^*, \dots \dots ., x_N^*] \in D^N = D_1 \cap D_2 \cap \dots \dots . . \cap D_N$

which will satisfy

$$c_i^=(\vec{x}) = 0, 1 \le i \le N_{c^=} \tag{3.105}$$

$$c_i^+(\vec{x}) \ge 0, 1 \le i \le N_{c^+} \tag{3.106}$$

$$c_i^-(\vec{x}) \le 0, 1 \le i \le N_{c^-} \tag{3.107}$$

and will optimize (maximize or minimize) the function $f(\vec{x})$, i.e.

$$\left.\begin{array}{c} f(\vec{x}^*) \le f(\vec{x})(minimization) \\ or \\ f(\vec{x}^*) \ge f(\vec{x})(maximization) \end{array} \quad \forall \vec{x} = [x_1, x_2, \dots x_N \in D^N] \right\} \tag{3.108}$$

where,

N is the number of optimization parameters, or the dimension of the optimization problem.

f is called objective function

D^N is called the search space.

The elements of N-dimensional search space D^N are called feasible solutions. A feasible solution is the N-dimensional vector of optimization variable $\vec{x} = [x_1, x_2, \ldots, x_N]$. A feasible solution $\vec{x}^*$ that minimizes (maximizes) the objective function is called an optimal solution. D_i, either continuous or discrete, is the search space of x_i of the i^{th} optimization parameter and D_i and D_j are not necessarily identical in terms of their type or size. $c_i^=$ is the i^{th} equality constraint function, $N_{c^=}$ is the number of equality constraint functions. c_i^+ is the i^{th} positive constraint function, N_{c^+} is the number of positive constraint functions, c_i^- is the i^{th} negative constraint function, N_{c^-} is the number of negative constraint functions. A choice of values for the set of parameters $\vec{x}$ that satisfy all constraints is called a feasible solution. Feasible solutions $\vec{x}^*$ with objective function value as good as the value of any other feasible solutions are called optimal solutions.

The solution obtained by optimization techniques can be classified on the basis of their quality as

- Global optimal solution
- Local optimal solution.

A global optimal solution gives the lowest output value among all inputs, whereas a local optimal solution gives the lowest output value among all nearby inputs. A global optimal solution $\vec{x}^*$ is the absolutely best set of parameters in entire search space D^N to optimize an objective function *f*. Mathematically for any minimization problem a solution x^* is called global optimal solution if and only if:

$$f(x^*) \leq f(x), \forall\, x \in D^N.$$

Global optimization problems are generally very difficult and are categorized under the class of nonlinear programming (NLP). A true global optimization algorithm will find x^* regardless of the selected starting point $x_0 \in D^N$. On the other hand, local optimal solution x^{**} is best solution in the neighbourhood L of x^{**} to optimize objective function f, L is the proper subset of D^N. Mathematically for any minimization problem a solution x^{**} is called local optimal solution if and only if:

$$f(x^{**}) \leq f(x), \forall\, x \in L \subseteq D^N.$$

Local optimization also depends on an initial point. Generally, a local optimization algorithm should guarantee that it will be able to find local optimal solution x^{**} if starting point $x_0 \in L$. An optimization algorithm that converges to a local optimal solution, regardless of the selected starting point $x_0 \in D^N$, is called a "globally convergent" algorithm.

3.3.1 Categories of optimization

Optimization algorithms can be categorized into six categories. The different categories are not necessarily mutually exclusive, for example a multi-objective optimization algorithm can be either constrained or unconstrained and their variables can be discrete or continuous.

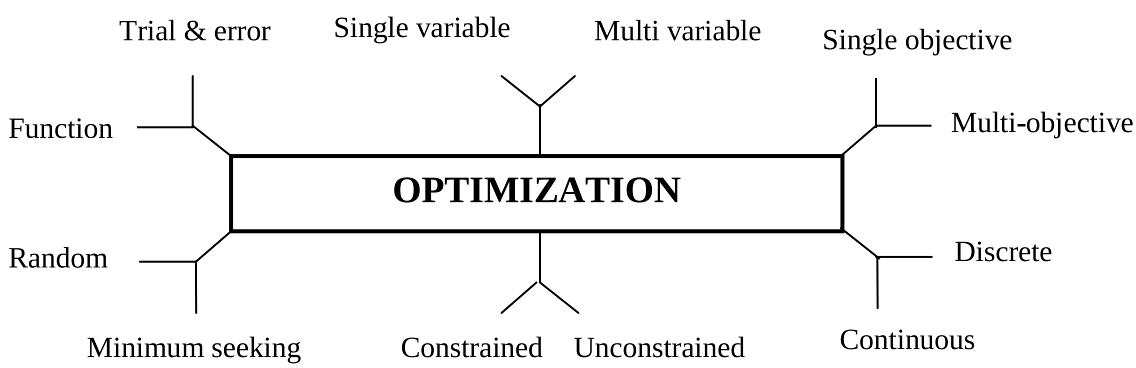


Fig. 3.5 Categories of optimization algorithms.

Category 1

This category of optimization is categorized as trial-and-error optimization and function optimization. In Trial-and-error optimization, the variables that affect the output are adjusted without knowing about the process that produces the output. A simple example that describes this approach is adjustment of television antenna by a lay man who adjust the position for best picture and audio reception while an antenna engineer can better judge and analyze the situation. In contrast, a mathematical formula describes the objective function in function optimization which can be mathematically solved to find the optimal solution.

Category 2

This category of optimization is categorized as one dimensional and multi-dimensional. One dimensional optimization problem has a single variable while multi-dimensional problem has more than one variable. Optimization becomes increasingly difficult as the number of dimension increases.

Category 3

This category of optimization is differentiated on the basis of number of objective of optimization problem. When the optimization problem deals with single objective then it is classified as single objective problem while when it has more than one objective than it is classified as multi-objective optimization problem. The multi-objective optimization problem are common in engineering applications, due to multi-carrier nature of most of the real world problems. In this thesis, multi-objective problem has been formulated in which side lobes are decreased in addition to increasing the SINR in ABF.

Category 4

This category of optimization is distinguished on the basis of permissible values of the design variables. The variables can take either discrete or continuous values. If the design variables are restricted to take on only finite number of discrete (integer) values, then the problem is called as integer programming problem. On the other hand if the design variables are permitted to take infinite number of real values, the optimization problem is called a real valued programming problem. Discrete optimization problem is also known as combinatorial optimization problem, since the optimum solution consists of a certain combination of variables from the finite pool of all possible variables.

Category 5

This category of optimization problem is classified as constrained or unconstrained optimization problem. Constrained optimization problem takes into account variables equalities and inequalities into the cost function while unconstrained optimization allows the variables to take on any value.

Category 6

This category is distinguished on the basis of the mode of operation of optimization algorithms. Some set of algorithms try to minimize the cost in a sequence of steps by starting from an initial set of variables values. These algorithms are the traditional optimization algorithms and are based on calculus methods. On the other hand random methods use some probabilistic calculations to find variable values. The minimum seeking algorithms are fast but can stuck in local minima while random algorithms are slower but have greater success in reaching out global minimum.

3.3.2 Classification of optimization techniques

There are different optimization techniques available in literature which can be selected based on the type of optimization problem. In the recent past, there has been an major advancement in optimization due to the progress of fast digital circuit technology. The optimization techniques are classified as classical optimization techniques and metaheuristic techniques which can be further divided into sub-classes as shown in figure 3.6.

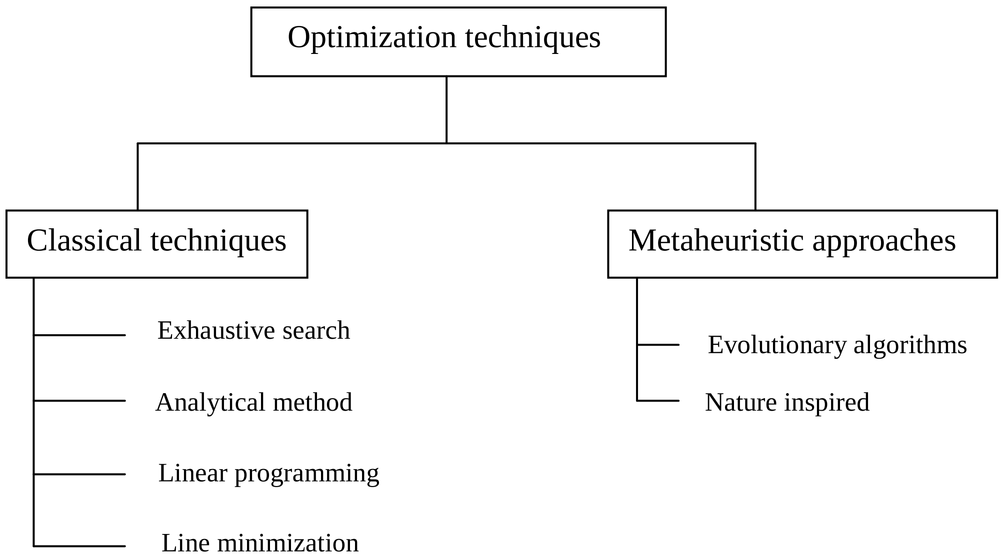


Fig. 3.6 Classification of optimization techniques.

Classical optimization techniques

The classical optimization techniques are useful for finding optimal solution of single and multi-dimensional problems. The brute force approach is an exhaustive approach where the algorithm tries every solution in the search so that the global optimum solution is found. In this approach, the search space is divided into sampled values and the function is evaluated at every point and then the list is searched for the optimum value. This is a slow approach and global optimum solution may be missed due to under sampling. However, with the increase of problem size only the small instances could be solved by this approach. Thus, this approach is not suitable for high dimension problems.

Another classical approach is the gradient method which uses the knowledge of derivatives to locate optimum point. The first derivative gives the information of the slope of the function being differentiated and at optima it become zero. In literature many numerical approaches are based on the gradient method. In these approaches the algorithm starts at picking a random point in the search space and gradient of the cost function is calculated and then it is headed downhill to the bottom. This approach has certain limitations as it can stuck in local minimum and there are certain functions which have no analytical gradients.

Later before and during World War II, Kantorovich, von Neumann and Leontief solved linear problems in the fields of transportation, game theory and input-output models **(Haupt, 2004)**. Linear programming concerns the minimization of linear mathematical function subject to constraints which are linear equations and equalities. Simplex method efficiently provide optimal solution for linear programming problems. In simplex method an external variable is added to convert inequality constraint into equality constraint. An extension of linear programming is an integer programming in which variables can take integer values. In 1951 Kuhn and Tucker popularized non-linear techniques in which Lagrange multiplier were used to constraints defined by equalities and inequalities which enables the solution of large category of non-linear and non-deterministic polynomial time (NP) hard problems.

The largest category of optimization methods fall under the domain of successive line minimization methods. In this approach initially a random point is chosen on the cost surface and a particular direction is chosen to move and it will be continued until the cost function begins to increase. A particular direction which is initially chosen is critical to algorithm convergence and has spawned a variety of approaches. Coordinate search method is a very simple approach to line minimization.

In real life applications, some of the optimization problems can be solved in reasonable time using classical optimization techniques. However, most of the problems will be too hard and it become difficult to solve these problems using classical techniques. Thus, classical techniques involves more computational effort and thus approximate method are used to find optimal solution in a reasonable amount of time. Metaheuristic approaches are approximate methods and are used to solve NP hard problems.

Metaheuristic approaches

The word "heuristic" is Greek which means "to know", "to find", "to discover" or "to guide an investigation". Heuristic are techniques which find near-optimal solution at a reasonable computational cost without able to guarantee either feasibility or optimality or even in many cases to state how close to optimality a particular solution can be found. These approaches solve difficult NP hard problems for which classical approaches failed to find optimal solution in a reasonable amount of time.

Heuristic algorithms are very specific and problem dependent means these approaches require knowledge about the structure of specific problems being undertaken to devise a strategy for solving it. However, there are some problems in which one have no idea of how the problem can be solved or one can't devise any strategy for solving the problem. In these cases it is best to use more general heuristics, often called metaheuristics. These approaches are problem independent algorithms. Metaheuristic are sometimes also called black box optimization algorithms or general purpose optimization algorithms.

Metaheuristic algorithms are population based algorithms which mimic physical or biological processes. Some of the most common of these algorithms are GA, SA, artificial immune system, ACO, PSO, bacteria foraging algorithm, GSA, DE algorithm. All of these algorithms have stochastic behaviour. However, Formato has proposed a deterministic heuristic search algorithm which is based on the metaphor of gravitational kinematics that is called central force optimization (CFO). In simulated annealing search starts from a single initial point while most of the metaheuristic approaches search in a parallel manner with multiple initial points like swarm based algorithms.

Metaheuristic approaches works with two key aspects i.e. exploration and exploitation. The exploration defines expanding the search space while exploitation is the ability of finding the optima around a good solution. In the initial iterations, the algorithm explores the search space means it uses exploration while by lapse of iterations exploration fades out and exploitation fades in, means the algorithm is reaching to the optimal solution. Different algorithms of optimization use different approaches and operators to employ exploration and exploitation. An algorithm to work successfully there should be a suitable tradeoff between exploration and exploitation. The principle behind this nature inspired metaheuristic approaches to realize the concept of exploration and exploitation is self-adaptation, cooperation and competition. Self-adaptation means each member of population improves its performance, cooperation means members cooperate by transferring information among each other while competition means members compete to survive. In the following section we will introduce some of the well known metaheuristic approaches.

Evolutionary algorithms

Evolution is an optimization process where the aim is to improve the ability of an organism to survive in dynamically changing and competitive environment (Engelbrecht). Evolutionary algorithms simulate the evolution of individuals via process of selection, reproduction and mutation. This evolution is guided by the fitness of individuals. Organisms have certain characteristics that influence their ability to survive and to reproduce. The characteristics of an individual are represented by a chromosome also referred to as genome which can be divided into two classes of evolutionary information i.e. genotype and phenotypes. A genotype describes the genetic composition of an individual as inherited from their parents and phenotype defines what an individual looks like. Evolutionary algorithms works in the following manner:

Step1. Randomly initialize a population of individuals where each individual represents a potential solution to the problem within a variable constraint range.

Step2. Evaluate the fitness of each individual in the current population.

Step3. Select the individuals having higher fitness and discard the bad ones in each iteration.

Step4. Individuals are altered through a process of crossover and mutation.

Step5. This procedure is repeated until the convergence criteria is reached.

The evolutionary algorithm have three main evolutionary operators:

- **Selection:** This is an important operator in the evolutionary approach and relates to the Darwinian principle of the survival of the fittest. This approach basically selects best individual on th basis of their fitness and eliminates the rest. This set of individual will reproduce new individuals such that the population size is constant. Tournament selection, roulette-wheel selection, ranking selection are some of the commonly used selection operators.
- **Crossover:** This operator creates new individuals through the exchange of genetic material randomly selected from two or more individuals. This process basically explore new areas in the search space. Single-point crossover, multi-point crossover and uniform crossover are some of the common crossover techniques.
- **Mutation:** This operator modifies an individual by creating a small random change to generate a new individual. The value of the gene is randomly changed with low probability. This process does not permit the algorithm to get stuck at a local minimum.

There are many evolutionary algorithm available in literature and some major algorithms are discussed in the following section.

Genetic algorithm

Genetic algorithm is the most commonly used optimization technique in the field of AI which is inspired by biological system and improves fitness through evolution. The algorithm was first proposed by Professor John Holland in the university of Michigan around 1975. The population of individuals in GA are chromosomes and each chromosomes consists of a string of cells called genes. The evolution starts from a randomly generated individual (chromosomes) and in successive iterations fitness of each individual is evaluated against an objective function. The individual having the highest fitness are stochastically selected (selection) and the genes are modified through recombination (crossover) and randomly mutated (mutation) to form individual for next generation. In successive generation the fitness of the individual improves and the algorithm terminates when either the maximum number of generations has been produced or satisfactory fitness has been reached for the population. The crossover is a natural process and traditionally given a rate of 0.6 to 1.0 while the rate of mutation is usually less than 0.1. The genetic algorithm find wide range of applications in diverse fields such as pattern recognition, image processing, machine learning, etc.

Differential evolution

Differential evolution is a stochastic, population based global optimization technique developed by Storn and Price in 1996. The algorithm was initially developed to optimize real parameters and real valued functions. The distance and direction information from the current population is used to guide the search process. The operation of this evolutionary technique is similar to the other evolutionary technique, since it incorporates the selection, crossover and mutation operators while generating the new offspring but the order of occurrence of operators are different. In this technique the mutation is applied first and a donor vectors is generated which is then used within the crossover operator to produce trial vectors. The trial and target vectors are compared based on their fitness and best vector is selected for next generation. The mutation step size is not sampled from a known probability distribution function as was done in other evolutionary techniques.

There is no proof of this technique to converge but Storn and Price shows that this technique performs better than other evolutionary techniques. The technique finds applications in design of digital filters, beamforming applications and other various fields.

Ant colony optimization

Ant colony optimization is a subset of swarm intelligence techniques in which collective intelligence emerges in decentralized and self-organized system with simple individuals to solve difficult engineering problems. The algorithm was inspired by the behaviour of ant species and was developed by M. Dorigo in 1992. He modeled the algorithm based on the behaviour of ant that have the ability of finding the shortest path between their next and the source of food. Ant have very limited visual and vocal perspective abilities and they communicate among each other through an indirect form of communication by modifying the environment.

Ant initially explores the area surrounding the nest in a random manner and they deposit same amount of pheromone (chemical) in all the direction. When an ant finds the food, they carry the food and return to the nest. During the return trip they deposit pheromone trial on the ground. The other ants sense the existence of higher concentration of pheromone and follow this path. As a result, shorter path accumulates more pheromone and eventually all the ants converge to the shortest path. This concept was utilized by M. Dorigo to solve optimization problems by simulating artificial ants searching the solution space similar to the real ants searching their environment.

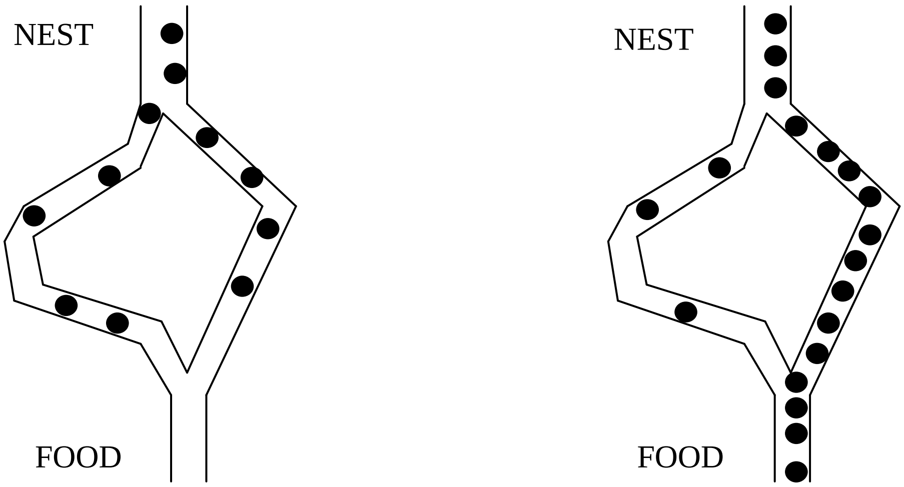


Fig. 3.7 The behaviour of real ant movements.

The algorithm is described on the following steps:

Step1. Initialize the population of agents.

Step2. Modeled the artificial ways representing possible solution for ants in the colony.

Step3. Length of all possible ways corresponding to the cost value of solution is calculated.

Step4. Depending on the cost value, the amount of artificial pheromone is attached to all possible ways.

Step5. Total pheromone of all the ways are modified and the shortest way is found representing the best solution and is stored in the memory.

Step6. All the steps are repeated until the convergence criteria is satisfied.

Simulated annealing

Simulated annealing is a probabilistic optimization technique which finds optimal solution for discrete optimization problems as well as exceedingly complex problems. The algorithm was proposed by Kirkpatrick, Gelett and Vecchi in 1983 to find global optimum solution for the problem that contains several local minima. The algorithm mimics the annealing process in metallurgy. Annealing is a process in metallurgy where metals are slowly cooled to reach a state of low energy state. By analogy with this physical process, each step of the SA algorithm replaces the current solution by a random neighbourhood solution, chosen with a probability that depends on the difference between the corresponding function values and on a parameter c_k. The control parameter c_k is gradually decreased with the iteration *k* of the simulated annealing algorithm. The current solution changes randomly when c_k is large which saves the algorithm to stuck in local minima and with the decrease in temperature the algorithm finds the global optimum. Analogous to simulated annealing achieving a global optimum is a kind of reaching the minimum energy state in the end.

Particle swarm optimization

Particle swarm optimization is a population based stochastic optimization technique which was proposed by Kennedy and Eberhart in 1995. The technique is inspired by the social behaviour of animals, such as bird flocking or fish schooling. It is a robust optimization technique which is based on the movement and intelligence of swarms searching for food. Typically, a flock of these animals having no leaders find their food by randomly moving in the search space and they follow one of the members in the group that has closest position with the food source. This will happen until all the members discover the food source. The process of PSO algorithm in finding optimal solution follows the food searching behaviour of such animals. The algorithm uses a number of particles (same as the chromosomes of GA) that constitute a swarm moving around in the search space looking for the best solution. Each particle in search space adjusts its “flying” according to its own flying experience as well as the flying experience of other particles. In this algorithm, the population of particles are updated by adding an operator based on the fitness information obtained from the environment which enables the individuals to move towards the better solution. The position and velocity of the population are updated using the following expressions:

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1), \tag{3.109}$$

$$v_i^d(t+1) = w(t)v_i^d(t) + c_1 r_{i1}\left(pbest_i^d(t) - x_i^d(t)\right) + c_2 r_{i2}\left(gbest^d(t) - x_i^d(t)\right). \tag{3.110}$$

where, x_i^d and v_i^d represents the position and velocity of the i^{th} particle in the d^{th} dimension of search space, *t* is the iteration number and r_{i1}, r_{i2} are two random variables lying in the interval [0,1]. These variables contribute in the stochastic nature of the algorithm.

The variables c_1 and c_2 are cognitive and social coefficient also known as constriction coefficients that weights the stochastic terms r_{i1}, r_{i2} so as to pulls each particles towards *pbest* and *gbest* position, *w* represents the inertia weight which controls the rate of convergence of the algorithm. $pbest_i$ represents the best position of the i^{th} particle and *gbest* represents the best position among all the particles in the population respectively.

Thus, the position of the particles are modified in each iteration and in each dimension using eq. (3.109, 3.110) until the termination criteria is met.

The general principle of the algorithm is shown in the figure 3.8 and is explained in steps which are as follows:

Step 1. Initialize swarm (a set of particles) in the search space. The position and velocity of the particles are randomly initialized as:

$$X_i = \left(x_i^1, \ldots x_i^d, \ldots\ldots x_i^n\right) fori = 1,2, \ldots\ldots N. \qquad (3.111)$$

$$V_i = \left(v_i^1, \ldots v_i^d, \ldots\ldots v_i^n\right) for\ i = 1,2, \ldots\ldots N. \qquad (3.112)$$

where *N* represents the space dimension.

Step 2. Compute the fitness (cost) of all the particles in each iteration and for a minimization problem the best fitness is the minimum value of the function.

Step 3. Compute the *pbest* and *gbest* value corresponding to all the particles in the population.

Step 4. Update the velocity and position of all particles in each dimension using eq. (3.109, 3.110).

Step 5. Update *pbest* and *gbest* if the current fitness value is smaller than the previous fitness value.

Step 6. Go to step 2 until the convergence criteria is met. The algorithm terminates when the maximum number of iterations or the minimum error criteria is attained.

Researchers have proposed several variants of PSO algorithm to improve speed of convergence and quality of the solution. They modify different parameters of the algorithm to get good results. Velocity of the particles is clamped to a maximum velocity V_{max} to prevent abrupt variation in the algorithm. If the velocity is too high, particles might fly past good solutions or if it is too small then particles may not explore beyond good solutions. Inertia weight *w* is controlled between certain limits usually it is decreased from 0.9 to 0.4 to maintain a balance between exploration and exploitation. Another important parameter is constriction coefficients c_1, c_2 which controls the rate of convergence of the algorithm. A constriction model describes a way of choosing the values of w, c_1, c_2 so that the convergence is ensured.

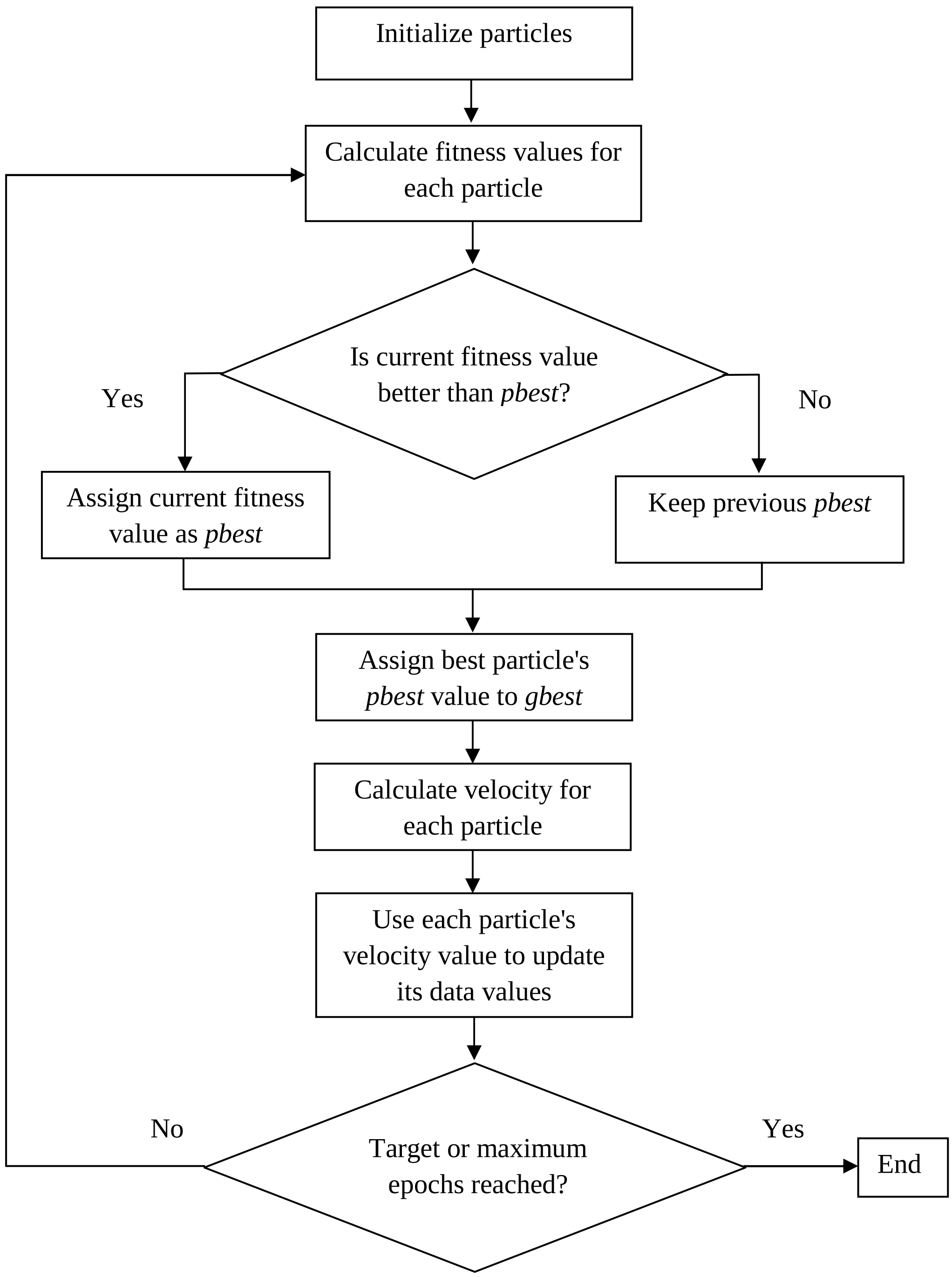


Fig. 3.8 General principle of PSO.

The algorithm is successfully used for solving various engineering optimization problems. In this thesis, we have used PSO in estimating DOA of narrow band deterministic and stochastic sources. PSO gives optimum results in estimating direction in multipath channel environment.

Gravitational search algorithm

Gravitational search algorithm is a novel metaheuristic optimization method proposed by Esmat Rashedi, Hossein Nezamabadi-pour, Saeid Saryazdi in 2009. The algorithm is constructed based on the law of gravity and mass interaction. The algorithm is inspired from the physical phenomena of Newton's which states that "Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them".

In this algorithm agents are considered as objects which have masses proportional to the value of their fitness function. All these objects attract each other with the gravitational force, which causes the global movement of all the objects towards the objects with heavier mass. All the agents in GSA are characterized by four parameters: position of the mass, inertial mass, active gravitational mass and passive gravitational mass. The position of the mass corresponds to a solution of the problem and the gravitational mass and inertial mass controls the velocity of the agent in a specified dimension. The algorithm is mathematically modeled as:

Step 1. Initialize agents in the search space. The position and velocity of the agents are randomly initialized as in eq. (3.111, 3.112) as:

$$X_i = \left(x_i^1, \dots x_i^d, \dots\dots x_i^n\right) for\ i = 1,2, \dots\dots N.$$

$$V_i = \left(v_i^1, \dots v_i^d, \dots\dots v_i^n\right) \quad for\ i = 1,2, \dots\dots N.$$

where, x_i^d and v_i^d represents the position and velocity of i^{th} agent in the d^{th} dimension and *N* represents the space dimension.

Step 2. Compute the fitness of all the agents in each iteration and for a minimization problem best and worst fitness are defined as:

$$best(t) = \min_{j\in\{1,\dots\dots,K\}} \quad fit_j(t) \tag{3.113}$$

$$worst(t) = \max_{j\epsilon\{1,.....,k\}} \quad fit_j(t) \tag{3.114}$$

where, $fit_j(t)$ represents the fitness value of the j^{th} agent at time *t* and *best(t)* and *worst(t)* represents the best and worst fitness value at iteration *t*.

Step 3. Evaluate the gravitational constant *G* at iteration *t* using the following equation:

$$G(t) = G_0 exp\ (-alpha * iter/maxit) \tag{3.115}$$

where, *alpha* and G_0 are the gradient constant and initial value of gravitational constant respectively, *iter* is the current iteration and *maxit* is the maximum number of iterations.

Step 4. Determine and update the mass of the agents in each iteration by the following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i, i = 1 \dots \dots . N \tag{3.116}$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)}, \tag{3.117}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)}, \tag{3.118}$$

where, M_{ai} is the active gravitational mass of i^{th} agent, M_{pi} is the passive gravitational mass of i^{th} agent and M_{ii} is the inertia mass of the i^{th} agent.

Step 5. Evaluate the force acting on agent '*i*' from agent '*j*' at d^{th} dimension and t^{th} iteration as:

$$F_{ij}^{d}(t) = G(t) \frac{M_{pi}(t) * M_{aj}(t)}{R_{ij}(t) + \epsilon} (x_j^d(t) - x_i^d(t)), \tag{3.119}$$

where, $R_{ij}(t)$ is the Euclidean distance between the two agents '*i*' and '*j*' at each iteration *t* and ϵ is a zero offset constant.

The total force that acts on agent '*i*' in a dimension *d* is calculated as:

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_j F_{ij}^d(t), \tag{3.120}$$

where, $rand_j$ is a random number in the interval [0,1].

Step 6. Determine the acceleration of the i^{th} agents at iteration t as:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{3.121}$$

Step 7. Evaluate the velocity and position at next iteration *(t+1)* using the following equations:

$$v_i^d(t+1) = \text{rand}_i v_i^d(t) + a_i^d(t), \tag{3.122}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1). \tag{3.123}$$

Step 8. Go to step 2 until the termination criteria is met. The algorithm terminates when the maximum number of iterations or the minimum error criteria is attained.

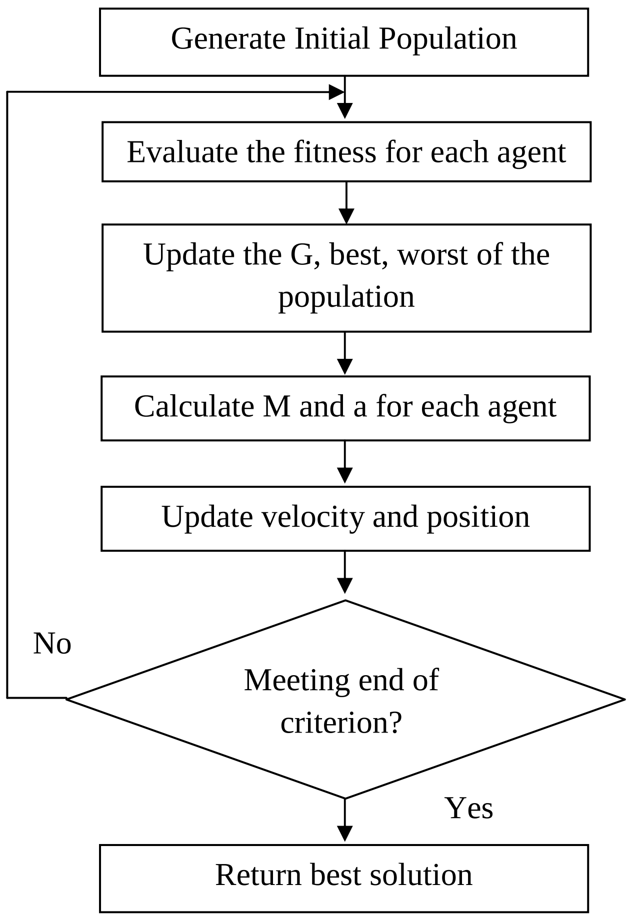


Fig. 3.9 General principle of GSA.

Although, GSA has been developed recently there has been a promising growth in the areas where GSA has been utilized to solve complex optimization problems. There are various variants of GSA that has been developed and finds applications in various diverse fields such as training of neural network, mechanical and power engineering applications, robotics, image processing, networking and communication. We have used GSA in this thesis for estimating direction of narrowband deterministic signals and in ABF by solving multi-objective optimization problem in which narrow beams are formed in the direction of desired signals and pattern nulls are formed in the direction of interfering signals.

Table 3.3 Difference between PSO and GSA.

Properties	**PSO**	**GSA**
Basic analogy of the algorithm	Inspired from the social behaviour of birds and fishes.	Inspired from the physical phenomena.
Agent movement strategy	Direction of movement is calculated based on two best positions $pbest_i$ and *gbest.*	Direction of movement is calculated based on the overall force obtained by all other agents.
Consideration of fitness function	Fitness value is not considered in the updating procedure.	Force between agents is proportional to fitness value thus it is considered in the updating procedure.
Use of memory	The algorithm uses the memory to store *pbest* and *gbest.*	Memory -less system.
Distance between the agents	The positions of agents are updated without considering the distance between solutions.	Force is reversely proportional to distance thus distance between agents are considered.

3.4 Brief introduction of MATLAB

MATLAB is an acronym which stands for MATrix LABoratory. It is a fourth generation high level programming language optimized to perform engineering and scientific calculations. MATLAB first version was written by a numerical analyst Cleve Moler in 1970 to provide easy access to matrix software developed by the LINPACK (linear system package) and EISPACK (Eigen system package) projects. Since 1984, the software package is commercially available by MathWorks and is used globally as a standard tool in universities and industries. The software was originally designed to perform matrix mathematics for example to solve linear algebra problems using matrices but later it has developed into a flexible computing system to solve any technical problem.

MATLAB is an interactive software whose basic data element is a matrix, thus if array based data have to be manipulated it provides fast and accurate results as compare to other software like C, C++, JAVA, FORTRAN. The software provides numerous built-in functions for a wide variety of computations that helps in solving complex mathematical problems, generating graphs and performing numerical techniques which require large number of iterations. Specific research disciplines are collected in packages referred to as toolboxes. There are several toolboxes which includes toolbox for signal and image processing, data analysis, optimization, artificial neural network, solution of partial differential equations, control system, curve fitting, communication and various other diverse fields of science and technology.

3.4.1 Basic features of MATLAB

- It is a special purpose computer program for numerical computation, visualization and application development.
- It is easy to use as the programs are easily written and modified with the built-in integrated development environment.
- It is a platform independent tool as the language is supported on all versions of Windows, Linux, Unix and Macintosh. Programs written on any platform can be run on any other platform without any modifications.
- It provides extensive library of mathematical functions for linear algebra, numerical differentiation and integration, solution of ordinary differential equations, statistics, Fourier transform and design of filters in signal processing.

- It has many integral plotting and imaging commands which can be displayed on any other graphical output device supported by the computer on which the software is installed. Thus the technical data can be efficiently visualized in the software.
- It provide tools through which a programmer can construct a graphical user interface (GUI) for their programs developed for certain applications.
- It provides functions through which MATLAB based algorithms can be integrated with external applications and languages for example: C, C++, Java, .Net, Microsoft Excel.

3.4.2 MATLAB environment

The fundamental unit of data in an MATLAB program is an array where the data is organized in rows and columns. The software displays different types of windows that accept commands and display results. The three most important types of windows are:

- Command window: Commands are entered.
- Figure window: It displays plots and graphs.
- Edit window: Programs are created and modified.
- Command history window: It displays the list of commands of command window.
- Workspace: It temporarily stores all the variables and arrays when a particular command, M-file or function is executed.

3.4.3 Applications of MATLAB

The algorithm finds wide range of applications in various fields of science and engineering. The fields where the algorithms has been widely used are as follows:

- Signal Processing and Communications
- Digital image and video processing
- Fuzzy Logic and artificial neural network
- Control system and optimization

3.5 Methodology for Deterministic maximum likelihood DOA estimation using GSA

In this section, we will describe the formulation of the GSA and PSO algorithm for DML optimization to estimate the direction of incoming signals. The methodology of DOA estimation for deterministic signals is as follows:

Step1. Initialize the input parameters i.e. number of array elements, SNR, number of snapshots, direction of incoming signals which is to be estimated.

Step2. Create BPSK signals and calculate array steering vector and array correlation matrix corresponding to incoming signals.

Step3. Initialize the population of agents in the search space with random positions and velocities in the interval $-\pi/2$ to $\pi/2$ in each dimension. The N dimensional positional vector of the j th probe takes the form $\theta_j = [\theta_1, \ldots \theta_N]$, where θ represents the source DOA's. A particle position vector is converted to a candidate solution vector in the problem space through a suitable mapping.

Step4. Calculate the array steering vector corresponding to all the random DOA defined in the interval.

Step5. Evaluate the fitness of all the agents as given in eq. (3.44) as:

$$V_{DML} = Tr\{(I - A(\theta)(A(\theta)^H A(\theta))^{-1} A(\theta)^H \ )\hat{R}\}$$

which is determined by taking the data from all the elements of the array for N_s number of snapshots.

Step6. Apply the GSA and PSO algorithm and in each iteration the position and velocity of the particles are updated. The optimization problem is a minimization problem so the position of the agent corresponding to minimum fitness represents the ML estimate of source DOA's.

Step7. Evaluate the root mean square error (RMSE) as:

$$RMSE = \sqrt{\frac{1}{N_n N_{runs}} \sum_{l=1}^{N_{runs}} \sum_{i=1}^{N_n} \left[\hat{\theta}_i(l) - \theta_i\right]^2}, \quad (3.124)$$

where N_n is the number of sources, N_{runs} are the independent Monte-Carlo trials, $\hat{\theta}_i(l)$ is the estimate of the *i* th DOA in the *l* th run, θ_i is the true DOA of the *i* th source.

Step8. Evaluate the Probability of resolution i.e. the ability to resolve closely spaced sources. The two sources are said to be solved in a given run if both $|\hat{\theta}_1 - \theta_1|$ and $|\hat{\theta}_2 - \theta_2|$ are smaller than $|\theta_1 - \theta_2|/2$.

3.6 Methodology for Maximum likelihood DOA estimation using PSO in multipath channel environment

In this part, we will describe the methodology of PSO algorithm for stochastic ML optimization to estimate the direction of incoming signals in multipath channel environment. The methodology of DOA estimation is explained in the following steps:

Step 1. Initialize the input parameters i.e. number of array elements, SNR, number of snapshots, direction of incoming signals which is to be estimated.

Step 2. Create uncorrelated, partially correlated, correlated random signals and calculate array steering vector and array correlation matrix corresponding to incoming signals.

Step 3. Initialize the population of agents in the search space with random positions and velocities in the interval *-π/2 to π/2* in each dimension. The *N* dimensional position vector of the *j*th particle takes the form θ_j=[θ_1,....... θ_N] where θ represents the source DOA's. A particle position vector is converted to a candidate solution vector in the problem space through a suitable mapping.

Step 4. Calculate the array steering vector corresponding to all the random DOA defined in the interval.

Step 5. Evaluate the fitness of all the agents using eq. (3.60)

$$V_{SML}(\theta) = \log\left|A(\theta)\hat{S}(\theta)A^H(\theta) + \hat{\sigma}^2(\theta)I\right|.$$

which is determined by taking the data from all the elements of the array for *N* number of snapshots.

Step 6. Apply the PSO algorithm and in each iteration the position and velocity of the particles are updated. The optimization problem is a minimization problem so the position of the agents corresponding to minimum fitness represents the ML estimate of source DOA's.

Step 7. Evaluate the RMSE using eq. (3.124) and probability of resolution.

3.7 Methodology for Adaptive beamforming using GSA

In this part, we will describe the formulation of GSA algorithm for ABF. The algorithm will iteratively tries to optimize the weights of linear antenna array so as to place pattern maxima in the direction of desired user and nulls in the direction of interference signals. The methodology of ABF using GSA is as follows:

Step 1. Initialize the number of array elements, SNR, direction of desired and interfering signals.

Step 2. Determine the array steering vector corresponding to desired and interfering signals.

Step 3. Initialize the population of agents in the search space with complex random position and velocities. The complex position of agents represents the amplitude and phase of weight vectors. The size of the weight vector represents the number of sensor elements.

Step 4. Determine the fitness of all the agents in GSA by taking inverse of the eq. (3.70) which is determined by taking data from all the elements of the array.

$$F = \frac{\bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w}}{\bar{w}^H \bar{a}_0 \bar{a}_0^H \bar{w}} \tag{3.125}$$

Step 5. Apply the GSA algorithm and in each iteration the position and velocity of the agents are updated. The optimization problem is a minimization problem, thus the position of agents corresponding to minimum fitness represents the optimum weights for ABF.

Step 6. Plot the linear and polar normalized array factor plot.

3.8 Methodology for Adaptive beamforming with reduced sidelobe levels using GSA

In this part, we will describe the formulation of GSA algorithm for ABF with reduced SLL. The algorithm will iteratively tries to optimize the weights of linear antenna array so as to place pattern maxima in the direction of desired user and nulls in the direction of interference signals with reduced SLL.

The methodology of ABF with reduced SLL using GSA is as follows:

Step 1. Initialize the number of array elements, SNR, direction of desired and interfering signals.

Step 2. Initialize the population of agents in the search space with complex random position and velocities. The complex position of agents represents the amplitude and phase of weight vectors. The size of the weight vector represents the number of sensor elements.

Step 3. Define the parameters γ_1, γ_2 which balance the minimization of two terms.

Step 4. Determine the fitness of all the agents in GSA using eq. (3.71) which is determined by taking data from all the elements of the array.

$$F = \gamma_1 \frac{\bar{w}^H \bar{A} \bar{R}_{ii} \bar{A}^H \bar{w} + \sigma^2 \bar{w}^H \bar{w}}{\bar{w}^H \bar{a}_0 \bar{a}_0^H \bar{w}} + \gamma_2 SLL \qquad (3.126)$$

SLL is determined using eq. (3.72),

$$SLL = 20 \log_{10} \left(\frac{AF(SL)}{\max{(AF)}} \right)$$

Step 5. Apply the GSA algorithm and in each iteration the position and velocity of the agents are updated. The optimization problem is a minimization problem, thus the position of agents corresponding to minimum fitness represents the optimum weights for ABF.

Step 6. Plot the linear and polar normalized array factor plot.

Results and Discussion

CHAPTER 4 RESULTS AND DISCUSSION

In this chapter, simulation results of DOA estimation and ABF techniques are presented and discussed. All the simulations are performed using MATLAB software. In the initial part of the chapter, the conventional DOA estimation techniques Bartlett, Capon and MUSIC algorithm are simulated and compared for narrow and wide angular separation of incoming signals. The DOA estimation techniques i.e. MUSIC, Root-MUSIC and ESPRIT are then compared on the basis of varying number of antenna elements, varying SNR and varying number of snapshots.

The DOA estimation of deterministic binary phase shift keying (BPSK) signals is performed using metaheuristic approaches namely PSO and GSA by optimizing DML function and compared with conventional techniques. The techniques are compared for narrow angular separation so as to determine the resolving capability of the algorithms. The DOA estimation of uncorrelated, partially correlated and coherent signals are performed for multipath channel environment and compared with conventional techniques by optimizing SML function using PSO. Since the GSA fails to optimize the SML function, thus the algorithm is not considered for DOA estimation in multipath channel environment. The techniques are compared on the basis of their resolving capability for narrow angular separation for uncorrelated and partially correlated random signals.

In the later part of the chapter ABF algorithms are simulated and compared on the basis of their beam pattern characteristics (beamwidth, maximum side lobe level and null depth) and their rate of convergence. The ABF problem is modeled as an optimization problem and the weights of antenna array are evaluated so as to direct major lobe in the direction of desired user and nulls in the direction of interfering signals using GSA and the results are compared with MVDR technique. The ABF problem is extended to multi-objective optimization problem in which the weights are evaluated such that the beam pattern is obtained with reduced SLL using GSA and the results are compared with MVDR and the previous published results.

In all the simulations uniformly spaced linear array of omnidirectional antenna elements has been considered as shown in figure 3.1. The space between two adjacent array elements is one half of the wavelength of the incoming signals. The impinging angles of the incoming signals are relative to the broadside of the array. The additive background noise is assumed to be spatially and temporally independent zero mean white Gaussian noise (AWGN). In Bartlett, Capon, MUSIC algorithm the search range is performed over [-90^o, 90^o] with the scanning interval of 0.1^o.

4.1 Performance analysis of DOA estimation algorithms

In the first simulation, performance of Bartlett, Capon and MUSIC algorithms are evaluated for narrow angular separation of incoming signals. Two independent uncorrelated narrowband signals have been considered which are impinging with DOA of -5^0 and 5^0, SNR of 10 dB and having 100 snapshots on an array of 10 antenna elements.

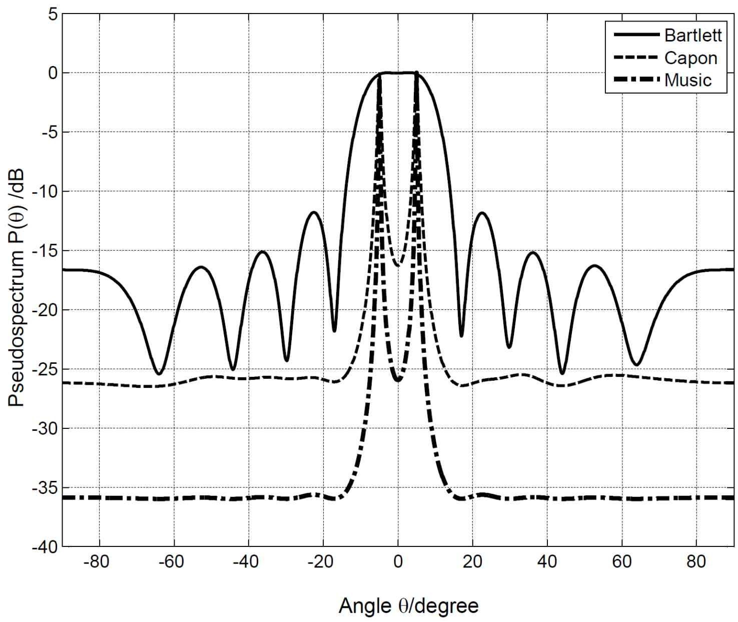


Fig. 4.1 Bartlett, Capon and MUSIC pseudospectrum in case of narrow angular separation.

In the second simulation, performance of Bartlett, Capon and MUSIC algorithm are evaluated for wide angular separation of incoming signals. Two independent uncorrelated narrowband signals have been considered impinging with DOA of -15^0 and 25^0, SNR of 10 dB and having 100 snapshots on an array of 10 antenna elements.

Figures 4.1 and 4.2 show the simulation results of conventional techniques for narrow and wide angular separation. The simulation results shows that for narrow angular separation of -5^0 and 5^0 Bartlett fails to detect the incoming signals while Capon and MUSIC accurately detect DOA whereas in case of wide angular separation of -15^0 and 25^0, Bartlett shows significant response while Capon and MUSIC shows sharp peaks in the spectrum. MUSIC algorithm shows the best response among the other two algorithms.

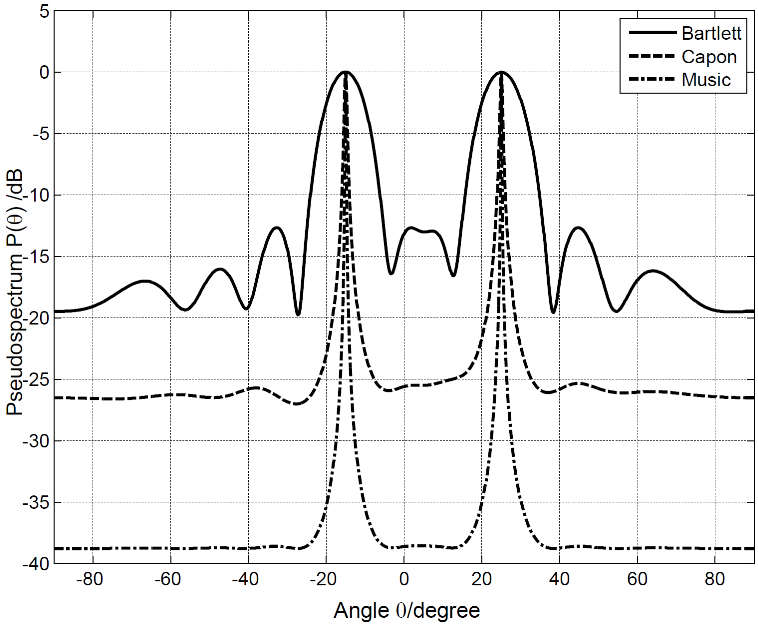
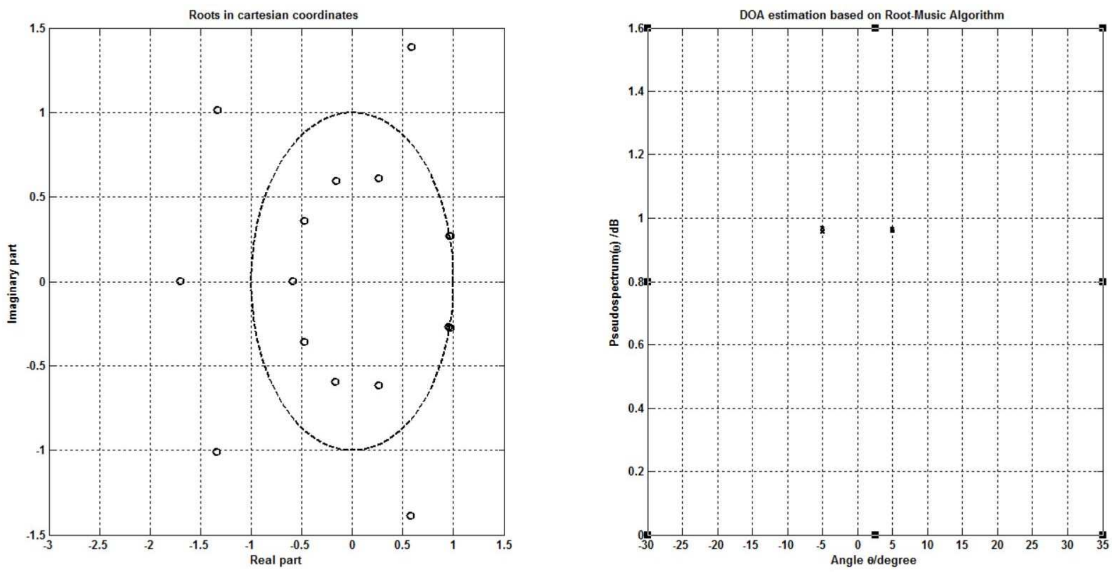


Fig. 4.2 Bartlett, Capon and MUSIC pseudospectrum in case of wide angular separation.

In the third simulation, the direction of incoming signals are estimated from Root-MUSIC algorithm. Two independent narrow band signals have been considered impinging with DOA of -5^0 and 5^0, SNR of 10 dB having 100 snapshots on an array of 10 sensor elements.



(a) (b)

Fig. 4.3 Root-MUSIC pseudospectrum.

Figure 4.3 shows the simulation results of Root-MUSIC algorithm for narrow angular separation. In Root-MUSIC algorithm, the direction of incoming signals is estimated by plotting the roots and roots which lie closest to the unit circle signifies the direction of incoming signals. The plot in figure 4.3 (a) locates all 18 roots in Cartesian coordinates and four roots are nearest to the unit circle and are thus close to expected direction of incoming signals. In the plot shown in figure 4.3 (b), the pseudospectrum shows the four roots signifying the direction of incoming signals. The numerical value of DOA of incoming signals are -4.9626, -4.9626, 5.0701, 5.0701.

In the three previous simulations, the performance of Bartlett, Capon, MUSIC and Root-MUSIC algorithms are evaluated. The simulation results show that MUSIC and Root-MUSIC algorithms show superior performance as compared to Bartlett and Capon algorithm. In the fourth simulation the performance of MUSIC, Root-MUSIC is evaluated along with ESPRIT algorithm on the basis of RMSE as a function of number of array elements, number of snapshots and SNR. All the simulation results are averaged over 100 independent Monte Carlo test runs. Two uncorrelated narrow band signals with DOA of -5^0 and 10^0 are assumed to impinge on antenna array elements.

4.1.1 Impact of number of array elements

In this case the comparison of MUSIC, Root-MUSIC and ESPRIT algorithms is made for DOA estimation for varying number of array elements {6, 8, 10, 12, 14, 16} with SNR of 10 dB and 100 snapshots.

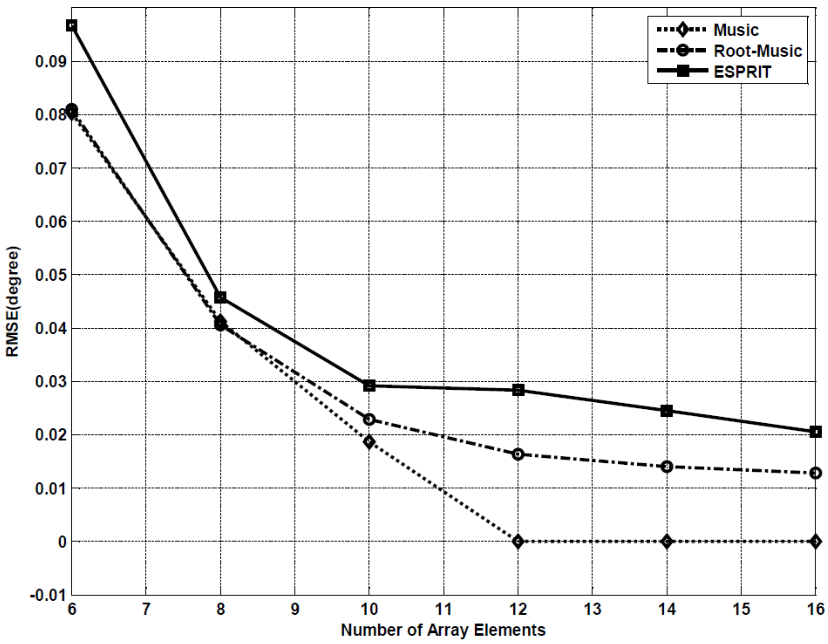


Fig. 4.4 RMSE of the DOA estimates versus number of array elements.

Figure 4.4 shows the performance of algorithms for varying antenna elements. It can be observed that MUSIC algorithm is proved to be more robust than Root-MUSIC and ESPRIT algorithms having its error reduced to zero above 12 array elements.

4.1.2 Impact of SNR

In this case the comparison of MUSIC, Root-MUSIC and ESPRIT algorithms is made for DOA estimation for varying SNR of {-10, -5, 0, 10, 20, 30} with 10 array elements and 100 snapshots.

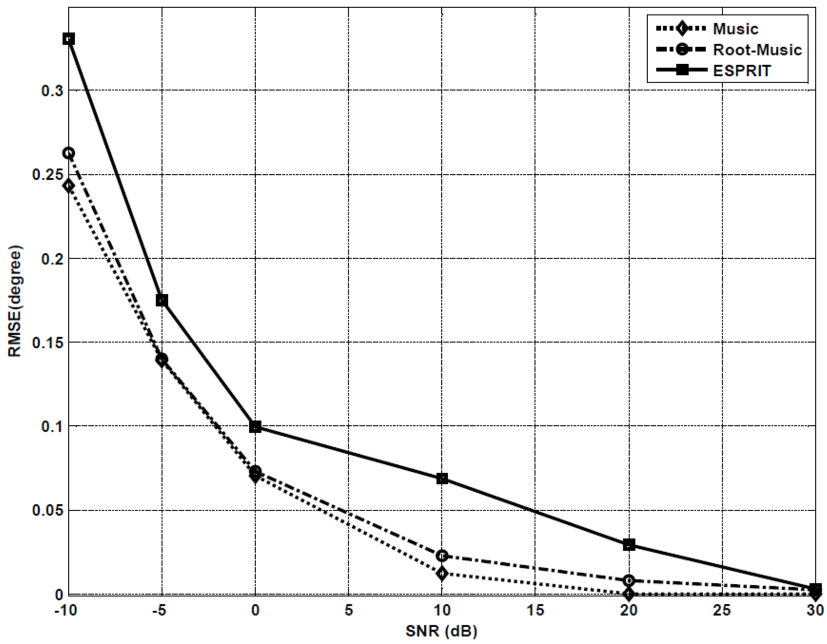


Fig. 4.5 RMSE of the DOA estimates versus input SNR.

Figure 4.5 shows the performance of MUSIC, Root-MUSIC and ESPRIT algorithm with varying SNR. In case of low SNR, MUSIC and Root-MUSIC algorithms show similar performance and have low RMSE while ESPRIT algorithm shows high RMSE. In high SNR conditions beyond 20 dB MUSIC algorithm shows zero RMSE as compared to other two algorithms. All the three algorithms show similar nature having their RMSE reduced to zero at 30 dB SNR.

4.1.3 Impact of number of snapshots

In this case the comparison of MUSIC, Root-MUSIC and ESPRIT algorithms is made for estimating direction of incoming signals for varying number of snapshots {10, 50, 100, 300, 700, 1000} with SNR of 10 dB on an array of 10 antenna elements.

Figure 4.6 shows the performance comparison of MUSIC, Root-MUSIC and ESPRIT algorithms in terms of RMSE for varying number of snapshots. MUSIC algorithm provides best accuracy of DOA estimates having its error reduced to zero above 300 snapshots. ESPRIT algorithm shows significant performance and its RMSE curve shows a sudden dip and performs better than Root-MUSIC algorithm after 700 snapshots.

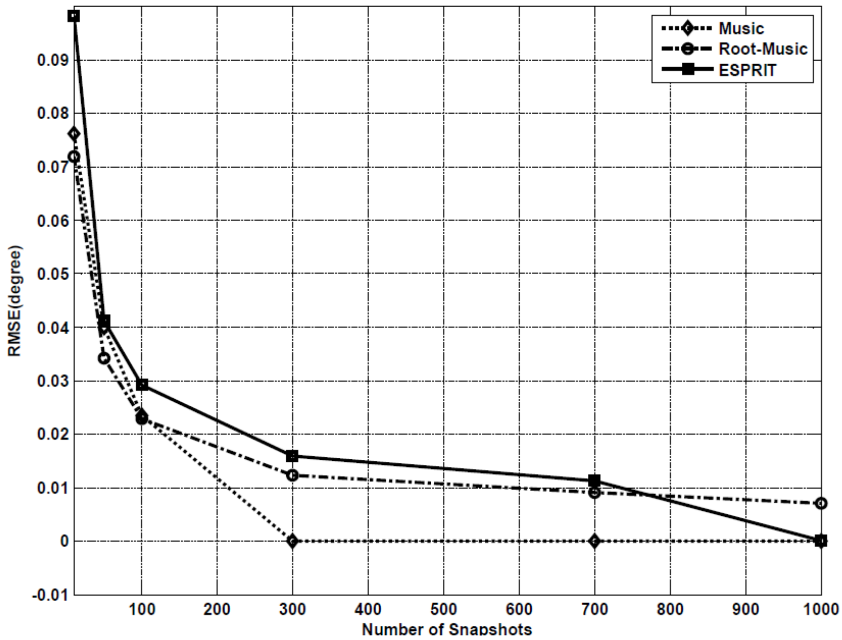


Fig. 4.6 RMSE of the DOA estimates versus number of snapshots.

4.2 Deterministic ML DOA estimation using GSA

In this section, the DOA of deterministic signals is estimated by optimizing DML function using proposed GSA and compared with earlier reported soft computing technique i.e. PSO and other conventional techniques. Let us consider that two narrowband plane waves impinge on the 10 element ULA from 40^o and 44^o. The two sources are BPSK signals that are uncorrelated with each other and the noise is additive and uncorrelated from sensor to sensor and with the signals. We have considered two sources with narrow angular separation so as to analyze the resolving ability of the algorithms when the sources are closely spaced. The total number of snapshots considered are 100 and the SNR is varied from -20 dB to 30 dB with the step size of 2 dB. The GSA and PSO parameters used in simulation are given in table 4.1. Figure 4.7 shows the DOA estimation RMSE obtained by GSA, PSO, MUSIC, Capon and ESPRIT with respect to different SNR. Figure 4.8 show the source resolution probability for the same methods with respect to different SNR.

Table 4.1 GSA and PSO parameters for deterministic ML DOA estimation.

	Population size	Cognitive & Social parameter	Gravitational Constant	Alpha	N_{runs}
GSA	30	N.A.	100	20	100
PSO	30	2, 2	N.A.	N.A.	100

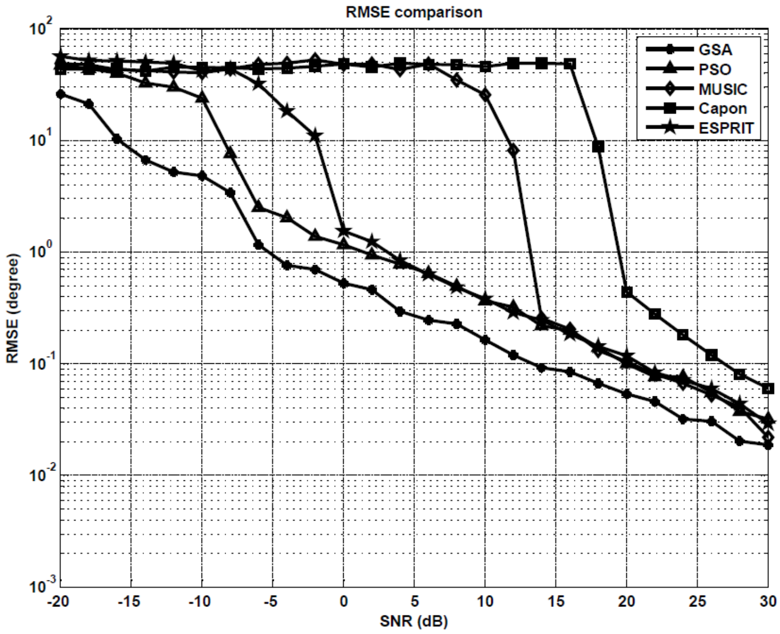


Fig. 4.7 DOA estimation RMSE versus SNR for GSA, PSO, MUSIC, Capon and ESPRIT algorithms.

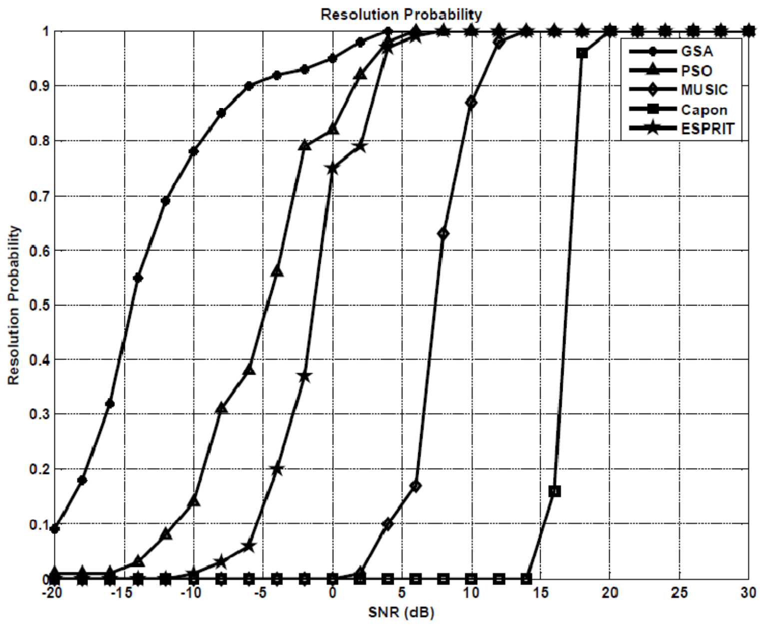


Fig. 4.8 Resolution probability versus SNR for GSA, PSO, MUSIC, Capon and ESPRIT algorithms.

As can be seen from the figures 4.7 and 4.8, the metaheuristic approaches, GSA and PSO show better statistical performance than MUSIC, Capon and ESPRIT in low SNR region. Capon and MUSIC algorithms have high RMSE till 16 dB and 8 dB, thus these algorithms fail to resolve closely spaced sources. ESPRIT algorithm applicable on linear arrays shows better performance compared to other classical approaches. Simulation results verify that GSA performs best among all the algorithms in terms of RMSE and can better resolve closely spaced sources.

In order to analyze the resolution capabilities of these algorithms we have simulated all these algorithms with varying angular separation. In the previous simulation, the two sources had angular separation of 4^0. Now we simulate all these algorithms with an angular separation of 3.5^0, 3^0, 2.5^0, 2^0, 1.5^0, 1^0 with an SNR at which a particular algorithm achieves resolution probability of 1 at 4^0 angular separation.

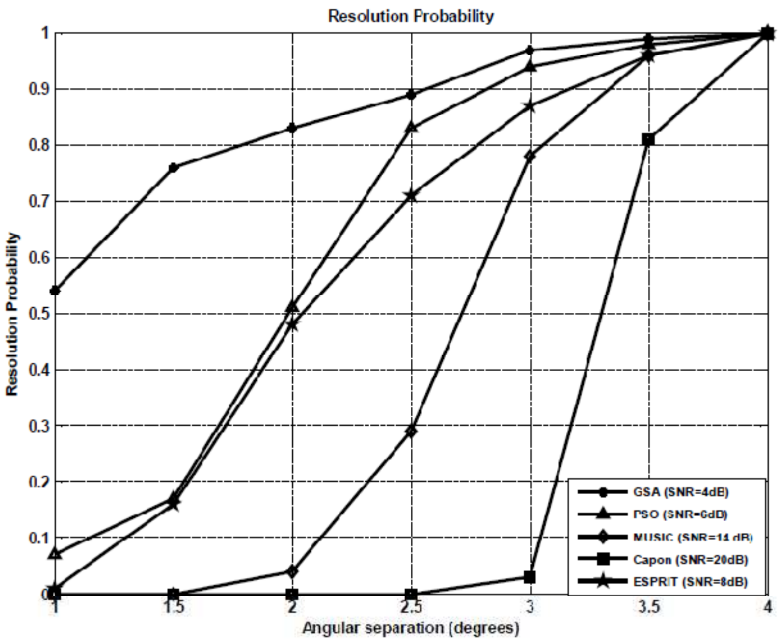


Fig. 4.9 Resolution probability versus angular separation for GSA, PSO, MUSIC, Capon and ESPRIT algorithms.

Figure 4.9 shows the resolution probabilities of all these algorithms as a function of separation angle between sources. The results show that GSA performs best among all the algorithms and proves to detect the incoming signal with narrow angular separation. Thus, GSA can accurately and precisely detect the incoming signals and found to be a appropriate DOA estimation technique for deterministic signals.

4.3 ML DOA estimation using PSO in multipath channel environment

In this section the direction of incoming signals are estimated in multipath channel environment using PSO by optimizing SML function and the results are compared with well known MUSIC, Capon and ESPRIT algorithms. The simulations are carried out in three different scenarios that are uncorrelated, partially correlated and coherent channel environment. All the simulation experiments are performed over 100 Monte-Carlo trials. The parameters used in PSO algorithm are summarized in table 4.2.

Table 4.2 PSO parameters for stochastic ML DOA estimation.

Parameter	Description	Values
N	Population size	30
d	Dimension of the search space	2, 3
t_{max}	Maximum iteration	1000
w	Weight	Start weight=0.9 End weight=0.5
c_1	Cognitive coefficient	2
c_2	Social coefficient	2

4.3.1 Scenario 1: Uncorrelated signal environment

In the first scenario, we consider that two equal power sources impinge the 10 element ULA from 40^o and 44^o. The incoming signals are random in nature and uncorrelated with each other and the noise is additive and uncorrelated from sensor to sensor and with the signals. The total number of snapshots considered are 100 and the SNR is varied from -15 dB to 30 dB with the step size of 1 dB. Figures 4.10 and 4.11 show the DOA estimation RMSE and resolution probability as a function of SNR for PSO, ESPRIT, MUSIC and Capon algorithm.

The simulation results show that ML-PSO algorithm shows better statistical performance than MUSIC, Capon and ESPRIT algorithm in low SNR region. This verifies the suitability of ML-PSO algorithm over high resolution techniques to detect closely spaced sources.

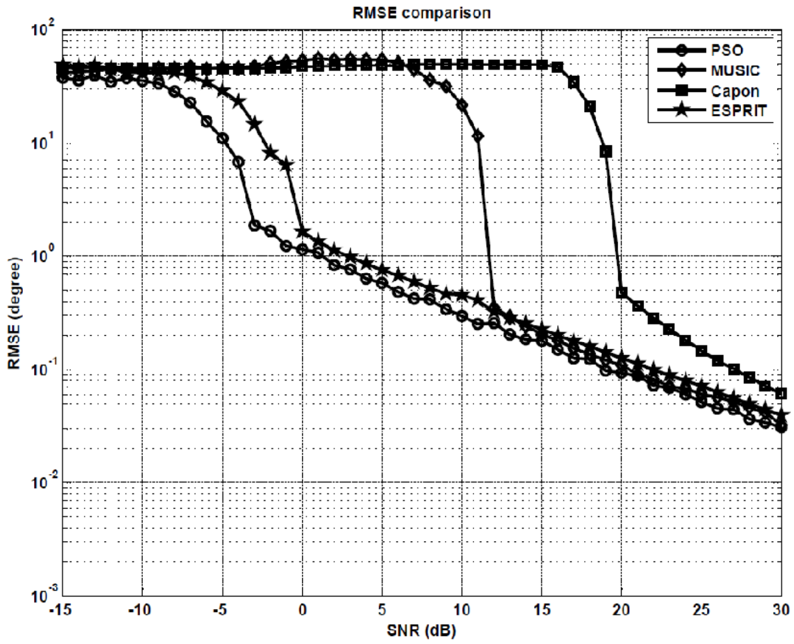


Fig. 4.10 DOA estimation RMSE versus SNR for PSO, MUSIC, Capon and ESPRIT algorithms in uncorrelated channel environment.

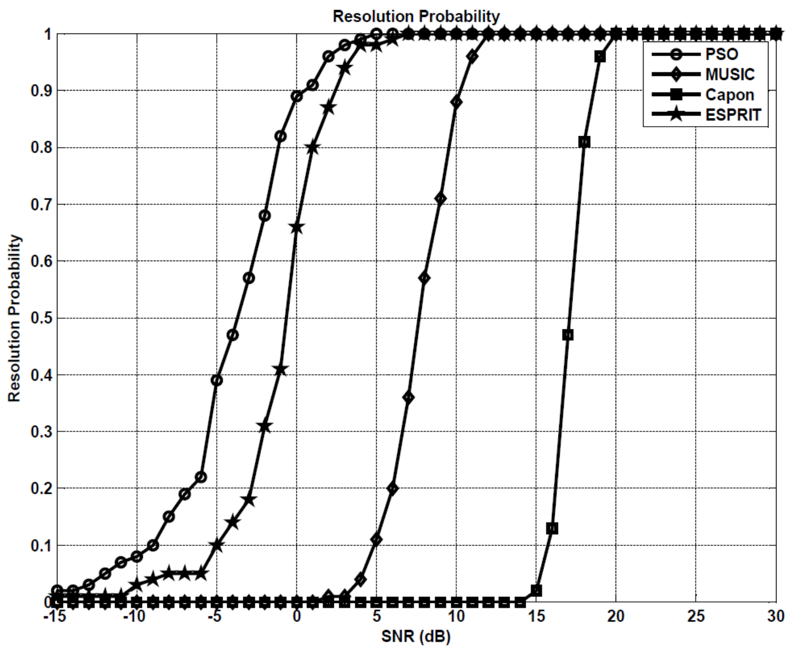


Fig. 4.11 Resolution probability versus SNR for PSO, MUSIC, Capon and ESPRIT algorithms in uncorrelated channel environment.

In order to investigate the resolution capabilities of these algorithms we have simulated all these algorithms with varying angular separation. In the previous simulation, the two sources have angular separation of 4^0. Now we have simulated all these algorithms with an angular separation of 3.5^0, 3^0, 2.5^0, 2^0, 1.5^0, 1^0 with an SNR at which a particular algorithm achieves resolution probability of 1 at 4^0 angular separation.

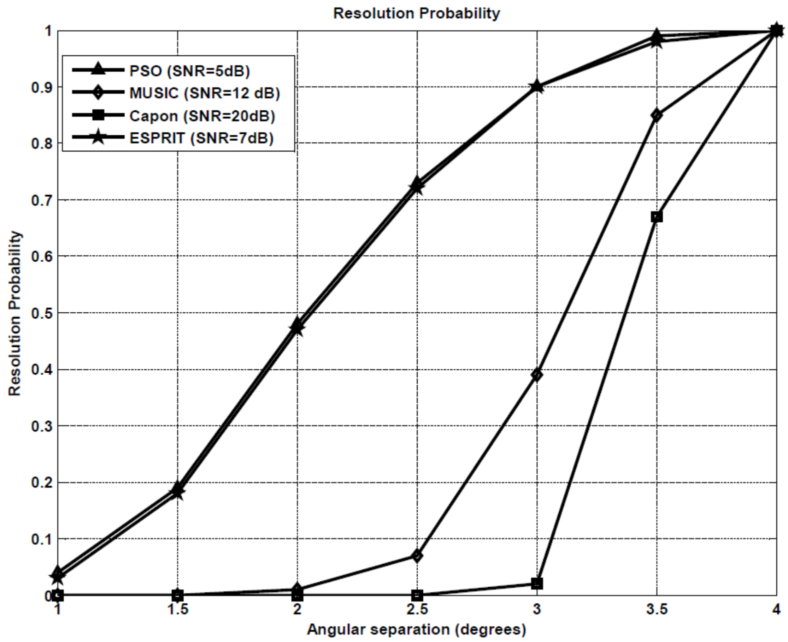


Fig. 4.12 Resolution probability versus angular separation for PSO, MUSIC, Capon and ESPRIT algorithms in uncorrelated channel environment.

Figure 4.12 shows the resolution probability of all these algorithms as a function of separation angle between sources. ML-PSO and ESPRIT show best performance among all these algorithms, but ESPRIT achieves the same resolution probability as achieved by ML-PSO at higher SNR value of 7 dB in comparison to PSO of 5 dB. Thus, ML-PSO performs best in terms of resolving closely spaced sources.

4.3.2 Scenario 2: Partially correlated signal environment

In the second scenario, we consider a multipath environment in which two partially correlated equal power sources impinge the 10 element ULA from 40^o and 44^o. The correlation coefficient between the two partially correlated sources is kept 0.8. The total number of snapshots that are considered in simulation are 100 and the SNR varies within the range [-15 dB to 30 dB] with a step size of 1 dB. Figures 4.13 and 4.14 demonstrate the DOA estimation RMSE and the resolution probability of PSO, MUSIC, Capon and

ESPRIT algorithms. The results show that ML-PSO estimator gives better performance in partially correlated environment within the large range of SNR. This proves that ML-PSO is a optimal technique for multipath channel environment.

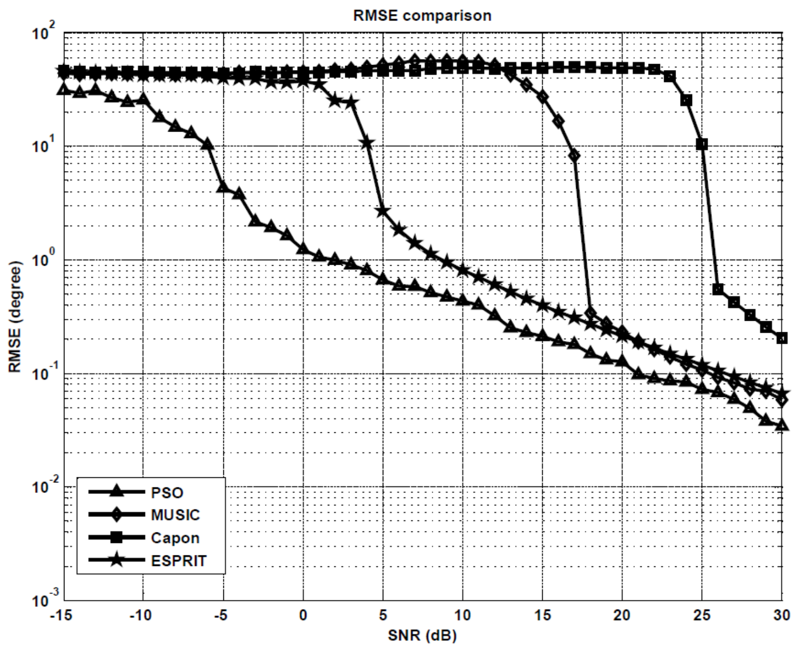


Fig. 4.13 DOA estimation RMSE versus SNR for PSO, MUSIC, Capon and ESPRIT algorithms in partially correlated channel environment.

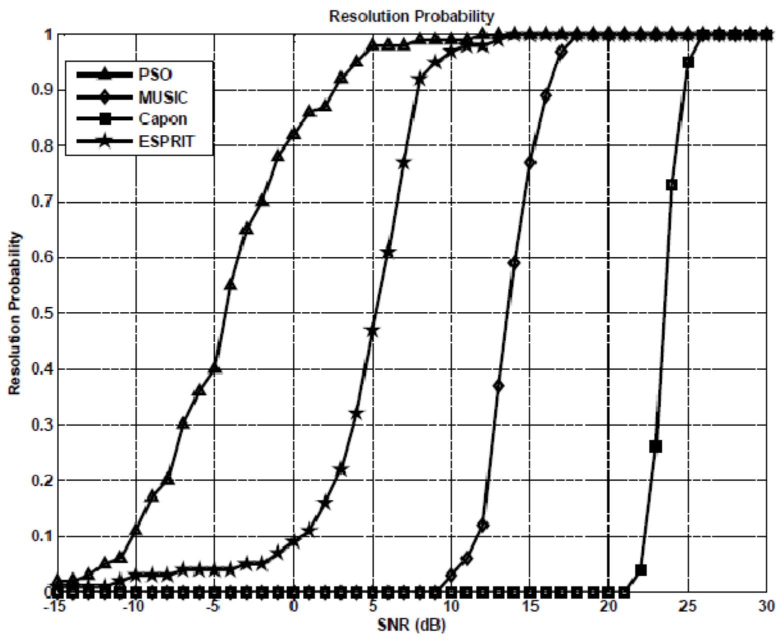


Fig. 4.14 Resolution probability versus SNR for PSO, MUSIC, Capon and ESPRIT algorithms in partially correlated channel environment.

The resolution probability of algorithms are also investigated in partially correlated environment. Simulations are carried out in varying angular separation of 0.5^0 between 40^0 and 44^0 with an SNR at which a particular algorithm achieves resolution probability of 1 at 4^0 angular separation. Figure 4.15 shows the resolution probability versus angular separation of all the algorithms.

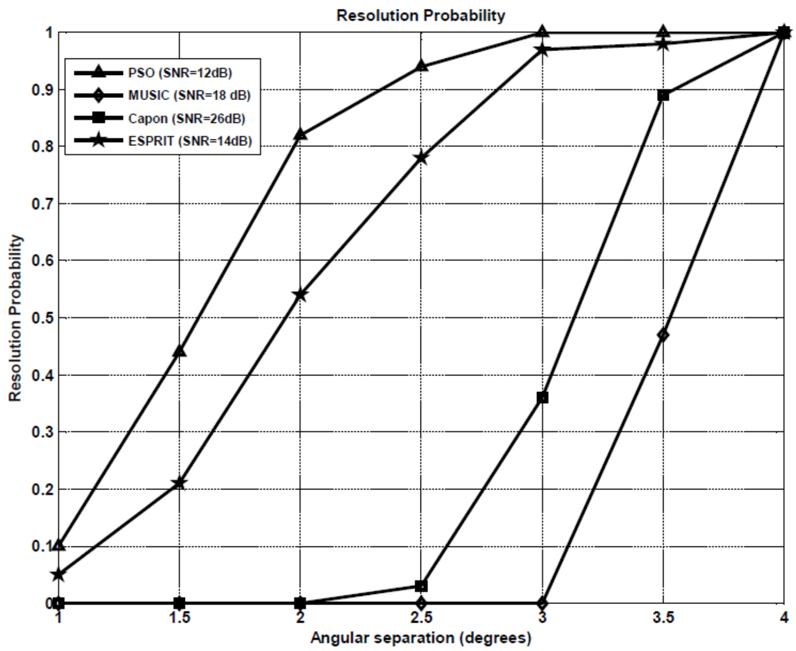


Fig. 4.15 Resolution probability versus angular separation values for PSO, MUSIC, Capon and ESPRIT algorithms in partially correlated channel environment.

Simulation result verifies that ML-PSO based DOA estimation gives best results among all other algorithms at minimum SNR. Thus, ML-PSO possesses the best capability to identify closely spaced sources in partially correlated environment.

Now in order to analyze the effect of varying correlation between the impinging signals, the performance of the Capon, MUSIC, ESPRIT and ML-PSO DOA estimation techniques is evaluated at different values of correlation coefficients ranging from 0.1 to 0.9 with a step size of 0.1. In this simulation two narrowband signals are assumed to impinge on a 10 element ULA from 40^0 and 44^0. Figures 4.16-4.18 show the variation of RMSE with SNR for Capon, MUSIC and ESPRIT algorithms at different values of correlation coefficient. The simulation results show that RMSE of Capon, MUSIC and ESPRIT algorithm for 0.1 correlation coefficient shows a sudden dip around 17 dB, 8 dB and -2 dB. The RMSE of all the algorithms decreases with an increase of SNR and correlation coefficient. Thus, the results show that with the variation of correlation coefficient ESPRIT algorithm gives better results than Capon and MUSIC algorithm.

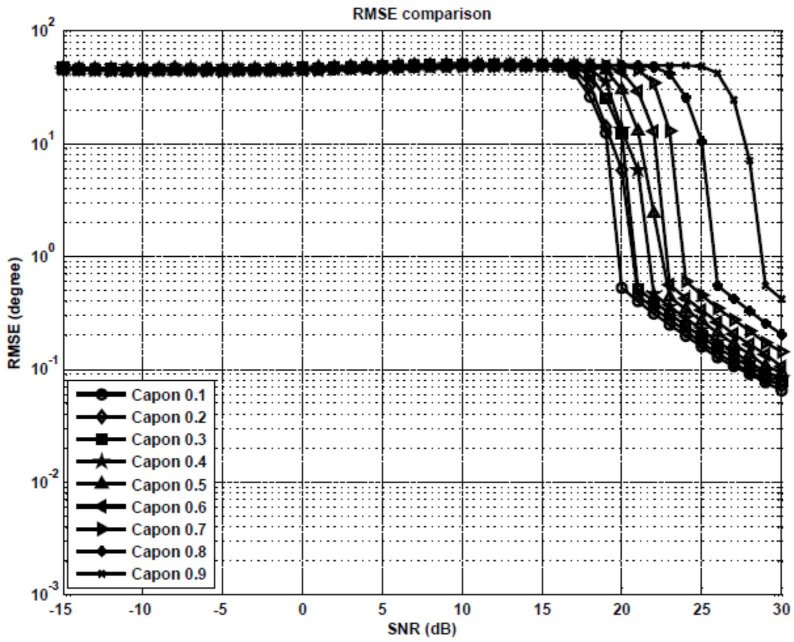


Fig. 4.16 Variation of DOA estimation RMSE for Capon algorithm.

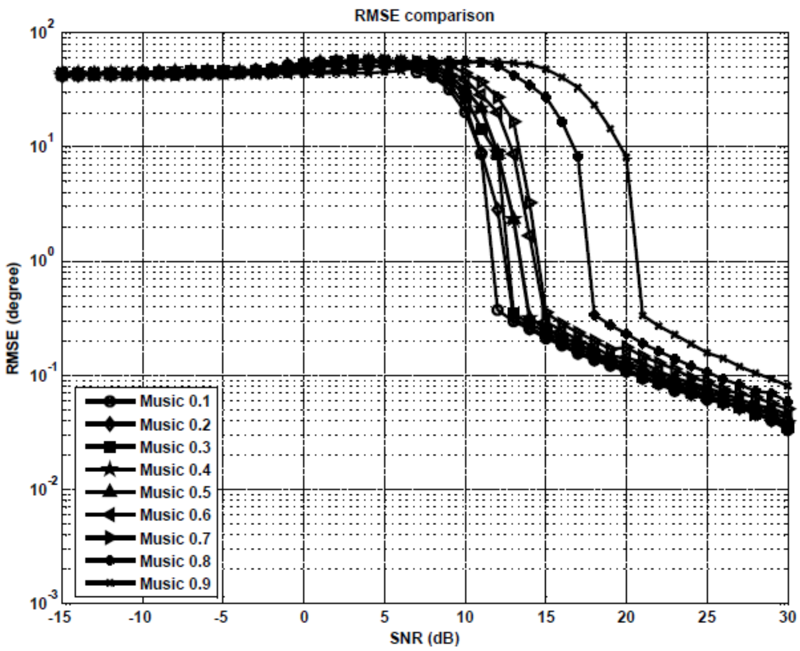


Fig. 4.17 Variation of DOA estimation RMSE for MUSIC algorithm.

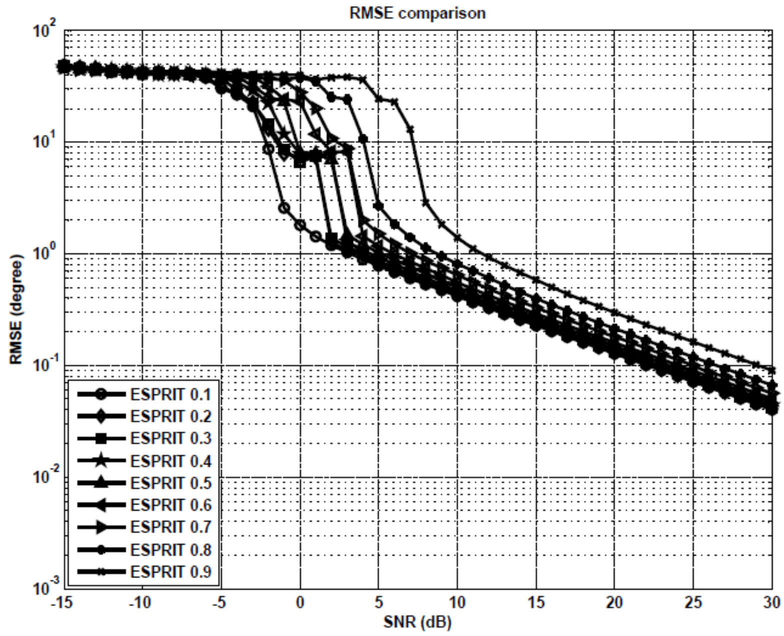


Fig. 4.18 Variation of DOA estimation RMSE for ESRIT algorithm.

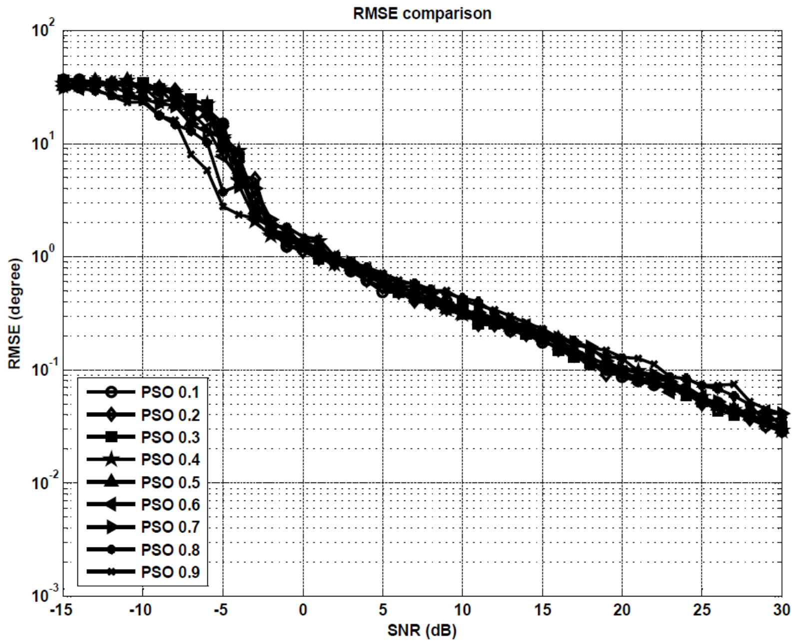


Fig. 4.19 Variation of DOA estimation RMSE for PSO algorithm.

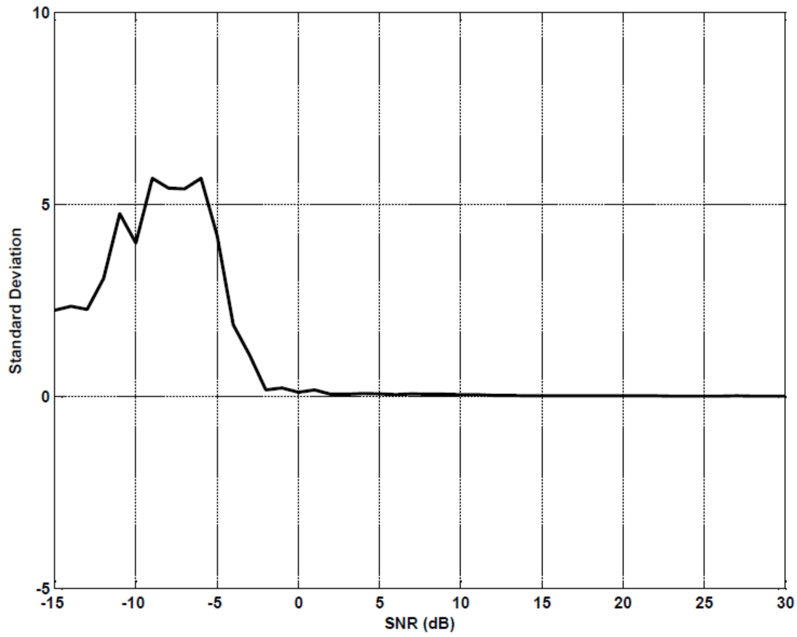


Fig. 4.20 Variation of standard deviation of RMSE at different correlation coefficient with SNR for PSO algorithm.

Figure 4.19 show the simulation results for ML-PSO estimator and RMSE curve shows a sudden dip at -8 dB for 0.1 correlation coefficient. Thus, the simulation result shows that ML-PSO estimator gives almost same performance in varying partially correlated channel environment whereas conventional algorithms suffers from deterioration in performance with increasing correlation coefficient.

Considering the stochastic nature of the ML-PSO estimator mean and standard deviation (SD) of RMSE at different correlation coefficient is determined for all values of SNR ranging from -15 dB to 30 dB. Figure 4.20 shows the variation of SD of RMSE at different correlation coefficient for all values of SNR. The results show that there is a variation in SD up to 4 dB and after that the SD is approximately zero.

Next, the performance of all four DOA estimation algorithms as considered above are analyzed with varying correlation coefficient ranging from 0.1 to 0.9 at an RMSE of 1 degree in order to estimate the required value of SNR. The Capon, MUSIC and ESPRIT algorithms are deterministic in nature, thus their RMSE can be directly analyzed from figures 4.16-4.18. ML-PSO estimator is a stochastic algorithm, thus the mean value of RMSE is calculated for different correlation coefficient at a particular value of SNR. Figure 4.20 shows the performance comparison of PSO, MUSIC, Capon and ESPRIT DOA estimation algorithms.

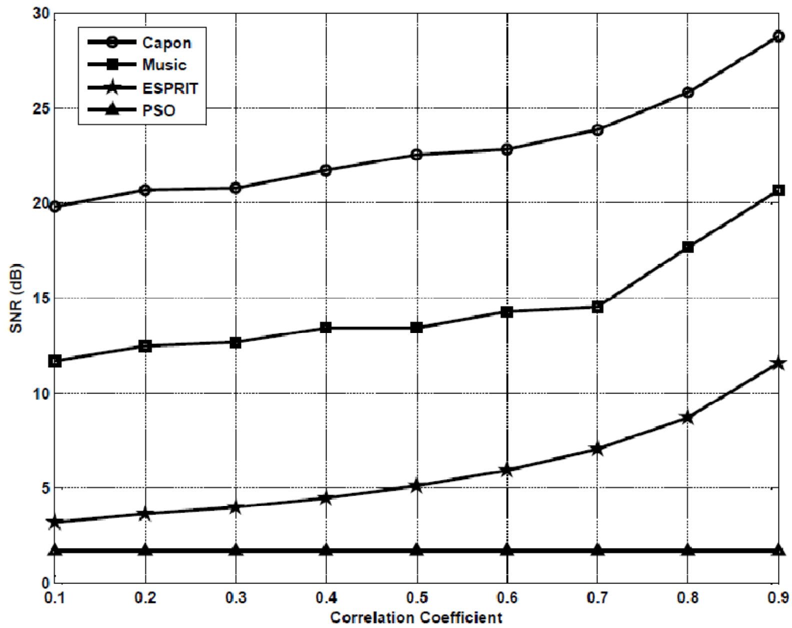


Fig. 4.21 Performance analysis of DOA estimation algorithms at an RMSE of one degree.

The simulation result as shown in figure 4.21 shows that as the value of correlation coefficient increases, the SNR required to maintain a RMSE of 1 degree proportionally increases. Capon algorithm requires highest value of SNR while ML-PSO estimator requires minimum value of SNR to maintain a RMSE of 1 degree as compared to Capon, MUSIC and ESPRIT algorithms.

4.3.3 Scenario 3: Coherent signal environment

In the third scenario, we consider multipath channel environment in which three coherent sources impinging the 10 element ULA from 20^o, 30^o, 40^o relative to the end-fire. The attenuation coefficient of the three coherent sources are 1, (0.4+0.8j), (-0.5-0.7j). The total number of snapshots are 100 and the SNR is varied in the range of [-15 dB 30 dB] with a step size of 1 dB. Figure 4.22 shows the simulation results of RMSE for all the four PSO, MUSIC, Capon and ESPRIT algorithms.

The simulation results show that in ML-PSO algorithm with an increase of SNR the RMSE decreases, while other techniques fail to estimate the DOA of incoming signals and shows that their RMSE remains constant in the whole range of SNR. Thus, ML-PSO approach proves to be a good alternative to eigen structure methods in multipath channel environment.

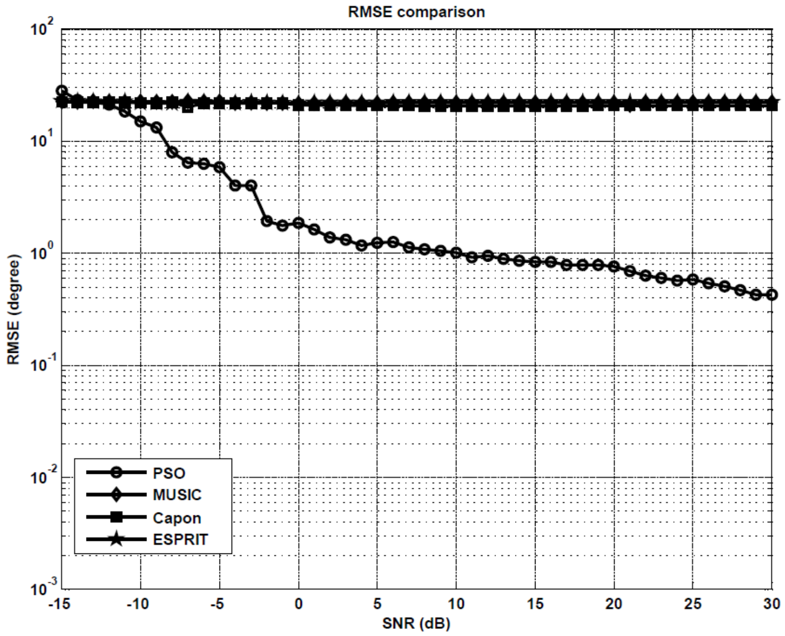


Fig. 4.22 DOA estimation RMSE versus SNR of PSO, MUSIC, Capon, ESPRIT algorithms in correlated channel environment.

In the sections 4.1-4.3 dealing with DOA estimation, the simulation results of conventional DOA estimation techniques, DOA estimation based on DML method using GSA and DOA estimation based on SML using PSO were discussed. The simulation results show that in narrow and wide angular separation MUSIC algorithm shows better results than Bartlett and Capon algorithm. The performance of the algorithms are judged on the basis of varying antenna elements, varying SNR and varying number of snapshots for wide angular separation. The simulation results show that MUSIC algorithm gives better results than Root-MUSIC and ESPRIT algorithms. The direction of deterministic BPSK signals are evaluated by optimizing non-linear function using metaheuristic approaches. The simulation results show that GSA gives better results in terms of RMSE and resolution probability than PSO and conventional techniques. The direction of signals are found in multipath channel environment by optimizing complex multimodal function using PSO. The simulation results show that ML-PSO algorithm gives good statistical performance and resolution probability than conventional techniques even for narrow angular separation of incoming signals.

4.4 Performance analysis of adaptive beamforming algorithms

In this section the performance of ABF algorithms are discussed. In the first simulation of beamforming algorithms, a comparative study of LMS, SMI, and RLS algorithm is presented. The beampattern of these algorithms are judged based on the variation of antenna elements. We have considered three signals, one desired and two interfering signals with DOA of 0^o, -40^o and 60^o. The number of antenna elements are chosen as{8, 12, 16}. The noise variance is 0.01, step size for LMS algorithm is 0.02 and the number of snapshots are 100.

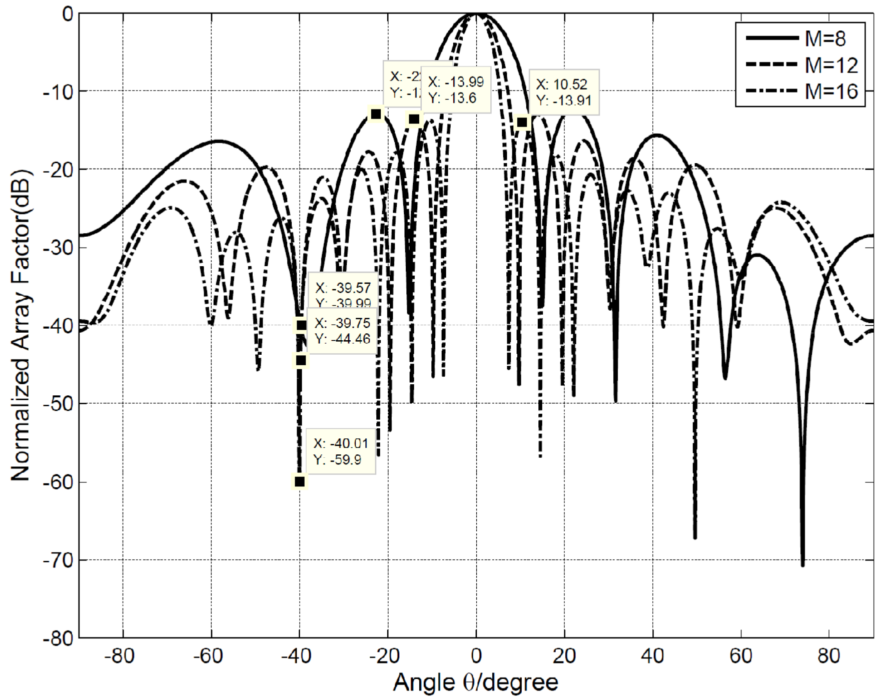


Fig. 4.23 Normalized array factor plot for LMS algorithm.

Table 4.3 Variation of antenna elements for LMS algorithm.

Sl.No.	Antenna Elements	Beamwidth (Degree)	Maximum Sidelobe Level (dB)	Maximum Null Depth(dB)
1.	8	13.26	-12.91	-39.99
2.	12	8.527	-13.6	-44.46
3.	16	6.465	-13.91	-59.9

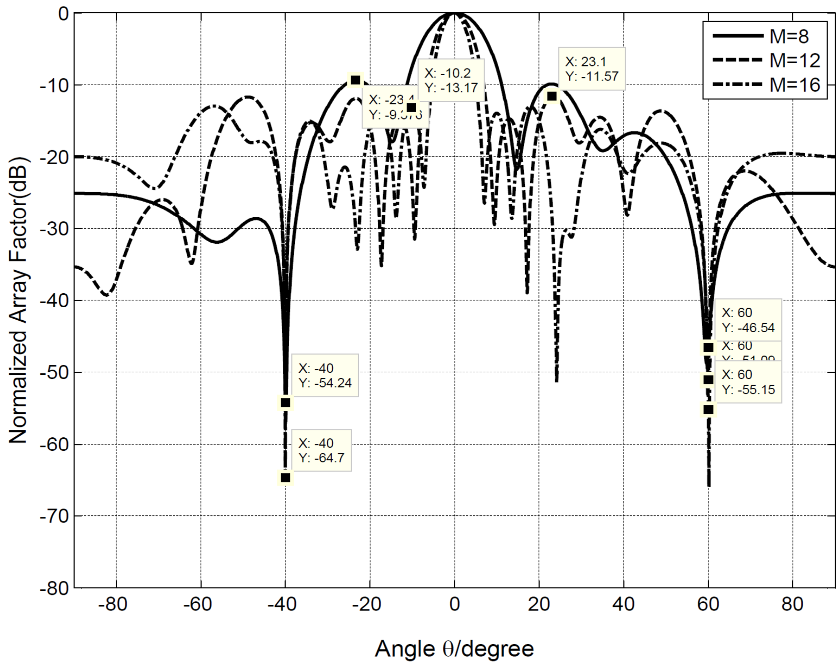


Fig. 4.24 Normalized array factor plot for SMI algorithm.

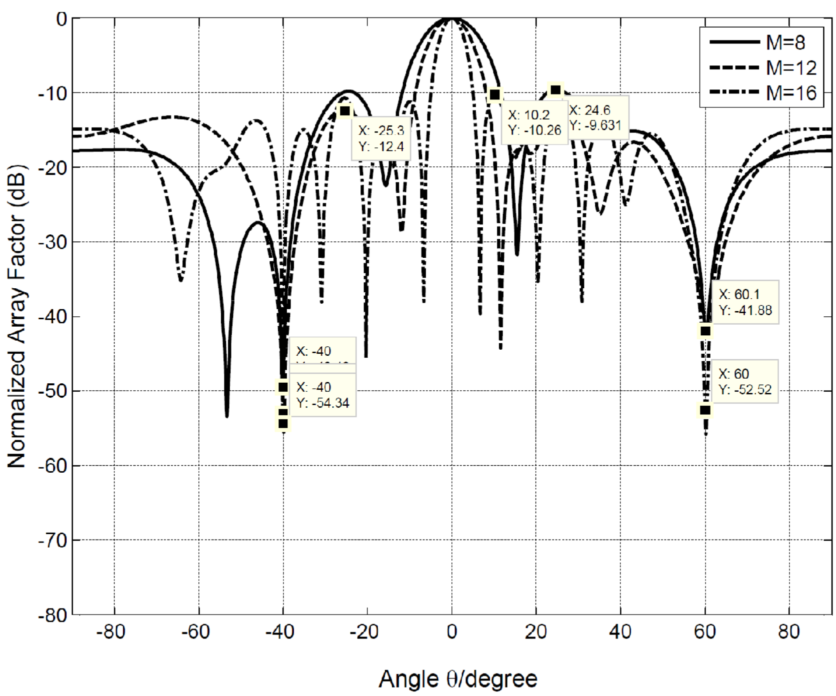


Fig. 4.25 Normalized array factor plot for RLS algorithm.

Table 4.4 Variation of antenna elements for SMI algorithm.

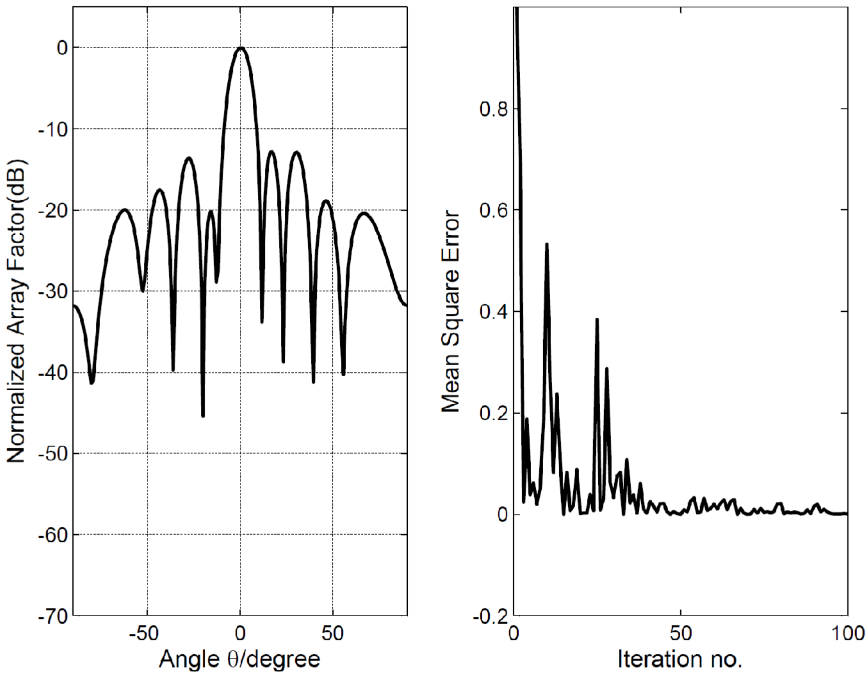
Sl.No.	Antenna Elements	Beamwidth (Degree)	Maximum Sidelobe Level (dB)	Maximum Null Depth(dB)
1.	8	13.4	-9.376	-46.54
2.	12	8.2	-11.57	-54.24
3.	16	6.3	-13.17	-64.7

Table 4.5 Variation of antenna elements for RLS algorithm.

Sl.No.	Antenna Elements	Beamwidth (Degree)	Maximum Sidelobe Level (dB)	Maximum Null Depth(dB)
1.	8	14.1	-9.631	-49.48
2.	12	9.8	-10.2	-53.06
3.	16	6.178	-12.4	-54.34

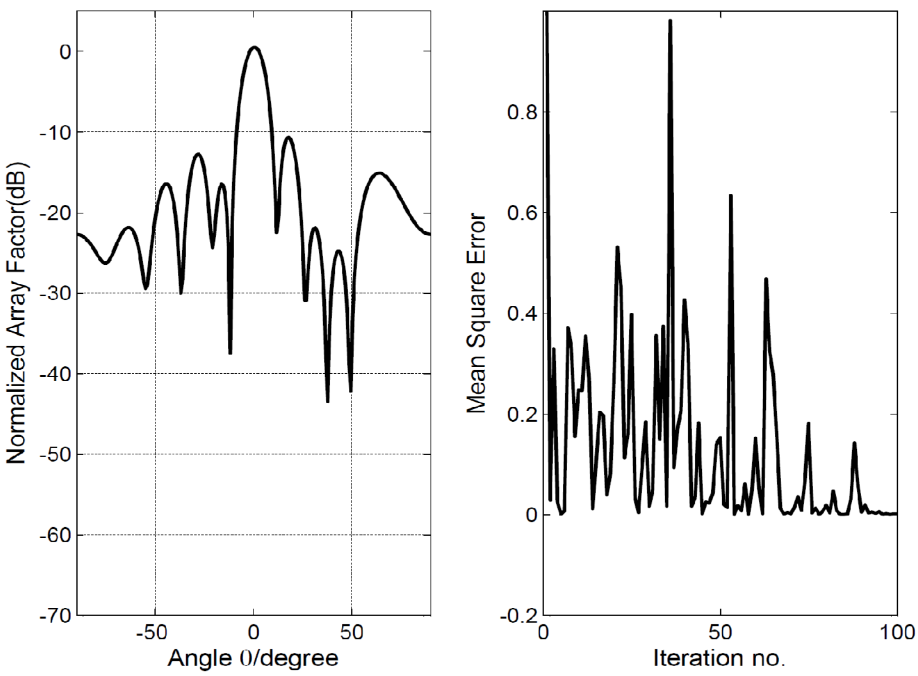
Figures 4.23-4.25 and tables 4.3-4.5 show the variation of the beamwidth, side lobe level and maximum null depth with the variation of antenna elements. The normalized array factor plots for LMS, SMI, and RLS algorithm indicate that the beamwidth, side lobe level and null depth decreases with the increase in the number of array elements. The variation of beamwidth and side lobe level with the increase in antenna elements is approximately same for all three algorithms. The SMI algorithm shows more null depth as compared to LMS and RLS algorithm.

In the second simulation, a comparative study of LMS, RLS and CM ABF algorithms based on their rate of convergence is presented. These algorithms are iterative beamforming algorithms where weights are updated on sample by sample basis. Thus, convergence rate is an important parameter for examining their performance. In this case, we have considered one desired and two interfering signals impinging on an array of 10 elements with DOA of 0^o, -20^o and 40^o. The noise variance is 0.01, step size for LMS and CM algorithm is 0.05, forgetting factor for RLS algorithm is 0.9 and number of snapshots are 100.



(a) (b)

Fig. 4.26 (a) Normalized array factor plot for LMS algorithm (b) Mean square error plot.



(a) (b)

Fig. 4.27 (a) Normalized array factor plot for CM algorithm (b) Mean square error plot.

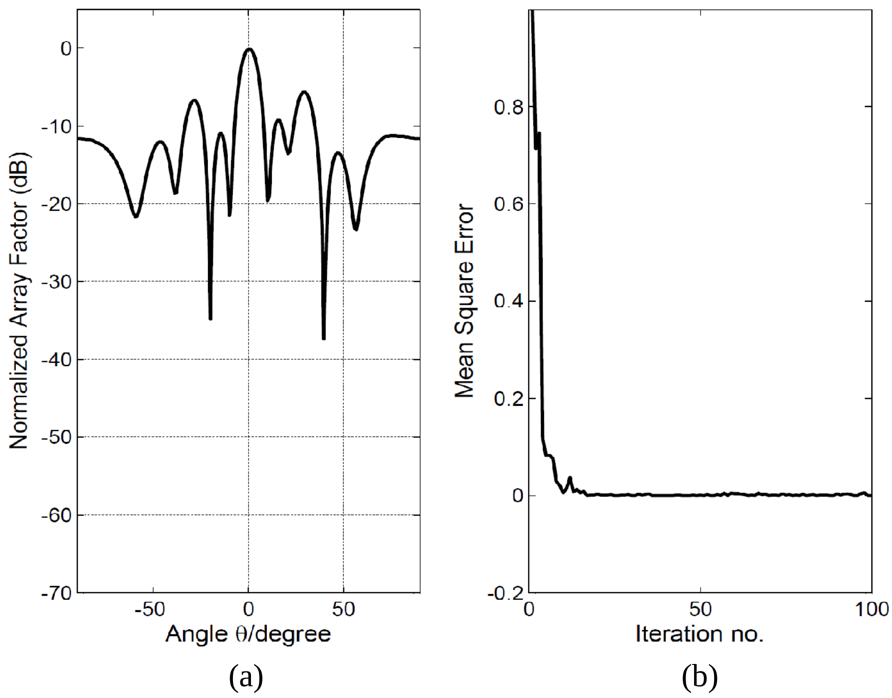


Fig. 4.28 (a) Normalized array factor plot for RLS algorithm (b) Mean square error plot.

Figures 4.26-4.28 show normalized array factor plot and mean square error plot for LMS, CM and RLS algorithms. It can be observed from the simulation results that the variation in mean square error of LMS and CM algorithm is more than the RLS algorithm. CM algorithm is less stable than the LMS algorithm which indicates that the CM algorithm is more sensitive to the step size parameter. This limits the usefulness of the CM algorithm in the dynamic environment where the channel conditions are rapidly changing. RLS algorithm has the best error performance and its convergence rate is typically an order of magnitude faster than the LMS algorithm. Thus, RLS algorithm is a best choice in the applications where fast tracking is required.

Thus, through simulation results of ABF algorithm it can be concluded that with the increase in number of array elements the beam pattern characteristics improves and the RLS algorithm has the best convergence rate among LMS and CM algorithms. Although, a comparison is done between different beamforming algorithms based on their beampattern characteristics and convergence rate, but with different signal formats, transmission schemes, channel conditions and operational requirements like hand-offs, it is difficult to compare all these algorithms on a single account.

4.5 Adaptive beamforming using GSA

In this section, the problem of ABF for linear antennas array is modeled as an optimization problem. The GSA is used for calculating the optimum weights by optimizing the fitness function of eq. (3.125) for ABF such that major lobe is directed in the direction of desired signal and nulls are formed in the direction of interfering signals. In this simulation a desired signal and several interfering signals from different angles are assumed to impinge on ULA at respective DOA relative to the broadside of the array. All the signals are uncorrelated with each other. The optimization algorithm is applied to calculate weights of linear arrays with different power levels so as to analyze the effectiveness of the algorithm.

The beamforming problem is a real time problem thus the optimization algorithm is investigated in terms of its rate of convergence. The convergence graph of mean fitness value versus number of iterations of GSA is shown in figure 4.29.

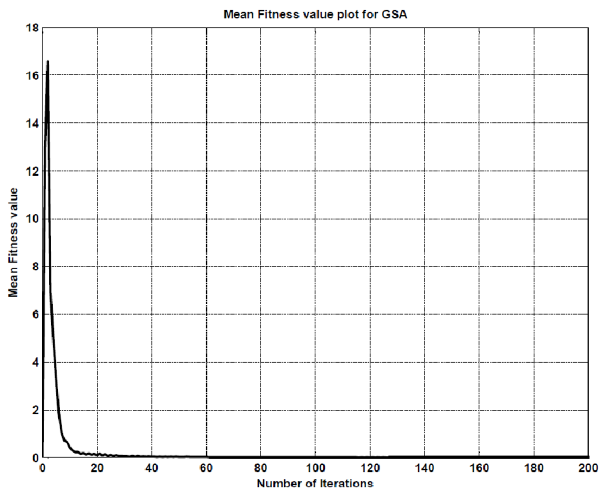
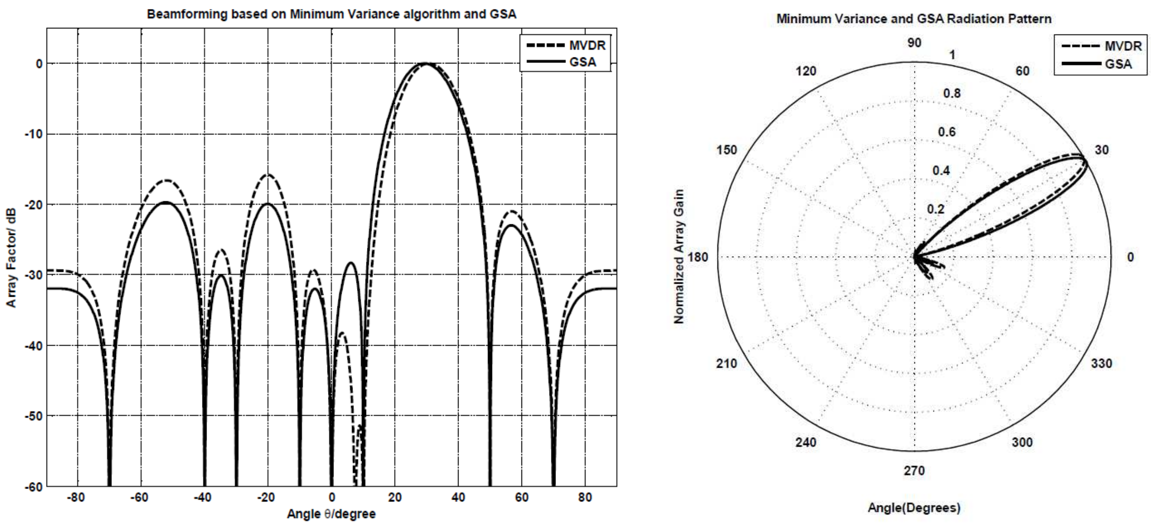


Fig. 4.29 Mean convergence plot of proposed GSA for ABF.

The convergence plot shows that the algorithm has a fast rate of convergence and it achieves better mean fitness values. The plot verifies that the algorithm converges before 100 iterations. Thus, due to the fast convergence rate GSA is found to be a suitable algorithm for beamforming. In the simulation, three different scenarios are studied with an SNR of 30 dB, 15 dB and 50 dB and the results are compared with well known robust MVDR ABF algorithm. Table 4.9 shows the parameters used in GSA for ABF with and without SLL reduction at certain angular region.

4.5.1 Scenario 1: Normal SNR case (30 dB)

In the first scenario, one desired signal arrives at the 10 element ULA from 30^0 and eight interfering signals arrives from -70^0,-40^0,-30^0,-10^0,0^0,10^0,50^0,70^0 with an SNR of 30 dB.



(a) (b)

Fig. 4.30 (a) Linear plot (b) Polar plot for first scenario of beamforming using GSA and MVDR algorithm.

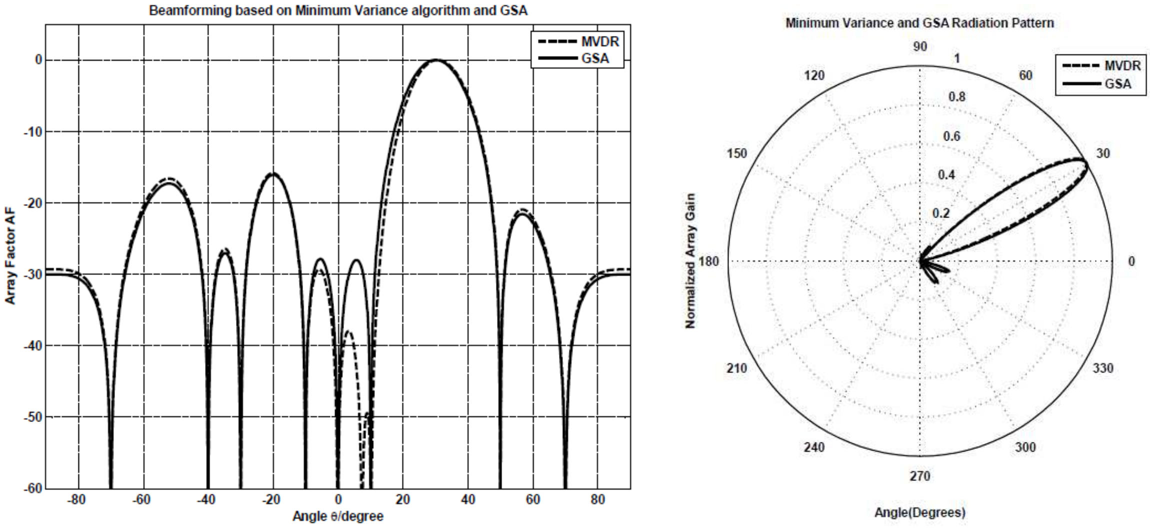
Figure 4.30 shows the radiation pattern and table 4.6 shows the weights for MVDR and GSA algorithm for the first scenario.

Table 4.6 Estimated weights of first scenario of beamforming using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	0.35245 - 0.22679j	0.41076 + 0.015865j
2	0.017253 + 0.40491j	-0.05278 + 0.50937j
3	-0.75025 + 0.15175j	-0.83341 - 0.026849j
4	-0.22043 - 0.89979j	-0.070759 - 1.0572j
5	1.0	1.0
6	0.34232 + 0.93958j	0.24772 + 1.0665j
7	-0.92089 + 0.10091j	-0.87421 + 0.15384j
8	-0.11425 - 0.75687j	-0.11762 - 0.66439j
9	0.38635 - 0.1224j	0.37239 - 0.19029j
10	-0.092439 + 0.4088j	-0.081946 + 0.19331j

4.5.2 Scenario 2: Low SNR case (15 dB)

In the second scenario, one desired signal arrives at the 10 element ULA from 30^0 and eight interfering signals arrives from $-70^0,-40^0,-30^0,-10^0,0^0,10^0,50^0,70^0$ with an SNR of 15 dB.



(a) (b)

Fig. 4.31 (a) Linear plot (b) Polar plot for second scenario of beamforming using GSA and MVDR algorithm.

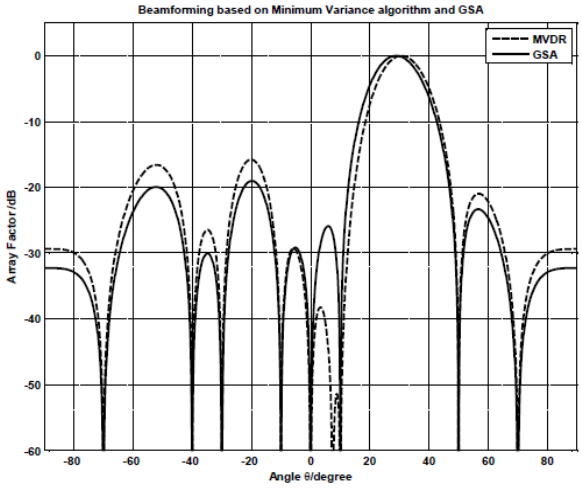
Figure 4.31 shows the radiation pattern and table 4.7 shows the weights for MVDR and GSA algorithm for the second scenario.

Table 4.7 Estimated weights of second scenario of beamforming using GSA and MVDR algorithm.

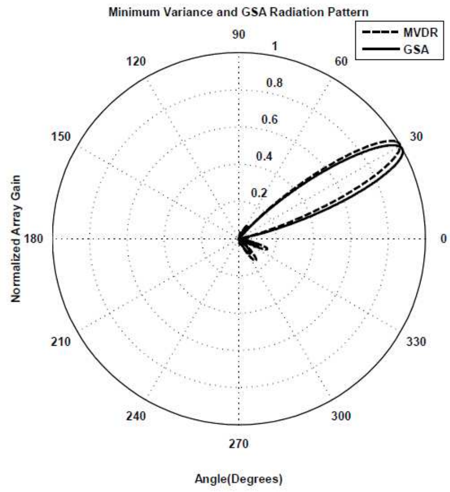
m	w_{MVDR}	w_{GSA}
1	0.35407 - 0.22629j	0.52486 - 0.041515j
2	0.018068 + 0.407j	0.0073994 + 0.5335j
3	-0.75082 + 0.15166j	-0.91186 + 0.035403j
4	-0.21946 - 0.90085j	-0.16494 - 1.113j
5	1.0	1.0
6	0.34045 + 0.94026j	0.32004 + 1.0984j
7	-0.92175 + 0.10035j	-0.8547 + 0.12385j
8	-0.11301 - 0.7576j	-0.071041 - 0.67482j
9	0.38884 - 0.12158j	0.34309 - 0.17439j
10	-0.092227 + 0.40996j	-0.18896 + 0.21582j

4.5.3 Scenario 3: High SNR case (50 dB)

In the third scenario, one desired signal arrives at the 10 element ULA from 30^0 and eight interfering signals arrives from $-70^0,-40^0,-30^0,-10^0,0^0,10^0,50^0,70^0$ with an SNR of 50 dB.



(a)



(b)

Fig. 4.32 (a) Linear plot (b) Polar plot for third scenario of beamforming using GSA and MVDR algorithm.

Figure 4.32 shows the radiation pattern and table 4.8 shows the weights for MVDR and GSA algorithm for the third scenario.

Table 4.8 Estimated weights of third scenario of beamforming using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	0.3524 - 0.22681j	0.43822 + 0.085019j
2	0.017227 + 0.40484j	-0.069042 + 0.54307j
3	-0.75023 + 0.15175j	-0.86553 - 0.076036j
4	-0.22046 - 0.89976j	-0.033305 - 1.11j
5	1.0	1.0
6	0.34238 + 0.93956j	0.22525 + 1.108j
7	-0.92086 + 0.10093j	-0.85816 + 0.1675j
8	-0.11429 - 0.75684j	-0.11475 - 0.63695j
9	0.38627 - 0.12243j	0.36534 - 0.2098j
10	-0.092446 + 0.40876j	-0.088023 + 0.1292j

Table 4.9 Parameters of GSA algorithm.

Parameter	GSA
Population size	30
Dimension of the search space	9, 10
Maximum iteration (*maxit*)	1000
Initial value of gravitational constant (G_0)	100
Gradient constant (alpha)	20
Zero offset constant (ϵ)	2.22e-16

The simulation results show that both the algorithms direct the major lobe in the direction of desired user and nulls in the direction of interfering signals. However, the GSA algorithm presents better SLL as compare to MVDR algorithm. But, in order to achieve specific value of SLL at certain angular region, a specific function of SLL must be added to the fitness function.

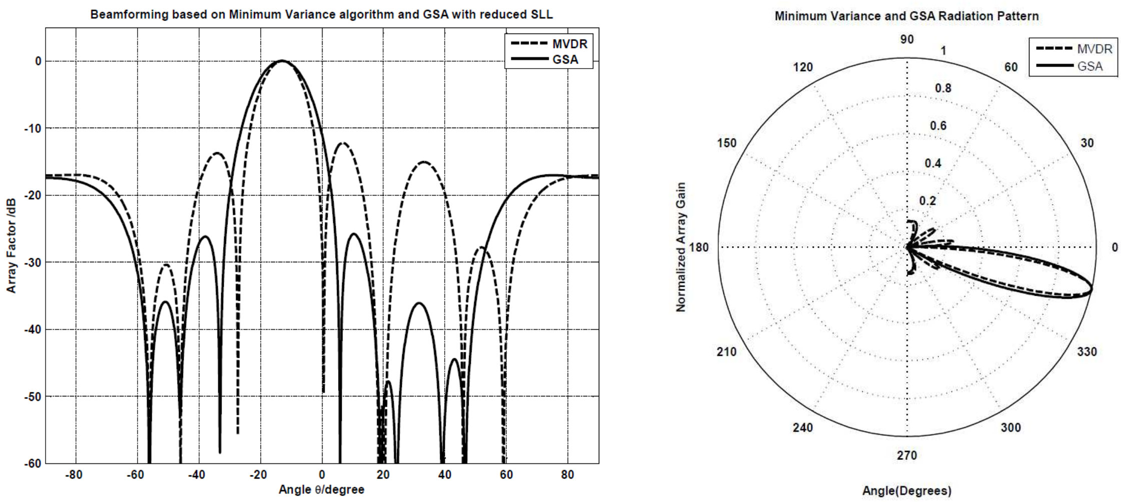
4.6 Adaptive beamforming with reduced SLL using GSA

In this section the problem of ABF which is a basic requirement of smart antenna technology is modeled such that the major lobe and nulls are not only controlled but also the specific value of SLL is achieved in the radiation pattern. The complex weights which controls the beampattern of linear array are calculated by optimizing the function in eq. (3.126) using GSA. The SLL are calculated by optimizing the function corresponding to the side lobes and the angle of the array factor represents the angle corresponding to the angle of side lobes. The parameters γ_1, γ_2 which balance the minimization of two terms in the complex expression of eq. (3.126) are randomly chosen so as to obtain the desired radiation pattern with reduced side lobes. The GSA doesn't need the knowledge of interference correlation matrix but only need the knowledge of direction of incoming signals in order to steer the main lobe towards the desired user and nulls in the direction of interfering signals. The direction of incoming signals are calculated using any of the techniques discussed in the previous section of DOA estimation.

Four different scenarios are considered in this section with different number of interference signals and different power levels of additive zero-mean Gaussian noise to compare results with the results reported earlier by **Zaharis *et al.*(2012, a, c)**. In the first two scenarios, three and in the last two scenario five interference signals which are uncorrelated with each other are considered to impinge on the linear array.

4.6.1 Scenario 1: One desired and three interfering signals at an SNR of 10 dB

In the first scenario, 9 element ULA receives one desired signal arriving at -13^0 and three interference signals arriving at -56^0, 20^0, 46^0 with an SNR of 10 dB.



(a) (b)

Fig. 4.33 (a) Linear plot (b) Polar plot for first scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

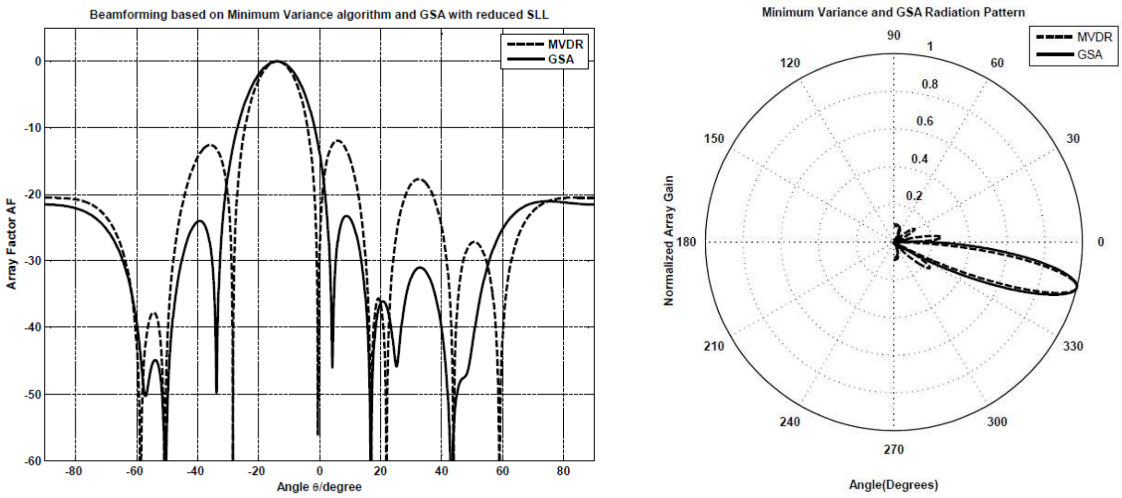
Figure 4.33 shows the radiation pattern and table 4.10 shows the weights for MVDR and GSA algorithm for the first scenario.

Table 4.10 Estimated weights of first scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	-0.056779 - 0.46736j	-0.0087043 - 0.38329j
2	-0.62684 - 0.17644j	-0.54208 - 0.23461j
3	-0.32364 + 0.44325j	-0.46449 + 0.42195j
4	0.19468 + 0.79576j	0.17684 + 0.88336j
5	1.0	1.0
6	0.19468 - 0.79576j	0.18943 - 0.86948j
7	-0.32364 - 0.44325j	-0.45735 - 0.47684j
8	-0.62684 + 0.17644j	-0.54934 + 0.23428j
9	-0.056779 + 0.46736j	0.023335 + 0.37018j

4.6.2 Scenario 2 One desired and three interfering signals at an SNR of 10 dB

In the second scenario, 9 element ULA receives one desired signal arriving at -14^0 and three interference signals arriving at -51^0, 17^0, 44^0 with an SNR of 10 dB.



(a) (b)

Fig. 4.34 (a) Linear plot (b) Polar plot for second scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

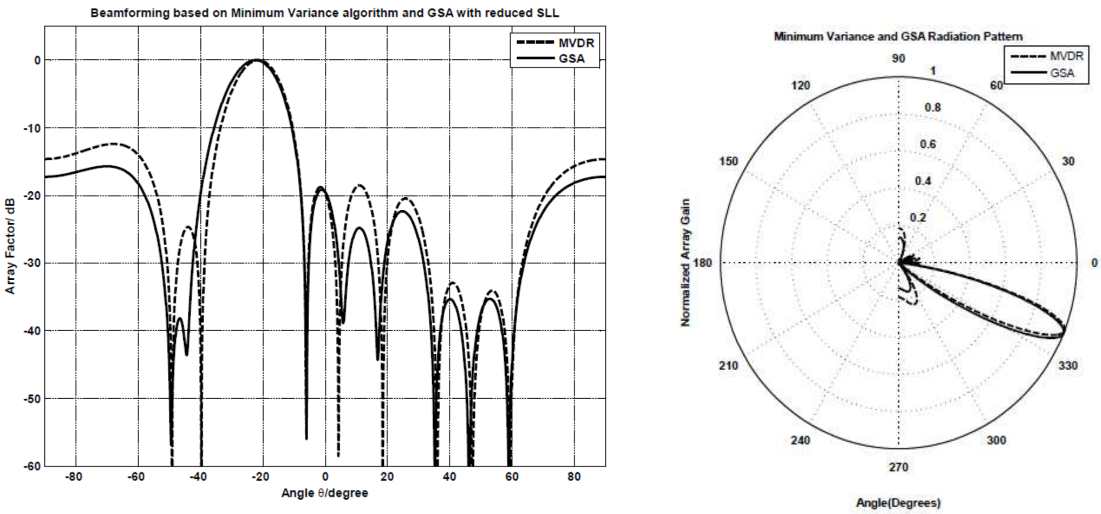
Figure 4.34 shows the radiation pattern and table 4.10 shows the weights for MVDR and GSA algorithm for the second scenario.

Table 4.11 Estimated weights of second scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	-0.067525 + 0.43092j	0.038927 + 0.16368j
2	0.42449 + 0.50037j	0.42803 + 0.30802j
3	0.92929 + 0.43968j	0.64103 + 0.3452j
4	0.80484 + 0.039045j	1.0735 + 0.13422j
5	1.0	1.0
6	0.80484 - 0.039045j	0.87202 - 0.34468j
7	0.92929 - 0.43968j	0.5086 - 0.20691j
8	0.42449 - 0.50037j	0.16986 - 0.26878j
9	-0.067525 - 0.43092j	0.059872 + 0.031493j

4.6.3 Scenario 3 One desired and five interfering signals at an SNR of 10 dB

In the third scenario, 9 element ULA receives one desired signal arriving from -22^0 and five interference signals arriving from -49^0, -6^0, 36^0, 47^0, 59^0 with an SNR of 10 dB.



(a) (b)

Fig. 4.35 (a) Linear plot (b) Polar plot for third scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

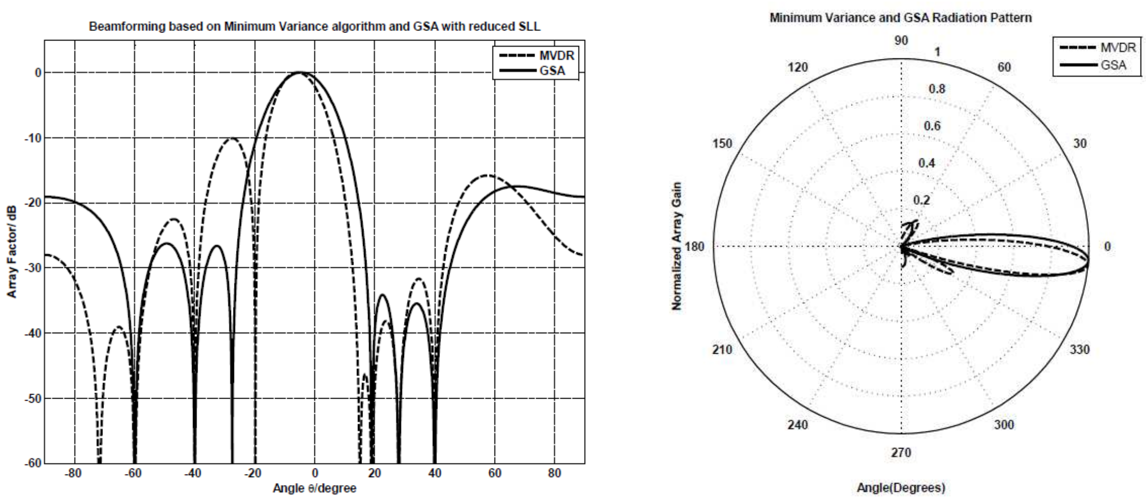
Figure 4.35 shows the radiation pattern and table 4.12 shows the weights for MVDR and GSA algorithm for the third scenario.

Table 4.12 Estimated weights of third scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	-1.0551 + 0.17039j	-0.3299 + 0.033961j
2	-0.96789 + 1.0791j	-0.34126 + 0.47771j
3	0.13351 + 1.7829j	0.011475 + 0.87807j
4	1.1538 + 1.1583j	0.83697 + 0.72181j
5	1.0	1.0
6	1.1538 - 1.1583j	0.79725 - 0.7355j
7	0.13351 - 1.7829j	-0.00096732 - 0.83001j
8	-0.96789 - 1.0791j	-0.33192 - 0.43911j
9	-1.0551 - 0.17039j	-0.27452 - 0.019345j

4.6.4 Scenario 4 One desired and five interfering signals at an SNR of 30 dB

In the fourth scenario, 9 element ULA receives a desired signal arriving from -5^0 and five interference signals arriving from -60^0, -40^0, 19^0, 28^0, 40^0 with an SNR of 30 dB.



(a) (b)

Fig. 4.36 (a) Linear plot (b) Polar plot for fourth scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

Figure 4.36 shows the radiation pattern and table 4.13 shows the weights for MVDR and GSA algorithm for the fourth scenario.

Table 4.13 Estimated weights of fourth scenario of beamforming with reduced SLL using GSA and MVDR algorithm.

m	w_{MVDR}	w_{GSA}
1	-1.1799 + 0.47872j	-0.30134 + 0.055405j
2	-0.7345 + 1.2544j	-0.2088 + 0.50954j
3	0.34361 + 1.8366j	0.052456 + 0.83488j
4	1.3781 + 1.3247j	0.91466 + 0.70145j
5	1.0	1.0
6	1.3781 - 1.3247j	0.90733 - 0.71886j
7	0.34361 - 1.8366j	0.049442 - 0.828j
8	-0.7345 - 1.2544j	-0.21948 - 0.51843j
9	-1.1799 - 0.47872j	-0.30212 - 0.044711j

The simulation result shown in figures 4.33-4.36 and tables 4.10-4.13 demonstrate that optimal weights calculated by GSA and MVDR algorithm provides sufficient steering ability of the main lobe towards desired signal and produce nulls in the direction of interfering signals. The weight vectors of GSA produce radiation pattern with lower SLL as compared to MVDR algorithm. The weight vectors are normalized with respect to the centre element of the array.

Table 4.14 Comparison of angular deviation and SLL values of MVDR, MBPSO and GSA algorithm.

	No. of array elements	**No. of Interfering signals**	**SNR**	$\Delta\theta_{main}$ **(degree)**	$\Delta\theta_{null}$ **(degree)**	**SLL (dB)**
			Scenario 1			
MVDR	9	3	10	0.2	0.7	-12.23
MBPSO	9	3	10	0.33	0.75	-16.73
GSA	9	3	10	0	1.5	-25.789
			Scenario 2			
MVDR	9	3	10	0.1	0.4	-11.922
MBPSO	9	3	10	0.329	0.752	-13.614
GSA	9	3	10	0.1	1.5	-23.239

Table 4.15 Comparison of angular deviation and SLL values of MVDR, ADIWO and GSA algorithm.

	No. of array elements	**No. of Interfering signals**	**SNR**	$\Delta\theta_{main}$ **(degree)**	$\Delta\theta_{null}$ **(degree)**	**SLL (dB)**
			Scenario 3			
MVDR	9	5	10	0.6	0.8	-12.396
ADIWO	9	5	10	0.64	0.2	-15.96
GSA	9	5	10	0.3	1.9	-15.704
			Scenario 4			
MVDR	9	5	30	0.8	0.10	-10.113
ADIWO	9	5	30	1.5	0.18	-21.18
GSA	9	5	30	0.5	0	-26.641

Tables 4.14 and 4.15 show the angular deviation and SLL values for MVDR, MBPSO trained by neural network, ADIWO algorithm and GSA. The stochastic nature of GSA results in greater angular deviation ($\Delta\theta_{main}$) of main lobe from its desired value θ_0 and null deviation ($\Delta\theta_{null}$) from their respective desired value θ_n (n= 1,..., N) compared to MVDR algorithm.

The simulation results show that the angular deviation of main lobe of GSA is less as compared to MVDR, MBPSO and ADIWO algorithms but the null deviation is comparatively more as compared to all the other three algorithms in all the four scenarios. The results also verify that in the first three scenarios, where the SNR is 10 dB the angular deviation of main lobe and the nulls are more while in the last scenario where the SNR is 30 dB, the angular deviation is less. Thus, it shows that with an increase of SNR, the performance of the ABF techniques gradually improves. The GSA also shows more SLL reduction relative to other three algorithms. Finally, it can be concluded that GSA provides sufficient steering ability regarding the main lobe and the nulls, and achieves better SLL, but there is a certain trade off in terms of more null deviation in comparison to all other three algorithms.

In the sections 4.4-4.6 dealing with beamforming, simulation results of ABF algorithms were discussed. The results show that with the increase of antenna elements the beampattern characteristics improves. SMI algorithm shows maximum null depth among LMS and RLS algorithm. RLS algorithm have the fastest rate of convergence and is suitable for tracking vehicles which are moving with fast speed. The GSA is used for ABF and the results are compared with MVDR algorithm. The results show that GSA steers the main lobe in the direction of desired signal and nulls in the direction of interfering signals with reduced SLL.

Summary and Conclusion

CHAPTER 5 SUMMARY AND CONCLUSIONS

The present research work focuses on adaptive array signal processing algorithms viz. DOA estimation and ABF algorithms for smart antennas. The present research work is divided into five chapters. The first chapter discussed the present scenario of wireless communication and the role of smart antenna in wireless communication system. The conventional adaptive array signal processing algorithms of smart antennas and their limitations were discussed. The chapter introduced the metaheuristic approaches and their application in the field of adaptive array signal processing. The second chapter presented a brief literature review of DOA estimation and ABF algorithms. It also reviewed the metaheuristic approaches which are used in this research work. The contribution of researchers to estimate direction of narrow band signals, solve pattern synthesis and ABF problems using metaheuristic approaches and other soft computed techniques were briefly discussed. The research gaps were identified on reviewing the literature in the field of array signal processing and the objectives for the present research work were formulated.

The third chapter introduces the data model and presents a step wise description of all the developed adaptive array signal processing algorithms. All the three categories of DOA estimation viz. spectral estimation methods, eigen structure methods and ML techniques were briefly discussed. A comparison of DOA estimation algorithms with neural network approach for DOA estimation was made. The conventional beamforming algorithms were addressed and compared with neural network based ABF algorithm. The different classes of optimization techniques viz. classical and metaheuristic approaches were presented and a brief introduction to MATLAB tool used in the simulation was addressed. The last section of the chapter discussed the methodology of present research work.

The fourth chapter presented and discussed the simulation results of DOA estimation and ABF algorithms. The simulation result shows that MUSIC algorithm accurately estimate the direction of incoming signals for narrow and wide angular separation of incoming signals. The performance of conventional DOA estimation techniques viz. MUSIC, Root-MUSIC and ESPRIT algorithms improves with the increase in number of array elements, SNR and number of snapshots. GSA accurately and precisely estimates the direction of deterministic BPSK signals as compared to PSO and conventional approaches. PSO algorithm estimates the direction of signals in uncorrelated, partially correlated and coherent channel environment. The radiation pattern characteristics of LMS, SMI and RLS algorithm improves with the increase in number of array elements. RLS algorithm has the fast rate of convergence. The GSA produces beampattern with reduced SLL as compared to MVDR and previous reported results. The present fifth chapter summarizes, concludes and proposed the future work in the field of array signal processing in smart antennas.

Findings and Conclusions

1. The eigen structure based DOA estimation techniques gives better resolution than spectral estimation techniques. The simulation results show that MUSIC algorithm is more accurate than Bartlett and Capon algorithm both in case of narrow and wide angular separation.

2. The performance of MUSIC, Root-MUSIC and ESPRIT algorithm improves with increase in number of antenna elements, SNR and number of snapshots.

3. The simulation results show that MUSIC algorithm proves to be more robust and stable than Root-MUSIC and ESPRIT algorithms having its error reduced to zero above 12 array elements, above 20 dB SNR and above 300 snapshots for wide angular separation of $-5^0$and 10^0.

4. The direction of deterministic BPSK signals are more accurately determined using metaheuristic approaches, GSA and PSO algorithm by optimizing DML function specially in low SNR region. The simulation results verify that GSA gives better performance than PSO and other conventional approaches like MUSIC, Capon and ESPRIT algorithm for narrow angular separation ranging from 4^0 to 1^0 with a step size of 0.5^0. GSA algorithm accurately and precisely estimates the direction of deterministic signals.

5. The direction of random signals with narrow angular separation ranging from 4^0 to 1^0 with a step size of 0.5^0 are more precisely determined by optimizing SML function using PSO algorithm. PSO algorithm gives better statistical performance than conventional approaches in estimating direction of incoming signals for uncorrelated, partially correlated and coherent channel environment.

6. The performance of Capon, MUSIC, ESPRIT and PSO algorithms are evaluated at different correlation coefficient. The RMSE of PSO based ML estimator decreases at -8 dB while RMSE for Capon, MUSIC and ESPRIT decreases at 17 dB, 8 dB and -2 dB. The performance of DOA estimation algorithms are analyzed with varying correlation coefficient ranging from 0.1 to 0.9 at an RMSE of 1 degree. The ML-PSO estimator achieves 1 degree of RMSE at minimum SNR among conventional algorithms.

7. The performance of LMS, SMI and RLS ABF algorithms are analyzed in terms of beampattern characteristics viz. beamwidth, maximum SLL and maximum null depth and the simulation results show that their performance improves with the increase in the number of array elements.

8. The variation of beamwidth and SLL is approximately same for LMS, SMI and RLS algorithm. The SMI algorithm shows more null depth as compared to LMS and RLS algorithm.

9. RLS algorithm has the best rate of convergence among LMS and CMA having its MSE reduced to zero above approximately 20 iterations and is the best choice in the applications where fast tracking is required.

10. The GSA shows fast rate of convergence having its error reduced to zero above 30 iterations, which is an essential requirement for ABF. The algorithm optimizes the single objective function and accurately directs major lobe and nulls in the direction of desired user and interfering signals and results in reduced SLL as compared to MVDR algorithm.

11. The antenna array weights obtained for beamforming by optimizing multi-objective function using GSA produce radiation pattern with reduced SLL at certain angular region as compared to MVDR, MBPSO and ADIWO algorithms. GSA shows sufficient steering ability regarding main lobe and the nulls but there is a trade off in terms of more null deviation as compared to other three algorithms.

Scope of Future work

The present research work covered various aspects of array signal processing which includes different approaches for DOA estimation and ABF. But still their remains certain opportunities to extend the scope of this thesis work. In this research work, the direction of incoming signals and weights for beamforming are estimated for linear arrays. The work can be extended for circular arrays, two-dimensional and three-dimensional array geometries. The performance of DOA estimation and ABF algorithms can be analyzed in the presence of modulated, jamming signals and in constantly changing environment conditions. The field of AI is growing at an exponential rate with the development of new NN structures, fuzzy rules and optimization techniques. New metaheuristic approaches such as coral reef algorithm, bee colony algorithm, cat swarm algorithm and monkey algorithm, etc. can also be explored for obtaining global optimum solution for DOA estimation and ABF problems. Recurrent neural network (RNN) with reduced structural complexity can be incorporated for DOA estimation and ABF. Adaptive neuro-fuzzy inference system (ANFIS) may be explored to get better robustness to the DOA and beamforming algorithms. The optimization capabilities of GSA can be combined with neural network for ABF. The generalization capabilities of GSA trained artificial neural network and with the advancement in the digital circuit technology, real time implementation of ABF with reduced SLL can be performed using field programmable gate arrays (FPGA's).

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VITA

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ABSTRACT

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Major	**:** Electronics & Comm. Engg.	**Advisor**	Dr. Sanjay Mathur
Minor	Computer Science		
Department	**:** Electronics & Comm. Engg.		
Thesis Title	**: Metaheuristic Approaches for Adaptive Array Signal Processing in Smart Antenna**		

In recent years, there has been a substantial growth in the broadband wireless services which results in the increase of traffic for mobile and personnel communication systems. The increasing demand of these services has motivated the researchers worldwide to explore new ways to maximize the spectral efficiency of the network. Smart antennas have emerged as one of the leading technologies which efficiently utilize the network and maximize the system capacity in urban populated areas. An antenna array combined with adaptive signal processing is termed as smart antenna. These antennas have the capability of estimating the direction of desired and interfering signals and accordingly optimizing its radiation pattern in response to changing signal environment. In this thesis, two primary aspects of smart antennas i.e. DOA estimation and ABF are studied, compared and explored using metaheuristic approaches for linear arrays. The direction of deterministic BPSK uncorrelated signals are estimated using GSA based DML estimator and it efficiently estimate direction as compared to PSO and conventional algorithms. The GSA also estimates the direction of signals having narrow angular separation. The direction of random signals are estimated using PSO based SML estimator in uncorrelated, partially correlated and coherent channel environment. The PSO-ML estimator gives best statistical performance in terms of RMSE and resolution probability as compared to conventional algorithms. PSO-ML estimator achieves 1 degree of RMSE at minimum SNR among conventional algorithms. The ABF problem is modeled as an optimization problem and the GSA is used for calculating the weights of linear arrays so as to direct major lobe in the direction of desired signal and nulls in the direction of interfering signals with reduced SLL. The GSA gives superior results than MVDR and previously reported results.

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Advisor

(Abhinav Sharma)
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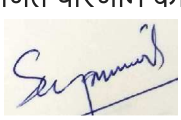
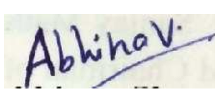
<u>सारांश</u>

नाम	:	अभिनव शर्मा	**परिचायांक**	:	३९३४४
सत्र एवं प्रवेश वर्ष	:	द्वितीय सत्र २०१२-१३	**उपाधि**	:	पीएच° डी°
प्रमुख विषय	:	इलेक्ट्रोनिक्स एवं संचार अभियांत्रिकी	**सलाहकार**	:	डॉ° संजय माथुर
सहायक विषय	:	संगणक विज्ञान			
विभाग	:	इलेक्ट्रोनिक्स एवं संचार अभियांत्रिकी			
शोध ग्रन्थ शीर्षक	:	म□ाहेुरिस्टिक अप्प्रोचेस फॉर अनुकूलक ऐरे सिगनल प्रसंस्करण इन स्मार्ट एंटीना			

हाल के वर्षों में, ब्रॉडबैंड वायरलेस सेवाओं में वृद्धि देखी गयी ह□जिसके परिणाम स्वरूप मोबाइल और पर्सनल संचार प्रणालियों के यातायात में वृद्धि हुई ह□ इन सेवाओं की बढ़ती मांग ने दुनिया भर के शोधकर्ताओं को नेटवर्क के वर्णक्रमीय क्षमता को अधिकतम करने के लिए नए तरीकों का पता लगाने के लिए प्रेरित किया ह□ स्मार्ट एंटेना प्रमुख प्रौद्योगिकियों में से एक के रूप में उभरा ह□जो नेटवर्क का कुशलता पूर्वक उपयोग और प्रणाली की क्षमता का अधिकतम प्रयोग शहरी आबादी वाले क्षेत्रों में करता ह□ अनुकूली सिग्नल प्रोसेसिंग के साथ संयुक्त एंटीना सरणी को स्मार्ट एंटीना के रूप में परिभाषित किया ह□ यह एंटेना वांछित और अवांछित की दिशा का आकलन करता ह□और तदनुसार संकेत माहौल बदल के जवाब में अपनी विकिरण प□र्न को बदलता ह□ इस शोध प्रबंध में, स्मार्ट एंटेना के दो प्राथमिक पहलुओं अर्थात दिशा आकलन और बीमफोर्मिंग का अध्ययन, तुलना और म□ाहेुरिस्टिक अप्प्रोचेस के द्वारा संकेत प्रसंस्करण का पता लगाया गया ह□ नियतात्मक BPSK असहसंबद्ध संकेतों की दिशा जीएसए आधारित डीएमएल आकलनकर्ता का उपयोग कर अनुमान लगाया गया ह□और इसकी तुलना पीएसओ और पारंपरिक एल्गोरिदम से की गयी ह□ जीएसए द्वारा संकीर्ण संकेतों की दिशा का अनुमान लगाया गया ह□ यादृच्छिक संकेतों की दिशा, असहसंबद्ध, आंशिक रूप से सहसंबद्ध और सुसंगत च□ल वातावरण में पीएसओ आधारित एमएल आकलनकर्ता से अनुमान लगाया गया ह□ पारंपरिक एल्गोरिदम की तुलना में पीएसओ आधारित एमएल आकलनकर्ता ने RMSE और संकल्प संभावना के मामले में सबसे अच्छा सांख्यिकीय प्रदर्शन दिया ह□ पीएसओ -एमएल आकलनकर्ता पारंपरिक एल्गोरिदम के बीच न्यूनतम SNR पर 1 डिग्री RMSE प्राप्त करता ह□ अनुकूलक बीमफोर्मिंग एक अनुकूलन समस्या के रूप में मॉडल की गयी ह□और जीएसए रेखीय सरणियों के वजन की गणना के लिए प्रयोग किया गया ह□ताकि सीधा प्रमुख पालि वांछित दिशा में एवं नल्स अमुख्य दिशा में कम SLL के साथ निर्देशित हो। जीएसए ने MVDR एवं पहले के वर्णित परिणाम की तुलना में बेहतर परिणाम दिए ह□



(संजय माथुर)
सलाहकार



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Performance Analysis of Adaptive Array Signal Processing Algorithms

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ABSTRACT

Adaptive antenna is an array of antenna elements with signal processing capability to optimize its radiation pattern in response to the changing signal environment. The adaptive antenna improves the performance of wireless communication by increasing channel capacity and spectrum efficiency, and reduces the cost for establishing new wireless networks. Adaptive antenna aims at increasing the gain in the direction of desired user and direct nulls in the direction of interfering signals. It involves processing of signals induced on an array of antennas that can estimate the direction of radiating sources and calculate optimum weights for adaptive beamforming. This paper presents a performance evaluation of different direction of arrival (DOA) estimation and adaptive beamforming (ABF) algorithms. The simulation results show that multiple signal classification (MUSIC) algorithm provides more accurate and stable results among other DOA estimation techniques while recursive least square (RLS) algorithm shows the fastest convergence rate among other beamforming algorithms. In order to reduce the computation complexity and to increase the convergence speed, the problem of DOA estimation and ABF are approached as a non-linear mapping which can be modelled using a suitable neural network. Artificial neural intelligence provides best solution for real-time implementation of beamforming algorithms in practical systems. The capabilities of adaptive antenna are easily employable to cognitive radio and orthogonal frequency division multiple access (OFDMA) system.

KEYWORDS
ABF; adaptive antenna; DOA; MUSIC; OFDMA; RLS

1. INTRODUCTION

The field of wireless communication has shown a rapid growth in the recent past, which requires higher channel capacities and data rates to satisfy increasing demand of smart phones, tablets, ipads, and other electronic gadgets. The increasing demand of these services without a corresponding increase in radio frequency (RF) spectrum allocation motivates the need for new techniques to improve spectrum utilization. Joseph Mitola first proposed the concept of cognitive radios [1] in 1998 that autonomously coordinate the usage of spectrum. They identify radio spectrum when it is unused by the incumbent (primary, license holding) users and use this spectrum in an intelligent way based on spectrum observation. Another approach which has shown promise in terms of capacity improvement is the use of adaptive array antenna [2]. This antenna has the capability of spatial filtering by steering its beam towards the direction of interest and placing nulls in the direction of interfering signals.

This outcome is achieved by proper weighting of each received signal according to an adaptive algorithm. This concept is called space division multiple access (SDMA). It is used in conjunction with time division multiple access (TDMA), frequency division multiple access (FDMA), and code division multiple access (CDMA) technologies in order to provide the latter with the additional ability of identifying a single user with its unique *spatial signature* [3]. Adaptive antennas were first introduced in 1960s and initially deployed in military communication systems, where narrow beams are used to avoid interference from noise and jamming signals.

The researchers extended the concept of adaptive antennas in wireless communication which offers many advantages like improvement in bit error rate (BER) and network throughput and reduction in fading due to multipath propagation [4], but it could be best explained by words of Andrew Viterbi, a pioneer in the global spread of wireless communication, "*Spatial processing remains as the most promising, if not the last frontier, in the evolution of multiple access systems*" [5].

Signal processing stage allows the antenna beam pattern to adapt to the changing RF environment. Signal processing combined with antenna array produces a directive beam that can be scanned electronically by varying the excitation of the individual elements. The objectives of the signal processing stage are estimation of the direction of arrival (DOA) estimation of all the impinging signals and the calculation of appropriate weights to steer the

Deterministic Maximum Likelihood Direction of Arrival Estimation using GSA

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***Abstract*— Direction of Arrival (DOA) estimation is an important problem in the field of array signal processing. There are many spectral and eigen structure algorithms for estimating direction of narrow band sources. Maximum Likelihood (ML) method is an efficient DOA estimation technique compared to other eigen structure based methods mainly due to its superior statistical performance. In this paper, we use Gravitational Search Algorithm (GSA) to find deterministic ML solution by optimizing a complex nonlinear multimodal function over a high dimensional space in linear arrays. Simulation results show that GSA algorithm gives better performance at lower SNR compared to other heuristic approaches like Particle Swarm Optimization (PSO) and conventional methods like Capon, MUSIC and ESPRIT. The performance of the algorithms are judged in terms of Root Mean Square Error (RMSE) and probability of resolution.**

***Keywords*— Direction of Arrival, Maximum likelihood, GSA, PSO, MUSIC, Capon, ESPRIT, RMSE.**

I. INTRODUCTION

In the field of sensor array signal processing, direction of arrival estimation of narrow band plane waves has been a major area of research due to its widespread applications in radar, sonar, seismic exploration, electronic surveillance and mobile communication. The field of wireless communication has shown a tremendous growth in recent past by employing adaptive antennas which increases the channel capacities and network throughput. These antennas require efficient DOA estimation algorithms for adaptive beamforming.

Several high resolution algorithms, such as multiple signal classification (MUSIC) [1], the minimum variance method of capon [2], estimation of signal parameters via rotational invariance technique (ESPRIT) [3], and many more have been proposed [4]. In [5], authors made a review and comparison of all related DOA estimation techniques. However, in multipath propagation environment where highly correlated or coherent sources are present, these high resolution algorithms fail, since they inherently require the signals to be uncorrelated or lowly correlated. In order to resolve coherency problems, various effective techniques have been developed [6]. Another limitation of these eigen structure based algorithms are that they perform better in high signal to noise ratio (SNR) environment.

DOA estimation techniques can be categorized on the basis of the data analysis into three categories, spectral estimation methods, eigen structure methods and ML techniques. The ML method is a standard technique in statistical estimation theory. This method gives superior statistical performance compared to other methods when the SNR is small, the number of samples are small and the sources are coherent. Initially, a parametric data model is specified and a likelihood function is formulated from the sampled data [7]. The ML estimate is computed by maximizing the likelihood function or minimizing the negative likelihood function with respect to all unknown parameters, which may include the source DOA's, the signal covariance and noise parameters. Due to large computational burden direct maximization of multimodal nonlinear likelihood function over the large parameter space is unrealistic. Thus, different optimization techniques have been considered to jointly estimate source DOA's and other parameters over a high dimensional space.

Many researchers have proposed different optimization techniques to optimize ML function. Alternating projection [8], simulated annealing [9] and data supported grid search [10] algorithm optimize ML function but they cannot guarantee global convergence. In [11], author proposed a modified and refined genetic algorithm to find exact solution of ML function for linear and circular arrays. Particle swarm optimization proposed by Kennedy and Eberhart [12] has already been used as a global optimization technique for DOA estimation [13, 14] and the algorithm also shows robust performance for estimating direction of incoming source in coloured noise fields [15]. In [16], author proposed bacteria foraging optimization (BFO) for accurate DOA estimation in circular arrays. In [17], author proposed GSA for DOA estimation in circular arrays and found that the algorithm gives more accurate results than MUSIC, PSO and BFO algorithms. In this paper, we explore GSA algorithm for optimizing ML functions to accurately estimate source DOA in linear arrays.

The paper is organized as follows. Section II briefly introduces the data model and ML estimation. In Section III and IV, GSA algorithm is briefly discussed and its implementation for deterministic ML DOA estimation has been made. In section V, simulation results of GSA algorithm for ULA are presented and the results are compared with