RELIABILITY ESTIMATION OF SOME SENs USING UGF METHOD

Thesis

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By

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Pantnagar
July, 2020

(Vaibhav Bisht)
Author

CERTIFICATE

This is to certify that the thesis entitled **“RELIABILITY ESTIMATION OF SOME SENs USING UGF METHOD”,** submitted in partial fulfillment of the requirements for the degree of **Master of Science** with major in **Mathematics** and minor in **Computer Science** of the College of Post-Graduate Studies, G.B. Pant University of Agriculture and Technology, Pantnagar, India, is a record of bona fide research carried out by **Mr. Vaibhav Bisht, Id. No. 53961** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been duly acknowledged.

Pantnagar
July, 2020

(S. B. Singh)
Chairman
Advisory Committee

CERTIFICATE

We, the undersigned, members of the Advisory Committee of **Mr. Vaibhav Bisht**, **Id. No. 53961**, a candidate for the degree of **Master of Science** with major in **Mathematics** and minor in **Computer Science,** agree that the thesis entitled **"RELIABILITY ESTIMATION OF SOME SENs USING UGF METHOD",** may be submitted in partial fulfillment of the requirements for the degree.

(S. B. Singh)
Chairman
Advisory committee

(Vinod Kumar)
Member

(A. K. Pal)
Member

CONTENTS

Introduction

1.1 Reliability: An outline

In today's era, nearly everyone depends upon the proper functioning of large number of machines and equipment for our day to day requirements and safety, mobility and mobility welfare. We expect our communication system, electrical appliances, computer networks, nuclear power plants, aerospace applications etc. to perform as expected. When they fail the results can be disastrous, injury or even loss of life. Thus it becomes very vibrant to assure their working, by carrying out the study of reliability.

We frequently listen and see the questions raised like wheather an automobile or a particular brand of air conditioner or refrigerator, or motor is reliable or unreliable. If an air conditioner manufactured by a particular firm is reliable, then we mean it is free from any problem but we don't say it is completely free from failure. In this case, it is concluded that probability of its failure in the brand is less. Furthermore, in the modern customers become very much demanding for the products and always look for reliable products. So, system and reliability analysts constantly work on the systems and network structures to form reliable products and minimize the failure rates to improve the reliability.

During the Second World War, reliability was considered as a very vital concept. In June 1943, the first steps in passage of reliability were taken by establishing Joint Navy and Army groups of USA. The department of defense of USA formed an adhoc committee, in 1950, to increase the reliability of military equipment. In 1952, this department constituted a group called Advisory Group on the Reliability of Electronic equipment (AGREE), which in 1957, made a document on the reliability of electronic equipments used by the military and the proposals of this report were accepted by NASA. Also in 1965, this department concerned reliability programs for the equipments, which were modified in 1980. Since then, reliability has turned out to be a challenge for reliability engineers in the field of research.

With the increasing needs of today's consumers, the mathematicians and engineers of various countries are working endlessly to build up reliability theory and improve the system's reliability. In research point of view, the term reliability is related to the term repeatability i.e., a test is said to be reliable if it provides identical results every time under some particular conditions.

The concept of reliability engineering is presently getting a marvellous attention from several researchers due to its wide applications in many real world problems, for example in inspecting the risk and safety to figure out the availability and applicability of systems. Nowadays, the uses of theory of reliability principles have made a considerable growth in several companies. A device having low reliability can lead to a decrement in the safety and also have adverse results such as increased cost, which is from repairing (or replacing) the components of device with failure.

Many authors have given definitions for reliability, but according to the E.E Lewis "Reliability is the probability of component, system or equipment which will perform its intended function adequately for a specified period of time under given set of conditions". The above definition gives a vivid depiction of the world "Reliability".

1.2 Reliability Engineering

Reliability engineering is a field of engineering which is based on reliability theory which emphasizes dependability in life-cycle management of a product or network. Engineering and analysis techniques are used to improve the reliability or dependability of a product or system.

The main objectives of reliability engineering are as follows:

- To apply engineering knowledge and specialist techniques to prevent or to reduce the likelihood or frequency of failures.
- To identify and correct the causes of failure that occur despite the efforts to prevent them.
- To determine ways of coping with failures that occur, if their causes have not been fixed.
- To apply methods for estimating the likely reliability of new software and for analysing the reliability data.

1.3 Reliability

Many authors have given definitions for reliability, but according to the E.E Lewis "Reliability is the probability of component, system or equipment which will perform its intended function adequately for a specified period of time under given set of conditions". The above definition gives a vivid depiction of the world "Reliability".

Reliability of any system may be defined as the probability of a device performing its purpose adequately for period intended under given operating conditions.

Mathematically, it can be expressed as:

$$R(t) = P(T \geq t) = 1 - \int_0^t f(t)dt = \int_0^\infty f(t)dt$$

where, $f(t)$ is the probability density function of time to failure, T is a random variable representing the time to failure of the item.

Reliability is the function of time t and its value lies between 0 and 1. Hence reliability is the probability based concept because value of probability also lies between 0 and 1.

Reliability mainly focuses on following four factors:

- **Probability:** The reliability of any system is always expressed in terms of probability. So there is always some chance of failure of the system.
- **Adequate performance:** It describes in unambiguous terms what is expected of a device or system.
- **Duration of adequate performance:** Time duration is one of the main factors in calculating the reliability. It represents a measure of the period for which the performance is satisfactory.
- **Operating conditions:** These are the conditions under which device is expected to give adequate performance. Temperature, humidity, shock, vibration etc. are such conditions.

All these four factors are interrelated to each other. Reliability may also be defined as probability of not occurring an unfavourable event, and expressed as:

$$R = 1 - P_f$$

where, P_f is the probability of failure.

1.4 Objectives of reliability theory

In the present era, the systems and networks are pretty more complex and getting advanced day by day. The main objective of reliability engineers/experts is to design highly reliable networks or systems within the specified operating condition. Reliability engineers can help in:

- Proper working of the network / system.
- Minimize the possibilities of failures in system.
- Obtain high reliability at a very minimal cost.
- Applications of various techniques for the reliability improvement of system.

1.5 Unreliability

Unreliability of the system may be defined as the probability of failure of systems and is denoted by $F(t)$. It is the ratio of number of failures during a unit interval to the total population.

The total sum of reliability and unreliability of a system is always equal to one. Hence it may be expressed as:

$R(t) + F(t) = 1$

1.6 Some concepts and definitions related to reliability:

Reliability theory can't be explained without understanding some basic concepts and definitions. These are mainly used in calculating the reliability of systems and networks:

1.6.1 Maintainability

Maintainability is a measure of the ease and rapidity with which a system or equipment can be restored to operational status following a failure. It is expressed as the probability that an item will be retained to a specific condition within a given period of time.

1.6.2 Availability

It may be defined as the probability that a system is performing adequately when it is require to be used i.e., it is not going through a repair or it is not failed at that time. The availability of an item at time t is denoted by $A(t)$and may be expressed as:

$A(t) = Pr(X(t) = 1)$

Availability is the function of reliability and maintainability of the product.

1.6.3 Mean time to failure

Mean time to failure (MTTF) of an item is the length of time interval between two successive failures of that item. Since reliability and MTTF of any item are directly related to each other, so MTTF plays a very crucial role in reliability analysis of several systems and networks.

MTTF is expectation of a random variable T. When time is discrete function and t_1, t_2 ,…….t_n are the time to failure of 1^{st} , 2^{nd} ,…….n^{th} item, then MTTF for n times is given as:

$$\text{MTTF} = (t_1 + t_2 + \ldots. t_n)/n$$

$$= (\textstyle\sum_{i=1}^{n} t)/n$$

When time is a continuous function, MTTF is expressed as:

$$\text{MTTF} = \int_0^\infty R(t)dt \qquad \ldots(1.1)$$

where, $R(t)$ is reliability of that item at any time t.

1.6.4 Mean time between failures

Mean time between failures (MTBF) is a measure of a reliability of any product. It may be calculated as the average time elapsed between two successive failures of a system and is given by:

MTBF = Number of operation hour / number of failures

1.6.5 Mean time to repair

Mean time to repair (MTTR) is a measure of the maintainability of items that can be repaired. MTTR of failed device gives the average time require to repair it and make it operational again.

1.7 Networks

A network is generally an arrangement of nodes and vertices. A network $G=(N, E)$ consists of a set N of nodes (or vertices) together with a set E of edges (or links). The objects are designated to as nodes and links are the line segments linking the nodes. In computer network, the nodes denote computers and the edges in which the information is transmitted over various transmission lines. At any instant the elements of the network (nodes and/or edges) will be in either of two possible states, working or failed.

The nodes having no incoming edge are called the **source.** The nodes having no outgoing edge are called the **sink.** The nodes which are not expected to fail are called **terminals.**

A network having 4 nodes, 3 terminals and 11 links is shown in Fig. 1.1. The terminals are drawn as bold circles and the links are drawn as line segments connecting two nodes.

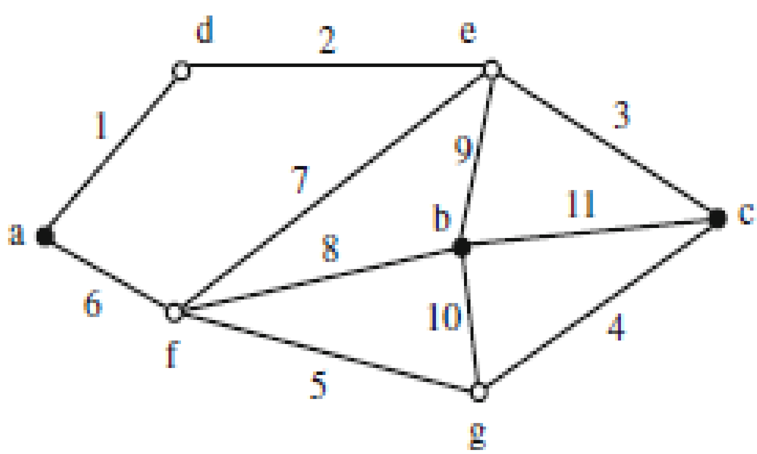

Fig. 1.1: Network

1.7 Reliability of Network

On the basis of flow of information / signals, the reliability of the networks are classified into three categories:

- **Terminal Reliability:** The terminal reliability (TR) of a network is defined as the probability of existence of at least one fault- free path between source and destination of the network.

- **Broadcast Reliability:** The broadcast reliability (BR) of a network is defined as the probability of transmitting signal from single source to all destination nodes.
- **Network reliability:** The network reliability (NR) of a network is defined as the probability of successful communication from all source nodes to all sink nodes.

1.7.1 Series Network

A network is said to be in series, if any one edge fails then complete network will also fail. A series network with two terminals s and t and having four links *A*=(*s,a*), *B*=(*a,b*), *C*=(*b,c*), *D*=(*c,t*) is shown in Fig. 1.2.

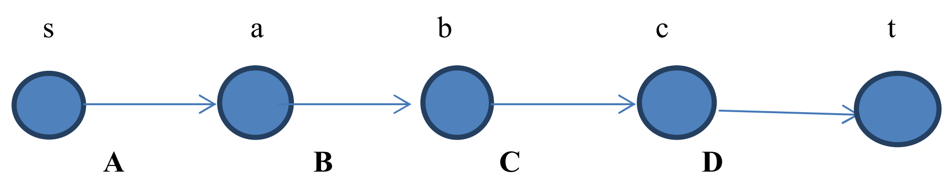

Fig. 1.2: Series Network

Mathematically, it may be expressed as:

$\emptyset(x) = \prod x_i = \min (x_A x_B x_C x_D)$

where, x_i is the probability of flowing of signal through edge *i*, where *i* varies from *A* to *D*.

The network reliability is inversely proportional to number of series node, i.e. network reliability decreases with increase in number of series node and vice versa.

1.7.2 Parallel Network

A network is said to be connected in parallel if whole of the network fails if and only if all of its edges fail. The parallel network does not fail even if any one of the edge is working. A parallel network having six nodes and six edges is shown in Fig. 1.3.

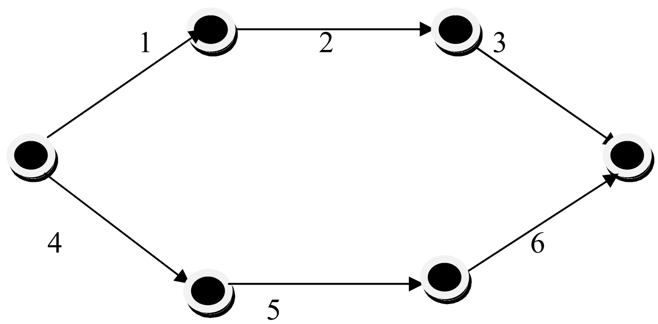

Fig. 1.3: Parallel Network

Mathematically, it may be stated as:

$$\phi(x) = 1- (1-\prod_{i=1}^{3} x_i)(1 - \prod_{i=4}^{6} x_i) = \max(x_1, x_2, x_3, x_4, x_5, x_6)$$

where, x_i is the probability of flowing of signal or message through edge *i*.

1.8 Interconnection Network

Interconnection Networks refer to the communication fabric interconnecting various components of a computer system. An interconnection network is simply the interconnection of various networks of connecting nodes. In the interconnection network, switching elements are used to connect the source node and destination node. The reliability of interconnection networks depends on the reliability of switching element and their interconnection network.

The main purpose behind the construction of interconnection network is that when the single processor is not able to handle the task involving a huge amount of data, then task is broken into different parallel tasks, which are performed simultaneously; resulting into reduction of processing time. So the interconnection network plays a very vital role in constructing large parallel processing system.

Interconnection networks are widely used in many real-life applications such as telephone switches, networks in industries, supercomputers and many more.

Interconnection network may be more beneficial if it is:

- Expandable
- Easy to use

- Cost Efficient
- Interoperable
- Adaptable
- Highly reliable
- Highly maintainable
- Scalable
- Highly modular

Interconnection networks are classified as:

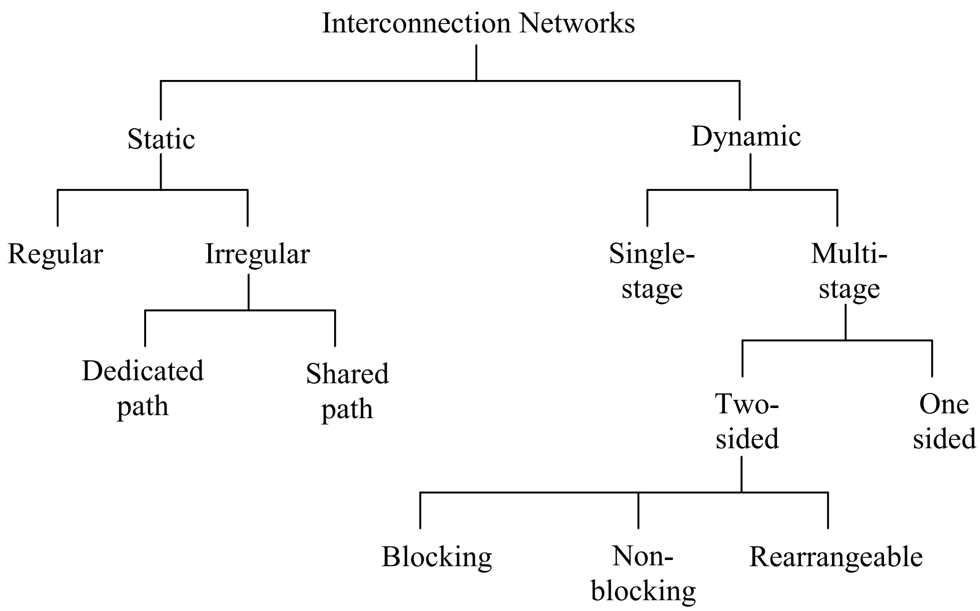

Fig. 1.4: Classifications of Interconnection Networks

1.8.1 Single-stage Interconnection Network

In a single stage interconnection network, there is only one stage of switching elements between input and output nodes of network.

As in the single stage interconnection network, all the source is not connected to all of the outputs, i.e. a signal or a message communicated from a source is not transmitted to all of the outputs. This idea helped engineers to develop the multistage interconnection network in which a signal from a source is transmitted to all of the outputs.

8×8 single stage interconnection network is shown in Fig. 1.5:

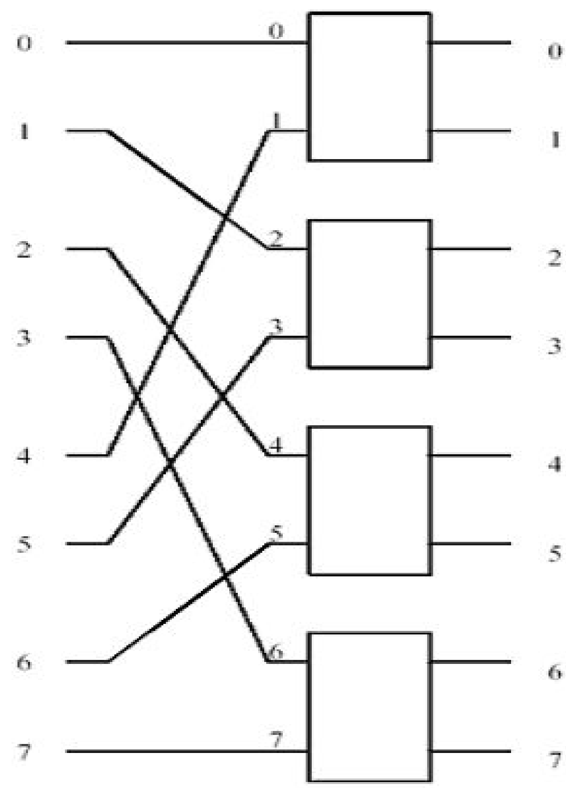

Fig. 1.5: 8×8 single stage interconnection network

1.8.2 Multistage Interconnection Network

Multistage Interconnection Networks (MINs) play a very critical role in the functioning of multiprocessors and parallel processing. The MINs connect the source node to all outputs by significant number of stages. The MINs may have same or distinct number of input and output. The MINs minimize the cost and path length, so they are widely used in real life. Initially, the MINs were constructed for telephone switches but now they are used widely in computer communication, integrated circuits and multiprocessor system.

Generally, a Multistage interconnection network consists of N inputs and N outputs and n (=log_2N) stages and N/2 switching elements per stage, where N is the size of network. Some of widely used multistage interconnection networks are Shuffle exchange network (SEN), SEN with additional stages, Gamma interconnection network (GIN), Extra-stage GIN, Omega network, Benes network, Clos network, Multistage cube network and many more.

The transmission of signal between p inputs and b outputs, through a number of stage in MIN is shown in Fig. 1.6:

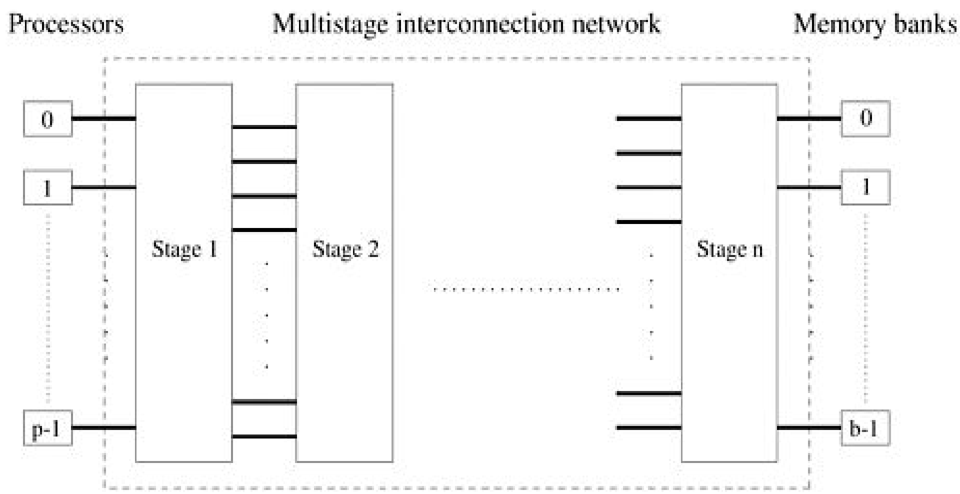

Fig. 1.6: Multistage Interconnection Network

Based on the path availability between stages, MINs are characterized into three categories:

- Non-Blocking
- Rearrangeable non-blocking
- Blocking

On the basis of kind of switches used, MINs can further be classified into:

- Single-path MINs
- Multiple-path MINs

1.8 Universal Generating Function (UGF)

Various techniques are used for calculating the reliability of complex networks and engineering systems. Out of these methods, UGF is one of the widely used method due to its less complexity and time reducing nature. The basic notion behind this method was given by Ushakov (1986).The UGF is based on the generalization of ordinary generating function. It is a form of the moment generating function in polynomial form representing the probability mass function of variables. If there are *m* possible values of *k* independent variables and r_m is the probability of *k* which is equal tok_m, then UGF of *k* is given by the polynomial as:

$$U(z) = \sum_{m=1}^{M} r_m \, z^{k_m} \qquad \ldots(1.2)$$

Consider p independent discrete random variables $X_1, X_2 \ldots X_p$. Let $U_1(z)$, $U_2(z), \ldots, U_p(z)$ be the UGF of random variable $X_1, X_2 \ldots X_p$ and $f(X_1, X_2 \ldots X_p)$ be an arbitrary function. Moreover, combination of *r* UGF is represented by composition operator, where the properties of the composition operator depend on properties of $f(X_1, X_2 \ldots X_p)$. So $U_r(z)$ is given in the following manner:

$$U_r(z) = \bigotimes_f (U_1(z), U_2(z) \ldots, U_p(z))$$

If in a network, two components are given then, their UGFs is given by-

UGF of a series system is given as:

$$(u_1(z) \underset{ser}{\otimes} u_2(z)) = \sum_{k_1=1}^{K_1} p_{1k_1} z^{g_{1k_1}} \underset{ser}{\otimes} \sum_{k_2=1}^{K_2} p_{2k_2} z^{g_{2k_2}}$$

UGF of a parallel system is given as:

$$(u_1(z) \underset{par}{\otimes} u_2(z)) = \sum_{k_1=1}^{K_1} p_{1k_1} z^{g_{1k_1}} \underset{par}{\otimes} \sum_{k_2=1}^{K_2} p_{2k_2}$$

where, $\underset{ser}{\otimes}$ and $\underset{par}{\otimes}$ denote the composition operators for series and parallel components respectively.

1.9 Reliability block diagram:

Reliability block diagram (RBD) is the pictorial representation of the system's components and the interactions between them, used to calculate the whole system's reliability. The RBD method can be used for evaluating the reliability of large-scale as well as complex systems. Due to this, most researchers found that using RBDs is an significant stage in determining the system's reliability.

Review of Literature

(a) In context of Reliability theory:

Cox, D. R. (1955) presented a technique called supplementary variable technique which was further applied by **Garg, R. C. (1963)** to calculate the reliability of different networks/systems. From this technique **Williams, H. L. (1958)** and **Smith, D. J. (1972)** have done a tremendous work to find the reliability of human component in the man-machine system reliability engineering.

When a system have considerable amount of load (like humidity, vibration, shock, temperature), which results into the failure of all components resulting into system failure. To control such type of failures, **Li *et al.*(2010)** framed an optimal model to calculate the reliability of multi-state series-parallel systems. To determine the reliability of the MSS, the UGF technique was taken and for solving the model, a genetic algorithm was used. In order to demonstrate the method, a numerical example was also given.

Tiwari *et al.* (2010) connected two complex subsystems M and N and determined their reliability. The subsystem M comprises of 1 unit while the subsystem N involves of two units N_1 and N_2. For the considered system a trainee and the supervisor repaired the failed components. Supervisor was always accessible whereas the trainee was at holiday and was called for the repair whenever required. Both the subsystems M and N followed exponential distribution system. The repair rate for the trainee followed exponential distribution while for supervisor it followed general distribution. This model was studied under “Head of line repair discipline”. The reliability features such as reliability, cost-effectiveness, transition state probabilities and availability were determined by using Laplace transform and supplementary variable technique.

Levitin *et al.* (2013) published a method to find the reliability and the performance distribution for the series-parallel multi-state systems, which could not be repaired, in which failures were initiated by the failure transmission in elements of system. They assumed that the failure propagation time has any random value with a specific distribution. The suggested algorithm is based on a generalized reliability block diagram method, and the UGF approach.

Ram (2013) did a study of reliability methods in various fields of physical sciences and engineering. He gave the past, present and future development of reliability methods and its uses in numerous firms.

Rebaiaia and Ait-Kadi (2013) discussed about the problems for inspecting and improving the reliability of networks by giving few new algorithms and methods. Both considered a stochastic network in which each node could be in only two states –operational or failed.

Gnedenko *et al.* (2014) discussed the basic perceptions of mathematical statistics, probability theory and depicted the relationships among the quantifiable characteristics experienced in the theory. They presented the various methods to find the approximations for reliability parameters on the basis of observations and method of testing hypothesis.

Gunawan (2014) discussed the basics of reliability engineering, distribution of probability and certain fundamentals of probability theory. He also described reliability methods with their applications in MINs.

Yu *et al.* (2014) assumed a repairable MSS which is having a common bus performance sharing and presented an instantaneous availability model. The MSS under observation consists of a common bus performance rearrangement system and various multi-state units, where every unit has multiple performance stages. Each component in the MSS has a certain random demand which must be satisfied by that component and the whole system falls if the demand of any of the element is not fulfilled.

Negi, S. and Singh, S.B. (2015) calculated reliability of a non-repairable complex system containing two subsystems P (weighted k-out-of-n: G) and Q (weighted l-out-of-m: G) which are joined in series. A and B have linear (v, f, e): G and circular (v, f, e): G components respectively. The MTTF, reliability and sensitivity of the system were calculated by UGF method. To explain this model, a numerical example was also considered.

Rawal *et al.* (2015) discussed a mathematical model for Linux operating system, connected to a local area network, to investigate its reliability parameters. The system under consideration have two topologies, viz., star topology and bus topology which were positioned at two different places and were linked to a server through a hub. With the help of Laplace transform and supplementary variable technique, the availability, non-availability, MTTF and cost-effectiveness of the considered system have been attained, when there were many types of failures and only two types of repairs.

Ahmed *et al.* (2017) calculated various modelling techniques which may be used in calculating the reliability of the communication networks. They gave required background on the modelling and analysis techniques such as reliability block diagrams, mathematical analytical methods, Markov chains, etc. They also estimated the pros and cons of various approaches.

Feizabadi and Jahromi (2017) suggested a new model for optimizing the reliability of the series and parallel systems, provided that the probability for the components of the subsystems to be non-homogeneous in the stated conditions. A genetic algorithm was introduced to explain the model under which was under consideration. These results showed the decent performance of the model under consideration to increase the reliability of the system and decreasing the costs.

Meenakshi *et al.* (2018) considered a non-repairable MSS consisting of two subsystems namely P and Q, arranged in series. These two subsystems P and Q are multi-state weighted *p*-out-of-*q*: G and *r*-out-of-*s*: G systems respectively. Every component of P consists of linear (*a*, *b*, *c*): G system and Q comprise of circular (*d*, *e*, *f*): G system and these components of P and Q are connected in parallel. The probability of the components of P and were calculated by applying Markov stochastic process and various reliability aspects like reliability, sensitivity analysis and MTTF were also calculated with the help of UGF method.

Palle *et al.* (2018) tried to optimize the series-parallel, parallel-series integrated redundant reliability model with the help of cost constraint. They modelled and explained the proposed systems with the help of Lagrangean multiplier from which a real solution for the various components and the reliability of system is obtained. They also analysed the model by using Dynamic Programming Method.

Ahlawat *et al.* (2019) considered the structure of a non series-parallel system containing six components to augment its reliability. They used the various logic diagrams to form the parallel pathways between IN and OUT terminals. The whole system would be functioning productively if the components in each route will operate effectively and the paths dominating other paths have been rejected. Weibull distribution law was considered by the rates of failure of these components. They also derived the expressions to calculate the system reliability and Mean time to system failure (MTSF).

Karimi *et al.* (2019) introduced a programming model to resolve the difficulties of redundancy allocations in the series-parallel configured systems which are having various distinct components with many failure rates. The main aim of this model was to calculate the number and types of these components such that the system reliability is maximized under various restrictions. To prove this, they suggested a weighted K-means clustering method in which to the component of each cluster they assigned some weight with the help of network process. A weed optimization procedure is used to find the solution of the model and the outcomes obtained were proved by the genetic algorithm.

Kvassay *et al.* (2019) showed that if we know the topology of the MSS then we can get its global reliability parameters in both, static and dynamic states. They also found how we can use the computations competently by using modular decomposition. To obtain the stochastic model of the MSS, they joined the Markov process with the methods used for calculating global reliability indices. They also obtained the series-parallel MSSs without maintenance.

Ling *et al*. (2019) calculated the redundancy distribution in the series (parallel) systems subjected to random shocks. They presented the strategy of allotting the redundancy for boosting the reliability of the parallel (series) system and also evaluated the result of the number of subsystems of the system under consideration on their reliability.

(b) In context of universal generating function:

Ushakov (1986) presented a new method called Universal generating function method to determine the reliability of different systems like *k*-out-of-*n***,** series and parallel, etc. Various engineers used the Universal generating function method to analyse the reliability of such different systems.

Levitin and Lisnianski (1999) gave a method based on UGF procedure to calculate the importance of element reliability in various Multi-State System (MSS). Both gave an effective tool for the importance analysis of complex series-parallel MSS and took a required performance (demand). This method was also extended for the analysing the sensitivity of MSS. They also gave numerical examples to verify this method.

Levitin (2004) extended the Universal generating function method to the situation when the performance distribution of certain elements of the MSS taken is dependent on the state of another element of the MSS considerd.

Yeh (2006) formed a new method, Universal Generating Function Method (UGFM), for the normal Multi-state Node Network (MNN) reliability problem. The UGFM recommended can be used to calculate the reliability of MNNs containing cycles i.e., cyclic MNNs. This UGFM was verified by the scientist by giving one example for the same.

Chun-Yang Li *et al.* (2010) defined a vector-UGF for the reliability examination of MSS with parameters of multiple performances. Both proved their method by considering a mathematical example and compared the consequences obtained by this method with the found from Monte Carlo simulation and conventional method of reliability. By these results we can see that the reliability of MSS may be exactly assessed by using vector-UGF.

Li *et al.* (2011) recommended a new process to estimate the reliability of MSS when there is insufficient component's data. In these cases rather than there are precise values of the probabilities, the interval valued probabilities of the components are obtained. To demonstrate the proposed technique a numerical example was also given. These results showed that this method is effective when the state probabilities of these components are imprecise.

For Markov process of discrete-state continuous time, **Lisnianski (2012)** gave a transform known as L_z-transform, to find out the reliability of various dynamic MSS. He defined this special transform mathematically and calculated its various properties. Numerical examples were also illustrated to verify this.

Meena and Vasanthi (2014) analysed how to estimate the MANET reliability from the Universal Generating Function Technique (UGFT). In their research, they both clarified there are two types of UGFs possible and also presented a new procedure to estimate the reliability of the system.

Kumar *et al.* (2016) determined the interval valued reliability of a 2-out-of-4 system consisting of two components which are configured in series. To evaluate the interval valued reliability of the system, interval universal generating function approach has been used. A numerical example has also been illustrated.

Pan G. *et al.* (2016) gave a method for the assessment of interval valued reliability of MSS considering epistemic uncertainty. They improved the traditional UGF approach and defined the algorithm for IUGF approach. They also verified and illustrated their method by taking examples.

Meenakshi, and Singh S.B. (2017) calculated the reliability and MTTF of a non-repairable MSS by using IUGF. The system under consideration has two subsystems P and Q, which were arranged in parallel, both of which are multi-state consecutive u-out-of-v-from-s: G systems. The reliability of the system is estimated by comprising the uncertainties in the probabilities and the failure rates of the components of the considered system.

Chacko, V.M. (2018) applied named UGF method for the fast reliability assessment for a set of continuous MSSs. This proposed new UGF method may evaluate the reliability factors quickly for many different series-parallel structures.

Ding and Han (2018) suggested a new method for examining the reliability of the rotor system. This recommended method was the blend of semi-Markov technique and the UGF approach. The reliability of the system taken for consideration at various performance stages was calculated.

Bisht, S. and Singh, S.B. (2019) suggested an efficient process to evaluate numerous reliability factors like reliability, MTTF and signature reliability of the complex bridge networks having independent and identically distributed lifetime components using UGF. It was found that a slight change in the complex bridge network significantly changes the reliability of the network.

Ding *et al.* (2019) proposed a method, called continuation discretization approximation (CDA) method, to find the reliability of big scale multistate series-parallel system (MSSPS) efficiently. The suggested CDA method consists of two procedures, namely, continuation and discretization. This method was also compared with several other methods and it was found out that the CDA has huge benefit in terms of computational competence and more accurate performance.

Kumar, A., and Ram, M. (2019) analysed a sliding window system and with the help of UGF techniques they calculated its interval valued reliability in upper and lower form.

(c) In context of Shuffle exchange network:

Blake and Trivedi (1989) studied the reliability of the unique-path MIN and focussed on fault-tolerant scheme for increasing the reliability of the network. Both obtained derivations for the reliability, depending upon the time, of the SEN+, 8×8 and 16×16 SEN. They also calculated arithmetic results for networks having size up to 1024×1024.

Bansal *et al.* (1994) proposed a new Augmented Baseline Networks (ABN), which belongs to the class of fault-tolerant MINs. They recommended a new technique which reduced the number of stages in the MIN to increase the reliability and make them more cost-effective. They also estimated their performance and reliability of ABNs which showed that ABNs are much better than the single-path MINs.

Fard and Gunawan (2005) calculated a modified SEN consisting of 1×2 switching elements (SEs) at the source, 2×2 at the intermediate stages and 2×1 at the terminal stage, and calculated the terminal reliability of modified SEN and usual shuffle exchange network (SEN). The terminal reliability of SEN was found lower for network sizes greater than 4×4.

Sharma *et al.* (2009) examined the reliability and path lengths of certain irregular MINs, in which there are different number of switching elements in each stage. The reliability of the new proposed networks on the basis of their MTTF have been calculated and compared with each other.

Yunus and Othman (2011) studied six various types of Shuffle exchange networks (SENs) having additional stages. The studied SENs are as: SEN+, Irregular Augmented Shuffle Exchange Network (IASEN), Generalized Shuffle Exchange Network (GSEN), Irregular Augmented Shuffle Network (IASN), Irregular Modified Alpha Network (ALN) and Improved Irregular Augmented Shuffle Multistage Interconnection Network (IIASN). They found that a more redundant path is attained by augmenting the stages, which would increase fault-tolerance by providing auxiliary links. However increasing the size of network increases the network complexity and the cost would also increase.

Bistouni and Jahanshahi (2014) suggested a new way to augment the reliability and fault-tolerance of one of the most often used MIN, Shuffle-exchange network (SEN) by increasing the number of switching stages. This was found that the reliability of SEN with one additional stage (SEN+) is better than that of SEN or SEN having two additional stages (SEN+2). Further, SEN+ was found more reliable than SEN+2. Thus, it was decided that adding one stage to SEN is much better than adding two stages in it.

Bistouni and Jahanshahi (2014) presented improved extra group network (IEGN), a new MIN which is more fault-tolerant, which was from Extra Group Network (EGN). The reliability aspects of the new proposed network like reliability, path-length, cost and fault-tolerance were estimated and they obtained that the results calculated by IEGN was way better than that of other networks like Replicated MIN, ASEN, EGN and Benes network.

Jahanshahi and Bistouni (2014) presented a new technique to improve the reliability of MINs, called reducing nodes. They also validated their method by analysing the terminal and broadcast reliability and the results showed that by reducing the nodes, it has a significant advantage over increasing the number of stages.

Yunus and Othman (2014) analysed two MINs, SEN- and SEN, and related the connection between the size of network and aspect of reliability of the two networks taken for consideration. The reliability performance of the networks was calculated on the basis of the three aspects, namely, terminal, broadcast and network reliability. The reliability of SEN- was found more than that of SEN due to less network size of SEN-.

Rajkumar and Goyal (2015) tried to relate and study several network topologies of MINs on the aspects of their reliability, fault-tolerance and cost-effectiveness.

Gupta, S., and Pahuja, G. L. (2016) developed a new SEN- using MUX and DEMUX. They calculated terminal, broadcast, network reliability and MTTF of SEN- using reliability block diagram.

Bistouni and Jahanshahi (2018) analysed the reliability significance of the switching elements (SE) in SEN, SEN+ and SEN+2. They found out that a high-reliable network can be obtained by substituting the sensitive switching elements and by using highly reliable switching elements.

Bistouni and Jahanshahi (2018) gave a new approach, called rearranging links, to increase the reliability of MINs. They applied this new approach on SEN+ and ASEN and then calculated their terminal reliability. They found that the procedure of reordering the links is an efficient one to increase the reliability of MINs. Moreover, this approach gave more cost efficient MIN than previous ones.

Yunus *et al.* (2018) examined two different topologies of MINs, namely, Gamma Interconnection Network (GIN) and SEN and examined their behaviour on the MINs reliability. It was obtained that the MIN having less number of stages have higher reliability than that which has more number of stages.

Chinnaiah (2019) suggested a new MIN named as replicated SEN and compared this replicated SEN with Benes network and SEN. They compared their three kinds of behaviours viz., replicated stage, basic stage and extra stage. The results obtained showed that the replicated SEN has highest reliability amongst all the networks.

PLAN OF THE PROPOSED WORK

In this modern era, the networks are becoming very complex due to large number of components are used to build them. In such complex networks/systems, it becomes very difficult to attain a satisfactory performance. Thus it becomes necessary to calculate the network reliability for the planning and designing of the networks. Reliability of any network depends on its performance rate.

In the proposed work, we have planned to study two multistage interconnection networks, viz., 8×8 SEN(-) and 4×4 SEN. The main objectives of the planned research work are as follows:

1. To calculate the terminal reliability, broadcast reliability and network reliability of 8×8 SEN(-).
2. To analyse the terminal reliability, broadcast reliability and network reliability of 4×4 SEN, 4×4 SEN(+), 4×4 SEN(+2).
3. To compare the results with the previously obtained results.

All the evaluations will be made with the help of universal generation function method.

Materials and Methods

Chapter 3 MATERIALS AND METHODS

This chapter has been classified into two different sections in which we will evaluate the reliability of two different models:

Model 1: Reliability assessment of 8×8 shuffle exchange network with one stage less (SEN-) by using universal generating function.

Model 2: Reliability assessment of 4×4 shuffle exchange network using universal generating function.

Model (3.1): Reliability assessment of 8×8 shuffle exchange network with one stage less (SEN-) using universal generating function.

Shuffle exchange networks are used as an appropriate interconnection network because their switching elements (SE) have small size and they have very simple arrangement. SENs are very high in demand in the present world, so many researcher have analyzed the reliability of SENs by using many different methods such as UGF method, reliability block diagram method etc.

Abbreviations and Notations:

BR : Broadcast reliability

$\underset{par}{\otimes}$: Composition operator for parallel configuration

$\underset{ser}{\otimes}$: Composition operator for series configuration

NR : Network reliability

SEN : Shuffle exchange network

SEN- : Shuffle exchange network with one stage less

SE : Switching element

TR : Terminal reliability

$u(z)$: UGF of the network

Assumptions:

1. Initially the network is in good condition i.e., all the nodes and links are functioning properly.
2. The network considered is an 8×8 SEN- in which each switching SE is of size 2×2.
3. All the network's components are either in working stage or in failed stage.
4. If the network fails completely then only will go for replacement and after replacement the network is as good as new.
5. The failure rates of different components are taken different while the replacement rate of all the components are supposed to be same.

3.1.1 Shuffle exchange network with one stage less (SEN-):

SEN is widely used multistage interconnection network in parallel processing communication system. Basically SEN is a single path MIN. The SEN of size $N\times N$ consist of log_2N stages and *N/2* switching elements per stage. SEN- is shuffle exchange network with one stage less than the usual SEN. The SEN− is a single path MIN. A SEN− of size $N\times N$ consists of $N/2$ SEs of size 2×2 in each stage and there are (log_2N-1) number of stages. The **switching complexity of SEN- network is $N/2(log_2N-1)$.** The SEN- of size 8× 8 is shown in Fig. 3.1.

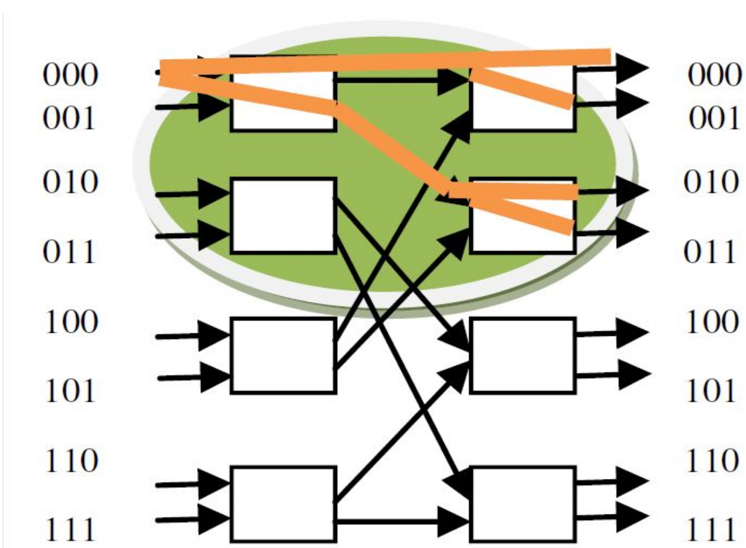

Fig. 3.1: 8× 8 SEN-

The SEN− MIN does not satisfy the basic condition of MIN of full connectivity i.e., some of the inputs nodes are not connected to all output nodes. For example input 000 is connected to outputs 000–011, but input node 000 is not connected to output nodes 100–111. As from the basic definition of MIN all input nodes should be connected to all output nodes at least by one of its path. So the network diminishes its claim to be approved as MIN.

3.1.2 Proposed SEN- :

In New SEN-structure MUX and DEMUX have been used at source end and destination end respectively to achieve full connectivity. For convenience, 2×1 MUX and 1×2 DEMUX have been used. To make system more redundant bigger size of MUX and DEMUX can be used. Presented new SEN− offers two paths between each source node and destination node. The main benefit of using this method to provide redundancy as it provides totally separate paths between source and destination, making the system fault tolerant at the source end and the destination end too, which has not been achieved in any of the regular topology to the best of our knowledge. In SEN− there are *N* MUX at source and *N* DEMUX at destination side. There are total $\boldsymbol{log_2} - \mathbf{1}$ stages in this network, which provide minimum path length and minimum latency than all other networks. In each stage there is *N/*2 SE. The new SEN− of size 8× 8 network is a shown in Fig. 3.2

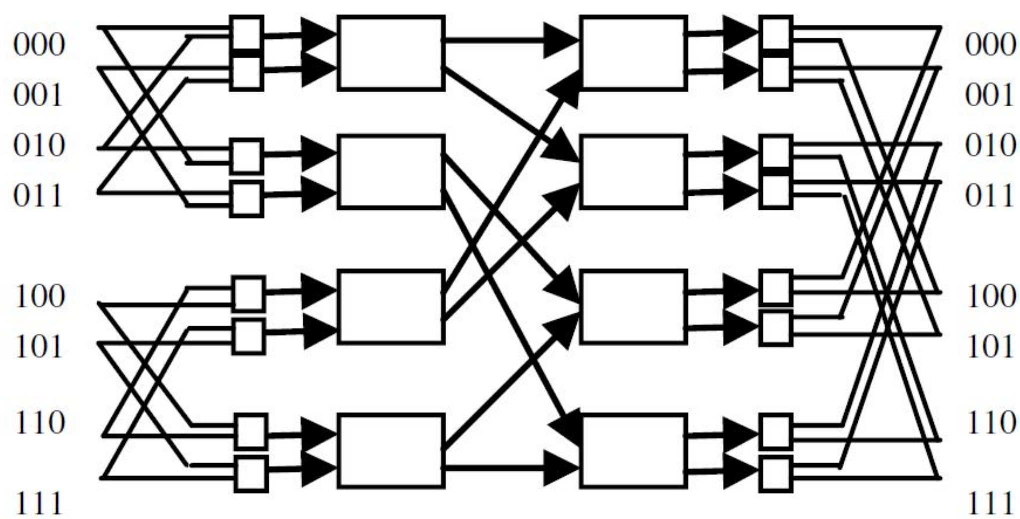

Fig. 3.2: New8× 8 SEN-

3.1.3 Reliability analysis of SEN- :

Reliability analysis of SEN- can be done on the basis of following three reliabilities, viz. terminal reliability, broadcast reliability and network reliability with the help of UGF. To calculate reliability, we make following assumptions:

- Reliability of 2 × 2 SE is taken as p.
- Reliability of 2 ×1 MUX and 1 × 2 DEMUX are taken as $p^{1/2}$.

3.1.3.1 Terminal Reliability

Terminal reliability block diagram for SEN- of size 8× 8 is shown in Fig. 3.3.

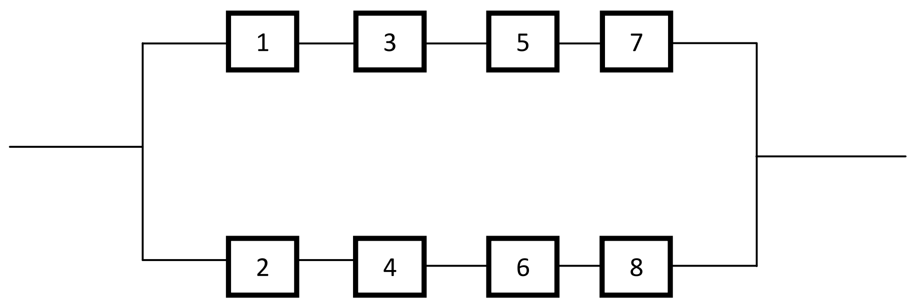

Fig. 3.3: TR of 8× 8 SEN (-)

The terminal reliability of SEN- with the help of UGF can be calculated as:

$R_{TR}(SEN-)$ = max (min (p_1, p_3, p_5, p_7) , min (p_2, p_4, p_6, p_8))

where, $p_1, p_2, \ldots, p_8$ are the probabilities of the switching element present in the network.

a) When the components of SEN- are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch s_j, $j = 1,2, \ldots,8$.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2,\ldots\ldots,8$ respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\,z^1 + 0.04z^0$

$u_{s_7}(z) = 0.93z^1 + 0.03z^0$

$u_{s_8}(z) = 0.92z^1 + 0.02z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_3}(z) \underset{\min}{\otimes} u_{s_5}(z) \underset{\min}{\otimes} u_{s_7}(z)$$

$$= 0.84842505\, z^1 + 0.15157495\, z^0$$

$$U_B(z) = u_{s_2}(z) \underset{\min}{\otimes} u_{s_4}(z) \underset{\min}{\otimes} u_{s_6}(z) \underset{\min}{\otimes} u_{s_8}(z)$$

$$= 0.81360384\, z^1 + 0.18639616\, z^0$$

Finally, the reliability of all terminal structure of SEN- is obtained as:

$$U(z) = U_A(z) \underset{\max}{\otimes} U_B(z)$$

$$U(z) = 0.971747011\, z^1 + 0.028252988\, z^0$$

b) When the components of the SEN- are identical and all switching elements has the same probabilities, then the structure function is expressed as:

$$R_{TR}(SEN-) = 2p^3 - p^6$$

3.1.3.2 Broadcast Reliability

Broadcast reliability block diagram for SEN- of size 8× 8 is shown in Fig. 3.4.

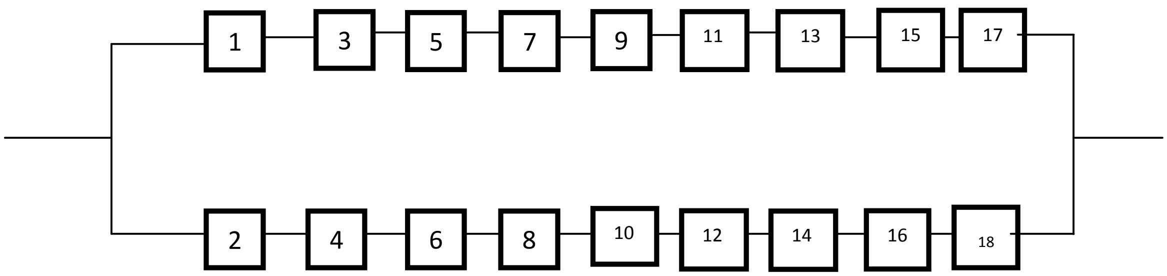

Fig. 3.4: BR of 8× 8 SEN-

The broadcast reliability of SEN- from the method of UGF can be calculated as:

R_{BR}(SEN-) = max $(\min(p_1, p_3, \ldots\ldots p_{17}), \min(p_2, p_4, \ldots\ldots p_{18}))$

where, $p_1, p_2, \ldots\ldots, p_{18}$ are the probabilities of the components present in the network.

a) When the components of SEN- are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch s_j, $j = 1,2, \ldots..18$.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2, \ldots\ldots 18$ respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\, z^1 + 0.06z^0$

$u_{s_7}(z) = 0.93z^1 + 0.07z^0$

$u_{s_8}(z) = 0.92z^1 + 0.08z^0$

$u_{s_9}(z) = 0.91z^1 + 0.09\, z^0$

$u_{s_{10}}(z) = 0.90\, z^1 + 0.10\, z^0$

$u_{s_{11}}(z) = 0.89\, z^1 + 0.11\, z^0$

$u_{s_{12}}(z) = 0.88z^1 + 0.12\, z^0$

$u_{s_{13}}(z) = 0.87\, z^1 + 0.13\, z^0$

$u_{s_{14}}(z) = 0.86\, z^1 + 0.14\, z^0$

$u_{s_{15}}(z) = 0.85\, z^1 + 0.15\, z^0$

$u_{s_{16}}(z) = 0.84\, z^1 + 0.16\, z^0$

$u_{s_{17}}(z) = 0.83\, z^1 + 0.17\, z^0$

$u_{s_{18}}(z) = 0.82\, z^1 + 0.18\, z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_3}(z) \underset{\min}{\otimes} u_{s_5}(z) \underset{\min}{\otimes} \ldots\ldots\ldots \underset{\min}{\otimes} u_{s_{17}}(z)$$

$$= 0.421755886\, z^1 + 0.578244114\, z^0$$

$$U_B(z) = u_{s_2}(z) \underset{\min}{\otimes} u_{s_4}(z) \underset{\min}{\otimes} u_{s_6}(z) \underset{\min}{\otimes} \ldots\ldots\ldots \underset{\min}{\otimes} u_{s_{18}}(z)$$

$$= 0.38170668\, z^1 + 0.61829332\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of SEN- is obtained as:

$U(z) = U_A(z) \underset{\max}{\otimes} U_B(z)$

$U(z) = 0.642475527\ z^1 + 0.357524473\ z^0$

b) When the components of the SEN- are identical and all switching elements has the same probabilities, then the structure function is expressed as:

R_{BR}(SEN-) = $2p^{13/2} - p^{13}$

3.1.3.3 Network Reliability

Network reliability block diagram for SEN- of size 8× 8 is shown in Fig. 3.5.

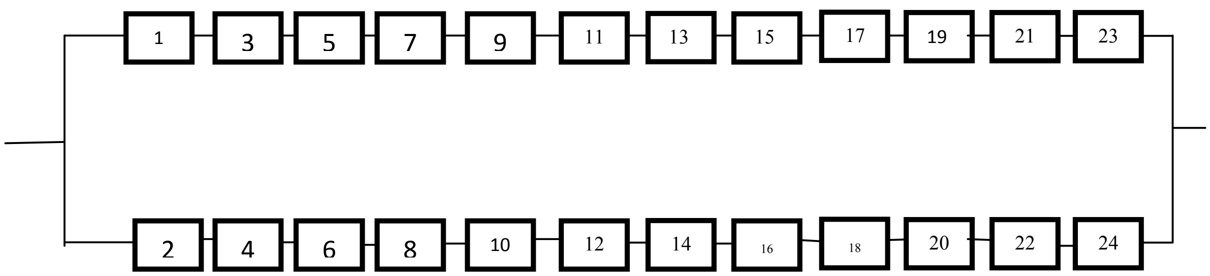

Fig. 3.5: NR of 8× 8 SEN-

The network reliability of SEN- from the method of UGF can be calculated as:

R_{NR}(SEN-) = max $(\min(p_1, p_3, \ldots \ldots p_{23}), \min(p_2, p_4, \ldots \ldots p_{24}))$

where, $p_1, p_2, \ldots\ldots, p_{24}$ are the probabilities of the components present in the network.

a) When the components of SEN- are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch s_j, $j = 1,2, \ldots\ldots 24$.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2,\ldots..24.$ respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94 z^1 + 0.06z^0$

$u_{s_7}(z) = 0.93z^1 + 0.07z^0$

$u_{s_8}(z) = 0.92z^1 + 0.08z^0$

$u_{s_9}(z) = 0.91z^1 + 0.09 z^0$

$u_{s_{10}}(z) = 0.90 z^1 + 0.10 z^0$

$u_{s_{11}}(z) = 0.89 z^1 + 0.11 z^0$

$u_{s_{12}}(z) = 0.88z^1 + 0.12 z^0$

$u_{s_{13}}(z) = 0.87 z^1 + 0.13 z^0$

$u_{s_{14}}(z) = 0.86 z^1 + 0.14 z^0$

$u_{s_{15}}(z) = 0.85 z^1 + 0.15 z^0$

$u_{s_{16}}(z) = 0.84 z^1 + 0.16 z^0$

$u_{s_{17}}(z) = 0.83 z^1 + 0.17 z^0$

$u_{s_{18}}(z) = 0.82 z^1 + 0.18 z^0$

$u_{s_{19}}(z) = 0.81 z^1 + 0.19 z^0$

$u_{s_{20}}(z) = 0.80 z^1 + 0.20 z^0$

$u_{s_{21}}(z) = 0.79 z^1 + 0.21 z^0$

$u_{s_{22}}(z) = 0.78 z^1 + 0.22 z^0$

$u_{s_{23}}(z) = 0.77\, z^1 + 0.23\, z^0$

$u_{s_{24}}(z) = 0.76\, z^1 + 0.24\, z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(\mathrm{z}) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_3}(z) \underset{\min}{\otimes} u_{s_5}(z) \underset{\min}{\otimes} \ldots\ldots \underset{\min}{\otimes} u_{s_{23}}(z)$$

$$= 0.207808825\, z^1 + 0.792191174\, z^0$$

$$U_B(\mathrm{z}) = u_{s_2}(z) \underset{\min}{\otimes} u_{s_4}(z) \underset{\min}{\otimes} u_{s_6}(z) \underset{\min}{\otimes} \ldots\ldots \underset{\min}{\otimes} u_{s_{24}}(z)$$

$$= 0.181020576\, z^1 + 0.818979423\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of SEN- is obtained as:

$$U(z) = U_A(\mathrm{z}) \underset{\max}{\otimes} U_B(\mathrm{z})$$

$$U(z) = 0.351211727\, z^1 + 0.648788272\, z^0$$

b) When all the components of the SEN- are identical then all switching elements has the same probabilities, then the structure function is expressed as:

$$R_{NR}(\text{SEN-}) = 2p^8 - p^{16}$$

Model (3.2): Reliability assessment of 4× 4 shuffle exchange network using universal generating function

3.2.1 4× 4 shuffle exchange network

The 4× 4 shuffle exchange network (SEN) is single path MIN. The SEN of size 4× 4 consist of *2* SEs of size 2×2 in each stage and total number of stages are 2. The 4× 4 SEN- is shown in Fig. 3.6.

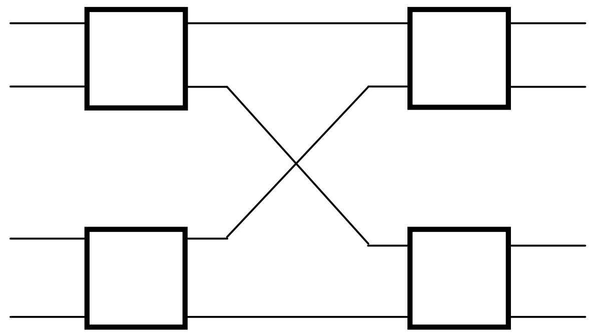

Fig. 3.6: 4×4 SEN

3.2.2 4× 4 shuffle exchange network with one stage (SEN+1)

4× 4 SEN is a single path MIN which becomes double path with the addition of one more stage and hence called SEN+1. The SEN+1 of size 4× 4 consists of 3 stages and each stage consists of 2 switching elements. The SEN+1 of size 4× 4 is depicted in Fig. 3.7.

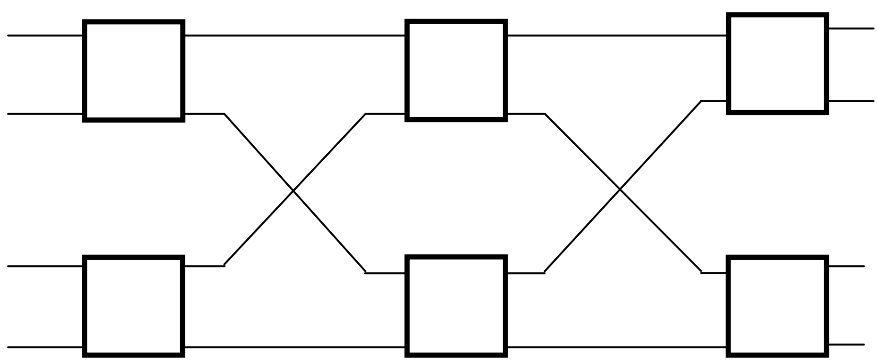

Fig. 3.7: 4×4 SEN+1

3.2.3 4× 4 shuffle exchange network with two additional stages (SEN+2)

4× 4 SEN becomes multipath when two additional stages are added in it and designated as 4× 4 SEN+2. The SEN+2 of size 4× 4 consist of 4 stages and each stage consists of 2 switching elements. The SEN+2 of size 4× 4 is shown in Fig. 3.8.

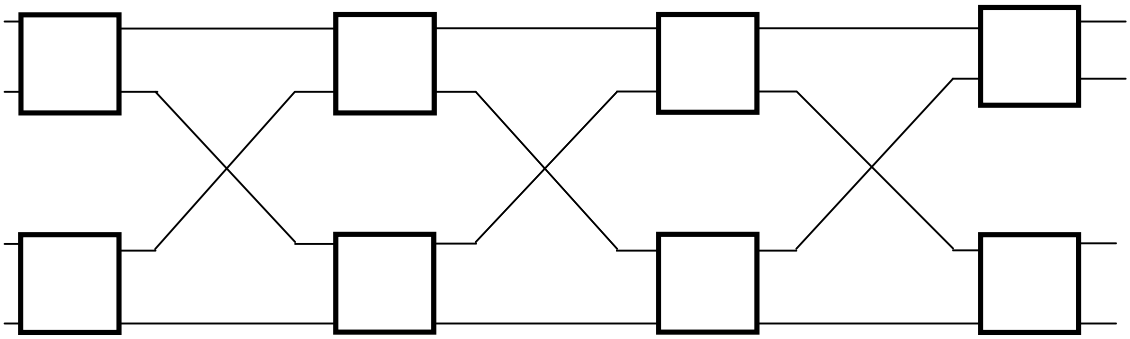

Fig. 3.8: 4×4 SEN+2

3.2.4 Reliability of 4×4 SEN, SEN+1, SEN+2

In this proposed work, reliability of different SENs is calculated on the basis of following three reliabilities: Terminal reliability, Broadcast reliability and Network reliability with the help of UGF unlike done before.

3.2.4.1 Terminal reliability of SEN, SEN+1, SEN+2

Terminal reliability is defined as probability of existence of at least one fault free path between source and sink. In this segment, terminal reliability of 4×4 SEN, SEN+1, SEN+2 are being calculated with the help of the universal generating function.

Terminal reliability of4×4 SEN

As described earlier, SEN is a unique path MIN, so all the switches in the path from the source to sink are essentially required to function. Terminal reliability block diagram of 4× 4 SEN is shown in Fig. 3.9.

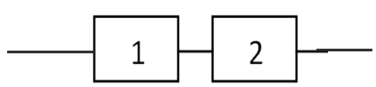

Fig. 3.9: TR of SEN

The terminal reliability of 4× 4 SEN with the help of UGF can be calculated as:

$$R_{TR}(\text{SEN}) = \min\,(p_1, p_2)$$

where, p_1, p_2 are the probabilities of the components present in the network.

a) When the components of 4× 4 SEN are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch $s_j, j = 1{,}2$

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1{,}2$ respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

Since both the SEs are connected in series, hence the UGF of both the switching elements can be calculated as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_2}(z)$$

$U(z) = 0.9702\ z^1 + 0.0298\ z^0$

b) When all the components of the 4× 4 SEN are same then all the switching elements has the same probabilities, then the structure function is expressed as:

$$R_{TR}(\text{SEN}) = p^2 \qquad \ldots(3.1)$$

Terminal reliability of 4× 4 SEN+1

SEN+1 is a double path MIN. Terminal reliability block diagram of SEN+1 of size 4× 4 is shown in Fig. 3.10.

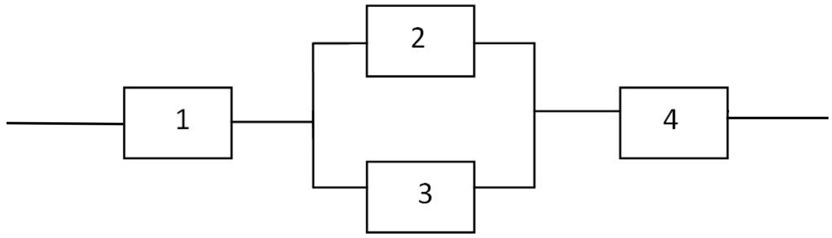

Fig. 3.10: TR of SEN+1

The terminal reliability of 4× 4 SEN+ with the help of UGF can be calculated as:

R_{TR}(SEN+1) = min (max $(p_2, p_3), p_1, p_4)$

where, $p_1, p_2,\ p_3,\ p_4$ are the probabilities of the switching elements present in the network.

a) If the components of 4× 4 SEN+1 are not similar, i.e. the probabilities of the components in the network are different, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = \ p_{s_j} z^1 + \left(1 - \ p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch$s_j, j\ = 1{,}2{,}3$ and 4.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1{,}2{,}3\ and\ 4$ respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

U_A(z) = $u_{s_2}(z) \underset{\max}{\otimes} u_{s_3}(z)$

$= \ 0.9994\ z^1 + 0.006\ z^0$

Finally, the reliability of the source-to-multiple terminal structure of 4× 4 SEN+1 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} U_A(z) \underset{\min}{\otimes} u_{s_4}(z)$$

$$U(z)=0.94982976\, z^1 + 0.05017024\, z^0$$

b) If the components of the 4× 4 SEN+1 are identical and all switching elements has the same probabilities, then the structure function is expressed as:

$$R_{TR}(\text{SEN+}) = 2p^3 - p^4 \qquad \ldots(3.2)$$

Terminal reliability of 4× 4 SEN+2

SEN+2 can transmit the signal through four different paths from input source to output sink. The terminal reliability block diagram of SEN+2 for the size 4× 4 is shown in Fig. 3.11.

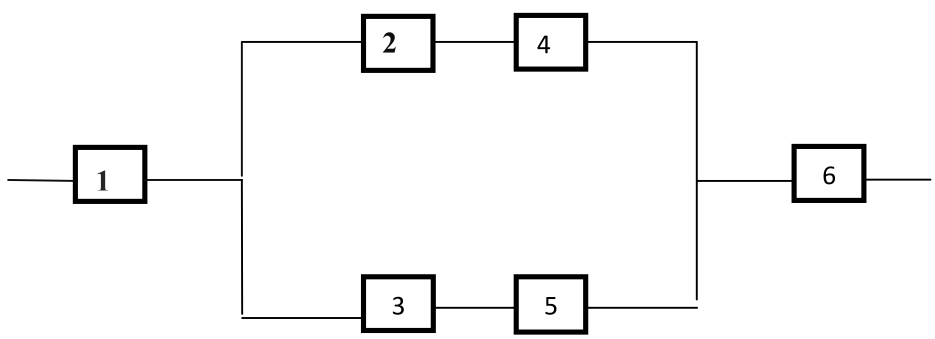

Fig. 3.11: TR of 4× 4 SEN+2

The terminal reliability of 4× 4 SEN+2 with the help of UGF can be evaluated as:

$$R_{TR}(SEN+2) = \min\left(\max(\min(p_2, p_4), \min(p_3, p_5)), p_1, p_6\right)$$

where, $p_1, p_2, p_3, p_4, p_5, p_6$ are the probabilities of the switching elements present in the network.

a) If the components of 4× 4 SEN+2 are not identical and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch $s_j, j = 1,2,3,4,5$ and 6.

The UGFs $u_{s_j}(z)$ of the network for the switches s_j, $j = 1, 2, 3, 4, 5$ and 6 respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\, z^1 + 0.06z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_2}(z) \underset{\min}{\otimes} u_{s_4}(z)$$

$$= 0.9408\, z^1 + 0.0592\, z^0$$

$$U_B(z) = u_{s_3}(z) \underset{\min}{\otimes} u_{s_5}(z)$$

$$= 0.9215\, z^1 + 0.0785\, z^0$$

$$U_C(z) = U_A(z) \underset{\max}{\otimes} U_B(z)$$

$$= 0.9953528\, z^1 + 0.00464z^0$$

Finally, the reliability of the source-to-multiple terminal structure of 4× 4 SEN+2 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_C(z) \underset{\min}{\otimes} u_{s_6}(z)$$

$U(z)$= 0.926275315 z^1 + 0.073724685 z^0

b) If all the components of the 4× 4 SEN+2 are identical ($p_i = p$)then the structure function of reliability is expressed as:

$$R_{TR}(SEN + 2) = 2p^4 - p^6 \qquad \ldots(3.3)$$

Broadcast Reliability

It is the probability of transmitting network from single source to all destination nodes. In this segment, Broadcast reliability of 4×4 SEN, SEN+1, SEN+2 are being calculated with the help of UGF and is compared with 8× 8 SEN, SEN+1 and SEN+2 respectively.

Broadcast Reliability of 4×4 SEN

Broadcast reliability block diagram of 4× 4 SEN is shown in Fig. 3.12.

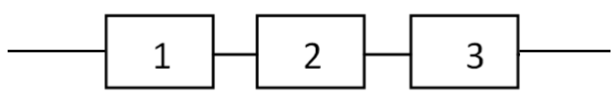

Fig. 3.12: BR of 4×4 SEN

The broadcast reliability of 4× 4 SEN with the help of UGF can be evaluated as:

R_{BR}(SEN) = min (p_1, p_2, p_3)

where, p_1, p_2, p_3 are the probabilities of the components present in the network.

a) If all the components of 4× 4 SEN are not identical and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch$s_j, j = 1,2,3$.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1, 2$ and 3 respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

Since all three SEs are connected in series, here the UGF of the switching elements can be calculated as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_2}(z) \underset{\min}{\otimes} u_{s_3}(z)$$

$U(z) = 0.941094\, z^1 + 0.058906\, z^0$

b) If the components of the 4× 4 SEN are same, then all switching elements has the same probabilities, then it's structure function is expressed as:

$R_{TR}(\text{SEN}) = p^3$ …(3.4)

Broadcast Reliability of 4×4 SEN+1

SEN+1 is a double path MIN. Broadcast reliability block diagram of SEN+1 of size 4× 4 is shown in Fig. 3.13.

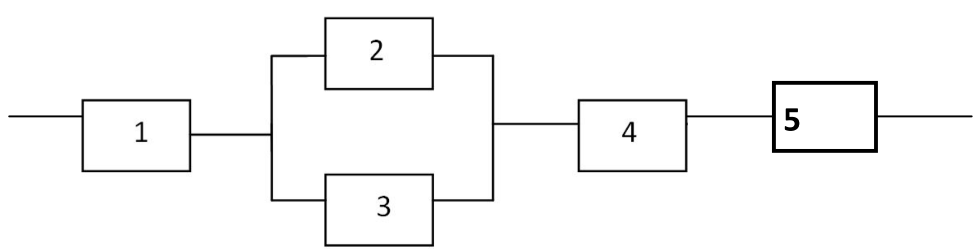

Fig. 3.13: BR of 4× 4 SEN+1

The broadcast reliability of 4× 4 SEN+ with the help of UGF can be calculated as:

$R_{BR}(SEN + 1)$ = min $(\max(p_2, p_3), p_1, p_4, p_5)$

where, p_1, p_2, p_3, p_4, p_5 are the probabilities of the components present in the network.

a) If all the components of 4× 4 SEN+1 are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switchs_j, $j = 1,2,3,4$ and 5.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2, \ldots .5$. respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_2}(z) \underset{\max}{\otimes} u_{s_3}(z)$$

$$= 0.9994\, z^1 + 0.006\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of 4× 4 SEN+1 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} U_A(z) \underset{\min}{\otimes} u_{s_4}(z) \underset{\min}{\otimes} u_{s_5}(z)$$

U(z) = 0.902338272 z^1 + 0.097661728 z^0

b) If all the components of the 4× 4 SEN+1 are same, i.e. ($p_i = p$) then the structure function is expressed as:

$R_{TR}(SEN + 2) = 2p^4 - p^5$ …(3.5)

Broadcast Reliability of 4× 4 SEN+2

SEN+2 can transmit the signal through four different paths from input source to output sink. The broadcast reliability block diagram of SEN+2 for the size 4× 4 is shown in Fig. 3.14.

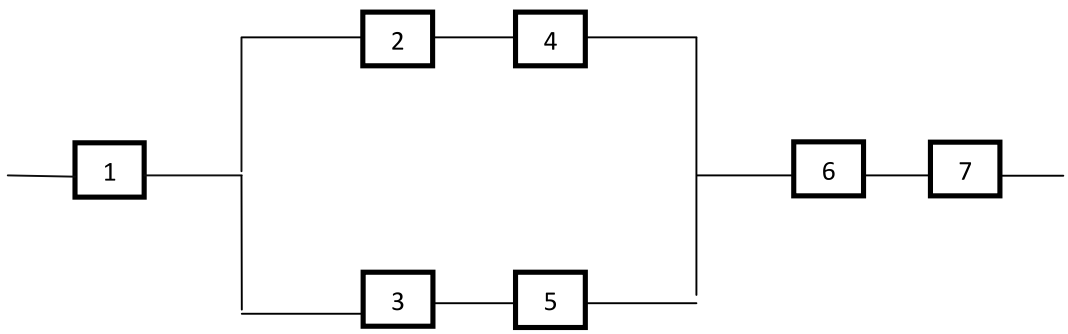

Fig. 3.14: BR of 4× 4 SEN+2

The broadcast reliability of 4× 4 SEN+2 with the help of UGF can be calculated as:

$R_{BR}(SEN + 2) = \min(max(min(p_2, p_4), min(p_3, p_5)), p_1, p_6, p_7)$

where, $p_1, p_2, p_3, p_4, p_5, p_6, p_7$ are the probabilities of the components present in the network.

a) If all the components of 4× 4 SEN+2 are not similar, then the probabilities of the components in the network will be different, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switchs_j, j = 1, 2, 3, 4, 5, 6 and 7

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2,\ldots.7$. respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\, z^1 + 0.06z^0$

$u_{s_7}(z) = 0.93z^1 + 0.07z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_2}(z) \underset{\min}{\otimes} u_{s_4}(z)$$

$$= 0.9408\, z^1 + 0.0592\, z^0$$

$$U_B(z) = u_{s_3}(z) \underset{\min}{\otimes} u_{s_5}(z)$$

$$= 0.9215\, z^1 + 0.0785\, z^0$$

$$U_c(z) = U_A(z) \underset{\max}{\otimes} U_B(z)$$

$$= 0.9953528\, z^1 + 0.0046472\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of 4× 4 SEN+1 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} U_C(z) \underset{\min}{\otimes} u_{s_6}(z) \underset{\min}{\otimes} u_{s_7}(z)$$

$$U(z) = 0.861436043\, z^1 + 0.138563957\, z^0$$

b) If the components of the 4× 4 SEN+2 are identical and all switching elements has the same probabilities, i.e. ($p_i = p$), then the structure function is expressed as:

$$R_{TR}(SEN + 2) = 2p^5 - p^7 \quad \ldots(3.6)$$

Network Reliability

It is defined as the probability of successful transmission of signals from all source nodes to all sink nodes. In this segment, network reliability of 4×4 SEN, SEN+1, SEN+2 are being calculated with the help of the universal generating function.

Network Reliability of 4×4 SEN

Network reliability block diagram of 4× 4 SEN is shown in Fig. 3.15:

Fig. 3.15: NR of 4×4 SEN

The network reliability of 4× 4 SEN with the help of UGF can be calculated as:

R_{TR}(SEN) = min (p_1, p_2, p_3, p_4)

where, p_1, p_2, p_3, p_4 are the probabilities of the components present in the network.

a) When the components of 4× 4 SEN are not same and the probabilities of the components in the network are distinct, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch$s_j, j = 1,2,3$ and 4.

The UGFs $u_{s_j}(z)$ of the network for the switches $s_j, j = 1,2,3$ and 4 respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

Since all the SEs are connected in series, then the UGF of the switching elements can be calculated as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_2}(z) \underset{\min}{\otimes} u_{s_3}(z) \underset{\min}{\otimes} u_{s_4}(z)$$

$U(z)$= 0.90345024 z^1 + 0.09654976 z^0

a) When the components of the 4× 4 SEN are identical and all switching elements has the same probabilities, then the structure function is expressed as:

$$R_{NR}(\text{SEN}) = p^4 \qquad \ldots(3.7)$$

Network Reliability of 4× 4 SEN+1

SEN+1 is a double path MIN. Network reliability block diagram of SEN+1 of size 4× 4 is shown in Fig. 3.16.

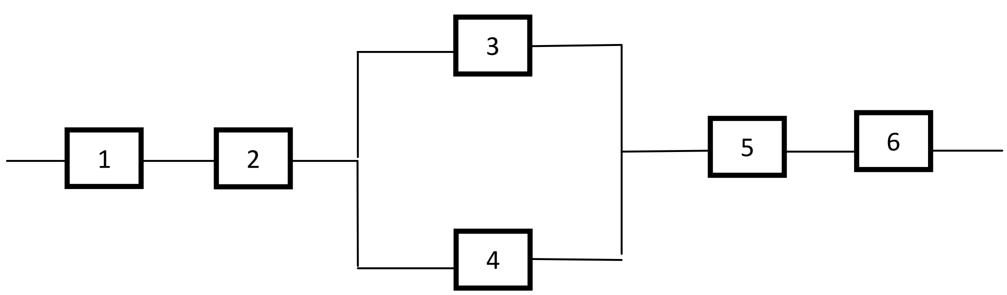

Fig. 3.16: NR of 4× 4 SEN+1

The network reliability of 4× 4 SEN+1 with the help of UGF can be calculated as:

$R_{NR}(SEN + 1) = \min(\max(p_3, p_4), p_1, p_2, p_5, p_6)$

where, $p_1, p_2, p_3, p_4, p_5, p_6$ are the probabilities of the components present in the network.

a) If the components of 4× 4 SEN+1 are not same, i.e. probabilities of all the components in the network are different, then UGFs of the various switching elements are given by:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch s_j, $j = 1,2,3,4,5$ and 6.

The UGFs $u_{s_j}(z)$ of the network for the switches s_j, $j = 1,2, \ldots\ldots 5$. respectively, are given as:

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\, z^1 + 0.06z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_3}(z) \underset{\max}{\otimes} u_{s_5}(z)$$

$$= 0.9988\, z^1 + 0.0012\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of 4× 4 SEN+1 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_2}(z) \underset{\min}{\otimes} U_A(z) \underset{\min}{\otimes} u_{s_5}(z) \underset{\min}{\otimes} u_{s_6}(z)$$

$$U(z) = 0.865348933\, z^1 + 0.134651066\, z^0$$

b) If all the SEs of the 4× 4 SEN+1 has the same probabilities, then the structure function is expressed as:

$$R_{TR}(SEN + 2) = 2p^5 - p^6 \qquad \ldots(3.8)$$

Network Reliability of 4× 4 SEN+2

SEN+2 can transmit the signal through four different paths from input source to output sink. The network reliability block diagram of SEN+2 for the size 4× 4 is depicted in Fig. 3.17.

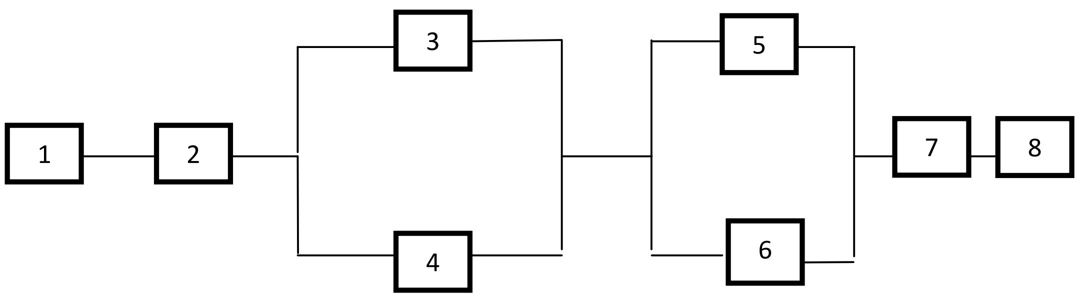

Fig. 3.17: NR 4× 4 SEN+2

The network reliability of 4× 4 SEN+2 with the help of UGF can be calculated as:

$$R_{NR}(\text{SEN+2}) = \min(max(min(p_3, p_4), min(p_5, p_6)), p_1, p_2, p_7, p_8)$$

where, $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8$ are the probabilities of the components present in the network.

a) If the components of 4× 4 SEN+2 are not similar, i.e. the probabilities of all the SEs are different, then UGFs of the various switching elements is expressed as:

$$u_{s_j}(z) = p_{s_j} z^1 + \left(1 - p_{s_j}\right) z^0$$

where, p_{s_j} is the probability of the switch s_j, $j = 1, 2, 3, 4, 5, 6, 7$ and 8.

$u_{s_1}(z) = 0.99z^1 + 0.01z^0$

$u_{s_2}(z) = 0.98z^1 + 0.02z^0$

$u_{s_3}(z) = 0.97z^1 + 0.03z^0$

$u_{s_4}(z) = 0.96z^1 + 0.04z^0$

$u_{s_5}(z) = 0.95z^1 + 0.05z^0$

$u_{s_6}(z) = 0.94\, z^1 + 0.06z^0$

$u_{s_7}(z) = 0.93z^1 + 0.07z^0$

$u_{s_8}(z) = 0.92z^1 + 0.08\, z^0$

Applying composition operators for different SEs as per their combination, we have UGFs as follows:

$$U_A(z) = u_{s_3}(z) \underset{\max}{\otimes} u_{s_4}(z)$$

$$= 0.9988\, z^1 + 0.0012\, z^0$$

$$U_B(z) = u_{s_5}(z) \underset{\max}{\otimes} u_{s_6}(z)$$

$$= 0.997\, z^1 + 0.003\, z^0$$

Finally, the reliability of the source-to-multiple terminal structure of 4×4 SEN+2 is obtained as:

$$U(z) = u_{s_1}(z) \underset{\min}{\otimes} u_{s_2}(z) \underset{\min}{\otimes} U_A(z) \underset{\min}{\otimes} u_B(z) \underset{\min}{\otimes} u_{s_7}(z) \underset{\min}{\otimes} u_{s_8}(z)$$

$$U(z) = 0.826619675\, z^1 + 0.173380325\, z^0$$

b) If all the components of the 4×4 SEN+2 are same then all the switching elements has the same probabilities, i.e. ($p_i = p$), then the structure function is expressed as:

$$R_{TR}(SEN + 2) = 4p^6 + p^8 - 4p^7 \qquad \ldots(3.9)$$

Results and Discussion

In this chapter we demonstrate the different results obtained from the proposed models discussed in the chapter 3.

4.1 Model [1]: Reliability assessment of 8× 8 shuffle exchange network with one stage less (SEN-) using universal generating function

In the model 1, firstly we have evaluated the reliability of SEN, SEN+1 and SEN+2 with the help of universal generating function. Then, we compared the result obtained here in this study with 8× 8 SEN.

4.1.1 Terminal reliability of 8× 8 SEN (-)

When the components of the SEN (-) are non-identical as shown in 3.1.3.1 and the probabilities of switching elements in the network are different, then the terminal reliability of the 8× 8 SEN (-) is:

R_{TR}(SEN-) = 0.971747011

Terminal reliability of 8×8 SEN (-) is evaluated with respect to different switching reliability with the help of proposed UGF method and compared with 8×8 SEN, as presented in Table 4.1.

Table 4.1: Terminal reliability of 8×8 SEN (-)

Switching Reliability	TR evaluation by UGF	TR of 8× 8 SEN [Rajkumar and Goyal(2016)]
0.9	0.926559	0.72900
0.95	0.979658	0.85737
0.96	0.986714	0.88473
0.98	0.99654	0.94119
0.99	0.999118	0.97029

4.1.2 Broadcast reliability of 8× 8 SEN (-)

When the components of the SEN (-) are non-identical as shown in 3.1.3.2 and the probabilities of switching elements in the network are different, then the broadcast reliability of the 8× 8 SEN (-) is:

R_{BR}(SEN-) = 0.642475527

Broadcast reliability of 8×8 SEN (-) is evaluated with respect to different switching reliability with the help of the proposed UGF method and compared with 8×8 SEN is given in Table 4.2.

Table 4.2: Broadcast Reliability of 8× 8 SEN (-)

Switching Reliability	BR evaluation by UGF	BR of 8× 8 SEN [Rajkumar and Goyal (2016)]
0.90	0.754152	0.478297
0.95	0.919616	0.698337
0.96	0.945684	0.751447
0.98	0.984856	0.868126
0.99	0.996000	0.932065

4.1.3 Network reliability of 8× 8 SEN (-)

When the components of the SEN (-) are non-identical as shown in 3.1.3.3 and the probabilities of switching elements in the network are different, then the network reliability of the 8× 8 SEN (-) is:

R_{NR}(SEN-) = 0.351211727

Network reliability of 8×8 SEN (-) is evaluated with respect to different switching reliability with the help of proposed UGF method and compared with 8×8 SEN, which is presented in Table 4.3.

Table 4.3: Network Reliability of 8× 8 SEN (-)

Switching Reliability	NR evaluation by UGF	NR of 8× 8 SEN [Rajkumar and Goyal (2016)]
0.90	0.675632	0.2824295
0.95	0.886714	0.540360
0.96	0.922376	0.612709
0.98	0.977728	0.7847147
0.99	0.994032	0.8863849

4.2 Model [2]: Reliability assessment of 4× 4 shuffle exchange network using universal generating function.

In the model 2, firstly, we have evaluated the reliability of 4× 4 SEN, SEN+ and SEN+2 with the help of universal generating function. Then, we compared the result obtained here in this study with the earlier results.

4.2.1 Reliability of SEN, SEN+, SEN+2

4.2.1.1 Terminal reliability of SEN, SEN+, SEN+2

When the components of the 4× 4 SEN are non-identical and the probabilities of switching elements in the network are different, then the terminal reliability of the 4× 4 SEN is:

R_{TR}(SEN) = 0.9702

Terminal reliability of 4×4 SEN is evaluated with respect to different switching reliability with the help of the proposed UGF method using equation (3.1) and compared with 8×8 SEN as given in Table 4.4.

Table 4.4: Terminal reliability of 4× 4 SEN

Switching Reliability	TR evaluation by UGF	TR of 8× 8 SEN [Rajkumar and Goyal (2016)]	TR of 8× 8 SEN [Bisht, S. and Singh, S.B. (2018)]
0.90	0.81	0.72900	0.72900
0.95	0.9025	0.85737	0.85737
0.96	0.9216	0.88473	0.88473
0.98	0.9604	0.94119	0.94119
0.99	0.9801	0.97029	0.97029

When the components of the 4× 4 SEN+1 are non-identical and the probabilities of switching elements in the network are different, then the terminal reliability of the 4× 4 SEN+1 is:

R_{TR}(SEN+1) = 0.94982976

Terminal reliability of 4×4 SEN+1 is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.2) and compared with 8×8 SEN, which is presented in Table 4.5.

Table 4.5: Terminal reliability of 4× 4 SEN+1

Switching Reliability	TR evaluation by UGF	TR of 8× 8 SEN+ [Rajkumar and Goyal (2016)]	TR of 8× 8 SEN+ [Bisht, S. and Singh, S.B. (2018)]
0.90	0.8019	0.780759	0.780759
0.95	0.90024	0.893920	0.893920
0.96	0.920125	0.915935	0.915935
0.98	0.960016	0.95889	0.95889
0.99	0.98000	0.979712	0.979712

When the components of the 4× 4 SEN+2 are non-identical and the probabilities of switching elements in the network are different, then the terminal reliability of the 4× 4 SEN+2 is:

R_{TR}(SEN+2) = 0.926275315

Terminal reliability of 4×4 SEN+2 is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.3) and compared with 8×8 SEN+2 as presented in Table 4.6.

Table 4.6: Terminal reliability of 4× 4 SEN+2

Switching Reliability	TR evaluation by UGF	TR of 8× 8 SEN+2 [Rajkumar and Goyal (2016)]	TR of 8× 8 SEN+2 [Bisht, S. and Singh, S.B. (2018)]
0.90	0.780759	0.7888415	0.591145
0.95	0.893921	0.8971944	0.755517
0.96	0.915935	0.9182251	0.7966417
0.98	0.958894	0.9595733	0.889761
0.99	0.979712	0.9798963	0.942558

4.2.1.2 Broadcast reliability of SEN, SEN+1, SEN+2

When the components of the 4× 4 SEN are non-identical and the probabilities of switching elements in the network are different, then the broadcast reliability of the 4× 4 SEN is:

R_{BR}(SEN) = 0.941094

Broadcast reliability of 4×4 SEN is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.4) and compared with 8×8 SEN as given in Table 4.7.

Table 4.7: Broadcast reliability of 4× 4 SEN

Switching Reliability	BR evaluation by UGF	BR of 8× 8 SEN [Rajkumar and Goyal (2016)]	BR of 8× 8 SEN [Bisht, S. and Singh, S.B. (2018)]
0.90	0.729	0.478297	0.58458
0.95	0.857375	0.698337	0.77184
0.96	0.884736	0.751447	0.81406
0.98	0.941192	0.868126	0.90359
0.99	0.970299	0.932065	0.95089

When the components of the 4× 4 SEN+1 are non-identical and the probabilities of switching elements in the network are different, then the terminal reliability of the 4× 4 SEN+1 is:

R_{BR}(SEN+1) = 0.902338272

Broadcast reliability of 4×4 SEN+1 is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.5) and compared with 8×8 SEN+1 as shown in Table 4.8.

Table 4.8: Broadcast reliability of 4× 4 SEN+1

Switching Reliability	BR evaluation by UGF	BR of 8× 8 SEN+ [Rajkumar and Goyal (2016)]	BR of 8× 8 SEN+1 [Bisht, S. and Singh, S.B. (2018)]
0.90	0.72171	0.5548722	0.56917
0.95	0.85523	0.7611920	0.76642
0.96	0.88332	0.8067559	0.81036
0.98	0.94081	0.9014617	0.90250
0.99	0.97020	0.9503338	0.95061

When the components of the 4× 4 SEN+2 are non-identical and the probabilities of switching elements in the network are different, then the broadcast reliability of the 4× 4 SEN+2 is:

$R_{BR}(SEN+2)$ = 0.861436043

Broadcast reliability of 4×4 SEN+2 is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.6) and compared with 8×8 SEN+2, which is presented in Table 4.9.

Table 4.9: Broadcast reliability of 4× 4 SEN+2

Switching Reliability	BR evaluation by UGF	BR of 8× 8 SEN+2 [Rajkumar and Goyal (2016)]	BR of 8× 8 SEN+2 [Bisht, S. and Singh, S.B. (2018)]
0.90	0.702683	0.5669980	0.5776400
0.95	0.849225	0.7668366	0.7697292
0.96	0.879298	0.8108211	0.8126635
0.98	0.939716	0.9027414	0.9042140
0.99	0.969915	0.9506918	0.9508393

4.2.1.3 Network reliability of SEN, SEN+1, SEN+2

When the components of the 4×4 SEN are non-identical and the probabilities of switching elements in the network are different, then the broadcast reliability of the 4×4 SEN is:

$R_{NR}(SEN)$ = 0.90345024

Network reliability of 4×4 SEN is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.7) and compared with 8×8 SEN as given in Table 4.10.

Table 4.10: Network reliability of 4× 4 SEN

Switching Reliability	NR evaluation by UGF	NR of 8× 8 SEN [Rajkumar and Goyal (2016)]	NR of 8× 8 SEN [Bisht, S. and Singh, S.B. (2018)]
0.90	0.6561	0.2824295	0.4219009
0.95	0.814506	0.540360	0.6601074
0.96	0.849347	0.612709	0.7190827
0.98	0.922368	0.7847147	0.8500825
0.99	0.960596	0.8863849	0.9225601

When the components of the 4×4 SEN+1 are non-identical and the probabilities of switching elements in the network are different, then the network reliability of the 4×4 SEN+1 is:

$R_{NR}(SEN+1) = 0.865348933$

Network reliability of 4×4 SEN+1 is calculated with respect to different switching reliability with the help of proposed UGF method using equation (3.8) and compared with 8×8 SEN+1 as given in Table 4.11.

Table 4.11: Network reliability of 4× 4 SEN+

Switching Reliability	NR evaluation by UGF	NR of 8× 8 SEN+ [Rajkumar and Goyal (2016)]	NR of 8× 8 SEN+1 [Bisht, S. and Singh, S.B. (2018)]
0.90	0.649539	0.388707	0.406669
0.95	0.812470	0.645470	0.653831
0.96	0.847988	0.708630	0.714663
0.98	0.921999	0.8468415	0.848744
0.99	0.960499	0.9216594	0.922203

When the components of the 4×4 SEN+2 are non-identical and the probabilities of switching elements in the network are different, then the network reliability of the 4×4 SEN+2 is:

$R_{NR}(SEN{+}2) = 0.826619675$

Network reliability of 4×4 SEN+2 is evaluated with respect to different switching reliability with the help of proposed UGF method using equation (3.9) and compared with 8×8 SEN+2, which is presented in Table 4.12.

Table 4.12: Network reliability of 4× 4 SEN+2

Switching Reliability	**NR evaluation by UGF**	**NR of 8× 8 SEN+2 [Rajkumar and Goyal (2016)]**	**NR of 8× 8 SEN+2 [Bisht, S. and Singh, S.B. (2018)]**
0.90	0.643044	0.406633	0.406628
0.95	0.810438	0.655175	0.653830
0.96	0.846631	0.715835	0.714668
0.98	0.921630	0.849251	0.848751
0.99	0.9604039	0.922354	0.922294

Summary and Conclusion

Chapter 5 SUMMARY AND CONCLUSION

Reliability plays a very important role in the modern engineering system. Reliability analysis of the systems and complex networks is very important because of their importance in our modern society. The need to study reliability theory is also growing because of the increasing cost and network complexity. Reliability theory has very wide applications, and used in many firms like transportations, nuclear, aerospace, medical sector, defence and many more.

In this research work, we have considered two different interconnection networks, one is 8×8 SEN (-) and another is 4×4 SEN. The reliability of both the networks are evaluated by using the universal generating function approach.

This research work has been divided into following four chapters:

Chapter 1 is introductory part of the thesis and it consists of fundamental important terms and concepts which are essential for the study. It also gives the brief idea about the origin of reliability theory and the introduction of interconnection networks. The UGF method to calculate the reliability of the various networks was also discussed.

Chapter 2 is “review of Literature” which discusses various researches done by the researchers in the field of reliability engineering. This chapter consists of the results and conclusions obtained from the researches done in the past on reliability theory, Interconnection networks, Network reliability, Shuffle exchange networks and UGF.

Chapter 3 is “Materials and Methods” which describes the two proposed models in detail. Reliability bounds of both the models are determined on the basis of three parameters viz., Terminal, Broadcast and Network reliability and the results obtained are also demonstrated numerically.

The two model considered in this thesis are:

Model [1]: Reliability assessment of 8×8 shuffle exchange network with one stage less (SEN-) by using universal generating function.

Model [2]: Reliability assessment of 4×4 shuffle exchange network using universal generating function.

In Model [1], we have constructed a new 8×8 shuffle exchange network with one stage less i.e., SEN (-) incorporating MUX and DEMUX. All perspectives of the reliability, viz. terminal reliability, broadcast reliability and network reliability of the 8×8mSEN (-) have been calculated with the help of the universal generating function.

In Model [2], we have analysed the reliability of the 4×4 shuffle exchange network. In this model 4×4 shuffle exchange network has been investigated by increasing the number of switching stages of 4×4 SEN, SEN+1, SEN+2. Reliability of 4×4 SEN, 4×4 SEN+1, 4×4 SEN+2 is calculated on the basis of three parameter viz., terminal, broadcast and network reliability with the help of UGF.

Chapter 4 entitled "Results and Discussion" deals with the numerical analysis and graphical representation of the reliability measures such as reliability bounds of the proposed models. All the results obtained in Chapter 3 have been presented in tabular form and are illustrated graphically. In this chapter, both the models are being compared with 8×8 SEN. The numerical examples are given to show the efficiency and applicability of the proposed method.

Literature Cited

LITERATURE CITED

Ahlawat, N., Chauhan, S. K., & Malik, S. C. (2019). Reliability evaluation of a non series–parallel system of six components with Weibull failure laws. *Life Cycle Reliability and Safety Engineering*, *8*(1), 91-97.

Ahmad, W., Hasan, O., Pervez, U., & Qadir, J. (2017). Reliability modeling and analysis of communication networks. *Journal of Network and Computer Applications*, *78*, 191-215.

Bansal, P. K., Joshi, R. C., & Singh, K. (1994). On a fault-tolerant multistage interconnection network. *Computers & Electrical Engineering*, *20*(4), 335-345.

Bisht, S. (2018). Reliability indices and signature analysis of complex networks. Thesis Ph.D. GBPUA&T Pantnagar,149-152.

Bisht, S., & Singh, S. B. (2019). Signature reliability of binary state node in complex bridge networks using universal generating function. *International Journal of Quality & Reliability Management*, *36*(2), 186-201.

Bistouni, F., & Jahanshahi, M. (2014). Analyzing the reliability of shuffle-exchange networks using reliability block diagrams. *Reliability Engineering & System Safety*, *132*, 97-106.

Bistouni, F., & Jahanshahi, M. (2014). Improved extra group network: a new fault-tolerant multistage interconnection network. *The Journal of Supercomputing*, *69*(1), 161-199.

Bistouni, F., & Jahanshahi, M. (2018). Rearranging links: a cost-effective approach to improve the reliability of multistage interconnection networks. *International Journal of Internet Technology and Secured Transactions*, *8*(3), 336-373.

Bistouni, F., & Jahanshahi, M. (2019). Determining the reliability importance of switching elements in the shuffle-exchange networks. *International Journal of Parallel, Emergent and Distributed Systems*, *34*(4), 448-476.

Blake, J. T., & Trivedi, K. S. (1989). Multistage interconnection network reliability. *IEEE Transactions on Computers*, *38*(11), 1600-1604.

Chacko, V. M. (2018). Continuous Multistate System Universal Generating Function. *Reliability: Theory & Applications*, *13*(4 (51)).

Chinnaiah, B. (2019). Multistage Interconnection Networks in Reliability Shuffle Exchange Networks. In *First International Conference on Artificial Intelligence and Cognitive Computing*. Springer, Singapore, 571-581.

Cox, D. R. (1955, July). The analysis of non-Markovian stochastic processes by the inclusion of supplementary variables. In *Mathematical Proceedings of the Cambridge Philosophical Society* , Cambridge University Press, 51(3), 433-441.

Ding, F., & Han, S. (2018). Multi-state reliability analysis of rotor system using Semi-Markov model and UGF. *Journal of Vibroengineering*, *20*(5).

Ding, Y., Lin, Y., Peng, R., & Zuo, M. J. (2019). Approximate Reliability Evaluation of Large-Scale Multistate Series-Parallel Systems. *IEEE Transactions on Reliability*.

Fard, N. S., & Gunawan, I. (2005). Terminal reliability improvement of shuffle-exchange network systems. *International Journal of Reliability, Quality and Safety Engineering*, *12*(01), 51-60.

Feizabadi, M., & Jahromi, A. E. (2017). A new model for reliability optimization of series-parallel systems with non-homogeneous components. *Reliability Engineering & System Safety*, *157*, 101-112.

Garg, R. C. (1963). Analytical study of a complex system having two types of components. *Naval Research Logistics Quarterly*, *10*(1), 263-269.

Gnedenko, B. V., Belyayev, Y. K., & Solovyev, A. D. (2014). *Mathematical methods of Reliability Theory*. Academic Press.

Gunawan, I. (2014). *Fundamentals of Reliability Engineering: Applications in Multistage Interconnection Networks*. John Wiley & Sons.

Gupta, Shilpa, and G. L. Pahuja. "A new SEN Minus: Design and reliability measures." *International Journal of Reliability, Quality and Safety Engineering* 23.04 (2016): 1650012.

Jahanshahi, M., & Bistouni, F. (2014). A new approach to improve reliability of the multistage interconnection networks. *Computers & Electrical Engineering*, *40*(8), 348-374.

Karimi, B., Niaki, S. T. A., Miriha, S. M., Ghare Hasanluo, M., & Javanmard, S. (2019). A weighted K-means clustering approach to solve the redundancy allocation problem of systems having components with different failures. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 1748006X19844127.

Kumar, A., & Ram, M. Computation Interval-Valued Reliability of Sliding Window System.

Kumar, A., Singh, S. B., & Ram, M. (2016). Interval-valued reliability assessment of 2-out-of-4 system. In *2016 International Conference on Emerging Trends in Communication Technologies (ETCT)* (pp. 1-4). IEEE.

Kvassay, M., Rusnak, P., & Rabcan, J. (2019). Time-Dependent Analysis of Series-Parallel Multistate Systems Using Structure Function and Markov Processes. In *Advances in System Reliability Engineering* (pp. 131-165). Academic Press.

Levitin, G., & Lisnianski, A. (1999). Importance and sensitivity analysis of multi-state systems using the universal generating function method. *Reliability Engineering & System Safety*, *65*(3), 271-282.

Levitin, G., Xing, L., Ben-Haim, H., & Dai, Y. (2013). Reliability of series-parallel systems with random failure propagation time. *IEEE Transactions on Reliability*, *62*(3), 637-647.

Li, C. Y., Chen, X., Yi, X. S., & Tao, J. Y. (2010). Heterogeneous redundancy optimization for multi-state series–parallel systems subject to common cause failures. *Reliability Engineering & System Safety*, *95*(3), 202-207.

Li, C. Y., Chen, X., YI, X. S., & TAO, J. Y. (2010). Reliability analysis of multi-state system with multiple performance parameters based on vector-universal generating function. *Acta Armamentarii*, *12*, 1604-1610.

Li, C. Y., Chen, X., Yi, X. S., & Tao, J. Y. (2011). Interval-valued reliability analysis of multi-state systems. *IEEE Transactions on Reliability*, *60*(1), 323-330.

Ling, X., Zhang, Y., & Li, P. (2019). Redundancy optimization for series and parallel systems exposed to random shocks. *International Journal of Performability Engineering*, *15*(5), 1427.

Lisnianski, A. (2012). L z-transform for a discrete-state continuous-time Markov process and its applications to multi-state system reliability. In *Recent advances in system reliability* (pp. 79-95). Springer, London.

Meena, K. S., & Vasanthi, T. (2016). Reliability analysis of mobile ad hoc networks using universal generating function. *Quality and Reliability Engineering International*, *32*(1), 111-122.

Meenakshi, & Singh, S. B. (2017). Reliability analysis of multi-state complex system having two multi-state subsystems under uncertainty. *Journal of Reliability and Statistical Studies*, *10*(1), 161-177.

Meenkashi, K., Singh, S. B., & Kumar, A. (2019). Reliability analysis of multi-state complex system with multi-state weighted subsystems. *International Journal of Quality & Reliability Management*, *36*(4), 552-568.

Negi, S., & Singh, S. B. (2015). Reliability analysis of non-repairable complex system with weighted subsystems connected in series. *Applied Mathematics and Computation*, 262, 79-89.

Palle, S., Akiri, S., & Subbara, P. (2018, October). Optimization of series-parallel and parallel-series integrated redundant reliability model with cost constraint. In *ICRTEMMS Conference Proceedings*,760(764), 760-764.

Pan, G., Shang, C. X., Liang, Y. Y., Cai, J. Y., & Li, D. Y. (2016, November). Analysis of interval-valued reliability of multi-state system in consideration of epistemic uncertainty. In *International Conference on P2P, Parallel, Grid, Cloud and Internet Computing* (pp. 69-80). Springer, Cham.

Qiu, S., & Ming, H. X. Reliability analysis of multi-state series systems with performance sharing mechanism under epistemic uncertainty. *Quality and Reliability Engineering International*.

Rajkumar, S., & Goyal, N. K. (2016). Review of multistage interconnection networks reliability and fault-tolerance. *IETE Technical Review*, *33*(3), 223-230.

Ram, M. (2013). On system reliability approaches: a brief survey. *International Journal of System Assurance Engineering and Management*, *4*(2), 101-117.

Ramdani, N., Meslem, N., & Candau, Y. (2010). Computing reachable sets for uncertain nonlinear monotone systems. *Nonlinear Analysis: Hybrid Systems*, *4*(2), 263-278.

Rawal, D. K., Ram, M., & Singh, V. V. (2015). Study of reliability measures of a local area network via copula linguistics approach. *International Journal of Quality & Reliability Management*, *32*(1), 97-111.

Rebaiaia, M. L., & Ait-Kadi, D. (2013). *Network reliability evaluation and optimization: Methods, algorithms and software tools*. CIRRELT.

Sharma, S., Kahlon, K. S., & Bansal, P. K. (2009). Reliability and path length analysis of irregular fault tolerant multistage interconnection network. *ACM SIGARCH Computer Architecture News*, *37*(5), 16-23.

Smith, D. J. (1972). *Reliability Engineering*. Barnes & Noble Books.

Tiwari, N., Singh, S. B., & Ram, M. (2010). Reliability analysis of a complex system, attended by two repairmen with vacation under marked process with the application of copula. *International Journal of Reliability and Applications*, *11*(2), 107-122.

Ushakov, I. A. (1986). A universal generating function. *Soviet Journal of Computer and Systems Sciences*, *24*(5), 118-129.

Utkin, L. V. (2004). Interval reliability of typical systems with partially known probabilities. *European Journal of Operational Research*, *153*(3), 790-802.

Williams, H. L. (1958). Reliability evaluation of the human component in man-machine systems. *Electrical Manufacturing*, *61*(4).

Yeh, W. C. (2008). A simple universal generating function method for estimating the reliability of general multi-state node networks. *IIE Transactions*, *41*(1), 3-11.

Yu, H., Yang, J., & Mo, H. (2014). Reliability analysis of repairable multi-state system with common bus performance sharing. *Reliability Engineering & System Safety*, *132*, 90-96.

Yunus, N. A. M., & Othman, M. (2011). Shuffle exchange network in multistage interconnection network: A review and challenges. *International Journal of Computer and Electrical Engineering*, *3*(5), 724.

Yunus, N. A. M., & Othman, M. (2014). Reliability evaluation and routing integration in shuffle exchange omega network. *Journal of Networks*, *9*(7), 1732.

Yunus, N. A. M., Othman, M., Hanapi, Z. M., & Lun, K. Y. (2018). Number of stage implication towards multistage interconnection network reliability. *Advanced Science Letters*, *24*(2), 1259-1262.

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ABSTRACT

In this research work, author has developed two different models which are given as: **1.** Reliability assessment of 8×8 shuffle Exchange network with one stage less (SEN-) by using universal generating function method. **2.** Reliability assessment of 4×4 shuffle exchange network (SEN) by using universal generating function method. In the first model, 8×8 SEN- MIN does not satisfy the basic condition of MIN of full connectivity as some of the inputs nodes are not connected to all output nodes. So 2×1 MUX and 1×2 DEMUX have been used at source end and destination end respectively of 8×8 SEN- to achieve full connectivity. The reliability block diagram of the terminal, broadcast and network reliability of 8×8 SEN- was presented and its reliability was calculated by universal generating function method. In the second model we have considered three 4×4 shuffle exchange networks, namely, 4×4 SEN, 4×4 SEN+ and 4×4 SEN+2. The reliability block diagram of terminal, broadcast and network reliability for each of these networks was presented. The reliability of 4×4 SEN, 4×4 SEN+ and 4×4 SEN+2 was calculated by using universal generating function method unlike done before.

Both the presented models are demonstrated by appropriate illustrative examples.

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सहायक विषय : संगणक विज्ञान

शोध का शीर्षक : **"यूजीएफ विधि से कुछ एसईएनएस की विष्वसनीयता का विष्लेषण"**

सलाहकार : **डॉ. एस. बी. सिंह**

सारांश

इस शोध में लेखक ने दो विश्वसनीय मॉडलों का निर्माण किया है, जो निम्नलिखित है। **1-**यूजीएफ विधि से एक स्तर कम 8×8 शफल एक्सचेंज नेटवर्क (एसईएन-) की विश्वसनीयता का आकलन। **2-** यूजीएफ विधि से 4×4 शफल एक्सचेंज नेटवर्क (एसईएन) की विश्वसनीयता का आकलन। **मॉडल 1** में 8×8 एसईएन बहुस्तरीय इन्टर कनैक्शन नेटवर्क की शर्त का अनुपालन नहीं करता है क्योंकि कुछ इनपुट सभी आउटपुट से नहीं जुडे हुए हैं। 8×8 एसईएन में 2×1 एमयूएक्स एवं 1×2 डीएमयूएक्स को स्त्रोत एवं गन्तव्य स्थान पर जोडकर पूर्ण क्नैक्टीविटी प्राप्त की गई है। 8×8 एसईएन- की टर्मिनल, प्रसारण और नेटवर्क विश्वसनीयता का ब्लाक आरेख प्रस्तुत किया गया है तथा यूजीएफ से इसकी विश्वसनीयता का आकलन किया गया है। **मॉडल 2** में 4×4 एसईएन, 4×4 एसईएन+ एवं 4×4 एसईएन+2 में शोध किया गया है। तीनों नेटवर्कों के लिये टर्मिनल प्रसारण एवं नेटवर्क विश्वसनीयता का ब्लाक आरेख किया गया है तथा यूजीएफ से इन तीनों नेटवर्कों की विश्वसनीयता का आकलन किया गया है।

सभी प्रस्तुत मॉडलों को उपयुक्त उल्लेखनीय उदाहरणों द्वारा प्रदर्शित किया गया है।

(एस. बी. सिंह)
सलाहकार

(वैभव बिष्ट)
लेखक