

Flow and Heat Transfer of Cu-water Nanofluid over Stretching Sheet with Stefan Blowing Effect

Thesis

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By

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"God wished to send his message to us and so he gave us teachers"

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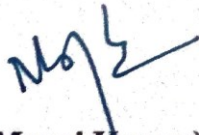

(Shubham Negi)
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CERTIFICATE

This is to certify that the thesis entitled “**Flow and Heat Transfer of Cu-water Nanofluid over Stretching Sheet with Stefan Blowing Effect**”, submitted in partial fulfilment of the requirements for the degree of **Master of Science** with major in **Mathematics** and minor in **Computer Science** of the College of Post-Graduate Studies, G.B. Pant University of Agriculture and Technology, Pantnagar, India, is a record of *bona fide* research carried out by **Mr. Shubham Negi, Id. No. 53951** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been duly acknowledged.

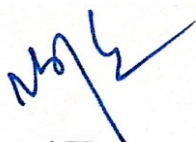
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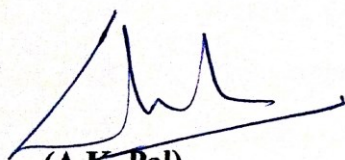
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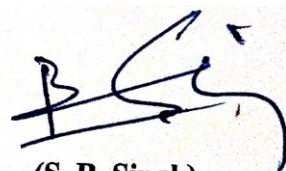
We, the undersigned, members of the Advisory Committee of **Mr. Shubham Negi, Id. No. 53951**, a candidate for the degree of **Master of Science** with major in **Mathematics** and minor in **Computer Science**, agree that the thesis entitled “**Flow and Heat Transfer of Cu-water Nanofluid over Stretching Sheet with Stefan Blowing Effect**”, may be submitted in partial fulfilment of the requirements & for the degree.



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ABSTRACT

RESEARCH PAPER

PLAGIARISM CHECK: ORIGINALITY REPORT

NOMENCLATURE

A_o, B_o	Constant
B	Magnetic field
C	Constant
C	Concentration of the fluid
C_f	Skin friction coefficient
C_o, D_o	Constant
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
E	Activation energy parameter
E_a	Activation energy
E_o, F_o	Constant
F_w	Blowing parameter
f'	Dimensionless velocity
Gr_x	Local Grashof number
G	Acceleration due to gravity
J	Mass flux
K	Thermal conductivity
k_o	Permeability
K_o	Chemical reaction parameter
Kr	Reaction rate
k^*	Mean absorption coefficient
K'_1	Slip length
M	Fitted rate constant
M	Magnetic parameter
Nb	Brownian motion parameter
Nr	Buoyancy ratio parameter
Nt	Thermophoretic parameter

Nu_x	Local Nusselt number
P_o	Porosity parameter
Pr	Prandtl number
q_r	Radiation heat flux
Q	Heat flux
Re_x	Local Reynolds number
Rd	Thermal radiation parameter
Sc	Schmidt number
T	Temperature of the fluid
U	Velocity component in x- direction
V	Velocity component in y- direction
V	Velocity
W	Slip parameter
X, Y	Cartesian coordinate system

Greek Symbols

β	Thermal expansion coefficient
κ	Boltzmann constant
δ	Temperature difference parameter
γ_t	Thermal relaxation time parameter
γ_c	Solutal relaxation time parameter
η	Similarity variable
Θ	Dimensionless temperature
λ	Mixed convection parameter
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Effective density
ρc_p	Heat capacity
σ	Electrical conductivity
τ	Ratio of the effective heat capacity of nanomaterial and base fluid

τ_t	Heat flux relaxation time
τ_c	Mass flux relaxation time
σ^*	Stefan-Boltzmann constant
φ	Dimensionless concentration
ϕ	Volume fraction of nanoparticle

Superscript

'	Derivative with respect to η
---	-----------------------------------

Subscript

F	Base fluid
Nf	Nanofluid
P	Solid particle
W	Wall
∞	Ambient condition

1.1 Fluids: An Overview

Matter are categories as solid and fluid. The solid comprises larger number of molecules in a given volume and the attraction between the molecules is very large whereas the fluid has a smaller number of molecules in a given volume as compared to solid and the attraction between the molecules is less as compares to solid and hence the molecules of fluid are in motion inside it. Fluid further classified as liquid and gaseous. Liquid has definite volume; it is held in the vessel but gases shows expansion nature and diffuses into atmosphere. Gravity is the factor which distinguish fluids as gases and liquids. Liquids are the fluids which flow due to the gravitational force while gases are those which flow independent to gravitational force.

Spacing between the molecules of fluid is continuum and uniformly distributed. Fluid statistics and fluid dynamics are the branches of Fluid mechanics. Fluid statistics is the study of fluid when it as at rest and fluid dynamics is the study of fluid, when it is in motion.

The fluid has the tendency to flow due to the shear stress applied on it. Examples – liquid, gas, vapours etc. For static fluid the shear force is zero and fluid is at rest. In the fluid state the matter is compressive at some extinct and deform after applying shearing force.

Generally, Fluids are classified as- **Ideal fluid** a fluid which doesn't contain any surface tension and viscosity and ideal fluids are incompressible in nature and **Real fluid** a fluid which possesses viscosity, surface tension and are compressible in nature. There is no existence of ideal fluids in real life because all fluids have viscosity and possesses surface tension. As the viscosity is responsible for the effect of temperature in fluids, hence, real fluids have effect of temperature on them. So only real fluids are existing.

In ancient times, the nanotechnology was used to make pottery in Mesopotamia in the ninth century AD. **Maxwell (1873)** gives an advanced idea of addition of solid particles in fluids to increase its thermal conductivity. The millimetre and micrometre size of solid

particles added to fluids. The foundation of development of nanotechnology is defined by the fact that the past people used nanotechnology without well knowledge of it. The advanced nanotechnology was first mentioned in the well-known lecture delivered in 1959 at the conference of the American Physical Society by Mr. R. Feynman, the professor of Californian institute of technology. He discussed in his lecture “There is a lot of space down there” the possibility to create nanosized particles by using the atoms as building elements. The beginning of nanotechnology in modern world is given in the research of Michael Faraday. Nanofluids were an outcome of research projected to advancement in thermal conductivity of liquids.

1.2 Nanofluids: An Overview

The modern developments in nanotechnology, the manufacture of nanometer sized particles (nanoparticles). The concept of nanoparticles suspension in the base liquid to improve its thermal conductivity has been initiated by **Choi (1995)**. Suspension of nanoparticles in base fluids demonstrated a new type of fluid known as nanofluids. Base fluids are usually used for heat transfer fluids such as water, ethanol, ethylene glycol etc. but they possess low thermal conductivity. Nanoparticles are nanometer sized particles of metals like Silver, Copper, Gold, Iron and metal oxides like Alumina, Silica, Titania, and Copper oxides nanoparticles are often used in various research in nanofluids. Carbon nanotubes are also frequently used for nanofluid research because of their excessive thermal conductivity. Nanofluids are the colloidal suspension of nanoparticles in a base fluid. Nanoparticles are small in size hence; they can easily suspend in the bulk of base fluid. To maintain the stability of nanoparticles in the base fluid, some small amount additives are added to the mixture of nanoparticles in base fluid. On the basis of many experimentations it can be seen that nanofluid is most appropriate to rise the thermal conductivity of base fluids.

Nanofluids are prepared by suspension of nanoparticles in base fluids. “Higher ratio of surface area to volume”, “greater stability” and “lower kinetic energy” are important properties of nanoparticles. **Choi et al. (2001)** initiate that “a small amount of copper (Cu) nanoparticles or carbon nanotubes (< 1% volume fraction) dispersed in ethylene glycol or oil is reported to increase the inherently poor thermal conductivity of the

liquid by 40% and 150% respectively”. The transport of energy in low concentration ($< 1\%$ volume fraction) fluids with metallic nanoparticles is higher than that of the oxide nanoparticles because thermal conductivity of oxide nanoparticles is less than thermal conductivity of metallic nanoparticle.

Based on various researches and experiments it can be seen that the nanofluids can improve the thermal conductivity and energy efficiency in several thermal systems. The nanofluids have enhanced properties like “thermal diffusivity, viscosity, mass diffusivity, thermal conductivity and convective heat transfer coefficients” as compared the properties base fluids Nanofluids has a vast application in the field of engineering and industries. Much of the work has been done on nanofluids in national laboratories and academic world and is at a stage beyond its exploration. Recently, there are many industries, that see the potential of nanotechnology, are increasing. In the transportation industry GM, Ford and many others, have enduring nanofluid research projects. Some other important applications of nanofluids are “Cancer Therapeutics, Nano cryosurgery, Sensing and Imaging, Cryopreservation, Nanofluid Detergent, intensify micro reactors, Nanofluids as vehicle brake fluids, Nanofluids based microbial fuel cell, Nanofluids as optical filters, use as coolant, biomedical application mainly in drug delivery”.

There is a lot of applications of thermal conductivity depends on concentration of nanoparticle. Chemical composition of base fluid doesn't change on adding nanoparticles in it. Thermal conductivity of nanofluids can control by varying the nanoparticle size. “Temperature, volumetric fraction, concentration of nanoparticles, material type, shape and size of nanoparticles, acidity” are some of the major factors which affects the thermal conductivity of Nanofluid. The effect of “nanoparticles interfacial layer, clustering of nanoparticles and Brownian motion of nanoparticles” are also significant aspects to be observed which affects the thermal conductivity of nanofluids.

1.3 Classification of Fluids

Newtonian fluid: - The fluid is said to be Newtonian fluid if “shear stress is directly proportional to the rate of shear strain or rate of angular deformation or velocity gradient.” Newtonian fluid obeys Newtonian’s law of viscosity.

Examples: - Air, water, petrol, diesel, kerosene etc.

Non-Newtonian fluid: - “The fluids which do not obey the Newtonian’s law of viscosity”, i.e., constant viscosity is not dependent on stress. Ketchup is a non-Newtonian fluid because it becomes thinner when shaken.

Non-Newtonian fluids are categorized into three fluid types, the Differential type fluid, the Rate type fluid and the Integral type fluid. The behaviour of relaxation time and retardation time is defined by Rate-type. Maxwell fluid is Rate type fluid. Maxwell fluid defines the characteristics of relaxation time.

Examples: - mud flow, butter, Custard, honey, toothpaste, starch suspensions, corn starch, paint, blood, and shampoo etc.

1.3.1 Classification of Non-Newtonian fluid:

1 Dilatant: - If the apparent viscosity of fluid increases with increase in rate of deformation these types of fluid are known as dilatant. These fluids are also known by the name shear thickening fluid.

Examples: - rice starch, sugar in water.

2 Pseudo-Plastic: - A fluid whose viscosity decreases with an increase in shear rate. Examples: - suspension of paper pulp or pigments.

3 Bingham-plastic fluids: - A fluid which have initial yield stress i.e. their linear relationship between shear rate and shear stress doesn’t pass by origin. Example: - toothpaste

4 Thixotropic fluids: - A fluid whose viscosity decreases with time. Examples: - paints and ink.

5 Rheopectic Fluids: - A fluid whose viscosity is dependent on time and increases with time.

Examples: - Gypsum suspension in water, Vanadium Pentoxide Solution etc.

1.4 Kinematics of Fluid Flow

It is the field of mechanical engineering that deals with the properties of fluid when it is in motion. Fluid have tendency to flow, because the fluid molecules moves from one phase to another phase as a function of time. Flow of fluid is because of the unbalanced forces between fluid layers and stresses between them. The fluid flows continuously until the unbalanced forces acting on it. The classification of fluid on the bases of Lagrangian and Eulerian hypothesis are described as: -

1.4.1 Steady and Unsteady flow

The properties of fluid flow such as velocity, pressure, temperature and density during the motion of the fluid flow remains constant with time i.e. independent of time is called the steady flow. The fluid flows whose flow properties are dependent on time i.e. varies with time is called unsteady flow.

1.4.2 Rotational and Irrotational flow

During flow of fluid when fluid particles rotate about their own axis while moving on their path is called rotational flow, although the fluid particles doesn't show any rotational motion about their own axis during the flow is called irrotational flow.

1.4.3 Laminar and Turbulent flow

The fluid flows whose particles moves along smooth and regular path is called laminar flow and the path of two different particles never intersect each other. While, in turbulent flow, each particle of fluid flows in irregular

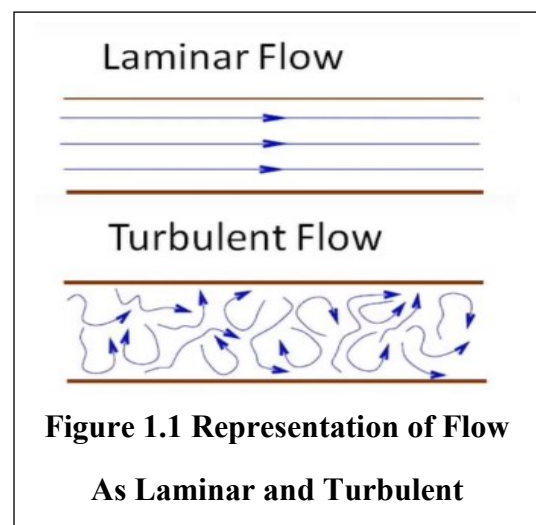


**Figure 1.1 Representation of Flow
As Laminar and Turbulent**

path or does not trace a certain path and these particles always interfere with each other's path.

1.4.4 Uniform flow and Non-uniform flow

The fluid flow in which each particle of fluid flows with uniform velocity at every segment of pipe is called uniform flow. The flow in which each particle of fluid moves with non-uniform velocity at every segment of pipe through which fluid is flowing is called non-uniform flow.

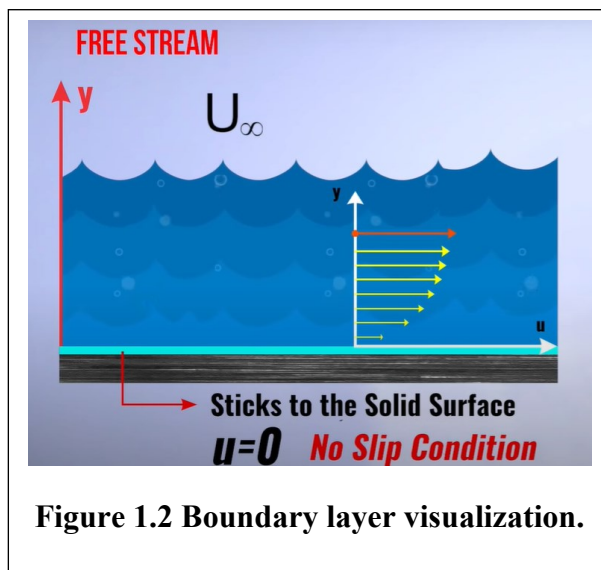
1.4.5 Compressible and Incompressible flow

If the variations in pressure in the fluid flow are large enough to bring a substantial change in fluid density then this type of fluid is known as compressible fluid whereas the flow of fluid in which there is no variation in the density by varying the pressure is called incompressible flow.

1.5 Concept of Boundary Layer Theory

Basic concept behind the boundary layer flow was initially given by German Mathematician **Ludwig Prandtl (1904)**. He described that when a real fluid flow over a solid body, then there exists a very thin section of fluid over the surface where the fluid sticks to the solid boundary due to which the velocity of fluid becomes approximately zero at boundary and that is why it is termed as “no slip condition”.

Since the solid body is at rest so, the fluid which is in contact with that surface also at rest. Thereafter, as we move away, along the normal to the surface, the fluid velocity will gradually increase and at a certain distance away from the solid surface fluid achieve the free stream velocity. It is noted that the velocity of fluid varies from zero at the surface to the velocity equivalent to the free stream velocity. This thin region at the surface of the body where velocity gradient exists is known as boundary layer and viscous flow



associated with it is known as hydrodynamic boundary layer flow. The fluid flow over a solid surface can be characterized into two different sections. The first section is in the near of the body surface, where friction plays an significant role and the other section is outside this layer, where friction is very small that it can be neglected and flow is governed by ideal fluid theory.

Boundary Layer Thickness. The thickness of Boundary layer is described as the width of region where fluid is viscous across boundary layer. At surface of the body, the fluid velocity is slow down due to viscosity and as we move normally above the surface the viscosity of fluid decreases and the velocity of is approximately equal to velocity of free stream. Thus, as we move normally above the surface of body the velocity of fluid asymptotically reaches to the velocity of free stream, in fact it never equivalents to the velocity of free stream. Hence, the point within the boundary layer region at which fluid velocity equals 99% of the free stream velocity is defined as the edge of the boundary layer.

1.6 Mass Transfer

It is defined as the movement of one constituent from one phase to another phase at molecular scale due to concentration difference or gradient to balance the difference in concentration inside the system. Concentration, temperature and pressure gradient are responsible for mass transfer.

Usually, the two approaches of mass transfer are defined as, first one is mass transfer by diffusion and another one is mass transfer by convection. Diffusion is a process of mass transportation due to the random motion of molecules in laminar flow, as the mass transfer between volume element of the fluid occurs only when different constituents of material have different average molecular velocities. Mass transfer by diffusion due to the hydrostatic pressure difference and concentration temperature gradient. The process of diffusion of molecule is same as the conductive heat transfer. Mass transfer by convection is allotted to the transfer of mass with the involvement of convective heat transfer. Various applications of mass transfer in manufacturing fields, such as final separation of products from by products and cleansing of substance, water evaporation from rivers, which accordingly increases the fraction of moisture in passing stream.

1.7 Heat Transfer

Heat transfer is defined as the phenomenon of thermal energy transmission from one substance to another substance due to temperature difference or gradient. When a fluid flow over the objects where, temperature difference exists in that medium, then heat is transferred from an element which has higher temperature to an element with lower temperature to attain the thermal equilibrium between the body and the surroundings. Heat transfer is caused by three elementary modes conduction, convection and radiation.

Conduction. The process of transfer of heat in which heat transmits by vibration of molecules of element from high temperature system to low temperature system. The systems through which energy is transferring, must be in contact with each other. Molecules basically vibrate about their mean position, when they conduct heat and transfer it to the adjacent molecules through vibration.

Convection. During the fluid flow, the heat is transferred through potential gradients and by motion of fluid particles and this procedure is called convection. The method of heat transfer which includes molecular diffusion. Basically, convection involves combination of conduction and motion of fluids. The convection process is governed by Newton's law of cooling. Heat transfer by convection is of two types, Natural convection and Forced convection. The natural convection is caused when fluid is forced in motion by buoyance force due to change in fluid density. When an external source causes convection flow it is called forced convection, which force the fluid to flow over the solid surface. Forced convection is superior than free convection as it gets results much faster. If both the natural and force convection is seen at same time, then this is called mixed convection.

Radiation. The radiation is the heat transfer process where, the matter emits thermal energy (radiation) in the form of E.M waves through space. It doesn't need any medium to transport whereas it transports through perfect vacuum. Black bodies are the systems which can absorb radiation incident on it and emits radiation at maximum rate at specific dimension and at certain temperature.

1.8 Chemical reaction

Chemical reaction is defined as a procedure where chemical reactants get altered to other chemical products. There are two types of chemical reactions. Reactions which don't need any energy to initiate are known as spontaneous reactions while the reactions in which the involvement of energy is essential to initiate the chemical reaction like heat, light and electricity are known as non-spontaneous reactions. The properties of products and reactants of chemical reaction are different. Generally, electrons are the important part of chemical reaction which play an important role in formation of chemical bonds. The theory of chemical reaction is appropriate for restoration of elementary particles and in nuclear reaction. Reaction chain tends to change in properties of flow. Chemical reaction in the flow is a significant way to alter the rate of heat and mass transfer.

1.9 Magneto Hydro-Dynamics

Magneto hydrodynamics is also known as hydro-magnetics, it is the study of behaviour and magnetic properties of electrically conducting fluids. The main concept behind Magneto hydrodynamics is- the current is induced in a moving conductive fluid by applying magnetic field. The equations which govern Magneto hydrodynamics are combination of the Maxwell's equations of electromagnetism and Navier-Stokes equations of fluid dynamics. It has an important role in manufacturing MHD power generators, liquid hydrocarbon fuels, cooling of fusion power reactor blankets and pumps.

1.10 Porosity and Permeability

Porosity and permeability control the movement and storage of fluids. A porous medium is categorized by its porosity, permeability and properties of its ingredients. The ratio of volume of pores (voids) to the total volume of material is known as void fraction or Porosity. Porosity mainly signifies the storage capacity of the fluids.

Permeability is a characteristic property of porous medium and it measures the rate of fluid flow transmitted through porous medium. A substance having high porosity i.e. pores are disconnected from each other, then the fluid cannot move through that medium. Unit of permeability is Darcy. Permeability is completely dependent on porosity.

1.11 Brownian Diffusion.

The unsystematic motion of particles in the base fluid is known as Brownian motion, due to this continual collision occurs between molecules of the base fluid and the particles. The nanoparticles are considered effectively as large molecules, whose average kinetic energy is same as the fluid molecules $\frac{1}{2}k_B T$, and thus they have lower velocity. k_B is the Boltzmann's constant and T is the nanofluid temperature. Brownian diffusion coefficient D_B , describes Brownian motion, which is given by the Einstein-Stokes's equation.

$$D_B = \frac{k_B}{3\pi\mu d_p}$$

k_B is the Boltzmann's constant, d_p is diameter of nanoparticle and μ is viscosity.

1.12 Thermophoresis

Particles diffuse due to the effect of temperature gradient. Physically, it can be explained as the migration of colloidal particles in a solution due to the impact of temperature gradient. This process is known as thermophoresis, and is equivalent to or familiar to the Soret effect for the mixtures of gaseous or mixtures of liquid. Thermophoresis or the Soret effect is a process that is seen in combinations of moving particles where the different particles show different behaviour towards the impact of a temperature gradient. Thermal diffusion coefficient D_T , describes Thermophoresis effect, which is given by,

$$D_T = \beta \frac{\mu}{\rho} \phi$$

β is proportionality factor, ρ is density, μ is viscosity, ϕ is volumetric fraction.

1.13 Cattaneo-Christov model

Fourier proposed, the heat flux model which was the most effective model for grasping the heat transfer mechanism in various circumstances but it tends to have a

parabolic energy equation that implies the initial disturbance is instantaneously experienced by the medium under consideration which is one of the limitations of this model. This physically unrealistic feature is also known as “Paradox of heat conduction”. In order to get rid of this limitation, **Cattaneo (1948)** altered this law by introducing a new term called “thermal relaxation time” which is described as the time needed to establish heat conduction after the temperature gradient is executed. **Christov (2009)** again altered the Cattaneo model by substituting the ordinary derivative with the Oldroyd’s upper-convected derivative. He also introduced the energy equation for arbitrary temperature and velocity fields.

1.14 Stefan Blowing Effect

In many practical applications, the situation arises, when there is transfer of mass due to evaporation like in paper drying process. Mass transfer can produce a “blowing effect” that depends upon the water content and the temperature within the wet paper surface. The concept of blowing effect originates from the Stefan problem for mass transfer. The extra or bulk motion of the fluid is caused by the diffusion of the species. A parameter is introduced to study the blowing effects for massive mass transfer flux and termed as blowing parameter. In this blowing effect the surface is considered as non-permeable surfaces and the blowing is caused by mass transfer flux from the surface to the ambient.

1.15 Dimensionless Parameters

A physical quantity with no units is known as dimensionless parameter. Dimensionless parameters are independent on system of units but depend upon the characteristic quantities of physical system. The parameters for nanofluids are used in the present study are defined as follows:

1.15.1 Prandtl number

The Prandtl number (Pr) is defined as the ratio of momentum diffusivity to thermal diffusivity. The parameter was named after German scientist Ludwig Prandtl. It controls

the relative thickness of momentum boundary layer and thermal boundary layer in heat transfer problems.

$$\text{Prandtl number}(\text{Pr}) = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\mu_{nf}(c_p)_{nf}}{k_{nf}}.$$

1.15.2 Reynolds number

Reynolds number (**Re**) is given by the ratio of inertial force to viscous force. The Reynolds number was named after scientist Osborne Reynolds.

$$\text{Reynolds number}(\text{Re}) = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho_{nf}UL}{\mu_{nf}}.$$

This made easier to understand the pattern of flow is laminar or turbulent while flowing through tube. When value of Reynolds number is greater than 2000 (**Re > 2000**), then the flow through the tube is said to be turbulent and When value of Reynolds number is less than 2000 (**Re < 2000**), then the flow through the tube is said to be laminar.

1.15.3 Schmidt number

Schmidt number is categorized as the ratio of momentum diffusivity to mass diffusivity. It characterizes simultaneous diffusion of mass and momentum in the flowing fluid. Basically, it relates the relative thickness of hydrodynamic layer to the mass transfer boundary layer.

$$\text{Schmidt number}(\text{Sc}) = \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = \frac{\mu_{nf} / \rho_{nf}}{D_{nf}}.$$

1.15.4 Skin Friction

Skin friction is related to viscous drag arises by the friction between fluid and object surface moving through it. It is due to the viscosity of fluid. It is generated from the friction when the fluid flows directly in contact with the surface of the object. As the fluid progress on the surface of a body. The flow in region of the boundary layer which lies just above the surface of the body is generally relatively thin and also laminar flow, but the flow becomes turbulent near the rearmost region of the body. Newton gives the law of viscosity for numerous types of fluid flow i.e. parallel, straight and uniform flow.

According to the Newton's law of viscosity- "shear stress (τ) between fluid layers is relative to velocity gradient $\left(\frac{\partial u}{\partial y}\right)$ in the direction perpendicular to the layers". At the surface of body, the viscous drag is specified as $\tau_w = \mu \frac{\partial u}{\partial y}$. The coefficient of friction associated with it is known as skin friction coefficient which is a non-dimensional quantity, which categorizes the frictional force at the boundary between fluid and a wall. The skin friction coefficient or drag coefficient is given by $C_f = \frac{2\tau_w}{\rho U_\infty^2}$.

1.15.5 Nusselt number

The Nusselt number (Nu) is a non-dimensional parameter, which is described as the ratio of coefficients of the convective heat transfer to conductive heat transfer perpendicular to direction of flow at the boundary. It is useful for the computation of the heat transfer between moving fluid and solid body. The Nusselt number is named after a German engineer K. Wilhelm Nusselt.

$$\text{Nusselt number}(Nu) = \frac{\text{Convective heat transfer coefficient}}{\text{Conductive heat transfer coefficient}} = \frac{h_w}{k_{nf}}.$$

1.15.6 Sherwood number

The Sherwood number (Sh) is applicable in those models which include the mass transfer. It is defined as the ratio of convective mass transport to diffusive mass transport. Sherwood number is given as:

$$\text{Sherwood number}(Sh) = \frac{\text{Convective mass transport}}{\text{Conductive Mass transport}} = \frac{h_m L}{D_{nf}}.$$

1.16 Governing Equations

The problems related to fluid mechanics are initially converted into a mathematical model of the actual world, similar to any other model. In the study related to fluid mechanics we have to make some elementary assumptions about the resources which are observed in this field. The equations which are defined below are the equation of motion of

the fluid flow that we get by applying Newton's second law of motion, along with the idea that the fluid stress can be defined as a sum of the diffusing viscous term which is relative to velocity gradient, and the pressure term. According to the Navier-Stokes equations the fluid under inspection is continuum and all fluids follow these assumptions of conservation of mass that produces continuity equation, conservation of momentum gives momentum equation and conservation of energy forms equation of energy.

The equations which describes the motion of fluid flow are also known as Navier-Stokes equations. The Navier-stokes equation in Cartesian coordinate system for incompressible nanofluid flow is:

$$\rho_{nf} \frac{D\vec{v}}{Dt} = \vec{F} - \text{grad } P + \mu_{nf} \nabla^2 \vec{V},$$

where $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$ is the velocity, $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ is the force, P is pressure and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ is the Laplace operator.}$$

According to **Rosenhead (1963)** the equations of the 3-D flow of an homogeneous, isotropic and incompressible Newtonian fluid in Cartesian coordinate system (x, y, z) are:

Equation of Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Equation of Motion:

$$\rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = F_x - \frac{\partial P}{\partial x} + \mu_{nf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

$$\rho_{nf} \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = F_y - \frac{\partial P}{\partial y} + \mu_{nf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right),$$

$$\rho_{nf} \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = F_z - \frac{\partial P}{\partial z} + \mu_{nf} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

Equation of Heat Transfer:

$$\rho_{nf} (C_p)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \mu_{nf} \zeta,$$

where ζ is the dissipation function is denoted as:

$$\zeta = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \left[\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right]$$

Since all the terms are quadratic in nature, therefore the value of viscous dissipation is above zero. Thus, as a adverse effect of second law of thermodynamics, there will be a loss of energy every time in a viscous flow because of dissipation.

Equation of Mass Transfer:

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} \right) = D_{nf} \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + D_c,$$

where D_{nf} mass diffusion of nanofluid and D_c is mass diffusion coefficient at constant concentration.

1.17 Objectives of the Study

The main aim of present study is to examine the flow and heat transfer in Cu-water nanofluid through stretching sheet using Cattaneo Christov double diffusion model with Stefan blowing effect and zero mass flux condition at the boundary. In the present study, author has also comprised the effects of some of the physical parameters i.e. magnetic parameter, mixed convection parameter, porosity parameter, activation energy parameter, thermal radiation parameter, solutal time relaxation parameter, thermal time relaxation

parameter, velocity slip parameter, nanoparticle volume fraction, chemical reaction parameter, Stefan blowing parameter, Brownian motion parameter, thermophoresis parameter, Prandtl number, Schmidt number etc. on velocity, temperature and concentration profiles. The effect of various physical parameters on skin friction coefficient and Nusselt number has also been included in this study.

The flow and heat transfer flow of several nanofluids through a stretching sheet with blowing effect and slip has been taken for discussion and analysis of present study. Flow and heat transfer of a steady 2-D several non-compressible nanofluid through the stretching surface with effect of slip velocity and blowing effect. The stream likewise includes the effects of Brownian motion together with thermophoresis in the vision of constant magnetic field with chemical reaction takes place at area of flow. The obtained equations which govern the fluid flow are get transformed using similarity transformations and then finally solving them by R-K-F scheme with shooting technique. Behaviour pattern of concentration, temperature and velocity for various values of Brownian motion parameter, chemical reaction parameter, Prandtl number, slip parameter, thermophoresis parameter, blowing parameter, Schmidt number, magnetic parameter and volume fraction of solid nanoparticles are represented by graphs. The local drag coefficient and the rate of flow and heat transfer at the surface of stretching surface are also organized for various values of nanofluid properties and the stream situations. The results and discussion of the present study have been described in chapter 4. The summary of entire work has been discussed in the chapter 5.

Nanofluids are made of nanoparticles suspended in a conventional fluid. Nanoparticles are comparatively close to the size of molecules of base fluids. Thus, they can form a very stable suspension. Taking into account the rising demand of modern technology, the researchers and scientists are finding the brilliant ideas towards the applications of the nanofluids in various fields. Nanofluids have a wide range of applications because of its enhanced thermal conductivity. **Schlichting (1979)** is a good literature to understand and study about fluids most importantly the boundary layer theory.

Choi (1995) demonstrated an innovative fluid known as nanofluids. Nanofluids are made of nanoparticles suspended in a conventional fluid, which are traditionally used for heat transfer applications. He originated the notion of nanoparticles in base fluid and by his experiments he exposed that the nanofluids are most appropriate to enhance the thermal conductivity. He takes copper as nanoparticle and presents a thermal conductivity model. Later on, many researchers started working on it by taking different nanofluids flowing over different geometries. **Choi et al. (2001)** deliberated the similar problem as **Choi (1995)** along with suspension of nanotube.

Analysis of non-dimensional equations suggest that transfer of energy due to dispersion of nanoparticles in base fluids is negligible as it can't clarify the irregular increment in coefficient of heat transfer. By comparing the nanoparticle length scales and turbulent eddy time shows that nanoparticles move with uniform velocity within the fluid with the existence of turbulent eddies. So, the impact on turbulence intensity is also uncertain. **Buongiorno (2006)** projected that the properties of nanofluids varies significantly across the boundary layer due to the influence of thermophoresis and temperature gradient.

Fourier proposed, the heat flux model which was the most effective model for grasping the heat transfer mechanism in various circumstances but it tends to have a parabolic energy equation that implies the initial disturbance is instantaneously

experienced by the medium under consideration which is one of the limitations of this model. This physically unrealistic feature is also known as “Paradox of heat conduction”. In order to get rid of this limitation, **Cattaneo (1948)** altered this law by introducing a new term called “thermal relaxation time” which is described as the time needed to establish heat conduction after the temperature gradient is executed. **Christov (2009)** again altered the Cattaneo model by substituting the ordinary derivative with the Oldroyd’s upper-convected derivative. He also introduced the energy equation for arbitrary temperature and velocity fields.

Tiwari and Das (2007) investigated mathematically the behaviour of nanofluids into the two-sided lid driven differentially heated square cavity to understand the convective circulation and process of flow induced by a nanofluid. A model is made to study the behaviour of nanofluids taking the effect of nanoparticles volume fraction. They found that the nanoparticles present in the fluid are accomplished to increasing the capacity of heat transfer of base fluid. The effect is more obvious as the solid volume fraction increases. The average Nusselt number varies nonlinearly with solid volume fraction.

Hayat et al. (2011) demonstrated the heat transfer analysis of two-dimensional magneto hydrodynamics flow of Maxwell fluid applying thermal radiation in porous medium subjected to suction/blowing.

Aziz et al. (2012) analyzed the free convection on nanofluid flow over a straight flat plate with porous medium comprising of gyrotactic microorganisms. They investigated that how the bioconvection parameters affect the properties of flow and density of mobile microorganisms along with the rates of heat transport and mass transport. They determined the effects of bioconvection parameters on the rate of transport of mass, heat and motile microorganism.

Kameswaran et al. (2012) investigated MHD nanofluid flowing on a shrinking or stretching sheet with the effects of viscous dissipation and chemical reaction. They studied the problem for two nanofluids, Copper nanoparticles in water and silver nanoparticles in water. Behaviour of temperature, velocity and concentration for various values of the governing parameters. They concluded that Ag–water nanofluid has lower rate of mass and wall heat transfer.

The boundary layer nanofluid flow over a stretching/shrinking sheet with a condition of convective boundary is explained by **Mansur and Ishak (2013)**. They concluded that the local Nusselt number is increases on increasing values of the convective parameter and reductions takes place on enhancing values of thermophoresis parameter and the Brownian motion parameter. However, Sherwood number enhances with rising values of Brownian motion parameter and reduces on enhancing convective parameter and thermophoresis parameter.

An unsteady boundary layer flow of Maxwell fluid on unsteady stretching surface in porous medium with thermal radiation investigated by **Mukhopadhyay et. al. (2013)**. They concluded that, the velocity of fluid decreases on enhancing suction at the boundary. Increasing porosity parameter leads to decreasing velocity and increment in temperature. Boundary layer thickness increases by enhancing the values of the thermal radiation parameter.

Fang and Jing (2014) examined the transfer of heat, flow and mass due to fluid flow over stretching surface by taking into account the double Stefan blowing effects of mass transfer. They were the first who considered the coupled blowing effect for transfer of heat, flow and mass for stretching plate. They determined that the blowing effect by mass transfer has great impact on velocity, temperature and concentration. Velocity, temperature and concentration are increases on rising the values of blowing parameter.

Hadded (2014) stated the numerical solution for thermal conductive flow of viscous fluid by applying Cattaneo-Christov heat flux model in energy equation.

Han (2014) suggested boundary layer flow of Maxwell nanofluid to study the C-C heat flux model in along with of slip velocity flowing on stretching surface.

Abbasi and Shehzad (2016) considered a model three-dimensional flow of Maxwell fluid flowing on stretching sheet applying the C-C heat flux model. The dependency of temperature on thermal conductivity for C-C heat flux model is key feature or their investigation. They conceded that for larger Deborah number velocity profile is smaller while temperature is higher. Cattaneo-Christov model contains heat flux relaxation time which lowers the temperature.

Hsiao (2016) examined the stagnation nano energy translation for conjugate mixed convection heat and mass transfer with electrical magneto hydrodynamic flow. Similarity transformation is applied and a modified Finite-Difference method is used to study the problem.

Mixed convection hydrodynamic flow of nanofluid draw attention of recent researchers **Hussain et al. (2016)** highlighted the effect of double stratification and magnetic field on the mixed convective radiative flow of Maxwell nanofluid. They concluded that enhancing magnetic parameter increases temperature and concentration but velocity is decreases while temperature and nanoparticle concentration strengthen for greater Deborah number.

The heat transfer in Magneto Hydro Dynamic flow in 3-D flow of Maxwell fluid applying Cattaneo-Christov heat flux model and fluid flowing on stretching surface is investigated by **Rubab and Mustafa (2016)**. Velocity is decreasing function of fluid relaxation time is same to both Fourier and Cattaneo-Christove models.

Sui et al. (2016) analysed the boundary layer Cattaneo-Christov double-diffusion model of heat and mass transfer in a Maxwell nanofluid on a stretching sheet, effects of thermophoresis, Brownian motion and the velocity slip boundary are applied. They determined that the temperature profile decreases on increasing temperature relaxation parameter but the velocity profile and concentration profile enhance to some extent. On increasing values of concentration relaxation parameter leads to decrement in the thickness of concentration profile but an increment in temperature profile.

Kundu et al. (2017) examined theoretically the effect of C-C heat flux model on C.N.T engine oil Maxwell nanofluid flowing over non-linear stretching sheet considering velocity and temperature slip conditions. They considered engine oil as primary fluid because it viscous in nature.

The non-steady laminar boundary layer 2-D flow, transfer of heat and mass of viscous non-compressible electric conducting Maxwell nanofluid on a porous stretching surface. The flow is subjected to non-uniform magnetic field and velocity is studied by **Madhu et al. (2017)**.

In the fluid dynamics, a stagnation point is the point in field of flow where the local velocity of fluid become zero. Stagnation point flow represents a fluid flow in the direct neighbourhood of solid surface at which fluid imminent the surface splits into different stream. **Rauf et al. (2017)** examined the M.H.D stagnation point flow of micro nanofluid with convection over shrinking sheet and applied zero mass flux condition.

Saleem et al. (2017) studied theoretically the 3-D mixed convection flow of non-compressible Maxwell fluid flowing along a stretching sheet. On applying magnetic field to any fluid, it enhances its viscosity until it become a viscoelastic solid. So, it is controlled by carefully varying the intensity of magnetic field.

Stefan blowing also known as wall injection, it has significant applications in various manufacturing like the process of drying and purifying where boundaries are perforated. The idea of blowing effect is given by the Stefan problem which is a substantial application of mass transfer of species. **Shukla and Rana (2017)** consider the non-steady MHD nanofluid flow flowing over a stretching surface under the concurrent effects of thermal slip, velocity slip, Stefan blowing with heat source. They found that Velocity is increases on enhancing magnetic parameter however it is decreases on increasing Stefan blowing parameter and the skin friction coefficient enhances on increasing the values of blowing parameter. They also observed that the heat transfer rate enhances with blowing parameter whereas it reduces on increasing heat source parameter.

Entire review on transfer of laminar flow and heat of pseudo-plastic non-Newtonian nanofluid flowing through the permeable circular concentric section is obtained by **Barnoon and Toghraie (2018)**. The results show that at a specified volume fraction, the effect of porous medium increases the transfer of heat is higher as compare to the enhancing values of Reynolds number. If the thickness of the porous layer increases on fixing the values of permeability and porosity then the nanofluid velocity is also increases at constant Reynolds number.

Gangadhar et al. (2018) observed the transfer of heat and flow in upper-convected Maxwell fluid applying C-C heat flux model along a quickly stretching surface with viscous dissipation. Maxwell fluid is a considerable viscoelastic fluid which generates the impact of fluid relaxation time and Cattaneo-Christov heat flux model has been used to

analyse the features of heat transfer. They found that the velocity distribution is greater in viscoelastic fluid as compare to the viscous fluid.

Imtiaz et al. (2018) explores the study of non-steady 3-D flow of Maxwell fluid which is flowing over stretching surface using C-C heat flux model and consider the impact of homogeneous and heterogeneous reactions. They found that for greater values of strength of parameter of homogeneous reaction, it decreases the concentration distribution and also, the temperature decreases on enhancing the thermal relaxation time.

Khanafer and Vafai (2018). Their study gives the review on the applications of nanofluid flowing over porous medium. Flow, heat and mass transfer of nanofluid flow on porous medium have gained a significant consideration of researchers in the last few years because its important applications in various industrial and engineering fields. These applications comprise of catalytic reactors, packed bed heat transfer, thermal insulation, drying technology, petroleum industries, geo-thermal systems and electronic cooling. Heat transfer by natural convection has applications in various industrial and engineering area for example geophysics, electronic cooling, thermal insulation in buildings, solar energy, and nuclear energy. They give their reviews on Natural convection in porous medium, MHD convection of nanofluids in porous media, Mixed and forced convection of nanofluids flowing over porous media.

Reddy et al. (2018) investigated of Magneto hydrodynamic Maxwell, Jeffery, and Oldroyd-B nanofluid with the mixed convection flow applying non-homogenous Buongiorno's model with heat source/sink over a cone. They revealed that the velocity, temperature, and concentration profile increase on enhancing the Brownian motion parameter. Thermophoresis parameter affects the concentration and temperature profiles and the concentration and temperature profiles increases on enhancing the magnetic parameter whereas it reduces the velocity profiles for all the models. The lower velocity and higher concentration profiles are observed in case of Jeffry nanofluid model because of a smaller amount of collisions occurs on increasing the Brownian motion parameter.

Prasannakumara et al. (2018) analysed the chemical reaction effect and nonlinear thermal radiation effect on Maxwell nanofluid flow through a porous media over

horizontal stretching surface. They observed that the velocity profile reduces and both the concentration and temperature profiles increase on rising values of velocity slip parameter. Both concentration and temperature profiles reduce on increasing values of thermal slip parameter and nanoparticles volume fraction is inversely proportional to chemical reaction parameter.

Ahmad et al. (2019) assumed the unsteady nanofluid flow due to bi-directional moving boundaries and Buongiorno model is used to study the behaviour of incompressible nanofluid flow with Cattaneo-Christov double diffusion expression. They concluded that temperature rises for greater value of thermal relaxation parameter, thermophoresis parameter and Brownian motion parameter. Concentration increases for greater values of concentration relaxation parameter and thermophoresis parameter and amount of heat transport increases for large value of concentration relaxation parameter however it reduces for higher values of thermal relaxation parameter.

Convective radiative flat Poiseuille nanofluid flow over porous medium with slip and Stefan blowing studied by **Alamri et al. (2019)**. They concluded that the Stefan blowing/suction decelerate velocity and temperature profiles however nanoparticle concentration increases. velocity profile decreases on increasing the porosity parameter. On increasing magnetic field parameter, the velocity profile however, temperature distribution and nanoparticle concentration increases. The buoyancy ratio as well as magnetic field both have the same effect on velocity profile. The nanofluid velocity profile enhances on increasing Prandtl number and Reynolds number but reduces the boundary layer thickness of temperature and concentration. The thickness of concentration and thermal boundary layer enhances on rising the thermophoresis parameter and radiation parameter. Radiation parameter, Brownian motion and Thermophoresis parameter shows similar enhancing impacts on thickness of thermal boundary layer but thickness of concentration boundary layer reduces with the rising Brownian motion parameter.

In 1889 a Swedish scientist introduced a term Activation energy, it is defined as the minimum amount of energy require, so that the chemical reaction initiated. Initially, when chemical reaction starts the system contains no activation energy. Activation energy act as barrier height. **Alsaadi et al. (2019)** considered non-linear mixed convective nanofluid

flow along the porous stretching surface Buongiorno model is applied with the effect of activation energy, viscous dissipation and nonlinear thermal radiation. They revealed that entropy enhances for greater values of Reynolds number, magnetic and radiation parameter. velocity decreases by increment in mixed convection parameter and porosity parameter. Concentration enhanced because of Brownian motion parameter and Activation Energy parameter.

Ramzan et al. (2019) designed a mathematical model to deliberate the 3-D magnetohydrodynamic couple stress nanofluid flowing on an exponential stretching surface with joule heating as well as viscous dissipation effects. The condition of zero mass flux are imposed for the foremost time for three-dimensional double stress nanofluid flow model. The nanoparticle concentration distribution decreases on enhancing the values of Brownian motion parameter while increases for thermophoresis parameter. The temperature profile increases on enhancing values of Hartmann number and Biot number.

Rana et al. (2019) present comprehensive mathematical and hypothetical analysis of Al-water and Ti-water nanofluid flow over stretching surface employed by Buongiorno model using the effects of Stefan blowing, velocity slip and thermal slip at the boundary. They analysed that Brownian diffusion parameter decelerate the friction and rate of transfer of heat at the surface. Fluid temperature reduces on enhancing the values of thermal slip parameter. It is also observed that the velocity of nanofluid flow is reduces on increasing velocity slip parameter.

Magnetohydrodynamic nanofluid flow of combination of three nanofluids Maxwell, Oldroyd-B and Jefferey along the stretching sheet using Cattaneo–Christov Heat Flux model with Homogeneous-Heterogeneous Reaction have been examined by **Saeed et al. (2019)**. The Cattaneo–Christov heat flux model essential to observe the thermal and temperature relaxation properties of the nanofluid flow. They concluded that the velocity reduces with enhancing the magnetic field parameter. The temperature profile decreases on enhancing Prandtl number and thermal relaxation time parameter. The concentration profile increases with the rising values of Schmidt number and reduces on enhancing the strength of homogeneous-heterogeneous reactions.

Sandeep and Sulochana (2018) investigated the transfer of heat and momentum of Maxwell, Oldroyd-B and Jeffrey nanofluids through the stretching surface and applying thermal radiation, non-uniform heat source/sink, suction and transverse magnetic field effects. They concluded that Jeffrey nanofluid shows improved performance of transfer of heat by comparing to the Oldroyd-B and Maxwell nanofluids. The work done by them was limited to the study of momentum and heat transfer in presence of magnetic field with non-uniform heat source/sink. In this study, author extended the problem with flow, heat and mass transfer in Cu-water nanofluid with the combination of Buongiorno's model and Tiwari and Das model. In literature the study based on Cattaneo-Christov double diffusion model and Stefan blowing effect are very rare. This motivates the author to explore the behaviour of nanofluid flow with C-C double diffusion model in presence of Stefan blowing effect. The effects of thermal radiation parameter, mixed convection parameter, activation energy parameter and slip parameter are also incorporated in the present study.

The governing equations applied to fluid flow problems are in the form of “coupled non-linear partial differential equations”. The non-linearity is subjected to input or output of the governing equations are “non-linear differential equation”. As the equations comprises of non-linear terms, thus the analytical methods give very few solutions. Usually, the analytical solution is attained only if these differential equations can be in the form of the linear equations by excluding the non-linear terms or neglecting the non-linear terms as the other terms should be comparatively very large to non-linear terms. Hence, before solving the differential equations analytically, it is important to check that the differential equations which is being solved should be linear because it is generally not possible to get the analytical solution of the non-linear equation. Therefore, some additional estimations are mandatory to solve them.

All well-known exact solutions of various problems are discussed by **Schlichting (1979)** and **Rosenhead (1963)**. For various fluid flow problems to attain the solution, foremost important step is to convert all governing equations in “coupled non-linear Partial Differentiation Equations” or “Ordinary Differential Equations” after this solve the attained equations by implementation of numerical method along with the initial and boundary situations. Since the governing equations contains the non-linear terms which can’t be ignored, thus, the numerical method is implemented to find the result to get the required solution. Henceforth, similarity transformations are used to transform the non-linear PDE into ODE. Then obtained equations are being solved by using “Runge-Kutta-Fehlberg method” and finally the shooting method is used to obtain the boundary conditions.

3.1 Runge-Kutta-Fehlberg Method

The Runge–Kutta methods are related to the group of implicit and explicit iterative scheme which are used to obtain the solution of ordinary differential equations. These methods are contained of single-step processes in which evaluation of first order derivative

is mandatory. The accuracy in outcomes is similar to higher order Taylor's method. The general form of the extrapolation equation related to all Runge-Kutta methods can be written as:

$$y_{n+1} = y_n + \text{slope} \times \text{interval size} = y_n + mh$$

where, m indicates the weighted averages of the slopes at several points in the interval h .

Let m be predicted by utilizing the slopes at r different points in the interval (x_n, x_{n+1}) ,

then m is given as:

$$m = k_1 m_1 + k_2 m_2 + k_3 m_3 + \dots + k_r m_r$$

where, k_i $1 \leq i \leq r$, represents the weights of slope at different points.

Several members of the family of R-K scheme are of the R-K method of 2^{nd} order as well as of R-K scheme of 4^{th} order. The R-K scheme of 4^{th} order method is written as follows:

$$y' = f(x, y), \quad y(x_0) = y_0 \quad \text{is}$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

The different values of k_1, k_2, k_3 and k_4 are given as follows:

$$k_1 = f(x_n, y_n)$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + h \frac{k_1}{2}\right)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + h \frac{k_2}{2}\right)$$

$$k_4 = f(x_n + h, y_n + hk_3)$$

The German mathematician Erwin Fehlberg developed R-K-F scheme to obtain the numerical solution of the ODE's. On the basis of Runge-Kutta method, the Fehlberg method implements 4^{th} and 5^{th} order.

The process contains several steps, at every step 2 dissimilar estimations are considered and then compared. If the calculated results are very close to each other or they are repeated then the estimation is accepted. Every step size uses these six values as follows:

$$k_1 = h f(x_n, y_n)$$

$$k_2 = h f\left(x_n + \frac{1}{4}h, y_n + \frac{1}{4}k_1\right)$$

$$k_3 = h f\left(x_n + \frac{3}{8}h, y_n + \frac{3}{32}k_1 + \frac{9}{32}k_2\right)$$

$$k_4 = h f\left(x_n + \frac{12}{13}h, y_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right)$$

$$k_5 = h f\left(x_n + h, y_n + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right)$$

$$k_6 = h f\left(x_n + \frac{1}{2}h, y_n - \frac{8}{27}k_1 + 2k_2 - \frac{3554}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)$$

Then an estimation to the solution of initial value problem (IVP) is made with the help of Runge-Kutta scheme of 4th order is as:

$$y_{n+1} = y_n + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5$$

And R-K scheme of 5th order is as:

$$z_{n+1} = y_n + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6$$

The optimum step size “sh” is obtained by taking product of the scalar s times the present step size h . The scalar s is given as:

$$s = \left(\frac{Tol h}{2|z_{k+1} - y_{k+1}|} \right)^{\frac{1}{4}} \approx 0.84 \left(\frac{Tol h}{2|z_{k+1} - y_{k+1}|} \right)^{\frac{1}{4}}$$

where Tol is stated as the error control tolerance.

Runge-Kutta-Fehlberg scheme provides very small error as compared to R-K-F scheme of 4th and 5th orders that was given by **Mathews and Fink (2004)**.

3.2 Shooting Method

By using the shooting technique one can solve the “boundary value problem” to the iterative solution by reducing it into “initial value problem”. This scheme generally includes the process of trial and error. The foremost step of this method is to find the boundary points which have the most favorable conditions is taken as the initial point and guesses are made for remaining missing initial conditions. Now, R-K scheme is used to solve the initial value problem. The initial conditions are changed accordingly and the problem is solved again and again using same method until the results, which are calculated, agrees to the known boundary conditions. This procedure is repeated till the supposed initial conditions gives a solution with the specified tolerance.

By using this method one can change the boundary value problem into an initial value problem with the requirement of a correct initial assumptions for the remaining initial conditions. The approximate value of the solution is decided should be very close to the solution that satisfy the final boundary conditions.

3.3 Flow and heat transfer of mixed convective flow of Cu-water nanofluid

The detailed methodology of computation of the problem as defined in aim of the flow problem given in chapter 1, is stated below.

This problem is related to the flow and heat transfer of a steady mixed convective flow of Cu-water nanofluid flowing on the stretching sheet and taking the effect of thermal radiation and activation energy effect. The stream includes impact of Brownian motion with thermophoresis and uniform transverse magnetic field is also applied during the flow. Moreover, the nanoparticles are spherical in shape and the sheet is porous in nature. The condition of zero normal mass flux of nanoparticles is imposed at the surface of the flow. Nanofluid is flowing over stretching surface with the impact of velocity slip and Stefan blowing.

Cattaneo-Christov heat and mass flux model is generalized form of Fourier's and Fick's model. C-C double diffusion model are applied to discuss the transfer of heat, thermal and solutal diffusion. It considers the heat and mass flux relaxation factor.

The obtained equations that governs the fluid flow are changed by means of similarity transformations and then solving them by R-K-F scheme with shooting technique. Graphically illustrate the result patterns of temperature, concentration and velocity profiles for various values of chemical reaction parameter, thermophoresis parameter, Brownian motion parameter, slip parameter, buoyancy ratio parameter, Prandtl number, mixed convection parameter, activation energy parameter, magnetic parameter, Thermal radiation parameter, Stefan blowing parameter, thermal time relaxation parameter, solutal time relaxation parameter and solid nanoparticle's volume fraction. The local drag coefficient and the heat transfer rate at the stretching surface are also arranged for different values of nanofluid parameters and the stream conditions.

3.3.1 Mathematical formulation

Study a steady incompressible mixed convective Cu-water nanofluid flowing on the stretching surface to observe heat and mass transfer. Nanoparticles are spherical in shape and nanofluid is flowing under the effect of activation energy and thermal radiation. The uniform transverse magnetic field is also applied during the flow. The surface of sheet chosen along x -axis and y -axis is along normal to direction of surface of stretching sheet. The value of concentration and temperature at the stretching sheet are taken as C_w and T_w respectively, whereas the nanofluid temperature and concentration are attained values T_∞ and C_∞ , respectively as y tend to ∞ . The Hall effects and viscous dissipation are neglected in this study.

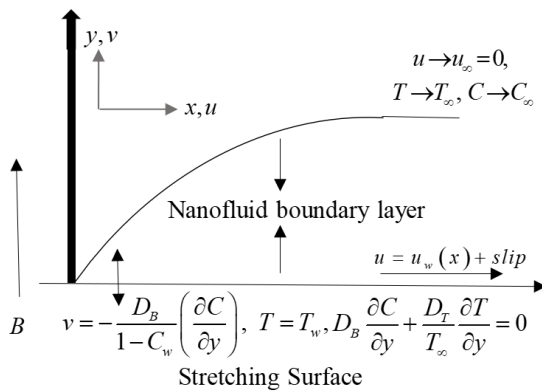


Fig. 3.1 Physical configuration and model of the modeled problem.

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.3.1)$$

Momentum Equation:

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B_o^2}{\rho_{nf}} u \\ & + \frac{1}{\rho_{nf}} \left[(1 - C_\infty) \rho_{f\infty} B_{nf} (T - T_\infty) \right] g - \frac{1}{\rho_{nf}} \left[(\rho_p - \rho_{f\infty}) (C - C_\infty) \right] g - \left(\frac{\mu_{nf}}{\rho_{nf}} \right) \frac{u}{K_o} \end{aligned} \quad (3.3.2)$$

Heat transfer Equation:

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \tau_t \left[u^2 \frac{\partial^2 T}{\partial x^2} + \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \frac{\partial T}{\partial x} + \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial y} + 2uv \frac{\partial^2 T}{\partial x \partial y} + v^2 \frac{\partial^2 T}{\partial y^2} \right] \\ = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial y^2} \right) + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y} \end{aligned} \quad (3.3.3)$$

Mass transfer Equation:

$$\begin{aligned} u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \tau_c \left[u^2 \frac{\partial^2 C}{\partial x^2} + \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \frac{\partial C}{\partial x} + \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \frac{\partial C}{\partial y} + 2uv \frac{\partial^2 C}{\partial x \partial y} + v^2 \frac{\partial^2 C}{\partial y^2} \right] \\ = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right) - K_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^m \exp \left(-\frac{E_a}{kT} \right) \end{aligned} \quad (3.3.4)$$

radiative heat flux term q_r , which defined as:

$$q_r = -\left(\frac{4\sigma^*}{3k_{nf}^*}\right)\left(\frac{\partial T^4}{\partial y}\right) \quad (3.3.5)$$

Since, the temperature difference is minimum. Hence, the term T^4 is linearized by applying Taylor's series w.r.t the temperature T_∞ and rejecting the second and higher order terms of $(T - T_\infty)$, we get $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$. (3.3.6)

Using Eqns. (3.3.5) and (3.3.6) in last term on right side of Eqn. (3.3.3),

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \tau_t \left[u^2 \frac{\partial^2 T}{\partial x^2} + \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \frac{\partial T}{\partial x} + \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial y} + 2uv \frac{\partial^2 T}{\partial x \partial y} + v^2 \frac{\partial^2 T}{\partial y^2} \right] \\ = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial y^2} \right) + \tau \left[D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{16}{3} \left(\frac{\sigma^* T_\infty^3}{K_{nf}^* (\rho C_p)_{nf}} \right) \left(\frac{\partial^2 T}{\partial y^2} \right) \end{aligned} \quad (3.3.7)$$

Where, u is the velocity along x -axis and v is the velocity along y -axis, T is the temperature, C is the nanoparticle concentration and D is the mass diffusivity. Further, the

effective dynamic viscosity is $\mu_{nf} = \left(\frac{\mu_f}{(1-\phi)^{2.5}} \right)$, the effective density

is $\rho_{nf} = ((1-\phi)\rho_f + \phi\rho_p)$, the effective heat capacity is

$(\rho C_p)_{nf} = ((1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p)$, the effective thermal conductivity is

$k_{nf} = \left(k_f \left(\frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + 2(k_f + k_p)} \right) \right)$ and the effective electric conductivity of the nanofluid is

$\sigma_{nf} = \sigma_f \left(1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} \right)$ where $\sigma = \frac{\sigma_p}{\sigma_f}$.

The boundary conditions of the equations which govern the fluid flow (3.3.1), (3.3.2), (3.3.4) and (3.3.7) are:

$$u = cx + K_1 \frac{\partial u}{\partial y}, \quad v = -\frac{D_B}{1-C_w} \left(\frac{\partial C}{\partial y} \right), \quad T = T_w, \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{at } y \rightarrow \infty \quad (3.3.8)$$

3.3.2 Method of Solution

The non-dimensional variables to transform equations into ODE's are as follows:

$$u = cx f'(\eta) \quad v = -\left(\nu_f c\right)^{0.5} f(\eta) \quad \eta = y \left(\frac{c}{\nu_f} \right)^{0.5}$$

$$\Theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)} \quad \varphi(\eta) = \frac{C - C_\infty}{C_\infty} \quad (3.3.9)$$

The equations (3.3.1), (3.3.2), (3.3.4) and (3.3.7) are transformed as:

$$A_o f''' - D_o (f'^2 - ff'') - F_o f' M + \frac{C_o}{D_o} \Theta \lambda - \lambda Nr \varphi - A_o P_o f' = 0 \quad (3.3.10)$$

$$\frac{E_o}{B_o} \frac{1}{Pr} \Theta'' \left(1 + \frac{4}{3} Rd \right) + (Nb \Theta' \varphi' + Nt \Theta'^2) + f \Theta' - \gamma_t (ff' \Theta' + f^2 \Theta'') = 0 \quad (3.3.11)$$

$$\varphi'' + Sc_f \varphi' - Sc \gamma_c (ff' \varphi' + f^2 \varphi'') + \frac{Nt}{Nb} \Theta'' - K_o Sc \varphi (1 + \delta \Theta)^m \exp \left(-\frac{E}{1 + \delta \Theta} \right) = 0 \quad (3.3.12)$$

where the values of dimensionless constant are as follows:

$$A_o = (1 - \phi)^{-2.5}; \quad B_o = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = (1 - \phi) + \phi \frac{(\rho C_p)_p}{(\rho C_p)_f};$$

$$C_o = \frac{(\rho \beta)_{nf}}{(\rho \beta)_f} = (1 - \phi) + \phi \frac{(\rho \beta)_p}{(\rho \beta)_f}; \quad D_o = \frac{\rho_{nf}}{\rho_f} = (1 - \phi) + \phi \frac{(\rho)_p}{(\rho)_f};$$

$$E_o = \frac{k_{nf}}{k_f} = \frac{(k_p + 2k_f) - 2\phi(k_f - k_p)}{(k_p + 2k_f) + \phi(k_f - k_p)}; \quad F_o = \frac{\sigma_{nf}}{\sigma_f} = \left[1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} \right];$$

Similarly, the boundary conditions in dimensionless form reduce to:

$$f = \frac{F_w}{Sc} \varphi'(0), \quad f' = 1 + Wf''(0), \quad \Theta = 1, \quad \varphi' = -\frac{Nt}{Nb} \Theta'(0) \quad \text{at } \eta = 0$$

$$f' \rightarrow 0, \quad \Theta \rightarrow 0, \quad \varphi \rightarrow 0 \quad \text{at } \eta \rightarrow \infty \quad (3.3.13)$$

The non-dimensional parameters appearing in Eqns. are expressed as such:

The magnetic parameter is $M = \left(\frac{\sigma_f B_o^2}{\rho_f c} \right)$, fluid relaxation time is represented by

$$\gamma_o (= \lambda_1 c), \quad \text{mixed convection parameter is } \lambda = \left(\frac{(1 - C_\infty)(T_w - T_\infty) \rho_{f_\infty} g \beta_f}{\rho_f c^2 x} \right) \equiv \frac{Gr_x}{Re_x^2}$$

where, $Gr_x = \left(\frac{g \beta_f (1 - C_\infty) x^3 \rho_{f_\infty} (T_w - T_\infty)}{\nu_f^2 \rho_f} \right)$, buoyancy ratio parameter is given by

$$Nr = \left(\frac{(\rho_p - \rho_{f_\infty}) C_\infty}{\beta_f (1 - C_\infty) (T_w - T_\infty) \rho_{f_\infty}} \right), \text{ Reynolds number is represented as } Re_x = \left(\frac{u_w x}{\nu_f} \right), \text{ porosity}$$

parameter is denoted by $P_o = \left(\frac{\nu_f}{k_o c} \right)$, Prandtl number is given by $Pr = \left(\frac{(\rho C_p)_f \nu_f}{k_f} \right)$,

thermal radiation parameter is denoted by $Rd = \left(\frac{4\sigma^* T_\infty^3}{k_{nf}^* k_{nf}} \right)$, Brownian motion parameter is

$$Nb = \left(\frac{D_B C_\infty \tau}{\nu_f} \right), \text{ Thermophoresis parameter is } Nt = \left(\frac{\tau D_T (T_w - T_\infty)}{T_\infty \nu_f} \right), \quad \gamma_t = \tau_t c \text{ is the}$$

thermal relaxation time parameter, $\gamma_c = \tau_c c$ is the solutal relaxation time parameter,

Schmidt number is given by $Sc = \left(\frac{\nu_f}{D_B} \right)$, $K_o = \frac{Kr^2}{c}$ is stands for chemical reaction

parameter, $\delta = \frac{T_w - T_\infty}{T_\infty}$ is temperature difference parameter, $E = \frac{E_a}{kT_\infty}$ is dimensionless

activation energy parameter, blowing parameter is denoted by $F_w \left(= \frac{C_\infty}{1 - C_w} \right)$ and slip

parameter is given by $W = \left(K_1 \sqrt{\frac{c}{\nu_f}} \right)$.

Skin friction coefficient C_f , and Nusselt number Nu_x are given by:

$$C_f = \frac{\tau_w(x)}{\rho_f u_w^2(x)} \quad \text{and} \quad Nu_x = \frac{x}{k_{nf}} \frac{(q_w + q_r)}{(T_w - T_\infty)} \quad (3.3.14)$$

$$\text{Where } \tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (3.3.15)$$

Now using equations (3.3.9) and (3.3.15) the skin friction coefficient and the Nusselt number are obtained as:

$$C_f^* = C_f \sqrt{Re_x} = \frac{f''(0)}{(1 - \varphi)^{2.5}} \quad \text{and} \quad Nu_x^* = \frac{Nu_x}{\sqrt{Re_x}} \frac{k_f}{k_{nf}} = - \left(1 + \frac{4}{3} Rd \right) \Theta'(0).$$

The PDE related to the boundary condition are explained by using R-K-F scheme and shooting technique. While finding the solution of modeled nanofluid flow problem select a suitable finite value of $\eta \rightarrow \infty$, say η_∞ and step-size is taken as 0.001. since there are no significant changes found in non-dimensional variables after the value of $\eta = 15$. So, the far field condition is replaced with $\eta_{\max} = 15$. We substitute $s_1 = f$, $s_2 = f'$, $s_3 = f''$, $s_4 = \Theta$, $s_5 = \Theta'$, $s_6 = \varphi$, $s_7 = \varphi'$, and get the system of ordinary differential equations consisting of seven first order equations and the following initial conditions:

$$s_1' = s_2$$

$$s_2' = s_3$$

$$s_3' = \frac{1}{(A_o)} \left[-D_o (s_1 s_3 - s_2^2) + M F_o s_2 - \left(\frac{C_o}{D_o} \right) \lambda s_4 + \lambda N r s_6 + A_o P_o g_2 \right]$$

$$s_4' = s_5$$

$$s_5' = \left\{ -Nb s_5 s_7 - Nt s_5 s_5 - s_1 s_5 + \gamma_t (s_1 s_2 s_5) \right\} \times \left\{ \left(\frac{E_o}{B_o Pr} \right) \left(1 + \frac{4}{3} Rd \right) - \gamma_t s_1 s_1 \right\}^{-1}$$

$$s_6' = s_7$$

$$s_7' = \left\{ -Sc s_1 s_7 - (Nt / Nb) s_5' + Sc K_o g_6 (1 + \delta s_4)^m \exp[-E / (1 + \delta s_4)] + \gamma_c Sc s_1 s_2 s_7 \right\} \times \{1 - \gamma_c Sc g_1 g_1\}^{-1}$$

Subject to the following initial conditions:

$$s_1(0) = \frac{F_w}{Sc} \left(-\frac{Nt}{Nb} g_2 \right); \quad s_2(0) = 1 + W g_1; \quad s_3(0) = g_1$$

$$s_4(0) = 1; \quad s_5(0) = g_2; \quad s_6(0) = g_3; \quad s_7(0) = -\frac{Nt}{Nb} g_2$$

In order to solve the above problem, the initial step is to find the values of missing slopes. In above problem, missing slopes are shown by g_1, g_2 and g_3 . The next step is to compute the remaining boundary conditions. When the residuals for the boundary conditions are within pre-decided error tolerance *i.e.*, $\max \{ |s_2(\eta_{\max}) - 0|, |s_4(\eta_{\max}) - 0|, |s_6(\eta_{\max}) - 0| \} \leq 10^{-5}$ then we stop the iterative process. A representative case is illustrated graphically to present the variation of dimensionless velocity, temperature and concentration with non-dimensional distance (see Fig. 3.1).

In this chapter we will investigate the Cu-water nanofluid flowing on porous stretching surface with activation energy and thermal radiation are applied to the flow and the nanoparticles mass flux is taken to be zero at the surface. The C-C double diffusion model is applied to study the concentration and thermal diffusion. we will discuss the result of numerical computations to solve linear and non-linear ODE's and PDE's that are found for fluid flow study which is explained in previous chapter. The equations governing fluid flow problem are being solved with certain initial and boundary situations with the help of the common software package, with computational schemes.

In the present chapter we will discuss the upshot of concentration, temperature and velocity distribution of Cu-water nanofluid flow over porous stretching sheet and the final outcomes are represented by graphs. The values of the skin-friction coefficient and Nusselt number are tabularized for varied values of different parameters. The effects are investigated for various parameters and discuss the graphical representation in detail. Effects on the velocity profile $f'(\eta)$, temperature profile $\Theta(\eta)$ and nanoparticle concentration profile $\phi(\eta)$ for various values of Brownian motion parameter Nb , thermophoresis parameter Nt , chemical reaction parameter K_0 , slip parameter W , magnetic parameter M , Prandtl number Pr , mixed convection parameter λ , buoyancy ratio parameter Nr , activation energy parameter Ea , Thermal radiation parameter Rd , Stefan blowing parameter Fw , thermal time relaxation parameter γ_t , solutal time relaxation parameter γ_c and volume fraction of solid nanoparticles ϕ are represented by graphs.

For validation of numerical method to solve the present study of mathematical model and to prove the exactness of our results, we made a comparison with existing case in literature, presented by **Sandeep and Sulochana (2018)**. They examined the Maxwell, Jeffrey and Oldroyd-B nanofluids flowing on a porous stretching sheet in presence of magnetic field and heat generation/absorption with suction and convective heating at the surface. We have compared the outcomes for the Maxwell nanofluid. From Table 4.1, we can see a close agreement with results of **Sandeep and Sulochana (2018)**.

4.1 Results

The equations (3.3.10) to (3.3.12) governing fluid flow problem with boundary conditions (3.3.13) are being solved by R-K-F scheme along with the shooting technique.

Default values of the various parameters are as follows:

$\gamma_o = 0.2$, $M = 0.2$, $\lambda = 0.4$, $Nr = 0.2$, $P_o = 0.2$, $Pr = 6.2$, $Rd = 0.2$,
 $Nb = Nt = 0.2$, $\gamma_t = 0.004$, $\gamma_c = 0.004$, $Sc = 0.5$, $K_o = 0.2$, $\delta = 0.2$, $E = 2$, $m = 0.5$,
 $W = F_w = 0.1$ and $\phi = 0.01$. Any deviation is noted for any parameter is stated at that particular Figure or Table. The outcomes are being represented by graphs and the detailed physical explanation for these graphs are as follows:

4.2 Discussion

Fig. 4.1 representing the curves for the behaviour of nanofluid velocity, as it moves away towards the far field on increasing the value of mixed convection parameter λ . It can be seen that the velocity of nanofluid reduces on rising the value of λ . It is also observed that the negative velocity near the surface, it is due to initially attaining the ambient value i.e. velocity of nanofluid was less than ambient value.

Fig. 4.2 shows the variation in nanoparticles concentration profile for the change in values of λ . It is observed that the value of concentration is negative at the surface ($\eta = 0$) and when nanofluid starts moving away from surface, it reaches to the maximum value and then falls down to achieve its ambient values as $\eta \rightarrow \infty$. This outcome similar for all cases. The value of $\varphi(\eta)$ increases on increasing the value of λ .

Fig. 4.3 represents the change in velocity distribution on changing the values of the buoyancy ratio parameter Nr . It is to be noted that the nanofluid velocity reduces on enhancing the value of parameter Nr . The graphs of Fig. 4.4 demonstrate the effect of activation energy parameter Ea on nanoparticle concentration field while Fig. 4.5 shows the impact of reaction rate constant K_o on nanoparticle concentration distribution. Activation energy is defined as the minimum amount of energy required to start a chemical

reaction. The modified Arrhenius function $\left(\frac{T}{T_\infty}\right)^m \exp\left(-\frac{E_a}{KT}\right)$ decreases on increasing the

values of activation energy. Concentration boundary layer thickness increases on increasing the value of activation energy parameter. Therefore, we observe an increment in concentration profile $\varphi(\eta)$ on enhancing the values of activation energy parameter.

As compared to the result from Fig. 4.4, in Fig. 4.5 the concentration profile decreases on increasing the reaction rate parameter. According to obtained results, it can be seen that the increment in value of reaction rate constant ($K_o > 0$) corresponds to larger rate of chemical reaction of destructive nature. Increasing reaction rate constant leads to destruction of species more effectively. Hence, decrease in value of $\varphi(\eta)$ is observed. The absolute maximum value of concentration also increases with increase in activation energy parameter; however, it decreases with increase in parameter K_o .

All three of the non-dimensional concentration, temperature and velocity are affected by thermal radiation parameter Rd . Figs. 4.6 and 4.7 show the impact of radiation parameter Rd on velocity and temperature distribution in nanofluid flow. It can be seen that, a significant increment in both temperature and velocity in nanofluid flow on increasing the radiation parameter Rd . These results for non-dimensional velocity $f'(\eta)$ and temperature $\Theta(\eta)$ with radiation parameter Rd can be explained with fact that under the considered physical conditions, radiation is more dominant over conduction during Cu-water nanofluid flow. Increment in the numerical value of radiation parameter indicates that more amount of radiative heat is transferred in the nanofluid flow. Due to addition of extra heat to the Cu-water nanofluid flow, thermal buoyancy forces also increase. Hence, graph shows rise in velocity $f'(\eta)$ and temperature $\Theta(\eta)$ on increasing radiation parameter.

Fig. 4.8 shows the graph of concentration $\varphi(\eta)$ for varied values of thermal radiation parameter Rd , for the fixed value of other parameters. It is seen that on rising the value of radiation parameter, we witness the decrement in nanoparticle concentration at the surface (when $\eta = 0$). The maximum value of $\varphi(\eta)$ during the flow is decreasing on rising the value of parameter Rd . After achieving the maximum value, the graphs of concentration $\varphi(\eta)$ falls to its ambient value. Whereas, the fall of concentration $\varphi(\eta)$ is more drastic for lower values of parameter Rd . It means that, after attaining maximum value, when Cu-water nanofluid moves towards ambient field, concentration value

enhances with increasing the value of parameter Rd . Increasing the numerical value of radiation parameter suggests that more amount of radiative heat is transferred in the nanofluid flow. Thermal buoyancy forces increase due to addition of extra heat to the Cu-water nanofluid flow. Hence, velocity $f'(\eta)$ and temperature $\Theta(\eta)$ are increases as radiation parameter is increased. Fig. 4.8 represents the graph of concentration $\phi(\eta)$ for varied values of parameter Rd , while other parameters are fixed. It can be seen that the nanoparticle concentration at the surface (when $\eta = 0$) decreases as the value of radiation parameter is increases. The maximum value of $\phi(\eta)$ during the flow also drop down on rising the values of parameter Rd . After attaining the maximum value, the graphs of concentration $\phi(\eta)$ falls and achieve the ambient value. However, decrease in concentration $\phi(\eta)$ is more drastic for lower values of thermal parameter Rd which implies that after reaching maximum value, when Cu-water nanofluid moves towards ambient field, concentration value enhances on rising values of thermal parameter Rd .

In the analysis of momentum, thermal and concentration equation together with Stefan blowing effect at the surface boundary, it is found that non-dimensional velocity, temperature and concentration in the Cu-water nanofluid flow are influenced by blowing parameter F_w , as represented via Figs. 4.9, 4.10 and 4.11 respectively.

Fang and Jing (2014), examined that the blowing effect is different in nature than the mass transpiration (suction or injection). The blowing effect rises due to transfer of species from wall to far field or vice-versa. Additionally, observing to the existing situation, as transfer of species is due to flow field and which in turn is influenced by mass blowing at the surface. The changes in velocity, temperature and concentration due to variation in blowing parameter F_w is due to coupling of equations. The positive value of blowing parameter F_w implies the physical situation of mass blowing, where mass transfer take place from surface to ambient field (*i.e.*, evaporation). From Fig. 4.9, we conclude that increase in mass flux from surface to free stream can decrease the speed of nanofluid at the surface and as well as when it moves further away from surface. The same behaviour is also observed for temperature $\Theta(\eta)$ in nanofluid flow via Fig. 4.10. Temperature decreases with increase in intensity of mass flux blown out from surface to free stream. Fig. 4.11 shows that with variation in value of parameter F_w , the nanoparticle concentration does not

follow similar behaviour in entire boundary layer. On increasing the value of blowing parameter F_w , value of concentration $\varphi(\eta)$ increases initially and attains the maximum value, then after when distance from surface increases, curves fall down asymptotically to attain the free stream value. Also, it is observed that when curves fall down, the value of nanoparticle concentration $\varphi(\eta)$ is higher for lower value of parameter F_w . This behaviour can be the characteristic of the presence of zero mass flux condition at the surface.

The graphs in Fig. 4.12, 4.13 and 4.14 establish the effect of slip applied at the boundary of surface on velocity, temperature and concentration distribution in Cu-water nanofluid flow. On rising the value of slip parameter, initially velocity decreases but after it moves far from surface *i.e.*, velocity rises with increase in slip parameter. The temperature of nanofluid is increase in entire boundary layer with the rise in slip parameter. The rise in slip parameter generate more friction in flow field and consequently, rise in temperature distribution across the flow. The rise in slip parameter causes increment in nanoparticle concentration near the surface but contrary results were obtained as nanofluid moves towards far field. This result indicates that increase in slip push more nanoparticles towards free stream.

The present study deals with thermal diffusion and concentration diffusion or Cattaneo-Christov double diffusion model *i.e.*, generalized Fourier's model and Fick's model, which deals with heat flux relaxation time and mass flux relaxation time. The numerical value of thermal relaxation parameter $\gamma_t = 0$ corresponds to Fourier's model and solutal relaxation parameter $\gamma_c = 0$ corresponds to Fick's model, whereas non-zero value of thermal relaxation parameter γ_t and solutal relaxation parameter γ_c and corresponds to Cattaneo-Christov double diffusion model. Figs. 4.15 and 4.16 show the dynamical behaviour of non-dimensional velocity $f'(\eta)$ and non-dimensional temperature $\Theta(\eta)$ due to variation in thermal relaxation parameter γ_t . Fig. 4.15 indicates that presence of time lag during heat transfer in the medium effect the velocity of nanofluid. The velocity of nanofluid is seen to decreasing with increasing value of parameter γ_t *i.e.*, fluid slow down with increase in thermal relaxation parameter. The temperature decreases with increment in value of parameter γ_t . The higher value of thermal relaxation parameter γ_t indicates that material takes more amount of transfer heat, which decays the temperature profiles. Since, the temperature profiles are more dominant for lower value of parameter γ_t , which

conclude that the material with less conducting nature has higher value of thermal relaxation parameter γ_t . Graph in Fig. 4.17 is illustrating the nature of concentration profiles as value of solutal relaxation parameter γ_c changes. Fig. 4.17 shows that on rising the value of parameter γ_c , nanoparticle concentration $\phi(\eta)$ enhances throughout the boundary layer to finally achieve the ambient value.

Figs. 4.18, 4.19 and 4.20 illustrate the impact of volume fraction of nanoparticles on non-dimensional profiles of velocity, temperature and concentration. Rising the value of ϕ implies addition of more nanoparticles into the base fluid. Velocity and temperature of Cu-water nanofluid increase with rising volume fraction. From Fig. 4.18 and 4.19, under the considered physical conditions, a greater number of nanoparticles in the base fluid cause increment in the velocity as well as temperature of the flow. Additionally, from Fig. 4.20, it is concluded that on increasing the volume fraction of nanoparticles, initially near the surface concentration decays but as fluid moves further away from the surface, rise in nanoparticle concentration to finally attain ambient value.

Tables 4.3 and 4.4 represent the calculated values of skin friction coefficient and heat transfer coefficient of for the present fluid flow problem. These tables show that on increasing the values of parameter thermal relaxation parameter γ_t , Stefan blowing parameter F_w and reaction rate constant K_o there is an increment in the values of both skin friction coefficient and heat transfer coefficient. However, skin friction coefficient enhances and heat transfer coefficient reduces on increasing value of buoyancy ratio parameter Nr . On increasing the values of parameters λ , Rd , W , γ_c , E_a and ϕ , the values of skin friction coefficient and heat transfer coefficient are decreases.

TABLE 4.1

Thermo-physical properties of water and Copper (Cu) nanoparticles.

	$\rho(\text{Kg/m}^3)$	$C_p(\text{J/KgK})$	$k(\text{W/mK})$	$\beta(1/\text{K})$	$\sigma(\text{S/m})$
Water	997.1	4179	0.613	21.4×10^{-6}	0.005
Copper (Cu)	8933	385	401	17×10^{-6}	5.98×10^7

TABLE 4.2Comparison of values of $f''(0)$ & $-\theta'(0)$ with results of **Sandeep and Sulochana (2018)** for Maxwell nanofluid when $Bi = 0.8$, $Pr = 6.8$, $\gamma_1 = 0.5$, $A^* = B^* = 0.1$, $M = 1$ and $R = 1$.

S	Present results		Sandeep and Sulochana (2018)	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.2	-1.742675	0.504524	-1.742481	0.504719
0.4	-2.070639	0.553311	-2.070379	0.553511

TABLE 4.3Numerical values of skin friction coefficient & heat transfer coefficient of Cu-water nanofluid for variation in λ , Nr , Rd , γ_t and F_w .

λ	Nr	Rd	γ_t	F_w	$-C_f^*$	Nu_x^*
0.2					2.32624	7.487087
0.4					2.31986	7.461338
0.8					2.31106	7.403717
	0.1				2.30685	7.521035
	0.2				2.31986	7.461338
	0.3				2.33451	7.400095
		0.2			2.31986	7.461338
		0.25			1.80713	5.744387
		0.3			1.62391	5.012826
		0.4			1.44941	4.285038
			0.001		2.06652	6.565247
			0.002		2.12433	6.784381
			0.004		2.31986	7.461338
			0.005		2.55560	8.164338
				0.1000	2.31986	7.461338
				0.1001	2.34295	7.527838
				0.1005	2.44518	7.810938

TABLE 4.4

Numerical values of skin friction coefficient & heat transfer coefficient of Cu-water nanof fluid for variation in E_a , K_o , W , γ_c and ϕ .

E_a	K_o	W	γ_c	ϕ	$-C_f^*$	Nu_x^*
1					2.32235	7.485205
2					2.31986	7.461338
3					2.31971	7.453738
	0.0				2.31845	7.445403
	0.2				2.31986	7.461338
	0.3				2.32137	7.471129
		0.05			2.94134	8.025765
		0.10			2.31986	7.461338
		0.15			1.95252	7.067531
			0		2.34804	7.557605
			0.004		2.31986	7.461338
			0.008		2.29487	7.373938
			0.016		2.25231	7.220443
				0.01	2.31986	7.461338
				0.02	2.03178	5.955271
				0.03	1.93386	5.115471

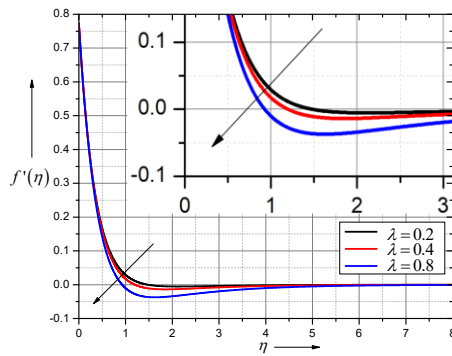


Fig. 4.1 Velocity profile with varying mixed convection parameter.

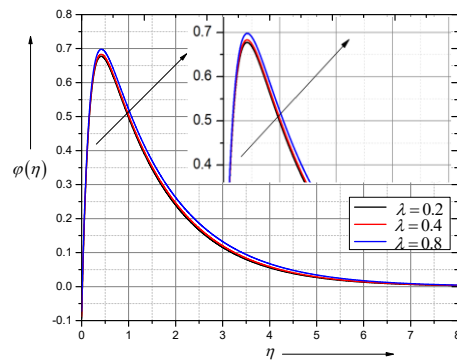


Fig. 4.2. Concentration profile with varying mixed convection parameter.

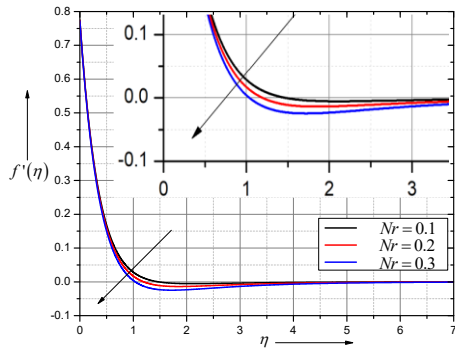


Fig. 4.3. Velocity profile with varying buoyancy ratio parameter.

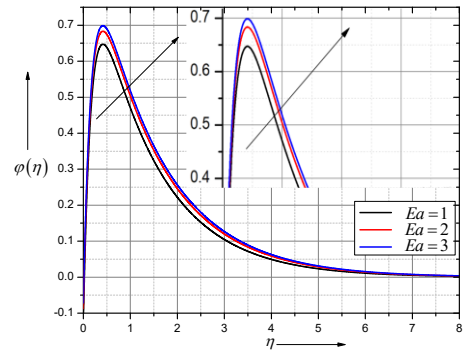


Fig. 4.4. Concentration profile with varying activation energy parameter.

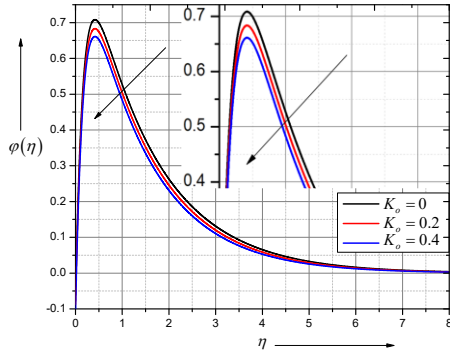


Fig. 4.5. Concentration profile with varying reaction rate constant.

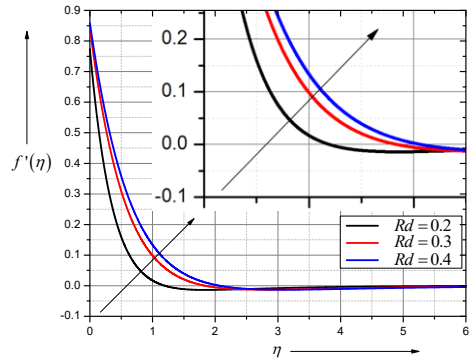


Fig. 4.6. Velocity profile with varying radiation parameter.

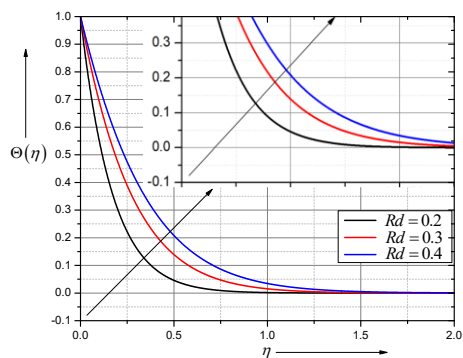


Fig. 4.7. Temperature profile with varying radiation parameter.

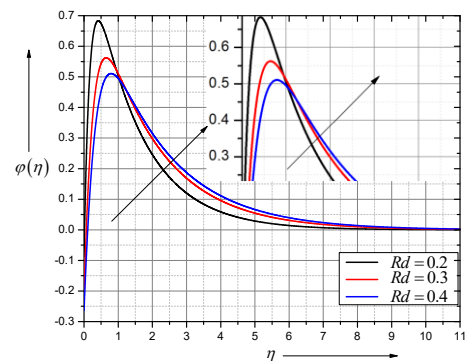


Fig. 4.8. Concentration profile with varying radiation parameter.

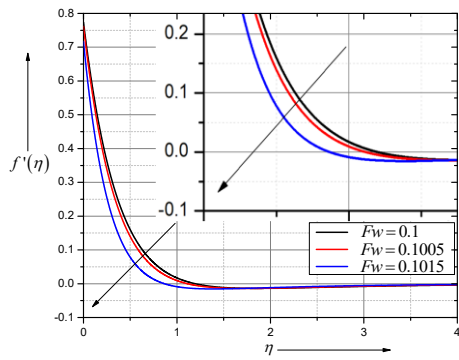


Fig. 4.9. Velocity profile with varying blowing parameter.

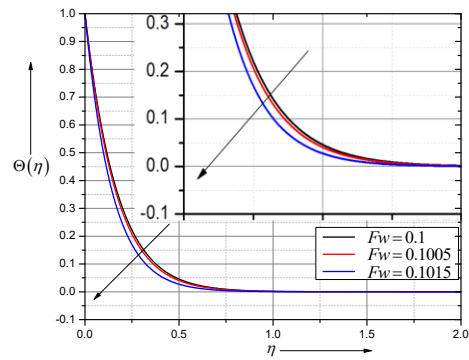


Fig. 4.10. Temperature profile with varying blowing parameter.

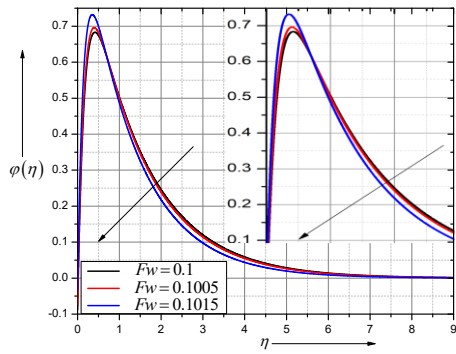


Fig. 4.11. Concentration profile with varying blowing parameter.

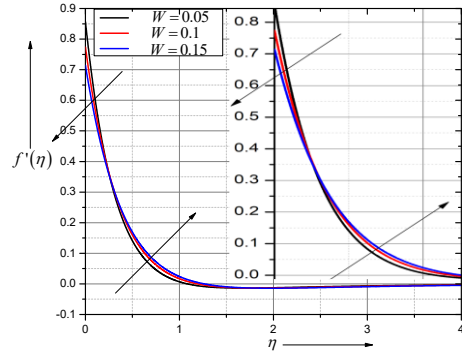


Fig. 4.12. Velocity profile with varying slip parameter.

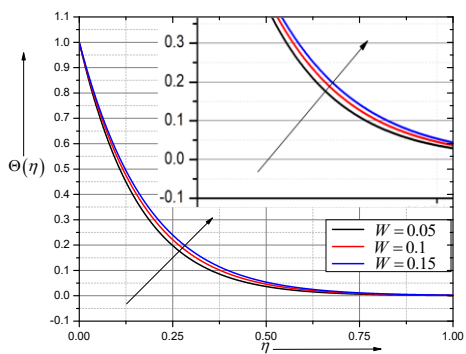


Fig. 4.13. Temperature profile with varying slip parameter.

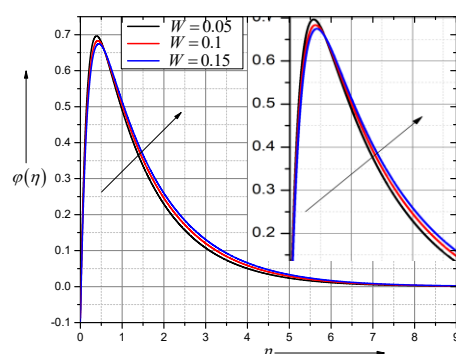


Fig. 4.14. Concentration profile with varying slip parameter.

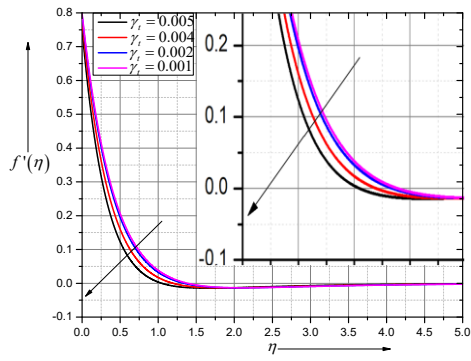


Fig. 4.15. Velocity profile with varying thermal relaxation parameter.

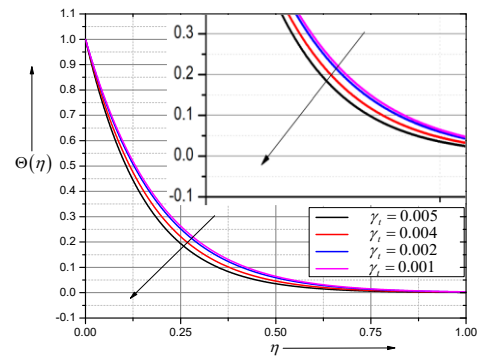


Fig. 4.16. Temperature profile with varying thermal relaxation parameter.

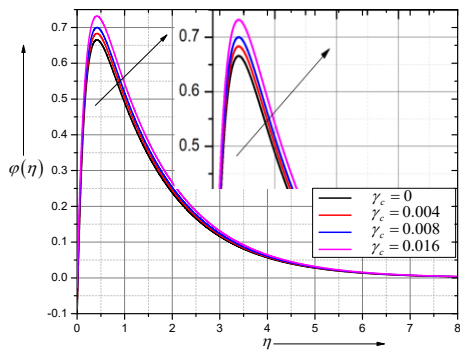


Fig. 4.17. Concentration profile with varying thermal solutal parameter.

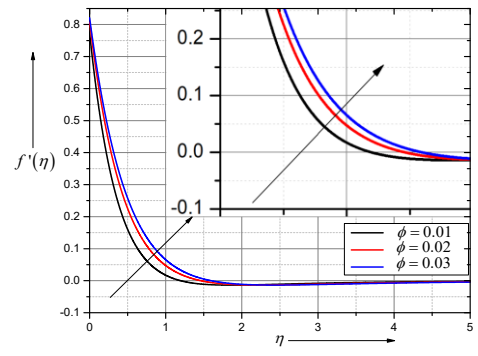


Fig. 4.18. Velocity profile with varying volume fraction.

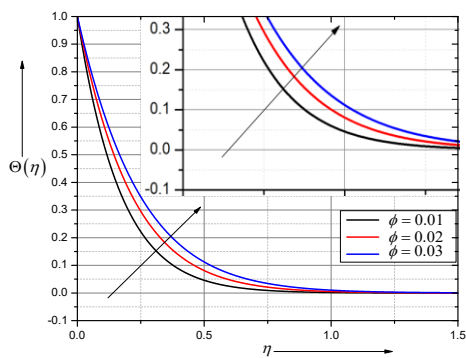


Fig. 4.19. Temperature profile with varying volume fraction.

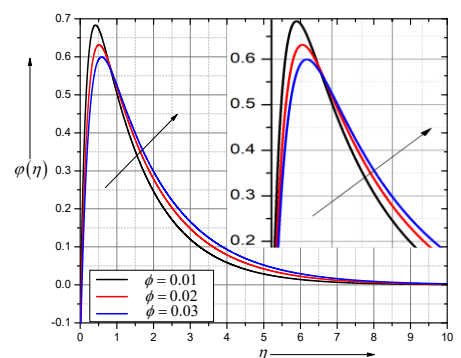


Fig. 4.20. Concentration profile with varying volume fraction.

In heat transfer application the traditional heat transfer fluids such as water and organic fluids such as ethanol, ethylene glycol etc. their thermal conductivity is very low as compare to solids because of the fact that millimeter or micrometer size of particles contained in the conventional fluids, block the minute passages of these gadgets. In this advanced modern technological era, the nanoparticles are manufactured in laboratories to study of flow, mass and heat transfer take place during the nanofluid flowing over different geometries, such as stretching or shrinking surfaces, cylinder, plate etc. The simultaneous solution of the continuity, momentum and energy equations are required to obtain the velocity, temperature and concentration distribution during the flow. The nanofluids has the vast area of applications in many science and engineering as in automotive industries, microchips cooling, delivery of nanodrugs etc. due to its enhancement property.

Past few decades, the various fluid flow problems have gathered a lot of attention of scientists and analysts around the world. Since, the researchers increase their research fields by adding wide range of parameters like magnetic, chemical reaction, heat generation/absorption, thermophoresis, Brownian diffusion, porosity, viscous dissipation, and suction and injection has applied in the problems of nanofluid flow. Mathematical modelling is inventive process for solving these flow problems and to get the numerical solution R-K-F scheme with shooting technique is applied. The truncation error is estimated at every step and retain them in the fixed limit and step size.

The present work titled with “Flow and Heat Transfer of Cu-water Nanofluid over Stretching Sheet with Stefan Blowing Effect” is made to study the effect of different parameters on nanofluid flowing over the stretching sheet. The graphical representation and the detailed study of variation of concentration, temperature and velocity on changing different parameters are discussed. Moreover, the Nusselt number, the skin friction coefficient all are calculated.

Chapter 1 comprises the detailed introduction of the concepts which are practically and theoretically important for the present study. The general ideas of fluids and its

classification are briefly explained in this chapter. After that the boundary layer theory and the equations which govern the nanofluid flow like the continuity equation, momentum, heat and mass equations are discussed. The physical quantities and non-dimensional quantities that appear in this study are also described. Furthermore, the last section is designed to give brief explanation of the objective of this study.

Chapter 2 consists of the review of literature, existing all previous research work available relative to this present work. It helps us to find out the application of nanofluids in different fields and how the parameters are used to study flow of nanofluid. This chapter also includes the summary of the review of research. It also shows that how the research field is grown in the field of nanofluid with passing time.

Chapter 3 includes the detailed explanation of mathematical formulation of the present problem. It contains the graphical models to study behaviour of different parameters discussed in this work. It also includes the explanation of included methodology, along with the details of mathematical solution. This chapter is subdivided into the five different sections. Sections 3.1 and 3.2 exhibit the specifics of methodology used i.e. the explanation of the R-K-F scheme and the shooting technique. The section 3.3 includes the flow and heat transfer of a steady mixed convective Cu-water nanofluid on stretching surface along with the impact of Stefan Blowing, thermal radiation and activation energy has been formulated.

Chapter 4 includes the results and discussion for attained linear and non-linear differential equations for different parameters along with the subjected boundary conditions contained during the formulation of the problems in chapter 3. Concentration, velocity and temperature distribution, Nusselt number and skin-friction coefficient are analysed for Cu-water nanofluid flowing on stretching surface. Graphs are depicted analyze the impacts of various parameters. The behaviour pattern of concentration, temperature and velocity for different values of slip parameter, Brownian motion parameter, chemical reaction parameter, thermophoresis parameter, magnetic parameter, mixed convection parameter, Prandtl number, buoyancy ratio parameter, Thermal radiation parameter, activation energy parameter, Stefan blowing parameter, thermal time relaxation parameter, solutal time relaxation parameter and nanoparticle's volume fraction are shown by graphs. For the validation of mathematical scheme to solve the problem, we compare it

with existing research literature. The numerical values of Nu_x^* and C_f^* for the Cu-water nanofluid and for different nanoparticle used in base fluid are calculated.

Nanofluid flow and heat transfer about the deep immersed objects are generally come across in various problems like in medical science, hydrodynamics, geophysics, chemical engineering etc. In all cases heat passes over the boundary layer. The flow of nanofluids over the boundary layer is significant, since it is important to predict the rate of transport of heat from nanofluid.

It is seen that the behaviour of nanofluid velocity decreases on progressive increase in value of mixed convection parameter λ . Physically, the increase in positive value of λ , corresponding to heating of nanofluid. The velocity of nanofluid decreases on increasing the value of parameter buoyancy ratio parameter Nr . Concentration profile increases on rising the activation energy parameter E , however, it reduces with rise in chemical reaction parameter K_o . On increasing the radiation parameter Rd it is seen that the temperature and velocity profiles are increases. But the behaviour is opposite in case of concentration profile, it reduces with rising the values of radiation parameter Rd . On increasing the values of blowing parameter F_w i.e. increase in mass flux from surface to free stream can decrease the velocity of nanofluid at the surface and as well as when it moves away from surface. Temperature profile shows the same behaviour during nanofluid flow. But concentration profile shows different behaviour, on increasing values of parameter F_w , value of concentration increases initially and achieves its peak value and then as the distance from surface increases, curves fall down asymptotically to attain the free stream value. Temperature and velocity reduce on increasing the thermal relaxation parameter γ_t . Concentration profile enhances with increase in solutal relaxation parameter γ_c . On increasing volume fraction, both temperature and velocity of Cu-water nanofluid increases. Form the Tables, it can be observed that values of both coefficient of heat transfer and skin friction increases on enhancing the values of thermal relaxation parameter γ_t , blowing parameter F_w and chemical reaction parameter K_o . On rising the values of parameter Nr , skin friction coefficient enhances but heat transfer coefficient decreases. It is also observed that values of coefficient of skin friction and coefficient of heat transfer drop on rising the

values of mixed convection parameter λ , radiation parameter Rd , slip parameter W , solutal relaxation parameter γ_c , activation energy parameter E_a and volume fraction ϕ .

The present study relative to transfer of flow and heat problems are applied to many scientific and industrial fields. To study the nanofluids the nanoparticles are much better as compared to the microparticles because nanoparticles are suspended in the bulk of base fluid for extended time due to the greater difference between the value of their surface area per unit volume. These enhance properties of nanofluids made them useful than other conventional heat transfer fluids. The outcomes of the present study may be useful to the exploration of nanofluids in the area of science and technology, where the fluid flow over stretching surface and porous medium. There is still much more to be discovered in the field nanofluids.

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RESEARCH PAPER

1. **Shubham Negi, Sawan K. Rawat and Manoj Kumar.**

Flow and heat transfer of cu-water nanofluid over stretching sheet with Stefan blowing effect.

Communicated to Journal of Thermal Analysis and Calorimetry.

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ABSTRACT

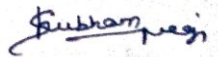
Name	: Shubham Negi	Id. No	: 53951
Sem. & year of admission	: I, 2018-19	Degree	: M.Sc.
Major	: Mathematics	Deptt.	: Mathematics, Statistics and Computer Science
Minor	: Computer Science		
Advisor	: Dr. Manoj Kumar		
Thesis Title	: “Flow and Heat Transfer of Cu-water Nanofluid over Stretching Sheet with Stefan Blowing Effect”		

Present work deals with the problem of flow and heat transfer in nanofluids past a stretching porous sheet mainly Cattaneo-Christov double diffusive model is applied to study the flow problem. The various flow conditions including different physical phenomenon like magnetic effect, activation energy effect, Stefan blowing effect, volume fraction and the condition of zero mass flux has been incorporated at the boundary, to study their effects on velocity, temperature and concentration profiles. The physical quantities of interest *viz.* skin friction coefficient and Nusselt number have also been calculated and their analysis is included in this study.

The study presented in the thesis comprises of five chapters. The first chapter is aimed to fulfill the basic need of introducing the various concepts and foundation needed for fluid mechanics. Chapter two throw light on majority of passed research work related with the present work. Chapter three covers the mathematical formulation of the problem and its detailed description. A proper description of used methodology, together with the details of the numerical solution is also presented. Chapter four describes the result and discussion of the considered problem taken in chapter three. The work has been summarized in chapter five. In along with the finding of the study, the literature used in the course of the study has been referred under the section of literature cited.

The related study may be helpful to applied scientific fields and different branch of engineering and applied mathematics.


(Manoj Kumar)
Advisor


(Shubham Negi)
Author

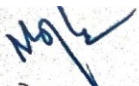
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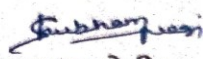
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सत्र एवं प्रवेश वर्ष	: प्रथम सत्र, २०१८-१९	उपाधि	: स्नातकोत्तर
प्रमुख विषय	: गणित	विभाग	: गणित, सांख्यिकी तथा संगणक विज्ञान
सहायक बिषय	: संगणक विज्ञान		
शोध शीर्षक	: “स्टेफेन ब्लोइंग इफेक्ट के साथ स्ट्रेचिंग शीट पर कॉपर-वॉटर नैनोफ्लुइड का फ्लो और हीट ट्रांसफर”		
परामर्शदाता	: डा. मनोज कुमार		

एक स्ट्रेचिंग पोरस शीट के पूर्व में नैनोफ्लुइड्स में प्रवाह और गर्मी हस्तांतरण की समस्या से संबंधित वर्तमान में काम करता है, मुख्य रूप से Cattaneo-Christov डबल डिफ्यूसिव मॉडल प्रवाह समस्या का अध्ययन करने के लिए लागू किया जाता है। विभिन्न भौतिक परिघटनाओं जैसे चुंबकीय प्रभाव, सक्रियण ऊर्जा प्रभाव, स्टीफन उड़ाने का प्रभाव, आयतन अंश और शून्य द्रव्यमान प्रवाह की स्थिति सहित विभिन्न प्रवाह स्थितियों को सीमा पर शामिल किया गया है, ताकि वेग, तापमान और एकाग्रता प्रोफाइल पर उनके प्रभाव का अध्ययन किया जा सके। ब्याज की भौतिक मात्रा। त्वचा घर्षण गुणांक और Nusselt संख्या की भी गणना की गई है और उनके विश्लेषण को इस अध्ययन में शामिल किया गया है।

थीसिस में प्रस्तुत अध्ययन में पाँच अध्याय शामिल हैं। पहले अध्याय का उद्देश्य द्रव यांत्रिकी के लिए आवश्यक विभिन्न अवधारणाओं और नींव को प्रस्तुत करने की बुनियादी आवश्यकता को पूरा करना है। अध्याय दो ने वर्तमान कार्य के साथ संबंधित पारित शोध कार्य के बहुमत पर प्रकाश डाला। अध्याय तीन में समस्या के गणितीय सूत्रीकरण और इसके विस्तृत विवरण को शामिल किया गया है। उपयोग की गई कार्यप्रणाली का समुचित विवरण, संख्यात्मक समाधान के विवरण के साथ भी प्रस्तुत किया गया है। अध्याय चार अध्याय तीन में ली गई समस्या के परिणाम और चर्चा का वर्णन करता है। अध्याय पाँच में काम को संक्षेप में प्रस्तुत किया गया है। अध्ययन की खोज के साथ-साथ, अध्ययन के दौरान उपयोग किए जाने वाले साहित्य को उद्धृत साहित्य के अनुभाग के तहत संदर्भित किया गया है।

संबंधित अध्ययन वैज्ञानिक क्षेत्रों और इंजीनियरिंग की विभिन्न शाखा और अनुप्रयुक्त गणित के लिए सहायक हो सकता है।


(मनोज कुमार)
परामर्शदाता


(शुभम नेगी)
लेखक

