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ORIGINAL ARTICLE

Evaluation of three unit hydrograph models to predict the surface runoff from a Canadian watershed

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Abstract The predictability of unit hydrograph (UH) models that are based on the concepts of land morphology and isochrones to generate direct runoff hydrograph (DRH) were evaluated in this paper. The intention of this study was to evaluate the models for accurate runoff prediction from ungauged watershed using the ArcGIS® tool. Three models such as exponential distributed geomorphologic instantaneous unit hydrograph (ED-GIUH) model, GIUH based Clark model, and spatially distributed unit hydrograph (SDUH) model, were used to generate the DRHs for the St. Esprit watershed, Quebec, Canada. Predictability of these models was evaluated by comparing the generated DRHs versus the observed DRH at the watershed outlet. The model input data, including natural drainage network and Horton's morphological parameters (e.g. isochrone and instantaneous unit hydrograph), were prepared using a watershed morphological estimation tool (WMET) on ArcGIS® platform. The isochrone feature class was generated in ArcGIS® using the time of concentration concepts for overland and channel flow and the instantaneous unit hydrograph was generated using the Clark's reservoir routing and S-hydrograph methods. An accounting procedure was used to estimate UH and DRHs from rainfall events of the watershed. The variable slope method and phi-index method were used for base flow separation and rainfall excess estimation, respectively. It was revealed that the ED-GIUH models performed better for prediction of DRHs for short duration (≤ 6 h) storm events more accurately (prediction error as low as 4.6–22.8%) for the study watershed, than the GIUH and SDUH models. Thus, facilitated by using ArcGIS®, the ED-GIUH model could be used as a potential tool to predict

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DRHs for ungauged watersheds that have similar geomorphology as that of the St. Esprit watershed.

Keywords GIUH · Unit hydrograph · Direct runoff hydrograph · Geomorphology · Clark's model · Isochrones · ArcGIS®

1. Introduction

Generation of hydrologic data for prediction of hydrologic responses of watersheds is not only cumbersome but also a costly affair. The better alternative of this lies in development of runoff prediction models for ungauged watersheds with acceptable accuracy (Garg *et al.*, 2003). Direct runoff from a watershed depends upon spatial and temporal distribution of rainfall as well as properties of soil and geomorphology (Lee, 1988). Geomorphology reflects the topographic and geometric properties of the watershed and its drainage channel network. It controls the hydrologic processes from rainfall to runoff, and the subsequent flow routing through the drainage network. The unit hydrograph (UH) is one of the widely used approaches for rainfall runoff analysis. Several studies (e.g. Chow *et al.*, 1988; Rodriguez-Iturbe and Rinaldo, 1997; Smart, 1972; Singh, 1988) intended to determine the parameters needed to define a UH based on the geomorphological properties, resulting in the establishment of a theoretical linkage between geomorphology and hydrology through the concept of geomorphological instantaneous unit hydrograph (GIUH) and the direct runoff response of the watershed to an unit impulse of excess rainfall. Gupta *et al.* (1980) improved the GIUH (Rodriguez-Iturbe and Valdes, 1979) to account for arbitrary holding time distribution for channels and overland planes. Wang *et al.* (1981) and Rodriguez-Iturbe *et al.* (1982) further extended the GIUH to consider nonlinear relation between flow velocity and peak discharge in terms of the kinematic wave theory. In these studies, the path probabilities, initial state probability and transition state probability, of a raindrop to reach the watershed outlet were quantified using the geomorphological parameters extracted from Strahler's stream network (Strahler, 1957). The initial state probability and the transition probability are functions of the watershed morphology and channel geometry. The probability density function (pdf) of the watershed holding time, called instantaneous unit hydrograph (IUH) (Gupta *et al.*, 1980), was computed as the multiplication of the path probability of certain path with the pdf of the random holding time for the same path and then summing these products for all possible paths that a raindrop may follow to reach the outlet.

In GIUH based approaches, the excess rainfall depth at a given time, and the path probabilities are completely specified by the watershed morphology. However, the holding time cannot be determined from geomorphologic considerations and need to be postulated. Therefore, the travel time of each individual water particle was considered as a Markov process controlled by the transition probabilities from one state to the other (Rodriguez-Iturbe and Valdes, 1979). This consideration resulted in an exponential holding time distribution for the particles in each channel segment, equivalent to the impulse-response of a linear reservoir (Zhang and Govindaraju, 2003). The advantage of using an exponential holding time is that it only involves one mean residence time (or velocity) parameter, which can be estimated from the basin area (McCuen, 1997). So, incorporation of exponential holding time concept besides the other GIUH techniques is termed as the exponentially distributed GIUH (ED-GIUH) approach for derivation of IUHs and UHs for the study watersheds. Zhang and Govindaraju (2003) developed Geomorphology-based Artificial Neural Network (GANN) and ED-GIUH model for prediction of watershed runoff and compared the performance of both the models.

These models when calibrated and validated for the Back Creek and Indian–Kentuck Creek watersheds in southern Indiana, U.S.A., revealed that there was no significant difference in the peak of the DRHs predicted by the ED-GIUH model and the observed DRHs for most of the rainfall events. However, the ED-GIUH model predicted a shorter tail of DRH recession limb as compared to the GANN model and observed discharges.

Given a GIUH, direct runoff hydrograph (DRH) from the watershed could be generated from the Horton's morphometric parameters (Horton, 1945; Ritter, 2002) and average channel flow velocity. Rodriguez-Iturbe and Valdes (1979) derived a set of basic equations for a 3rd order watershed. This quantitative conceptualization made it possible to generate a GIUH for an ungauged small watershed. Jain *et al.* (2000), for instance, derived peak runoff rate (q_p) and time to peak (t_p) using the GIUH formulas for the rivers in western India. The morphometric parameters required by the formulas were prepared using ArcInfo® GIS software and were used to develop the complete shape of the IUHs using Clark model (Clark, 1945) through a non-linear optimization procedure. Sorman (1995) applied the GIUH model to estimate the peak discharges resulting from various rainfall events for basins in Saudi Arabia. A hydraulic approach was used to estimate both kinematic and dynamic wave velocities. The prediction error of peak discharge varied from 18–34%, and the prediction error of time to peak varied from 22–32%.

As an alternative to the pdf approach, some researchers used the time-area diagram for generation of spatially distributed unit hydrograph (SDUH) (Muzik, 1996; Jain *et al.*, 1997, 2000; Martinez *et al.*, 2001; Kumar *et al.*, 2002). Muzik (1996) applied the SDUH concept on a 229-km^2 forest dominant watershed located on the eastern slopes of the Rocky Mountains in Alberta, Canada. The watershed was divided into 1-km^2 grid cells and the SCS Curve Number (CN) associated with each of the cells was estimated using the soil and land use data. The CN was used to estimate excess rainfall with uniform rainfall intensity of one hour. Each cell was assigned a CN value and hence the estimated excess rainfall was spatially distributed. The incremental one-hour spatially distributed excess rainfalls were then spatially averaged to obtain a representative uniform excess hyetograph for the watershed. The results were in close agreement with the observed hydrographs of the study watershed. Martinez *et al.* (2001) used the ARC Macro Language (AML) of ArcInfo® GIS to generate the coverage of isochrones using uniform flow rate and variable flow rate in each grid in terms of concentration time. The isochrone coverages obtained with different approaches for variable rainfall excess were used to generate the unit hydrograph for two ungauged small rural watersheds near Madrid, Spain. It was revealed that the use of variable flow rate of overland and channel flow concepts could more accurately characterize watersheds and thus could result in a more accurate prediction of the DRHs.

It is a gap in research literature to compare the different concepts and techniques used in the GIS-assisted UH models and to evaluate the accuracy of these models for runoff prediction over ungauged watershed. To fill out this gap, this paper evaluates three GIS-assisted UH models with different conceptual frameworks, including exponential distribution geomorphologic instantaneous unit hydrograph (ED-GIUH) model, GIUH-based Clark model, and spatially distributed unit hydrograph (SDUH) model, in terms of predicting the direct runoff from the St. Esprit watershed, to achieve the objectives of: (1) comparing the concepts and techniques used by the three models; (2) demonstrating how to use ArcGIS® as a tool to facilitate preparation of the model input data, including estimation of travel time, generation of isochrones, and computation of geomorphological parameters; and (3) assessing predictability of the three models in accordance with the recorded flow data of a gauged watershed.

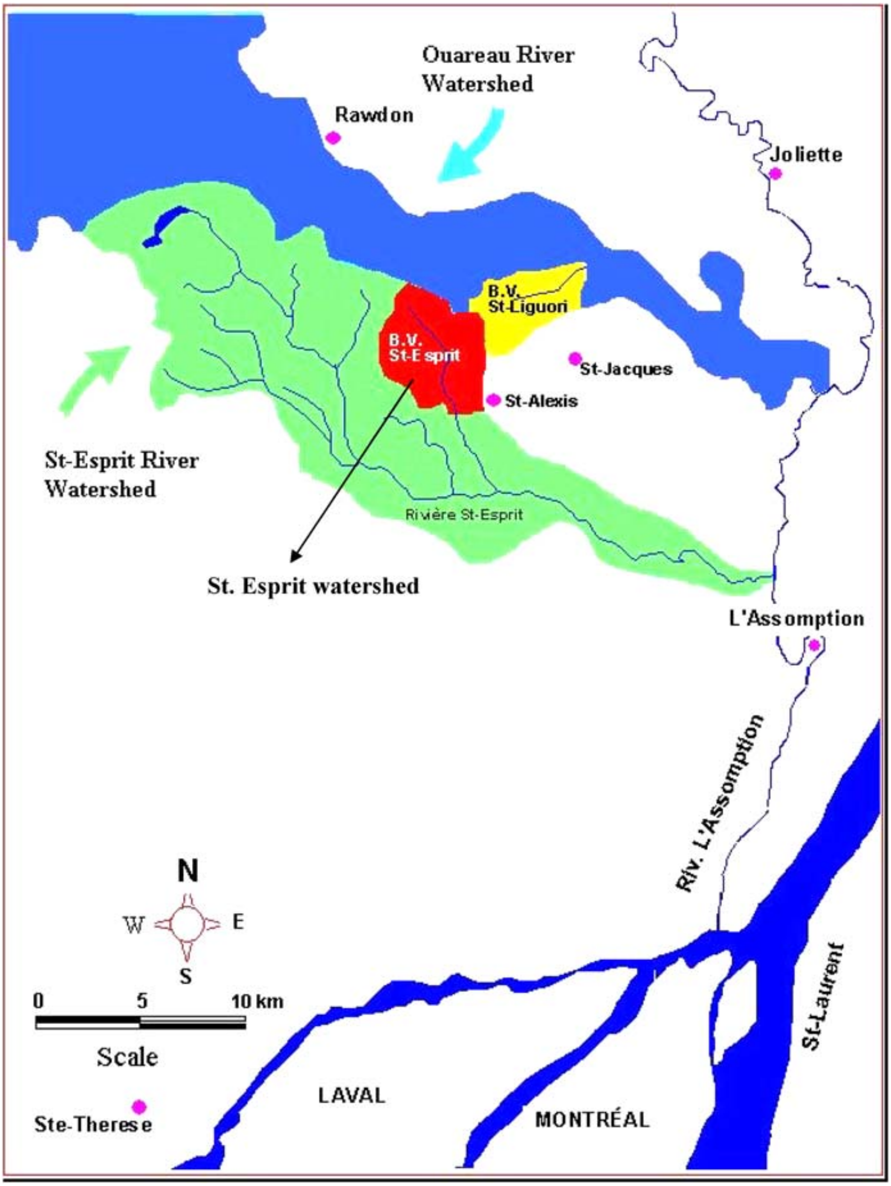

Fig. 1 Location map of the St. Esprit watershed

2. Materials and methods

2.1. Study watershed

The study was conducted in the 26.1-km^2 St. Esprit watershed located approximately 50 km north of Montreal, Canada (Figure 1). The St. Esprit watershed is part of the 210-km^2 St. Esprit river basin that is drained into the L'Assomption River. It is located between 45°55′00″ and

46°00′00″N latitude, and 73°41′32″ and 73°36′00″ W longitude. The maximum difference in elevation from the outlet to the highest point of the watershed is 40 m. The watershed is drained by the 9-km St. Esprit River and its 60.3-km tributaries (Romero *et al.*, 2002). The climate is temperate and the frost-free growing season varies from 122 to 138 days. The mean annual precipitation is 998 mm, approximately 20% of which is snowfall. The mean annual temperature is 5.2 °C and the daily mean temperature in July varies between 18 and 21°C. Soils formed from glacial tills (sandy loams and loams) are located in the upland areas that occupy approximately 37% of the watershed. Soils formed from marine sediments (clay, clay loam) occupy 38% of the watershed, and the balance of the soils (sand to loamy clay) is formed mostly from alluvial deposits (Romero *et al.*, 2002). About 64% of the watersheds are planting corn, cereal, soybean, vegetable, hay, and pasture.

The rainfall and runoff for the years of from 1994–1997 were recorded at an interval of 15 min. The four-year data, recorded during the months from May to September when snowfall was negligible, were used to set up the models, to eliminate the uncertainty in estimation of runoff rate and channel flow behavior that might be caused by ice cover over the watershed. Twenty runoff producing events of varying intensities and durations were used for analysis, out of which the detailed analysis of five distinct rainfall events representing the most possible variations of intensities and durations are presented in this paper. The Manning's roughness coefficient "n" was estimated to be 0.03 (Lapp *et al.*, 1998) and used in this study.

2.2. Estimation of Horton's parameters

The watershed morphology estimation tool (WMET), written in VBA macro on ArcGIS® platform by Sarangi *et al.* (2004), was used for watershed and stream network delineation from the digital elevation model (DEM) of the study watershed. The delineated stream network was also compared with the existing drainage pattern of the study watershed acquired from the Canadian Digital Elevation Data (CDED) sources. It was revealed that the drainage network obtained from the WMET tool was in line with the CDED drainage map. The delineated streams were grouped and labeled as 1–4 on the Strahler's ordering scheme (Strahler, 1957; Figure 2). Each of the four groups may include 1–40 streams, which in average are 0.4–2.7 km long and drain 0.3–26.1 km^2 (Table 1). Table 1 also shows the values of the Horton's geomorphological parameters, including bifurcation ratio (R_B), stream length ratio (R_L), and area ratio (R_A). The representative values of R_B, R_A, and R_L for the study watershed were determined by plotting the estimated values on a semi-logarithmic paper to be 3.6, 4.7, and 1.9, respectively (Sarangi *et al.*, 2005). These representative values are within the ranges reported by Ritter *et al.* (2002) for natural watersheds.

Table 1 Geomorphological and Horton's parameters of St. Esprit watershed

Stream order	No of streams	Mean area $\bar{A}_i$ (km^2)	Mean length $\bar{L}i$ (km)
1	40	0.3	0.4
2	17	1.0	1.5
3	3	5.9	2.4
4	1	26.1	2.7

$R_A = 4.7, R_B = 3.6, R_L=1.9$

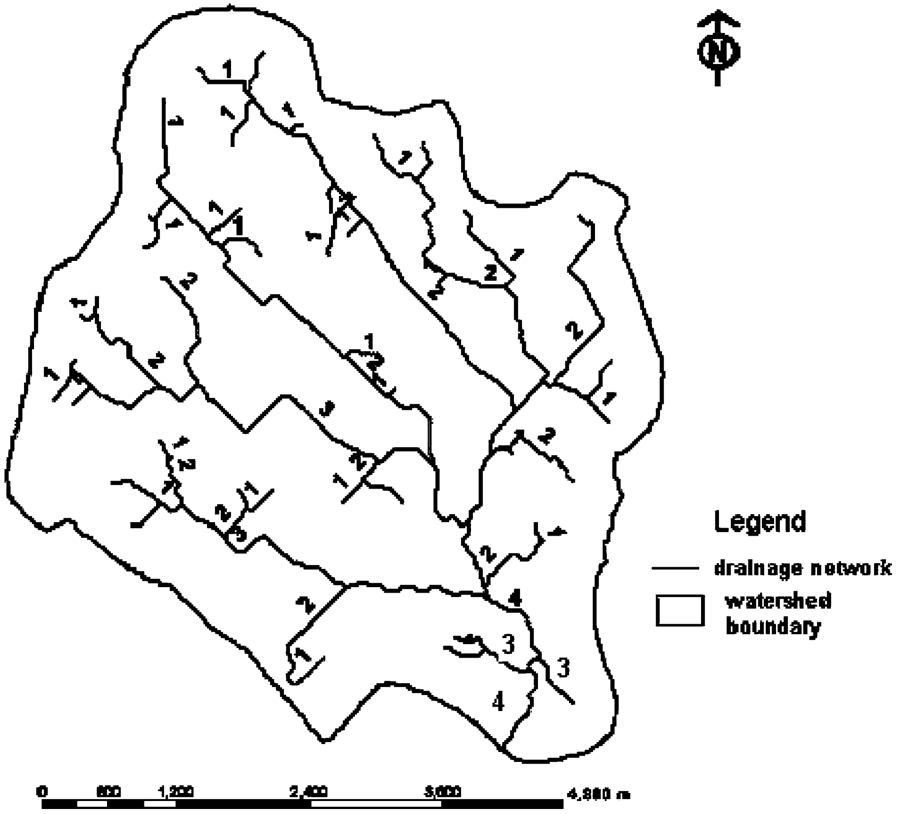

Fig. 2 The St. Esprit watershed, showing the streams delineated using ArcGIS® and ordered on the Strahler's ordering scheme

2.3. The ED-GIUH model

The ED-GIUH model is based on the probability density function (pdf) of a random raindrop falling on the watershed to reach the watershed outlet. The model can be expressed as (Jain and Sinha, 2003):

$$\frac{\mathrm{d}p_{\Omega+2}(t)}{\mathrm{d}t} = \sum_{i=1}^{\Omega} p_i(0) \frac{\mathrm{d}\Phi_{i(\Omega+2)}(t)}{\mathrm{d}t} \tag{1}$$

where, Ω = the highest Strahler's order; $p_{\Omega+2}(t)$ = the state probability, the probability that the random raindrop is on state $(\Omega + 2)$ at time t; $p_i(0)$ = the initial state probability, the probability that the random raindrop is on state i at time zero; $\Phi_{i(\Omega+2)}(t)$ is the transition probability from state i to state $\Omega + 2$, the probability that the random raindrop on state i at time zero will be transmitted to state $\Omega + 2$ at time t.

Applied to the St Esprit watershed, Equation (1) can be rearranged as:

$$\frac{\mathrm{d}p_6(t)}{\mathrm{d}t} = p_1(0) \cdot \frac{\mathrm{d}\Phi_{16}(t)}{\mathrm{d}t} + p_2(0) \cdot \frac{\mathrm{d}\Phi_{26}(t)}{\mathrm{d}t} + p_3(0) \cdot \frac{\mathrm{d}\Phi_{36}(t)}{\mathrm{d}t} + p_4(0) \cdot \frac{\mathrm{d}\Phi_{46}(t)}{\mathrm{d}t} \tag{2}$$

Table 2 Estimation of transition probabilities (p_{ij}) of St. Esprit watershed

Formula	Value
$p_{12} = \frac{2}{R_B} + \frac{(2R_B-1)(R_B^2-2R_B)}{R_B^2(2R_B-1)+R_B(R_B^2-1)+(R_B^2-1)(R_B-1)}$	0.7869
$p_{13} = \frac{(R_B^2-1)(R_B-1)}{R_B^2(2R_B-1)+R_B(R_B^2-1)+(R_B^2-1)(R_B-1)}$	0.2012
$p_{14} = \frac{(R_B^2-1)(R_B-1)(R_B-2)}{R_B^3(2R_B-1)+R_B^2(R_B^2-1)+R_B(R_B^2-1)(R_B-1)}$	0.0893
$p_{23} = \frac{R_B-2}{2R_B-1} + \frac{2R_B}{R_B^2}$	0.8138
$p_{24} = \frac{(R_B-1)(R_B-2)}{R_B(2R_B-1)}$	0.1861
p_{34}	1.0000

The transition state probability $\Phi_{i(\Omega+2)}$ and initial state probability $p_\Omega(0)$ need to be evaluated for solving Equation (2).

A given path is composed of one overland plane (o_i) and one or more channels (c_i). The number of possible paths is less than or equal to $2^{\Omega-1}$ ($2^{4-1} = 8$ for the St. Esprit watershed). The probability of a raindrop to be transmitted from o_i to c_i is given by the initial state probability $p_i(0)$, whereas the probability that the raindrop moves from a stream of order i to a stream of order j is designated as transition probability p_{ij}. The transition state probability $\Phi_{i(\Omega+2)}$ is a function of p_{ij} and the inverse of mean waiting time λi in a stream of order i. The relationship for calculation of $\Phi_{i(\Omega+2)}$ was obtained through inverse exponential transformation (Jain and Sinha, 2003), which for the St. Esprit watershed is given as:

$$\Phi_{i6}(t) = a_{i6} + e^{-\lambda_1 t} \cdot b_{i6} + e^{-\lambda_2 t} \cdot c_{i6} + e^{-\lambda_3 t} \cdot d_{i6} + e^{-2\lambda_4 t} \cdot t \cdot e_{i6} + e^{-2\lambda_4 t} \cdot f_{i6} \qquad (3)$$

where, a_{i6}, b_{i6}, c_{i6}, d_{i6}, e_{i6}, and f_{i6} are coefficients.

The p_{ij} is the function of the Horton's R_B (Rodriguez-Iturbe and Rinaldo, 1997; Gupta *et al.*, 1980; Jain and Sinha, 2003), and was estimated to be 0.0893–1.0 for the St. Esprit watershed (Table 2). The $\lambda_i{}^{-1}$ includes both the time spent as overland flow and channel flow. The importance of overland mean waiting time appears to be rather lower than the stream waiting time, as the raindrops draining directly by the overland flow are less in number (Jain and Sinha, 2003; Zhang and Govindaraju, 2003). The λ_i for the St. Esprit watershed is determined by:

$$\begin{cases} \lambda_4 = \dfrac{\nu}{L_4} \\ \lambda_i = \lambda_4 R_L^{\mathrm{int}\left(\frac{4}{i}\right)} \quad i = 1, 2, 3 \end{cases} \qquad (4)$$

where, v = the average velocity in the watershed; L_4 = the length of the 4th order stream.

The $p_i(0)$ is the function of the Horton's R_A and R_B, and p_{ij}. It was estimated to be 0.1797–0.386 for the St. Esprit watershed (Table 3). Given the $p_i(0)$, p_{ij}, and λi, the probabilities for a raindrop falling on the overland plane o_i to be transmitted to the 4th order stream c_4 in the St. Esprit watershed could be estimated as 0.034–0.247 (Table 4).

Table 3 Estimation of initial state probabilities ($p_i(0)$) of St. Esprit watershed

Formula	Value
$p_1(0) = \frac{N_1 \bar{A}_1}{A_4}$	0.386
$p_2(0) = \left(\frac{R_B}{R_A}\right)^2 - \left(\frac{R_B}{R_A}\right)^3 p_{12}$	0.2319
$p_3(0) = \left(\frac{R_B}{R_A}\right) - \left(\frac{R_B}{R_A}\right)^2 p_{23} - \left(\frac{R_B}{R_A}\right)^3 p_{13}$	0.2024
$p_4(0) = [1 - p_1(0) - p_2(0) - p_3(0)]$	0.1797

Table 4 Estimation of path probabilities $p(s)$ of St. Esprit watershed

Path number	Path	Path probability
1	$o_1 \rightarrow c_1 \rightarrow c_2 \rightarrow c_3 \rightarrow c_4$	0.247344
2	$o_1 \rightarrow c_1 \rightarrow c_2 \rightarrow c_4$	0.056575
3	$o_1 \rightarrow c_1 \rightarrow c_3 \rightarrow c_4$	0.077710
4	$o_1 \rightarrow c_1 \rightarrow c_4$	0.034501
5	$o_2 \rightarrow c_2 \rightarrow c_3 \rightarrow c_4$	0.188752
6	$o_2 \rightarrow c_2 \rightarrow c_4$	0.043173
7	$o_3 \rightarrow c_3 \rightarrow c_4$	0.202446
8	$o_4 \rightarrow c_4$	0.129177

2.4. The GIUH-based Clark model

The GIUH-based Clark model can be expressed as (Jain *et al.*, 1997):

$$\begin{cases} u_i = \mathrm{CI}_i + (1 - C)u_{i-1} \\ C = \dfrac{\Delta t}{R + 0.5\Delta t} \end{cases} \tag{5}$$

where, u_i = the ith ordinate of the IUH; I_i = the ith ordinate of the time-area diagram; C = the routing coefficient; R = the storage coefficient; and Δt = the time interval.

The time-area diagram (Figure 3) was generated by determining the drainage areas within the watershed corresponding to the various lengths of concentration time (t_c). The t_c in min was computed by using the Kirpich's formulae (Kirpich, 1940), which is given as,

$$t_c = 0.0195 L^{0.77} S^{-0.385} \tag{6}$$

where L is the maximum length of the watershed in *m*; and S is the average overland slope of the watershed in m/m.

The corresponding area with respect to the estimated time was determined using ArcGIS®. The R is estimated using a non-linear optimization procedure (Jain *et al.*, 2000; Kumar *et al.*, 2002).

2.5. The SDUH model

The SDUH model is based on the concept that the unit hydrograph ordinate ($U(t)$) at time t equals to the slope of the cumulative watershed time-area curve over the time interval [$t - \Delta t$, t] (Maidment, 1993). The model can be expressed as:

$$U(t) = \frac{Q_{\mathrm{DRH}}(t)}{i_e \cdot \Delta t} = \frac{A(t) - A(t - \Delta t)}{\Delta t} \tag{7}$$

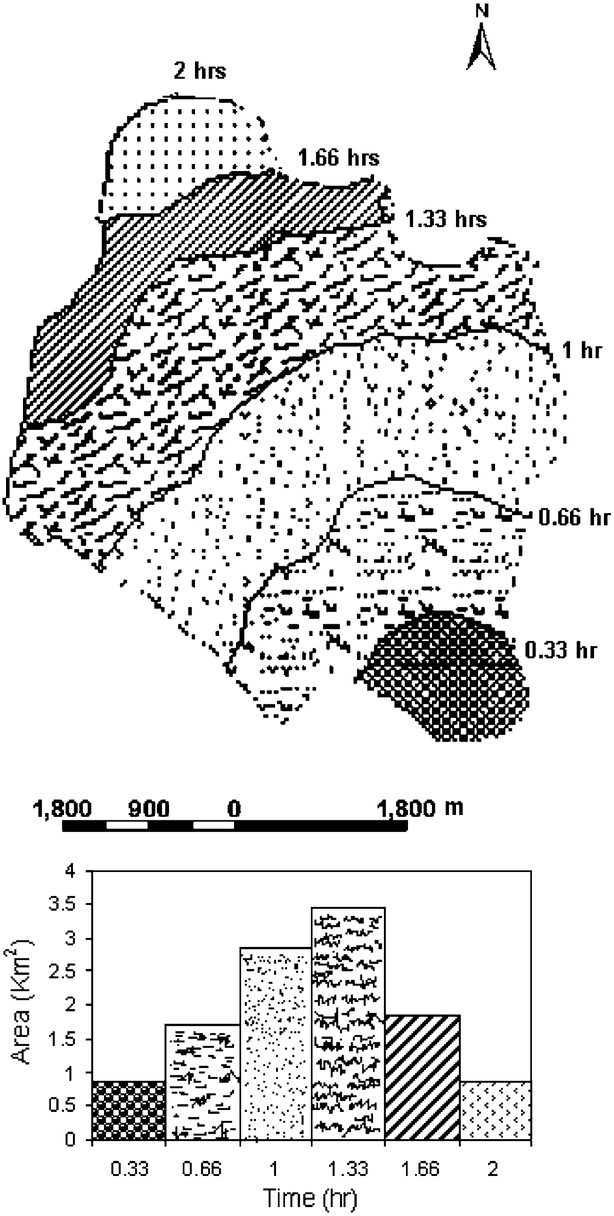

Fig. 3 The Isochrone coverage and time-area diagram for the GIUH-based Clark Model

where, $A(t)$ and $A(t-\Delta t)$ = the areas contributing to the runoff at time t and $t-\Delta t$, respectively; $Q_{\mathrm{DRH}}(t)$ = the DRH ordinate at time t; i_e = excess rainfall intensity; and $i_e\Delta t$ = the excess rainfall depth within the time interval Δt, which is the difference in S-hydrograph ordinate at time t and its value lagged by time Δt (Muzik, 1996).

Unlike the GIUH-based Clark model, the t_c for the SDUH model was estimated using the variable flow method, in which the times taken by the raindrop to flow over the land surface

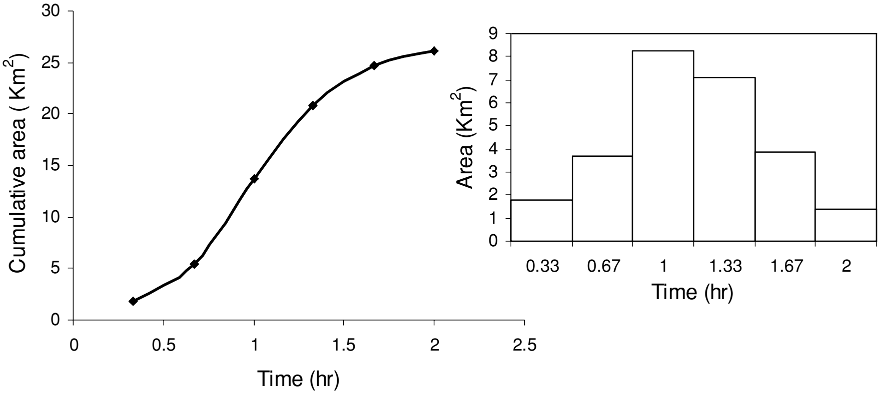

Fig. 4 The cumulative time area curve for the SDUH model

and channel are computed independently, and then summed to get the total travel time T_t. The travel time over the land surface was estimated using the Kirpich's formulae, whereas the travel time through the channel was estimated as the ratio of the channel length to the average flow velocity. The average flow velocity was estimated to be 0.4 m/s in terms of the stage-discharge data observed from 1994–1997 of the study watershed.

The values of T_t for different locations over the watershed were estimated by using the line coverage feature of the delineated stream network and by estimating the length to overland flow and channel flow for each individual raindrop to reach the outlet. The drainage area within the watershed corresponding to T_t was determined using the ArcGIS® geostatistical extension, and was cumulated to generate the cumulative time-area curve in Figure 4.

2.6. Prediction of direct runoff

While the SDUH model gave the UH, the outputs of the ED-GIUH model and the GIUH-based Clark model were the IUHs. To predict the DRH, the IUHs need to be converted to the corresponding UHs for a given time interval Δt using Equation (8) (Jain *et al.*, 1997):

$$U_i = \frac{1}{N}(0.5u_{i-N} + u_{i-N+1} \ldots + u_{i-1} + 0.5u_i) \tag{8}$$

where, U_i = the ith ordinate of the UH; N = the number of computational intervals; and u_i = the ith ordinate of the IUH.

The DRH was generated by multiplying $U(t)$ in Equation (7) or U_i in Equation (8) by the excess rainfall depth of an evaluation rainfall event. In this study, five rainfall events, occurred on June 13, June 27, and August 4, 1994 and July 15 and July 19, 1996, respectively, were used to evaluate the models. The excess rainfall intensities corresponding to these five rainfall events were estimated using a uniform infiltration rate (i.e. a constant Φ-index). The Φ-index was evaluated using the DRH (HEC, 2000) that equals to the observed flow hydrograph less the base flow that was determined using the variable-slope method (Chow *et al.*, 1988).

2.7. Model evaluation criteria

The three models have different theoretical bases. The ED-GIUH model is based on mathematical association of the geomorphological parameters, the initial state probability of a raindrop, and the transition probability of the raindrop. The GIUH-based Clark model uses a time-area diagram defined from isochrones generated from the Kirpich's formulas. The SDUH model is based on a cumulative time-area curve defined in terms of variable flow concept and an S-hydrograph. So, these three UH models incorporate different degree of complexities by associating the morphological parameters and water translation patterns over the land surface. Physically, the ED-GIUH method considers the probability of water movement both as overland and channel flow along with exponential travel time to estimate the runoff at watershed outlet. The GIUH-based Clark method considers single flow based travel time where as the SDUH concept considers variable flow concept for estimation of travel time of runoff water to reach the outlet. However, the SDUH method does not consider the geomorphologic parameters as in case of other two GIUH based methods. All the three models can be applied to the ungauged watersheds having geomorphological details. However, the recoded hydrological data will be useful for model calibration and validation for future runoff predictions. Though, both the GIUH-based Clark method and SDUH methods employ the time-area concept, but the former estimates lumped travel time parameter using Kirpich's formulae without explaining the spatial variation of the travel time from overland and channel sections. However, the SDUH method considers the spatial variation of travel time but neglects the geomorphological aspects of the watershed. Where as, the ED-GIUH method considers the geomorphological parameters, the flow path transitional probabilities and the exponential travel time in predicting the IUHs of the watersheds. When compared with other two models, the SDUH model was observed to be based on simpler mathematical concepts and required less input data. The GIUH-based Clark model is more data intensive than the SDUH model as it requires the Horton's morphological parameters besides the input parameter for estimation of time-area information. Therefore, considering the merits and the limitations of these three models with respect to the watershed hydrological processes occurring in reality, conceptually, the ED-GIUH model may approximate the watershed hydrologic responses in generation of runoff more accurately than the other two models.

In this study, visualization and error function (Equation (9); Lee *et al.*, 1972) were used to evaluate the predictability of the three models for the St. Esprit watershed. The purpose of visualization is to assess the overall similarity between the observed and generated DRHs, whereas the error function is an indicator of the model performance to predict an observed peak discharge and the time to peak.

$$\mathrm{ERR} = \left[\left(\frac{q_p - \hat{q}_p}{q_p}\right)^2 + \left(\frac{t_p - \hat{t}_p}{t_p}\right)^2\right]^{1/2} \times 100 \tag{9}$$

where, ERR = the error function; q_p and t_p = the observed peak discharge and time to peak; and $\hat{q}_p$ and $\hat{t}_p$ = the generated peak discharge and time to peak.

3. Results and discussion

Five distinct rainfall events, occurred on June 24, June 13, and August 4, 1994, July 15, and July 19, 1996, respectively, were used to evaluate the three models. The duration of

Table 5 Summary statistics of the five evaluation rainfall events

Rainfall event	Duration (h)	Peak flow (m^3/s)	Time to peak (h)	Total rainfall (mm)	Excess rainfall (mm)
June13, 1994	5.0	2.8	5.5	19.6	3.9
June 27,1994	5.5	9.6	6.0	40.2	12
August 4, 1994	5.25	1.9	5.5	19	2.7
July 15, 1996	4.75	2.2	5.0	16.4	2.3
July 19, 1996	10.0	1.78	11.0	21.5	3.6

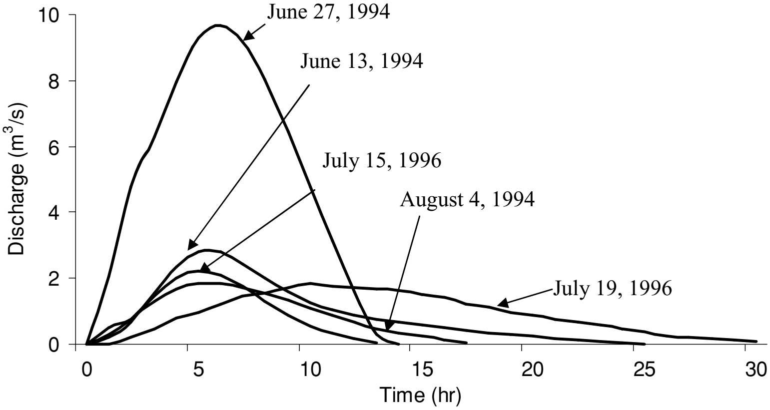

Fig. 5 The Direct Runoff Hydrographs (DRHs), generated using the variable slope method from the observed flow hydrograph for the five evaluation rainfall events of the St-Esprit watershed

these events varied from 4.75 to 10 h with total rainfall depth ranging from 16.4–40.2 mm, resulting in peak discharges of ranging from 1.78 to 9.6 m^3/s (Table 5). The DRHs of these events were derived from the observed flow hydrographs using the variable-slope method (Figure 5). The excess rainfall depths were estimated to be 2.3–12 mm, with excess rainfall intensities of ranging from 3.0–12.0 mm/h. The reprehensive excess rainfall intensity was determined as 8.0 mm/h and used in the SDUH model (Equation 6). The IUH generated by the ED-GIUH model has a peak of 18.11 m^3/s and a time to peak of 1.85 h (Figure 6). The 1-h UH associated with 10-mm excess rainfall was derived from the IUH in terms of Equation (7). The UH has a peak of 18.78 m^3/s and a time to peak of 2.0 h.

Using the linear optimization procedure, the R value for the GIUH-based Clark model was determined to be 1.22 h. Incorporating the value of R in Equation (5), the ordinates of IUH were generated and converted to 1-h UHs associated with 10-mm excess rainfall (Figure 7). The generated UH has a peak of 26.21 m^3/s and a time to peak of 3.0 h. Compared with the ED-GIUH model (Figure 6), the UH peak obtained from GIUH-based Clark model concept is 8.1 m^3/s higher but 1.01 h later. Again, using the SDUH model, a 1-h UH (Figure 7) with a peak of 20.12 m^3/s and a time to peak of 3.0 h was obtained. The peak of this UH is 2.01 m^3/s higher than the ED-GIUH model (Figure 6) but 6.09 m^3/s lower than the GIUH-based Clark model (Figure 7). And the time to peak is 1.01 h later than the ED-GIUH model but coincident with the GIUH-based Clark model.

The DRHs resulting from the five evaluation rainfall events were generated using the three models and were visually assessed in accordance with the corresponding observed

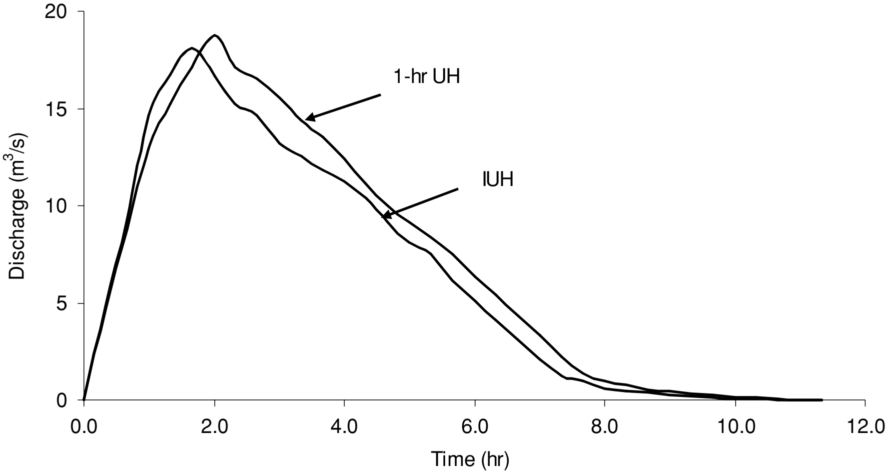

Fig. 6 The Instantaneous Unit Hydrograph (IUH) generated using the ED-GIUH model and the corresponding 1-h Unit Hydrograph (UH) associated with 10-mm excess rainfall

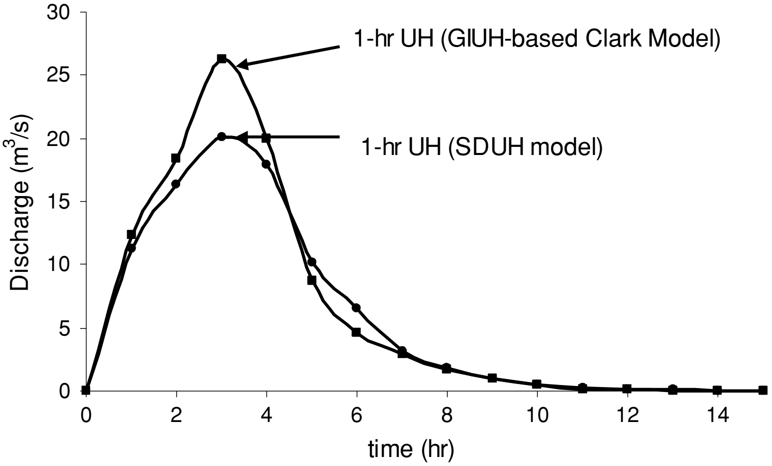

Fig. 7 The 1-h Unit Hydrographs (UH) generated using GIUH-based Clark Model and SDUH models associated with 10-mm excess rainfall

DRHs shown in Figure 5. The performance of all the three discussed models in prediction of the DRHs corresponding to the events of June 27, 1994 and July 19, 1996 are presented in Figures 8 and 9, respectively. The event of June 27, 1994 might be characterized to be short-duration but high-intensity, whereas the event of July 19, 1996 might be characterized reversely. Both the ED-GIUH model and SDUH model could predict the DRH with ERR value of 4.6 and 16.7% respectively for the event of June 27, 1994, but the GIUH-based Clark model failed (ERR = 42.1%) to predict within acceptable accuracy. And none of the three models could satisfactorily predict the DRH from the event of July 19, 1996. However, the ERR value for the ED-GIUH model was 36.8% in comparison to 62.1% for GIUH based Clark Model and 52.5% for SDUH model (Table 6). Above all, the shorter tail of the recession

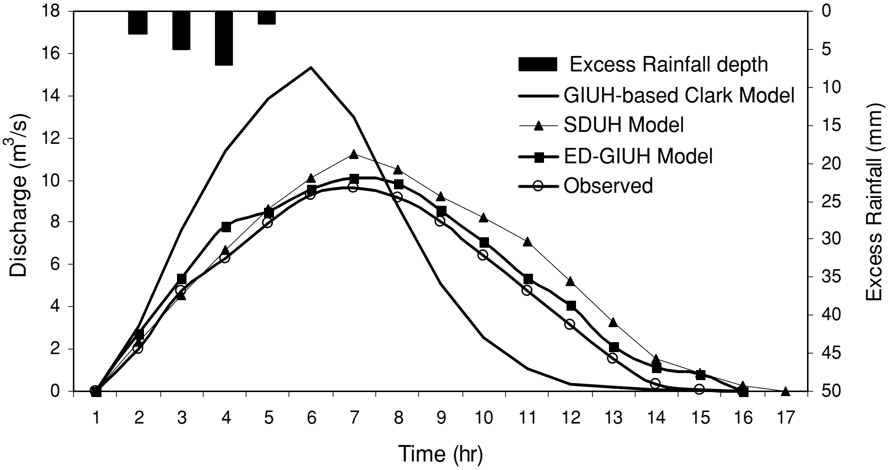

Fig. 8 The observed and generated Direct Runoff Hydrographs (DRHs) from the rainfall event of June 27,1994

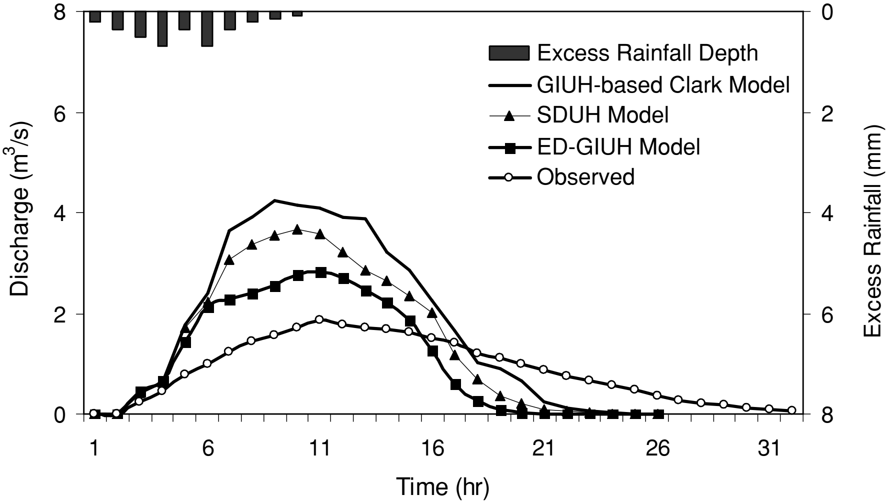

Fig. 9 The Observed and generated Direct Runoff Hydrographs (DRHs) from the rainfall event of July19, 1996

limb of the DRH obtained from ED-GIUH model in this study was in conformity with the observation of Zhang and Govindaraju (2003) for rainfall events of low intensity and longer durations. These rainfall events also affect the antecedent moisture condition of the watershed resulting in improper estimation of path probability and travel time values. So, the failure of the models could be attributed to that the concepts of path probability and travel time on which the models are based could not approximate the hydrologic conditions of long duration and low intensity rainfall events. The computed error functions to predict the five evaluation events for three different models revealed that the ERR value ranged from 4.6 to 36.8% for the ED-GIUH model, 40.1–62.1% for the GIUH-based Clark model, and 16.7–52.5% for

Table 6 Error functions using the three models to predict the direct runoffs from the five evaluation rainfall events

	Observed		Predicted using the ED-GIUH model			Predicted using the GIUH-based clark model			Predicted using the SDUH model		
Rainfall event	q_p	t_p	$\hat{q}_p$	$\hat{t}_p$	ERR (%)	$\hat{q}_p$	$\hat{t}_p$	ERR (%)	$\hat{q}_p$	$\hat{t}_p$	ERR (%)
June 13,1994	2.8	5.5	3.2	5.5	9.8	4.7	5	40.1	3.5	5	20.2
June 27, 1994	9.7	6.0	10.1	6	4.6	15.3	5	42.1	11.2	5.5	16.7
August 4, 1994	1.9	5.5	2.1	5.0	13.4	3.3	6.0	43.3	2.9	5.5	34.5
July 15, 1996	2.2	5.0	2.9	5.0	22.8	4.2	5.5	47.9	3.3	5.25	32.7
July 19, 1996	1.78	11.0	2.8	11.0	36.8	4.23	9.0	62.1	3.68	10.0	52.5

Note: q_p and $\hat{q}_p$: Observed and predicted peak discharges, respectively (m^3/s); t_p and $\hat{t}_p$: Observed and predicted times to peak, respectively (h) the SDUH model (Table 6). Overall, the ED-GIUH model, which is based on the concept of path probability and exponential distribution of travel time, could satisfactorily predict the DRH from a short-duration but high-intensity rainfall event for the St. Esprit watershed. The better predictability of ED-GIUH model ($4\% \geq \text{ERR} \leq 22.8\%$) was also observed in all available runoff producing events ranging from 0.5 to 6 h durations occurring over the St-Esprit watershed during snow less periods. The SDUH model also performed better ($10\% \geq \text{ERR} \leq 35\%$), while the GIUH-based Clark model performed poorly ($40\% \geq \text{ERR} \leq 48\%$) for the same events. However, for a long-duration and low-intensity rainfall event, while the ED-GIUH model still had a better performance than the other two models, none of the three models could satisfactorily predict the DRH as indicated by a high error function for the rainfall event of July 19, 1996 in Table 6. The poor performance of the other two models could be attributed to the lumped approach for estimating the velocity in the GIUH-based Clark model and to the inappropriate representative rainfall intensity (8.0 mm/h) in the SDUH model. This can also be attributed to the morphological nature of the study watershed, which is relatively flat having sandy and loamy soils with localized depressions, which reduces the flow velocity and increases the travel time of the water to reach the outlet for low intensity and longer duration events.

4. Summary and conclusions

The three models evaluated in this study, ED-GIUH, GIUH-based Clark, and SDUH, are based on morphology, flow path probability, and travel time responses over land surfaces and through channels. In this study, ArcGIS® was used to estimate the watershed morphological parameters, prepare the model input data (e.g. isochrones), which is an innovative approach to apply these models not only to the St. Esprit watershed but also to the others if needed. The validation of successful prediction of the ED-GIUH model can be applied to any ungauged watershed with similar morphology of the study watershed. Moreover, incorporation of the vector based approach in this study for estimation of travel time in SDUH method and use of built-in macros within ArcGIS® for calculation of model parameter values and subsequent analysis for model predictions proved to be more efficient and useful. Finally, in terms of predicting the DRHs from the five evaluation rainfall events, the ED-GIUH model had a better performance than the other two models. However, for a long-duration and low-intensity rainfall event (e.g. July 19, 1996 event), none of the three models could satisfactorily predict the DRH. Failure of the models for this event could be attributed to either an inappropriate method to estimate the model parameters or a poor representative parameter value (e.g. representative rainfall intensity in the SDUH model and the exponential time distribution in ED-GIUH model for long duration rainfall events). It can also be concluded that, SDUH model may be used as an alternative to ED-GIUH model in predicting the DRH, when limited input data is available for analysis and slightly higher prediction error is acceptable.

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