Construction of a Series of Balanced Ternary Designs Using Balanced Incomplete Block Designs and Their Robustness Against the Loss of Some Observations

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CERTIFICATE – I

This is to certify that the thesis entitled **"*Construction of a Series of Balanced Ternary Designs Using Balanced Incomplete Block Designs and Their Robustness Against the Loss of Some Observations*"** *submitted in partial fulfilment of the requirements for the degree of* ***MASTER OF SCIENCE (AGRICULTURE) in AGRICULTURAL STATISTICS*** *of the Jawaharlal Nehru Krishi Vishwa Vidyalaya, Jabalpur is a record of the bonafide research work carried out by* ***Mr. Anurag Gupta,*** *under my guidance and supervision. The subject of the thesis has been approved by the Student's Advisory Committee and the Director of Instructions.*

All the assistance and help received during the course of the investigation have been acknowledged by him.

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I, Anurag Gupta S/o Mr. Prahlad Gupta Certify the work embodied in thesis entitled "Construction of a Series of Balanced Ternary Designs Using Balanced Incomplete Block Designs and Their Robustness Against the Loss of Some Observations" is my own first hand bonafide work carried out by me under the guidance of Dr. H.L. Sharma Professor and Head, Department of Mathematics & Statistics, College of Agriculture, Jawaharlal Nehru Krishi Vishwa Vidyalaya, Jabalpur during 2017-2018.

The matter embodied in the thesis has not been submitted for the award of any other degree/diploma. Due credit has been made to all the assistance and help.

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbols/ Abbreviations	Stand for
BIBD	: Balanced incomplete block design
BPTD	: Balanced part ternary design
BTD	: Balanced ternary design
NBTD	: Nested balanced ternary design
v	: Treatments in BIBD
b	: Blocks in BIBD
r	: Replication in BIBD
k	: Block size in BIBD
λ	: Pair of treatments in BIBD
V	: Treatments in BTD
B	: Blocks in BTD
R	: Total multiplicity in BTD
K	: Block size in BTD
ρ_1	: Multiplicity of '1' treatment
ρ_2	: Multiplicity of '2' treatment
Λ	: Total pair of treatments in BTD.

Chapter-I

INTRODUCTION

INTRODUCTION

Design of Experiments forms a fascinating branch of Statistics, which owes its origin to agricultural experiments. During 1930's to 1950's there have been major contributions from Fisher and his co-workers in the development of classical theory of design of experiments. During last five decades there have been a quantum jump in the development of this branch of Statistics. A careful glance in classical theory reveals that the designs developed have inherent simplicity, which helps in analyzing the data easily. The principal reason for this was lack of computational facilities available in the earlier days. The emphasis on simplicity, with the advent of high-speed computers, shifted to designs, which have some statistical properties or are optimal in some statistical sense.

Balanced incomplete block (BIB) designs, are important experimental designs for several reasons, including many applications in organoleptic testing, which were introduced by Yates (1936). It is the general opinion of food research workers that, for most foods, a taster cannot differentiate among more than five to eight samples at one sitting and perhaps not more than four or five. This limitation is caused primarily by the inability of the judge to taste more than a few samples without becoming fatigued. The senses become dulled rather rapidly and many impressions are obtained if one attempts to taste too many samples. Therefore, when there are more treatments that can be tested at one sitting, an incomplete block design seems to be more appropriate for this purpose.

Balanced ternary designs can be constructed by using Balanced incomplete block designs. A block design which has an arrangement of V treatments in B blocks, each of size K in such a way that each treatment appears 0, 1 or 2 times in R blocks, where each treatment occurs alone in ρ_1 blocks and is repeated two times in ρ_2 blocks, where ρ_1 and ρ_2 are multiplicity of the treatments for the design, each of the distinct pairs of treatments appears Λ times, is called balanced ternary (BT) design.

Recently, there has been keen interest in investigating the robustness properties of BIB designs and BT designs. If there are loss of plots or entire blocks in the experimental design, does the resulting residual design still behave well in a statistical sense? A BIBD remains connected after removal of any *r-1* observations, or of any r-1 blocks.

Robustness Against Missing Data

In the field experimentation once the experiment has been laid in the field the observation can be destroyed or lost during the course of experimentation. Missing data may cause serious problems and may render even a well-planned experiment useless. This is one of the reasons for the popularity of this area of research over last two decades.

Objectives:

Keeping in view of the above facts, this investigation has been decided to plan with the following objectives:

1. To test the robustness of Balanced Incomplete block designs used in Balanced ternary designs,
2. To construct balanced ternary designs, and
3. To test the robustness of constructed Balanced ternary designs.

Chapter-II

REVIEW OF LITERATURE

REVIEW OF LITERATURE

During the last few decades the researchers and scientists working for the construction of incomplete blocks designs had been concentrated about the methodology and analysis of the designs. A number of methods was suggested to construct BIB, balanced ternary designs along with their efficiency.

For the purpose of this chapter, the literature in relation to the construction of above mentioned designs are given below:

2.1 Construction of a series of balanced ternary designs based on BIB designs

Tocher (1952) introduced initially balanced n-ary designs giving examples of ternary designs and discussed their uses based on incomplete block designs and also obtained some balanced ternary designs by trial and error.

Tyagi and Rizwi (1976) studied the construction of balanced ternary designs and advocated the addition of first row elements of the identity matrix of the order vxv to first treatment, second row elements to second treatments and last row elements to the last treatments in the design.

Wynn (1977) constructed a BIB design with $v = 8$, $b = 56$, $k = 3$ and $b^* = 24$ with repeated blocks. He also discussed the selection of a sample of k distinct elements from a set of v elements (varieties). He was led in particular to consider balanced incomplete block designs in which some of the blocks are repeated.

Elizabeth (1985) studied a balanced ternary design on V elements is a collection of B blocks (which are multisets) of size K, such that each element occurs 0, 1 or 2 times per block and R times altogether, and such that each unordered pair of distinct elements occurs Λ times. It is straightforward to show that each element has to occur singly in a constant number of blocks, say P_l, and so each element also occurs twice in a constant number of blocks, say p~, where $R = P_l + 2p_2$. If $P_2 = 0$ the design is a balanced incomplete block design (binary design), so we assume $P_2 > 0$, and $K < 2V$

(corresponding to incompleteness in the binary case), Necessarily A > 1 if p_2>0 (and K > 2).

Sarvate (1990) gave a construction for balanced ternary designs based on the designs so obtained, a construction of partially balanced ternary designs was given, which gave balanced ternary designs and series of symmetric balanced ternary designs in special cases.

Francel and Sarvate (1994) constructed ternary designs where the replication numbers of the design are the boundary values L_R and U_R and analyzed the situation where the replications numbers of the design cover all or all but one of the values in the range L_R and $U_{R.}$.

Mitra and Mandal (1998) constructed a self complementary BIB designs with the existing BIB design, v = 2k+1, b, r, k, λ of the parameters V=2(k+1), b' = 2b, r' = b, k' = k+1, λ' = r and obtained balanced ternary group divisible designs by taking the combination of two blocks of the self complementary design together at a time.

Abel *et al*. (2001) gave the necessary conditions for the existence of a balanced incomplete block design on v ≥ *k* points, with index λ and block size *k*, for *k* = 8, these conditions were known to be sufficient when λ = 1, with 38 possible exceptions, the largest of which is v = 3,753 and for these 38 values of v, showed (v, 8, λ) BIBDs exist whenever λ > 1 for all but five possible values of v, the largest of which was v = 1,177, and these five v's are the only values for which more than one value of λ is open. For λ>1, showed the necessary conditions which were sufficient with the definite exception of two further values of v, and the possible exception of 7 further values of v, the largest of which is v=589. In particular, showed the necessary conditions which were sufficient for all λ> 5 and for λ = 4 when v ≠ 22.

Mishra (2002) proposed partially balanced part n-ary block designs that was related to the design with a specified number of blocks, presented methods of construction of such designs.

Sharma *et al*. (2002) developed PB arrays of strength two and three in three symbols using Hadamard matrices which was converted in the form of BIBD parameters.

Kaski and Ostergard (2004) enumerated the balanced ternary design with the range of parameters $V \leq 10$, $B \leq 30$, and $R \leq 15$using a computer backstracksearch and presented non-isomorphic balanced ternary designs with given parametersby enumerating the corresponding incidence matrices.

Aoulmi and Bousseboua (2005) developed a recursive method for the construction of balanced n-ary block designs which was based on the analogy between a balanced incomplete binary block design and the set of all distinct linear sub-varieties of the same dimension extracted from a finite projective geometry.

Francel and Hurd (2008) reported balanced ternary designs (BTDs), in which every block contains one element singly and the rest doubly and also generated classes of packed BTDs that are nested with BIBD or partially balanced incomplete block (PBIB) design. Counting arguments were used to establish the necessity of the relationships among the parameters of any BTD.

Acha (2012) discussed the balanced incomplete–block design (BIBD's) and further compared two methods of analyzing the classical and vector space analysis of variance (ANOVA) methods. The classical ANOVA method was found to be preferable to the vector space ANOVA method in this research work.

Gupta and Prasad (2012) demonstrated the applications of factorial structure in agronomic experiments, particularly in the crop sequences experiments by using the incomplete block designs giving maximum efficiency as reducing the heterogeneity.

Singh and Jain (2012) developed some new methods of construction of balanced incomplete block designs with nested rows and columns and also concluded with a numerical illustration including the parameters of some BIB-Rows and Column designs with their efficiencies and efficiency factors in appendices.

Sharma *et al*. (2013) proposed the construction of balanced ternary design based on BIB design by taking the two blocks taken together at a time.

Awad and Banerjee (2013) presented a review of the available literature on balanced incomplete block designs with repeated blocks.It was useful in various problems of experimental designs and increasing its efficiency by reducing the heterogeneity and also verified that such a design was necessarily orthogonal and variance balanced.

Sharma *et al*. (2013) constructed a series of balanced ternary, ternary group divisible and nested ternary group divisible designs based on BIB designs. They obtained the designs by taking two blocks together at a time.

Awad and Banerjee (2015) gave some new construction methods of the optimum chemical balance weighing designs, which were based on the incidence matrices of the known balanced incomplete block designs, balanced bipartite block designs and ternary balanced block designs. Also they gave conditions under which the constructed chemical balance weighing designs to be A-optimal.

Biswas and Sharma (2016) studied recursive construction of a series of balanced ternary designs and their applications in agriculture. Also, they gave some examples which are used for the intercropping experiments.

2.2 Robustness of BIB design and BTD design against the unavailability of two blocks :

Ghosh (1982) discussed method for finding robustness of design against the unavailability of data.

Mukerjee and Kageyama(1990) showed that Group Divisible Designs arealso robust against the unavailability of one block.

Das and Kageyama (1992) showed that BIB designs and extended BIB designs are fairly robust against the unavailability of s ($s \leq k$) observations in any block. While any Youden design and Latin Square design are found to be fairly robust against the loss of any one column.

Dey (1993) studied robustness of block designs against missing data.

Toman and Gastwirth (1994) studied the efficiency robust experimental design and estimation using a data based prior.

Batra and Prasad (1997) discussed the robustness of block designs against the interchange of treatments and found them robust.

Ghosh and Gosai (1998) found that Singular Group Divisible Designs are also fairly robust against the unavailability of one replication.

Ghosh and Desai (1998,1999) obtained the robustness of Complete Diallel Crosses Plan against the unavailability of one block and also for those plans, which have unequal number of crosses in a block.

Ghosh and Biswas (2000) also pointed out the robustness for complete Diallel Crosses Plan, which are binary balanced against the loss of oneblock.

Shrivastava (2003). It is noticed that, design is fairly robust against the loss of one block. Here, the efficiencies of 33 BIBD with repeated block were worked out. In fact, all design satisfies $e \geq 0.95$. Thus it seems that design is fairly robust against loss of one block. Most of the robustness criteria against the unavailability of data are: (i) to get the connectedness of the residual design; (ii) to have the variance balance of the residual design; (iii) to consider the A–efficiency of residual design for the robustness study.

Reck and Morgan (2005) studied the necessary conditions for a balanced incomplete block design (BIBD) are satisfied, but no BIBD exists, there is no simple answer for the optimal design problem. In such an irregular BIBD setting, identification of an A-optimal or D-optimal design requires a delicate interplay of combinatorics and optimality tools. Here the known theory is extended, giving a more comprehensive picture of the set of potentially optimal designs and affording a better understanding of the relationship between optimality and simple combinatorial measures of symmetry. The theory in conjunction with an intricate search leads to the A- and D-optimal design for D(15,21,5); this is the first known optimal design for an irregular BIBD setting. Insight is also gained on resolvable members of D.

Morgan and Parvu (2008) studied balanced incomplete block designs had led to a great many designs with the same numbers of treatments, blocks, and block size. While the basic analysis does not differentiate among different BIBDs with the same parameters, they do differ in their capacity to withstand loss of experimental material. Competing BIBDs are compared here for their robustness in terms of average loss and worst loss.

Shunmugathai and Srinivasan (2011) studied robustness of doubly balanced incomplete block designs against unavailability of two blocks. The investigation shows that Doubly Balanced Incomplete Block Designs are fairly robust in terms of efficiency. As a special case, we can also show the robustness of Doubly Balanced Incomplete Block Design when two blocks are lost.

Godolphin and Warren (2011) investigated conditions for a binary block design, *D*, to be maximally robust such that every eventual design obtained from *D* by eliminating $r[v]-1$ blocks is connected, where $r[v]$ is the smallest treatment replication. Four new results for the maximal robustness of *D* with superior properties are given. An extension of these results to widen the assessment of robustness of the planned design is also presented.

Bhar (2014) The robustness of block designs against missing observations is revisited. It has been shown that A-efficiency criterion is not an appropriate measure to judge the efficiency of the residual design. As an alternate to this, E-efficiency criterion is defined. A lower bound of this criterion for the loss of any t observations in binary variance balanced block design is obtained. Balanced incomplete block designs (BIBD) that are robust as per E-efficiency criterion are identified.

Chapter-III

MATERIAL AND METHODS

MATERIAL AND METHODS

In this chapter some basic definitions and notations are giving which are used in the construction of a series of balanced incomplete block designs and balanced ternary designs.

3.1 Definitions and Notations

3.1.1 Incomplete Block Design (IBD)

An incomplete block design is one having v treatments and b blocks each of size k such that each of the treatment is replicated r times and each pair of treatments occurs once and only once in the same block .v, b, r and k are known as the parameters of the Incomplete Block Design (IBD).

3.1.2 Balanced Incomplete Block Design (BIBD)

Itis an arrangement of v treatments belonging to a set $\Omega = \{ 1,2,\ldots,v\}$ (say), into b blocks of k plots each (k<v) is known as BIBD,if

1. Each block contains k (k<v) distinct treatments.
2. Each treatment appears in r number of blocks.
3. Every pair of treatments appears together in λ blocks.

Parameters of BIBD

The integers v,b,r,k and λ are called the parameters of the BIBD, where

v = number of treatments

b = number of blocks

r = number of replicates for each treatment

k = block size

λ = number of blocks in which each pair of treatments occurs together

The following parametric relations serve as a necessary condition for the existence of a BIBD:

1. $vr = bk$
2. $\lambda(v-1) = r(k-1)$
3. $b \geq v$ (Fisher's Inequality).

The inequality $b \geq v + r - k$ is sharper than Fisher's inequality

3.1.3 Incidence matrix of a design

We can always associate with any design D a matrix $N = (n_{ij})$, $i = 1,2,\ldots,v$ and $j = 1,2,\ldots,b$ where v = number of treatments and b = number of blocks in the design D and n_{ij} = number of times i^{th} treatment appears in the j^{th} block of D. Thus,

$$N = \begin{bmatrix} n_{11} & n_{12}\ldots & n_{1b} \\ n_{21} & n_{22}\ldots & n_{2b} \\ n_{v1} & n_{v2}\ldots & n_{vb} \end{bmatrix}$$

In special cases such as:

a) When $n_{ij} = 1$, for all i and j i.e. N = matrix of unities only then D is called a complete block designi.e. orthogonal design

b) When,

$$n_{ij} = \begin{cases} 1, \text{ for } i^{th} \text{treatment appears in the } j^{th} \text{ block,} \\ 0\,, \text{ otherwise} \end{cases}$$

then D is called incomplete (binary) block design. Binary in the sense of just two symbols 0 and 1 are in N.

c) When n_{ij} can take values 0 or 1 or …,or n-1 then it is called n- ary block design.

We observe that for a BIBD,

i. $\sum_{j=1}^{b} n_{ij} = r = \sum_{j=1}^{b} n_{ij}^2$ for each fixed $i = 1,2,\ldots,v$(that is treatment appears exactly in r blocks of thedesign)

ii. $\sum_{i=1}^{v} n_{ij} = k = \sum_{i=1}^{v} n_{ij}^2$ for each fixed $j = 1,2,\ldots,b$(that is each block has exactly k treatments)

iii. $\sum_{j=1}^{b} n_{ij} n_{tj} = \lambda$, $i \neq t$, $i , t = 1,2,\ldots,v$(since $n_{ij} . n_{tj} = 1$ only when i^{th} and t^{th} treatments appear together in a block and in a BIBD they appear in exactly λ blocks).

iv. If N' (b × v) denotes the transpose of matrix of N then

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j}n_{2j} & \dots & \sum n_{1j}n_{vj} \\ \sum n_{2j}n_{1j} & \sum n_{2j}^2 & \dots & \sum n_{2j}n_{vj} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ \sum n_{vj}n_{1j} & \sum n_{vj}n_{2j} & \dots & \sum n_{vj}^2 \end{bmatrix}$$

$$= \begin{bmatrix} r & \lambda & \dots & \lambda \\ \lambda & r & \dots & \lambda \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ \lambda & \lambda & \dots & r \end{bmatrix} = (r-\lambda)I_v + \lambda E_v ,$$

Which is a v × v matrix and summation is from j = 1 to v. Here I_v is the identity matrix, E_v is matrix of unitities, both of dimensions v.

The C matrix is simplified as follows:

$$C = r\,I - \frac{N'N}{k}$$

$$C = \lambda v[\, I_V - \frac{E_v E'_v}{v}\,]$$

3.1.4 Symmetric BIBD

A BIB design in which v = b or r = k is called a symmetric BIB design. In symmetric BIB designs any two blocks have λ treatments in common.

3.1.5 Complementary BIBD

Given a BIB design with parameters v,b,r,k,λ, another BIB design, called the complementary design of d exists. We denote the complementary design of D by D', wnich is obtained from D by taking in the j^{th} block of D' all those treatments which do not occur in the j^{th} block of D, for j = 1,2,…,b. Obviously, if $v_1,b_1,r_1,k_1,\lambda_1$ are the parameters of D',we have $v_1 = v$, $b_1 = b$. The number of treatments in each block is $k_1 = v-k$, and the treatments within a block are distinct. In D, a given treatment θ appears in r blocks and does not occur in (b-r) blocks. In D', these (b-r) blocks each contain θ, and thus r_1 = b-r. Consider now a pair of treatments α, β. This pair of treatments occurs together in λ blocks of D. Further, there are $(r-\lambda)$ blocks which contain α but not β and another $(r-\lambda)$ blocks which contain β but not α. Thus the number of blocks in D, which contain neither α nor β, is $b-2(r-\lambda)-\lambda = b-2r + \lambda$. Now, we have the following result:

The complementary design of a BIB design with parameters v,b,r,k,λ is also a BIB design with parameters

$$v_1 = v,\ b_1 = b,\ r_1 = b-r,\ \ k_1 = v-k,\ \lambda_1 = b-2r +\lambda.$$

3.1.6 Balanced n- ary incomplete block design

A balanced n-ary design is a collection of B multi sets each of size K, chosen from a set of size V in such a way that each of the V elements occurs R times altogether and 0,1,2,……or n-1 times in each block and each pair of distinct elements occurs Λ times. So the inner product of any two distinct rows of the $V\times B$ incidence matrix of the balanced n-ary design is Λ.

A balanced n-ary design where n=2 is the well known balanced incomplete block design(binary design).

3.1.6 (a)Balanced ternary design

A balanced n-ary design where n=3 is known as the balanced ternary design(BTD) which is a collection of B blocks, each of cardinality $K(K\leq V)$, chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in a precisely ρ_1 blocks and twice in precisely ρ_2 blocks, and with incidence matrix having inner product of any two rows Λ is denoted by BTD $(V,B,\rho_1,\rho_2,R,K,\Lambda)$.

The parameters of BTD follow the following relationship

(i) $VR = BK$, (ii) $\Lambda(V-1) = R(K-1)-2\rho_2$ (iii) $R = \rho_1+2\rho_2,$

The incidence matrix N of such a design has elements equal to 0,1 or 2 and moreover,

$NN^{l} = (\rho_1+4\rho_2-\Lambda)I_V+\Lambda 1_V 1'_V$.

The C matrix is simplified as follows:

$$C = R\,I - \frac{N'N}{K}$$

$$C = \Lambda V\,[I_V - \frac{E_v E'_v}{V}]$$

3.1.6 (b) Balanced (part) ternary design

A balanced part ternary design (BPTD) (V; b_1, b_2, B; ρ_1, ρ_2, R; K, Λ) is a balanced ternary design (BTD) with a specified number of blocks, say b_1, each having no repeated treatments and say b_2, having repeated treatments that also satisfies.

There exist exactly $b_2 = B - b_1$ blocks each containing atleast one treatment of multiplicity two. Hence, every BTD is a BPTD for some choice of b_1 and b_2.

Efficiency of balanced ternary designs is computed as

$$E = \frac{\Lambda V}{RK}$$

Robustness:

A fundamental requirement for a design to be robust is that it remain connected after the removal of either a small number of plots or a small number of blocks. It was shown by Ghosh, that a BIBD remains connected after the removal of any *r* - 1 observations, where *r* is the replication of the original BIBD.

In any case, when a small number of observations or blocks are lost, BIBDs remain connected. Another approach for judging robustness is to measure the efficiency of the residual design with respect to the original design. A-efficiency of the residual design is given by:

$$Eff = \frac{\textit{Sum of reciprocals of non-zero eigen values of } C_o}{\textit{Sum of reciprocals of non-zero eigen values of } C_t}$$

where C_o and C_t are the information matrices of the original design and the residual design respectively. The following represents the eigenvalues of a residual design formed by removing two blocks from a BIBD. Here, q denotes the number of treatments in common in the two blocks. Obviously, $0 \leq q \leq k$.

Case i). $q = 0$

Eigen values	With Multiplicity
$\lambda v/k$	v-2k + 1
$\lambda v/k -1$	2(k —1)

Case ii.) $0 < q < k$

Eigen values	With Multiplicity
$\lambda v/k$	$v - 2k + q$
$\lambda v/k - q/k$	1
$\lambda v/k -1$	$2(k - q - 1)$
$\lambda v/k -2 + q/k$	1
$\lambda v/k -2$	$q - 1$

Case iii) $q = k$

Eigen values	With Multiplicity
$\lambda v/k$	v-k
$\lambda v/k -2$	$k-1$

C-matrix: C matrix of an incomplete block design is defined as

$C= \text{diag}\ (r1,r2,\ldots,rv)-N\text{diag}(k^{-1}{}_1,k^{-1}{}_2,\ldots,k^{-1}b)N'$.

Connected design: A design is said to be connected if every block and every treatment of the design are connected to every other block and treatment. Two treatments, two blocks or a block and a treatment are said to be connected if it is possible to pass from one to the other by means of a chain consisting alternately of blocks and treatments such that any two consecutive of the chain are such that the treatment occurs in the block. A block design is said to be connected if R(c) = v-1, which is necessary and sufficient condition.

Robust design: A block design is said to be robust against unavailability of any n, a positive number $\leq r - 1$ observations, if the block design obtained by omitting any n observations remains connected with respect to treatment contrasts.

Robustness of Balanced Incomplete Block Designs against Unavailability of Two Blocks

Consider a Balanced Incomplete Block Design d having parameters v,b, r, k, λ, Suppose two blocks of a Balanced Incomplete Block Design with two blocks are lost. Under this situation, the following three cases will occur:

Case i: Unavailability of two blocks where the number of common treatments between two blocks are zero.

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

For all three cases when two blocks are lost from Balanced Incomplete Block Design, efficiency factor is depending upon the common number of treatments between two lost blocks. The efficiency for all three cases when common number of treatments between two lost blocks are 0,1,2,3,...,(k − 1),k respectively are studied. Here, the robustness criterion of Balanced Incomplete Block Design was further discussed for the different value of common number of treatments between two blocks.

Case (i): Unavailability of two blocks where the number of common treatments between two blocks are zero

Consider a Balanced Incomplete Block Design D with parameters v, b, r, k, λ. It follows that C matrix of design D is always given by

$C = \theta\{Iv - 1/vEvv\}$, where $\theta = \lambda v/k$ is the eigenvalues of C matrix of design D with multiplicity $(v - 1)$.

Let two blocks be lost. Call this design as a residual design assuming a residual design D∗ is a connected design. Let the blocks be bi and bj, let their zero treatment be common between two lost blocks i.e. $\eta(bi \cap bj) = 0$. Each treatment that is present in the two lost blocks will be replicated $(r-1)$ times. All remaining treatment will be replicated r times in design.

Let C∗ be the information matrix of design D∗. For this design D∗, the diagonal element of C∗ matrix are as follows,

1. $C_{jj} = (r-1)(k-1) / k$, where j denotes those treatments which are present in both the lost blocks but are distinct.

2. $C_{ll} = r(k-1) / k$, where l denotes the remaining treatments.

Similarly, in the residual design, pair of treatments occurs together in following ways, which we say λ_1, λ_2. Pattern of λ_i, (i = 1,2) are as follows,

1. $\lambda_1 = (\lambda - 1)$, for those treatments, which are present in two lost blocks.

2. $\lambda_2 = \lambda$, for remaining pair of treatments.

The C∗ matrix of design D can be written as,

$$kC* = \begin{bmatrix} (\lambda v-k)I_k-(\lambda-1)J_{kk} & -\lambda J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{kk} & (\lambda v-k)I_k-(\lambda-1)J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{k(v-2k)} & -\lambda J_{k(v-2k)} & \lambda v(I_{(v-2k)}-v^{-1}J_{(v-2k)(v-2k)}) \end{bmatrix}$$

The non zero eigenvalues of C∗ matrix with their corresponding multiplicities are,

1. $(\lambda v-k) / k$, with multiplicity $2(k - 1)$.

2. $\lambda v/ k$, with multiplicity $(v - 2k + 1)$.

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Consider a Balanced Incomplete Block Design D with parameters v , b, k, r, λ. The C matrix of the design is given by $C = \theta\ (I_v - 1 /vE_{vv})$, Where $\theta = (\lambda v/k)$ is the eigenvalues of C matrix of design D with multiplicity $(v-1)$. Let two blocks be lost. Call this design as a residual design and assume that the residual design D∗ is a connected design. Let the blocks be bi and bj, let one treatment be common between two lost blocks i.e.$\eta(b_i \cap b_j) = 1$. Here, this treatment is repeated $(r - 2)$ times. Similarly, those treatments that are present in the two lost blocks but are not common will be replicated $(r - 1)$ times. The remaining treatments will be replicated r times in design. Let C∗ be the information matrix of design D∗. For this design D∗, the diagonal element of C∗ matrix are as follows,

1. C_{ii} = (r−2)(k−1) /k , where i denotes those treatments which are present in both the lost blocks but are distinct.

2. C_{jj} = (r−1)(k−1) /k , where j denotes those treatments which are present in both the lost blocks but are distinct.

3. C_{ll} = r(k−1) /k , where l denotes the remaining treatments.

Similarly, in the residual design, pair of treatments occurs together in following three ways, which we say $\lambda_1,\lambda_2,\lambda_3$. Pattern of λi(i = 1,2,3) are as follows,

1. $\lambda_1 = (\lambda - 1)$, for those treatments that are present in two lost blocks.

2. $\lambda_2 = (\lambda - 1)$, for those treatments that are present in two lost blocks.

3. $\lambda_3 = \lambda$, for remaining treatment.

The C∗ matrix of design D can be written as, $kC^* =$

$$\begin{pmatrix} (\lambda v-2k+1)I_1-(\lambda-1)J_{11} & -(\lambda-1)J_{1(k-1)} & -(\lambda-1)J_{1(k-1)} & -\lambda J_{1(v-2k+1)} \\ -(\lambda-1)J_{1(k-1)} & (\lambda v-k)I_{k-1}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -(\lambda-1)J_{1(k-1)} & -\lambda J_{(k-1)(k-1)} & (\lambda v-k)I_{k-1}-(\lambda-1)J_{(k-1)(k-1)} & \lambda J_{(k-1)(v-2k+1)} \\ -\lambda J_{1(v-2k+1)} & -\lambda J_{(k-1)(v-2k+1)} & -\lambda J_{(k-1)(v-2k+1)} & \lambda v(I_{v-2k+1}-v^{-1}J_{(v-2k+1)(v-2k+1)}) \end{pmatrix}$$

The non-zero eigenvalues of C∗ matrix with their corresponding multiplicities are,

1. (λv−2k+1)/ k, with multiplicity 1.

2. (λv−k)/ k, with multiplicity 2(k − 2).

3. λv/ k, with multiplicity (v − 2k + 1).

4. (λv−1)/ k, with multiplicity 1.

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

Consider a Balanced Incomplete Block Design D with parameters v, b, k, r, λ. The C matrix of the design is given by C = θ[Iv − 1 vEvv],Where, θ = (λv/k) is the eigenvalues of C matrix of design D with multiplicity (v − 1). When two blocks are lost, call this design as a residual design and assume that the residual design $D*$ is a connected design. Let the blocks be bi and bj, let number of common treatment between two blocks be two, i.e.$\eta(bi \cap bj) = 2$. Here, these two blocks of the same treatment are repeated $(r - 2)$ times. Similarly, those treatments which are present in the two lost blocks but are not common will be replicated $(r - 1)$ times. The remaining treatments will be replicated r times in design.

Let $C*$ be the information matrix of design $D*$. For this design $D*$, the diagonal element of $C*$ matrix are as follows,

1. $C_{ii} = (r-2)(k-1) / k$, where i denotes those treatments which are present in both the lost blocks but are distinct.

2. $C_{jj} = (r-1)(k-1) /k$, where j denotes those treatments which are present in both the lost blocks but are distinct.

3. $C_{ll}= r(k-1) / k$, where l denotes the remaining treatments.

Similarly, in the residual design, pair of treatments occurs together in following three ways, which we say $\lambda_1,\lambda_2,\lambda_3$. Pattern of $\lambda i(i = 1,2,3)$ are as follows,

1. $\lambda_1 = (\lambda - 2)$, for those treatments that are present in two lost blocks.

2. $\lambda_2 = (\lambda - 1)$, for those treatments that are present in two lost blocks.

3. $\lambda_3 = \lambda$, for remaining treatment.

The $C*$ matrix of design D can be written as,

$$kC*= \begin{pmatrix} (\lambda v-k+1)I_2-(\lambda)J_{22} & -(\lambda-1)J_{2(k-1)} & -\lambda J_{2(v-k-1)} \\ -(\lambda-1)J_{2(k-1)} & (\lambda v-2k)I_{(k-1)}-(\lambda-2)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-k-1)} \\ -\lambda J_{2(v-k-1)} & -\lambda J_{(k-1)(v-k-1)} & \lambda v(I_{(v-k-1)}-v^{-1}J_{(v-k-1)(v-k-1)}) \end{pmatrix}$$

The non-zero eigenvalues of their corresponding multiplicities are,

1. $(\lambda v-k+1) /k$, with multiplicity 1.

2. $(\lambda v-2k) /k$, with multiplicity $(k - 2)$.

3. $(\lambda v-k-1)/k$, with multiplicity 1.

4. $\lambda v/k$, with multiplicity $(v - k - 1)$.

Chapter-IV

RESULTS

RESULTS

This chapter is concerned with the testing of Robustness of Balanced Incomplete Block Designs and balanced ternary designs against Unavailability of two blocks and the constructions of balanced ternary designs by using balanced incomplete block designs in the form of theorems which are given below:

4.1 Robustness of Balanced Incomplete Block Designs against Unavailability of Two Blocks :

Theorem 4.1. Balanced Incomplete Block Designs and with parameters v,b,k,r,λ, are fairly robust against the unavailability of two blocks, where the number of common treatment between two lost blocks are zero, provided the over all efficiency of the residual design is given by,

$$e(s) = \frac{(\lambda v - k)(v-1)}{(\lambda v - k)(v-2k+1) + 2\lambda v(k-1)}$$

Proof : Without loss of generality, let two blocks be lost from design D where the number of common treatment between two blocks is zero i.e., matrix of the residual design is given by,

$$kC* = \begin{bmatrix} (\lambda v - k)I_k & -(\lambda-1)J_{kk} - \lambda J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{kk}(\lambda v - k)I_k & -(\lambda-1)J_{kk} & -\lambda J_{k(v-2k)} \\ -\lambda J_{k(v-2k)} & -\lambda J_{k(v-2k)} & \lambda v(I_{(v-2k)} - v^{-1}J_{(v-2k)(v-2k)}) \end{bmatrix}$$

The non zero eigenvalues of C∗ matrix with their corresponding multiplicities are,

1. $(\lambda v - k)/k$, with multiplicity $2(k-1)$.

2. $\lambda v/k$, with multiplicity $(v-2k+1)$.

Further, over all A-efficiency is calculated as,

$$e(s) = \frac{\Phi 2(S)}{\Phi 1(S)}$$

Where, φ2(s) = sum of reciprocals of non-zero eigenvalues of C matrix of design D and φ1(s) = sum of reciprocals of non-zero eigenvalues of C∗ matrix of design D∗. That is,

$$\Phi_2(s)=\frac{k(\nu-1)}{\lambda\nu}$$

$$\text{and } \Phi_1(s)=\frac{k(\nu-2k+1)}{\lambda\nu}+\frac{2(k-1)k}{(\lambda\nu-k)}$$

Finally, A- efficiency is given by,

$$e(s)=\frac{(\lambda\nu-k)(\nu-1)}{(\lambda\nu-k)(\nu-2k+1)+2\lambda\nu(k-1)}$$

Example4.1.1 : Let D represent the Balanced Incomplete Block Design with parameters v = 10, b = 30, r = 12, k = 4, λ = 4. Design D is given by,

Table: Treatments of BIB Design

Blocks	Treatments			
B_1	**7**	**8**	**9**	**10**
B_2	**1**	**2**	**4**	**5**
B_3	1	3	4	6
B_4	2	3	5	6
B_5	2	3	4	7
B_6	3	6	9	10
B_7	1	5	9	10
B_8	2	4	9	10
B_9	2	5	8	10
B_{10}	3	4	8	10
B_{11}	1	6	8	10
B_{12}	1	4	7	10
B_{13}	3	5	7	10
B_{14}	2	6	7	10
B_{15}	4	5	6	10
B_{16}	1	2	3	10
B_{17}	1	4	8	9
B_{18}	2	3	8	9
B_{19}	5	6	8	9
B_{20}	1	3	7	9
B_{21}	2	5	7	9
B_{22}	4	6	7	9
B_{23}	3	4	5	9
B_{24}	1	2	6	9
B_{25}	1	2	7	8
B_{26}	3	6	7	8
B_{27}	4	5	7	8
B_{28}	2	4	6	8
B_{29}	1	3	5	8
B_{30}	1	5	6	7

Two blocks containing treatments (7 8 9 10) and (1 2 4 5) are lost and number of treatment common two blocks is zero.

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{36}{4}$, with multiplicities 6.

2. $\frac{40}{4}$, with multiplicities 3.

The over all A-efficiency of the design is, e(s) = 0.9375

Theorem 4.2: Balanced Incomplete Block Designs and with parameters v, b, k, r, λ, are fairly robust against the unavailability of two blocks, where the number of common treatment between two blocks is one, provided the over all efficiency of the residual design is given by

$$e(s) = \frac{(v-1)(\lambda v-2k)(\lambda v-1)(\lambda v-2k+1)}{(\lambda v-2k)(\lambda v-1)((v-2k+1)(\lambda v-2k+1)+\lambda v)+(\lambda v-2k+1)\lambda v(2(k-2)(\lambda v-1)+(\lambda v-2k))}$$

Proof: Without loss of generality, let two blocks be lost from design D where the number of common treatment between two blocks is one

i.e., $\eta(b_i \cap b_j) = 1$, C_* matrix of the residual design is given by, $kC_* =$

$$\begin{pmatrix} (\lambda v-2k+1)I_1-(\lambda-1)J_{11} & -(\lambda-1)J_{1(k-1)} & -(\lambda-1)J_{1(k-1)} & -\lambda J_{1(v-2k+1)} \\ -(\lambda-1)J_{1(k-1)} & (\lambda v-k)I_{k-1}-(\lambda-1)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-2k+1)} \\ -(\lambda-1)J_{1(k-1)} & -\lambda J_{(k-1)(k-1)} & (\lambda v-k)I_{k-1}-(\lambda-1)J_{(k-1)(k-1)} & \lambda J_{(k-1)(v-2k+1)} \\ -\lambda J_{1(v-2k+1)}) & -\lambda J_{(k-1)(v-2k+1)} & -\lambda J_{(k-1)(v-2k+1)} & \lambda v(I_{v-2k+1}-v^{-1}J_{(v-2k+1)(v-2k+1)}) \end{pmatrix}$$

The non-zero eigenvalues of their corresponding multiplicities are,

1. $(\lambda v-2k+1)/k$, with multiplicity 1.

2. $(\lambda v-k)/k$, with multiplicity $2(k-2)$.

3. $\lambda v/k$, with multiplicity $(v-2k+1)$.

4. $(\lambda v-1)/k$, with multiplicity 1.

Further, over all A-efficiency is calculated as,

$$e(s) = \frac{\Phi 2(S)}{\Phi 1(S)}$$

Where, φ2(s) = sum of reciprocals of non-zero eigenvalues of C matrix of design D and φ1(s) = sum of reciprocals of non-zero eigenvalues of C∗ matrix of design D∗. That is,

$$\Phi_2(s) = \frac{k(\nu - 1)}{\lambda\nu}$$

$$\varphi 1(s) = \frac{k(v - 2k + 1)}{\lambda v} + \frac{k}{(\lambda v - 2k + 1)} + \frac{2(k - 2)k}{(\lambda v - k)} + \frac{k}{(\lambda v - 1)}$$

Finally, A- efficiency is given by, e(s) =

$$\frac{(v - 1)(\lambda v - 2k)(\lambda v - 1)(\lambda v - 2k + 1)}{(\lambda v - 2k)(\lambda v - 1)((v - 2k + 1)(\lambda v - 2k + 1) + \lambda v) + (\lambda v - 2k + 1)\lambda v(2(k - 2)(\lambda v - 1) + (\lambda v - 2k))}$$

Example 4.2.1: Let D represent the Balanced Incomplete Block Design with parameters v =10, b = 30, r = 12, k = 4, λ = 4, Design D is given by,

Table: Treatments of BIB Design

Blocks	Treatments			
B_1	7	8	9	10
B_2	1	2	4	5
B_3	1	3	4	6
B_4	2	3	5	6
B_5	2	3	4	7
B_6	3	6	9	10
B_7	1	5	9	10
B_8	2	4	9	10
B_9	2	5	8	10
B_{10}	3	4	8	10
B_{11}	1	6	8	10
B_{12}	1	4	7	10
B_{13}	3	5	7	10
B_{14}	2	6	7	10
B_{15}	4	5	6	10
B_{16}	1	2	3	10
B_{17}	1	4	8	9
B_{18}	2	3	8	9
$\mathbf{B_{19}}$	**5**	**6**	**8**	**9**
$\mathbf{B_{20}}$	**1**	**3**	**7**	**9**
B_{21}	2	5	7	9
B_{22}	4	6	7	9
B_{23}	3	4	5	9
B_{24}	1	2	6	9
B_{25}	1	2	7	8
B_{26}	3	6	7	8
B_{27}	4	5	7	8
B_{28}	2	4	6	8
B_{29}	1	3	5	8
B_{30}	1	5	6	7

Two blocks containing treatments (5 6 8 9) and (1 3 7 9) are lost and number of treatment common two blocks is one.

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{33}{4}$, with multiplicities 1.

2. $\frac{36}{4}$, with multiplicities 4.

3. $\frac{40}{4}$, with multiplicities 3.

4. $\frac{39}{4}$, with multiplicities 1.

The over all A- efficiency of the design is, e(s) = 0.9345.

Theorem 4.3: Balanced Incomplete Block Designs D and with parameters v,b, k, r, λ, are fairly robust against the unavailability of two blocks, where the number of common treatment between two blocks are two, i.e. $\eta(b_i \cap b_j) = 2$, provided the over all efficiency of the residual design is given by,

$$e(s) = \frac{(v-1)(\lambda v-2k)(\lambda v-k+1)(\lambda v-k-1)}{(\lambda v-k+1)(\lambda v-k-1)((v-k-1)(\lambda v-2k)+\lambda v(k-2))+2\lambda v(\lambda v-k)(\lambda v-2k)}$$

Proof: Without loss of generality, let two blocks be lost from design D where the number of common treatment between two blocks is two i.e. $\eta(b_i \cap b_j) = 2$, C∗ matrix of the residual design is given by,

$$kC_* = \begin{pmatrix} (\lambda v-k+1)I_2-(\lambda)J_{22} & -(\lambda-1)J_{2(k-1)} & -\lambda J_{2(v-k-1)} \\ -(\lambda-1)J_{2(k-1)} & (\lambda v-2k)I_{(k-1)}-(\lambda-2)J_{(k-1)(k-1)} & -\lambda J_{(k-1)(v-k-1)} \\ -\lambda J_{2(v-k-1)} & -\lambda J_{(k-1)(v-k-1)} & \lambda v(I_{(v-k-1)}-v^{-1}J_{(v-k-1)(v-k-1)}) \end{pmatrix}$$

The non-zero eigenvalues of their corresponding multiplicities are,

1. $(\lambda v-k+1)/k$, with multiplicity 1.

2. $(\lambda v-2k)/k$, with multiplicity $(k-2)$.

3. $(\lambda v-k-1)/k$, with multiplicity 1.

4. $\lambda v/k$, with multiplicity $(v-k-1)$.

Further, over all A-efficiency is calculated as,

$$e(s) = \frac{\Phi 2(S)}{\Phi 1(S)}$$

Where, φ2(s) = sum of reciprocals of non-zero eigenvalues of C matrix of design D and φ1(s) = sum of reciprocals of non-zero eigenvalues of C∗ matrix of design D∗. That is,

$$\Phi_2(s)=\frac{k(\nu-1)}{\lambda\nu}$$

$$\Phi_1(s)=\frac{k(v-k-1)}{\lambda v}+\frac{k}{(\lambda v-k+1)}+\frac{(k-2)k}{(\lambda v-2k)}+\frac{k}{(\lambda v-k-1)}$$

Finally, A- efficiency is given by,e(s) =

$$\frac{(v-1)(\lambda v-2k)(\lambda v-k+1)(\lambda v-k-1)}{(\lambda v-k+1)(\lambda v-k-1)((v-k-1)(\lambda v-2k)+\lambda v(k-2))+2\lambda v(\lambda v-k)(\lambda v-2k)}$$

Example 4.3.1: Let D represent the Balanced Incomplete Block Design with parameters v =10, b = 30, r = 12, k = 4, λ = 4, Design D is given by,

Table: Treatments of BIB Design

Blocks	Treatments			
B_1	7	8	9	10
B_2	1	2	4	5
B_3	1	3	4	6
B_4	**2**	**3**	**5**	**6**
B_5	**2**	**3**	**4**	**7**
B_6	3	6	9	10
B_7	1	5	9	10
B_8	2	4	9	10
B_9	2	5	8	10
B_{10}	3	4	8	10
B_{11}	1	6	8	10
B_{12}	1	4	7	10
B_{13}	3	5	7	10
B_{14}	2	6	7	10
B_{15}	4	5	6	10
B_{16}	1	2	3	10
B_{17}	1	4	8	9
B_{18}	2	3	8	9
B_{19}	5	6	8	9
B_{20}	1	3	7	9
B_{21}	2	5	7	9
B_{22}	4	6	7	9
B_{23}	3	4	5	9
B_{24}	1	2	6	9
B_{25}	1	2	7	8
B_{26}	3	6	7	8
B_{27}	4	5	7	8
B_{28}	2	4	6	8
B_{29}	1	3	5	8
B_{30}	1	5	6	7

Two blocks containing treatments (2 3 5 6) and (2 3 4 6) are lost and number of treatment common two blocks is two

The non-zero eigenvalues with their corresponding multiplicities are,

1. $\frac{37}{4}$, with multiplicities 1.
2. $\frac{32}{4}$, with multiplicities 2
3. $\frac{35}{4}$, with multiplicities 1.
4. $\frac{40}{6}$, with multiplicities 5.

The over all A-efficiency of the design is, e(s) = 0.925.

4.2 Construction of Balanced Ternary Designs by using balanced incomplete block design:

Theorem 4.4: The existence of BIB designs with parameters v,b,r,k and λ employ the existence of BTD with the following parameters –

V =v, B =$\binom{v}{2}\lambda$,

$\rho_1 = \frac{(k-2)(v-1)\lambda}{2}$,

$\rho_2=(v-1)\lambda$,

R =ρ_1 +2ρ_2, K =k+2,

$\Lambda = 4\lambda+4(k-2)\lambda+\binom{k-2}{2}\lambda$.

Proof:

With the existing BIB design the pair of treatments λ, are added in blocks of BIB design, hence it is obvious to have the same number of treatment as in the original BIB design.

In order to get a number of blocks in BTD, the pairs of the treatments $\binom{v}{2}$ are develop & if it is multiplied by λ then certainly we have the total number of blocks in BTD.

Let us consider the occurrence of the treatment of the multiplicity 1 it is obvious to have $\frac{(k-2)}{2}$ times, the multiplicity of the two treatment, on the basis of the examples it is true for all.

Now the multiplicity of the treatment two times, since we are adding the pair of treatments with the particular treatment λ then it is obvious to have $\rho_2 = (v-1)\lambda$, then replication number R for treatment x

$R = \rho_1 + 2\rho_2$, and K = (k+2) since we are adding the pairs of the treatment of each block, increasing the block size as (k+2)

Λ, this parameter will consist of (2,2), (1,1), (1,2), (2,1) order pair of treatment,

For order pair (2,2) we consider 2's of total λ therefore it is equal to 4λ,

Order pair (2,1) & (1,2) we consider 1 λ's out of total λ's therefore it is equal to 4(k-2)λ, and for order pair (1,1) we consider the λ's in the form of the combination which is certainly equal to $\binom{k-2}{2}\lambda$

Thus $\Lambda = 4\lambda+4(k-2)\lambda+\binom{k-2}{2}\lambda$.

Hence, the theorem proved.

Corollary 4.4: The existence of a BIB design with parameters V = b= 4t-1,r =k=2t-1,λ = t-1,implies the existence of a BTD with parameters, V = 4t-1, B=(4t-1)(2t-1)(t-1), ρ_1 =(2t-1)(2t-3)(t-1),ρ_2 =(4t-2)(t-1), R =$(4t^2-1)(t-1)$, K =2t+1 ,$\Lambda = 8(t-1)^2 + (t-1)(t-2)(2t-3)$.

Example 4.4.1: Let us consider BIB design with parameters V=b=7, r=k=3,λ=1. On applying Theorem 4.1it is developed as BTD.

Table 4.4.1: The number of blocks in BTD with parameters V = 7, B = 21, ρ_1 = 3, ρ_2 = 6, R = 15, K = 5, Λ = 8.

B_1	1	1	2	2	6
B_2	1	1	3	3	4
B_3	1	1	3	4	4
B_4	1	1	5	5	7
B_5	1	1	2	6	6
B_6	1	1	5	7	7
B_7	2	2	3	3	7
B_8	2	2	4	4	5
B_9	2	2	4	5	5
B_{10}	1	2	2	6	6
B_{11}	2	2	3	7	7
B_{12-}	1	3	3	4	4
B_{13}	3	3	5	5	6
B_{14}	3	3	5	6	6
B_{15}	2	3	3	7	7
B_{16}	2	4	4	5	5
B_{17}	4	4	6	6	7
B_{18}	4	4	6	7	7
B_{19}	3	5	5	6	6
B_{20}	1	5	5	7	7
B_{21}	4	6	6	7	7

Efficiency of this design is: $\frac{\Lambda V}{RK}$ =**0.746**

Robustness of constructed Balanced ternary design:

Balanced Ternary Design with parameters 7 21 3 6 15 5 8 are fairly robust against the unavailability of two blocks,if two blocks of a Balanced Ternary Design are lost. Under this situation, the following three cases will occur:

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Two blocks containing treatments (1 1 2 2 6) and (3 3 5 5 6) are lost and number of treatment common two blocks is one.

The non-zero eigenvalues with their corresponding multiplicities are,

1. 47/5, with multiplicity 1.
2. 51/5, with multiplicity 6.
3. 56/5, with multiplicity -2.
4. 55/ 5, with multiplicity 1.

Further, over all A-efficiency is calculated as,

The over all A-efficiency of the design is, e(s) = 0.8826

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

Two blocks containing treatments (1 1 3 3 4) and (1 1 5 5 7) are lost and number of treatment common two blocks is two.

The non-zero eigenvalues with their corresponding multiplicities are,

1. 52/5, with multiplicity 1.
2. 46 /5, with multiplicity 3.
3. 50/5, with multiplicity 1.
4. 56/ 5, with multiplicity 1.

Further, over all A-efficiency is calculated as,

The over all A-efficiency of the design is, e(s) = 0.8763

Corollary 4.5: The existence of a BIB design with parameters V = 2t-1, b = 4t-2, r = 2t-2, k = t-1, λ = t-2, implies the existence of a BTD with parameters, V=2t-1, B = (2t-1)(t-1)(t-2), ρ_1 = (t-1)(t-2)(t-3), ρ_2 = (2t-2)(t-2), R = $(t^2$-1)(t-2), K = t+1, $\Lambda = \frac{8(t-2)+(t-2)(t-3)(t+4)}{2}$.

Example 4.5.1: Let us consider BIB design with parameters V=b=14, r=6,k=3,λ=2. On applying Theorem 4,it is developed as BTD.

Table 4.5.1: The number of blocks in BTD with parameters V = 7, B = 42, ρ_1 = 6, ρ_2 = 12, R = 30, K = 5, Λ = 16.

B_1	1	1	2	2	3
B_2	1	1	2	2	3
B_3	1	1	2	3	3
B_4	1	1	2	3	3
B_5	1	1	4	4	5
B_6	1	1	4	4	5
B_7	1	1	4	5	5
B_8	1	1	4	5	5
B_9	1	1	6	6	7
B_{10}	1	1	6	6	7
B_{11}	1	1	6	7	7
B_{12}	1	1	6	7	7
B_{13}	1	2	2	3	3
B_{14}	1	2	2	3	3
B_{15}	2	2	4	4	6
B_{16}	2	2	4	4	6
B_{17}	2	2	5	5	7
B_{18}	2	2	5	5	7
B_{19}	2	2	4	6	6
B_{20}	2	2	4	6	6
B_{21}	2	2	5	7	7
B_{22}	2	2	5	7	7
B_{23}	3	3	4	4	7
B_{24}	3	3	4	4	7
B_{25}	3	3	5	5	6

B_{26}	3	3	5	5	6
B_{27}	3	3	5	6	6
B_{28}	3	3	5	6	6
B_{29}	3	3	4	7	7
B_{30}	3	3	4	7	7
B_{31}	1	4	4	5	5
B_{32}	1	4	4	5	5
B_{33}	2	4	4	6	6
B_{34}	2	4	4	6	6
B_{35}	3	4	4	7	7
B_{36}	3	4	4	7	7
B_{37}	3	5	5	6	6
B_{38}	3	5	5	6	6
B_{39}	2	5	5	7	7
B_{40}	2	5	5	7	7
B_{41}	1	6	6	7	7
B_{42}	1	6	6	7	7

Efficiency of this design is: $\frac{\Lambda V}{RK}$ =0.7466

Robustness of constructed Balanced ternary design:

Balanced Ternary Design with parameters 7 42 6 12 30 5 16are fairly robust against the unavailability of two blocks,if two blocks of a Balanced Ternary Design are lost. Under this situation, the following three cases will occur:

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Two blocks containing treatments (1 1 2 3 3) and (2 4 4 6 6) are lost and number of treatment commons in two blocks is one.

The non-zero eigenvalues with their corresponding multiplicities are,

1. 103/5,with multiplicity 1.

2. 107 /5,with multiplicity 6.

3. 112/5,with multiplicity -2.

4. 111/ 5,with multiplicity 1.

Further, over all A-efficiency is calculated as,

The over all A-efficiency of the design is, e(s) = 0.9409

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

Two blocks containing treatments (1 1 2 2 3) and (1 1 4 4 5) are lost and number of treatment common two blocks is two.

The non-zero eigenvalues with their corresponding multiplicities are,

1. 108/5, with multiplicity 1.

2. 102 /5, with multiplicity 3.

3. 106/5, with multiplicity 1.

4. 112/ 5, with multiplicity 1.

Further, over all A-efficiency is calculated as,

The over all A- efficiency of the design is, e(s) = 0.9399

Chapter-V

DISCUSSION

DISCUSSION

In the present investigation attempts have been made to findout robustness of BIBD, to construct the balanced ternary designs as well as their robustness which are utilized for the blocking of the intercropping experiments with various intercrops as well as for the fractional factorial plans.

5.1 Robustness of Balanced Incomplete Block Designs against Unavailability of Two Blocks :

Theorem 4.1, revealed about the testing of Robustness of Balanced Incomplete Block Designs and with parameters v, b, k, r, λ, are fairly robust against Unavailability of Two Blocks, under the three situations:

Case i: Unavailability of two blocks where the number of common treatments between two blocks are zero.

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

Case i: Unavailability of two blocks where the number of common treatments between two blocks are zero.

Example 4.1.1, represents the Balanced Incomplete Block Design with parameters $v = 10$, $b = 30$, $r = 12$, $k = 4$, $\lambda = 4$, shows robust design against the removal of the two blocks containing treatments (7 8 9 10) and (1 2 4 5) and number of treatment common in two blocks is zero.

And the over all efficiency of the design becomes, $e(s) = 0.9375$.

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Theorem 4.2, Balanced Incomplete Block Designs and with parameters v, b, k, r, λ, are fairly robust against the unavailability of two blocks, where the number of common treatment between two blocks is one.

Example 4.2.1, Balanced Incomplete Block Design with parameters v =10, b = 30, r = 12, k = 4, λ = 4 shows robust design against the removal of two blocks containing treatments (5 6 8 9) and (1 3 7 9) and number of treatment common in two blocks is one.

And the over all efficiency of the design becomes, e(s) = 0.9345.

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

Theorem 4.3, Balanced Incomplete Block Designs D and with parameters v,b, k, r, λ, are fairly robust against the unavailability of two blocks, where the number of common treatment between two blocks are two.

Example 4.3.1, Balanced Incomplete Block Design with parameters v =10, b = 30, r = 12, k = 4, λ = 4, shows robust design against the removal of two blocks containing treatments (2 3 5 6) and (2 3 4 6) and number of treatment common in two blocks is two.

And the over all efficiency of the design becomes, e(s) = 0.925.

5.2 Construction of Balanced Ternary Designs by using balanced incomplete block design:

Theorem 4.4, revealed about the construction of BTD with parameters V =v, B $=\binom{v}{2}\lambda$, $\rho_1 =\frac{(k-2)(v-1)\lambda}{2}$, $\rho_2=(v-1)\lambda$, R $=\rho_1 +2\rho_2$, K =k+2,$\Lambda = 4\lambda+4(k-2)\lambda+\binom{k-2}{2}\lambda$.

The construction of BTD with the existing BIB design the pair of treatments λ, are added in blocks of BIB design, In order to get a number of blocks in BTD, the pairs of the treatments $\binom{v}{2}$ are developed and if it is multiplied by λ then certainly we have the total number of blocks in BTD.

The occurrence of the treatment of the multiplicity 1 it is obvious to have $\frac{(k-2)}{2}$ times, the multiplicity of the two treatment, on the basis of the examples it is true for all. Since we are adding the pair of treatments with the particular treatment λ then it is obvious to have $\rho_2 =(v-1)\lambda$, then replication number R for treatment x, $R = \rho_1 + 2\rho_{2,}$ and K = (k+2) since we are adding the pairs of the treatment of each block, increasing the block size as (k+2)

Λ, this parameter will consist of (2,2), (1,1), (1,2), (2,1) order pair of treatment,

For ordered pair (2,2) we consider 2's of total λ therefore it is equal to 4λ,

Ordered pair (2,1) & (1,2) we consider 1 λ's out of total λ's therefore it is equal to 4(k-2)λ, and for ordered pair (1,1) we consider the λ's in the form of the combination which is certainly equal to $\binom{k-2}{2}\lambda$

Thus $\Lambda = 4\lambda + 4(k-2)\lambda + \binom{k-2}{2}\lambda$.

Example 4.4.1, Shows the construction of BTD design with the parameters V= 7, B = 21, $\rho_1 = 3$, $\rho_2 = 6$, R = 15, K = 5, Λ = 8, by the application of Theorem 4.4 on a BIB design with parameters V=b=7, r=k=3, λ =1, and we get constructed BTD design's efficiency is 0.746. Constructed BTD design still shows robustness against the removal of two blocks under the two cases, case (ii) and case (iii).

Example 4.5.1, Shows the construction of BTD design with the parameters V= 7, B = 42, $\rho_1 = 6$, $\rho_2 = 12$, R = 30, K = 5, Λ = 16, by the application of Theorem 4.4 on a BIB design with parameters V=b=14, r=6,k=3,λ=2, and we get constructed BTD design's efficiency is 0.7466. Constructed BTD design still shows robustness against the removal of two blocks under the two cases, case (ii) and case (iii).

Chapter-VI

SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

6.1 Summary

It happens rather frequently in physical and occasionally biological science experimentation, one of the important problems in experimental research is to determine how the treatments should be allocated to the experimental units to achieve the best precision of the comparisons between the treatments? This requires some criteria for design selection. That is, one is concerned with most efficient designs estimating differences in the effects between the treatments. In such situation the experiments with animals, testing of the variety in breeding trials, bio-assays and industrial experiments, led to the generation of balanced incomplete block (BIB) designs.

The robustness of balanced incomplete block design, construction of balanced ternary designs using balanced incomplete block designs as well as the robustness of constructed balanced ternary designs, were given in this thesis . The thesis consists of 6 chapters including a list of alphabetical order bibliography at the end. The first chapter was introductory which advocated about importance of the robustness of BIBDs and BTDs designs and the construction of balanced ternary designs.

Chapter II reviewed the literature during last few decades by the scientists and researchers in relation to the robustness of the Balanced incomplete block designs and balanced ternary designs as well as the construction of various types of incomplete block designs. In this context, the several BIB designs parameters have been extensively used to construct balanced ternary designs.

Chapter III described with the material methods. In this chapter the various types of definitions and notations were given specially BIBD, complementary BIBD, incidence matrix, balanced ternary designs, Efficiency formula in the case of balanced ternary designs had also been added.

C matrix in case of BIBD and BTD had also given, and the formula to get robustness against the removal of two blocks from the designs had also given, eigenvalues and multiplicities of treatments of residual designs had given.

Chapter IV was concerned with the results of robustness of BIBD and BTD designs and construction of balanced ternary designs. In this chapter it has proved that BIBDs and BTDs designs are still robust against the removal of some observations. And Balanced ternary designs can be construct using balanced incomplete block designs and the Theorem which has given in this chapter is true for all, on the basis of this theorem some examples have included.

Chapter V was related to the discussion on the robustness of BIBDs and BTDs and construction of balanced ternary designs.

Chapter VI was concerned to the summary, conclusions and suggestions for further work. All the three divisions were given separately as sub-headings.

6.2 Conclusions:

Robustness of BIBD and Constructed BTD :

This results conclude about the robustness of BIBD and BTD designs when two blocks are removed from the design under three situations –

Case i: Unavailability of two blocks where the number of common treatments between two blocks are zero.

Case ii: Unavailability of two blocks where the number of common treatments between two blocks are one.

Case iii: Unavailability of two blocks where the number of common treatments between two blocks are two.

The designs are found to be still robust under the above situations, it indicated that these types of designs can be further used in several kinds of breeding program where the experiment are based on BIBD's and BTD's. These blocks can also be considered for the intercropping experiment.

Construction of Balanced ternary designs :

When the number of treatments are appeared in the experiment two times then we need the balanced ternary design. In this study the construction of BTD designs has been performed with the help of BIBD designs which can be used in the intercropping experiment also considering main plot treatment along with 2 or 3 subplot treatment.

6.3 Suggestions :

The suggestions for further research work on the basis of present investigation are as follows:

1. The balanced ternary designs for the smaller block size should be developed so that its applications can be utilized for the intercropping experiments.
2. The robustness and construction of balanced ternary designs should be advocated for the smaller number of treatments for the agricultural field experimentation.
3. The analysis procedure of constructed designs viz., balanced ternary designs and their robustness should be tested with the help of other methods.
4. On the basis of similar lines of construction of balanced ternary designs, suitable for any number of treatments and blocks, the construction of n-ary designs may be developed.
5. The analysis of variance for the constructed n-ary designs on the basis of BTD can also be generated specially for the agricultural data.

Chapter-VII

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CURRICULUM VITAE

The author of this manuscript, Mr. Anurag Gupta S/o Shri Prahlad Gupta was born on 6th, May 1994 at Gulabganj Vidisha District of Madhya Pradesh.

He passed his High School Examination of Board of Secondary Education, M.P. with First Division (84.5% marks) in 2009 from Gyanodaya Higher Secondary School, Gulabganj and passed Higher Secondary School Examination from Government Higher Secondary School, Gulabganj in 2011 with First Division (70.2% marks).

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For the partial fulfillment of Master's Degree, he was assigned a research problem entitled "**Construction of a Series of Balanced Ternary Designs Using Balanced Incomplete Block Designs and Their Robustness Against the Loss of Some Observations**" which is being submitted in the form of thesis.