NUMERICAL SOLUTION OF SOME FLOW PROBLEMS OF NON-NEWTONIAN FLUIDS

Thesis

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TO

THE OMNISCIENT

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Authoress

Certificate

This is to certify that thesis entitled **"Numerical Solution of Some Flow Problems of non-Newtonian Fluids"** submitted in partial fulfilment of the requirements for the degree of **Doctor of Philosophy** with major in **Mathematics** of the College of Post-Graduate Studies, G.B.Pant University of Agriculture and Technology, Pantnagar, is a record of *bona fide* research carried out by **Ms. Anjali Pant, Id. No. 29737** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been acknowledged.

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Certificate

We, the undersigned, members of the Advisory Committee of, **Ms. Anjali Pant, Id.No. 29737**, a candidate for the degree of **Doctor of Philosophy** with major in **Mathematics,** agree that the thesis entitled **"Numerical Solution of Some Flow Problems of Non-Newtonian Fluids"** may be submitted in partial fulfilment of the requirements for the degree.

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Member	**Member**	**Member**

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INTRODUCTION

Chapter 1 INTRODUCTION

The origin of fluid mechanics lies in the history of human use of wind, river, ponds and ocean for the practical advantages of transportation, irrigation, potable water supply etc. The field of fluid mechanics is so vast that it has manifold interests for engineers, scientists, industrialists and mathematicians. These interests arise in various branches of engineering: water-works, engines, power-plants, aircraft technology, transportation of sediments, action of waves on beaches and harbours, seepage etc.

The field of non-Newtonian fluid has gained new importance in engineering techniques, their applications in mathematical as well as physical foundations due to growing industrial demands on high polymers, paints, plastics, new alloys and synthetic fibers for textiles etc. These materials have much more complicated structure than other simple materials of daily use and give rise to new physical situations to be explained. Most of such situations arising in process industry will be concerned with deformation or flow of materials when they are subjected to stresses.

For engineering perspective, viscous effects can be very important since their existence accounts for a loss of efficiency of a driver or system. This is the stuff of precise practical design and analysis. The test goes on to consider some aspects of multidimensional inviscid fluid flow that will give us an opportunity to see how a flow field is analyzed in detail and introduce us to the concept employed in computational fluid dynamics. The role of Mathematics is to give mathematical foundations to the physical flows occurring in general, in bounded regions, in pipes and channels, over bodies with different geometrical configurations, in compressors and turbines. A number of them are interested in plasma physics and magneto-hydrodynamics because of their applications in astronautics and in power generations.

The vector algebra and calculus are role-model and act as a shorthand in describing the three dimensional flow of fluid. The vector calculus is utilized in deriving the differential and integral expression for the conservation of mass, momentum and energy. The derivation will help us to understand the physical meaning of the terms utilized in the conservation laws and are included for our information.

One of the most successful, fascinating and useful applications of mathematics has been in the study of motion of fluids. The subject of Fluid Dynamics has made tremendous advances since Euler gave his famous equations of fluid flow for perfect (non-viscous incompressible and compressible) fluids in 1755. The perfect fluids are characterized by the assumptions that for them the stress tensor is a linear function of the rate of strain tensor and the stress vector on a plane surface in contact with a fluid is normal to the surface.

The ideal fluid theory, however, led to the conclusion that when a solid moves through a fluid at rest at infinity it experiences no drag. This was a contradiction with observations (D'Alembert's Paradox). To explain drag, the concept of a viscous Newtonian fluid was introduced. Navier in 1821 and Stokes in 1845 obtained the equations of motion for these fluids independently and formed different consideration.

A great break-through came in 1904 when Prandtl proposed his assumptions that when viscous fluid flows past a surface, the viscous effect is dominant in a thin layer (called boundary layer) near the surface and outside thin layer, the flow may be regarded as that of ideal fluid. The boundary layer theory had tremendous achievements to its credit and transformed Fluid Dynamics into a discipline of great engineering importance.

About forty years ago some phenomena were observed which could not be explained on the basis of viscous fluid theory. When the assumptions of this theory were examined, it was considered feasible to relax the assumptions of linearity between stress and the rate of strain tensors. More general non-linear relations between stress and rate of strain were postulated. It immediately led to the development of the theory of non- Newtonian fluid flows.

The classical theory of rational mechanics of deformable media is due to Newton who suggested the hypothesis, that "The resistance which arises from the lack of slipperiness of the parts of liquid is proportional to the velocity with which the parts of the liquid are separated from one another." For Newton's concept of the internal stresses which arises in the flow the stress tensor τ_{ij} is related to what has been variously named rate of strain, rate of deformation, velocity strain or flow tensor d_{ij} by the rheological equation of a Newtonian fluid, namely, the Newton- Cauchy-Poisson Law.

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij} + \lambda d_n^m \delta_{ij} \qquad \text{--- (1.1)}$$

Where $d_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$,

p is the pressure, μ and $\lambda\left(= -\frac{2\mu}{3}\right)$ being material constants, also termed as coefficients of viscosity and δ_{ij} is kronecker delta tensor. The fluids satisfying the relation (1.1), are called Newtonian fluids e.g. honey, glycerin and certain thick oils. For incompressible fluids the relation (1.1) becomes

$$\tau_{ij} = -p\delta_{ij} + 2\mu\, d_{ij} \qquad \text{--- (1.2)}$$

Though certain phenomenon like strain-friction, form drag separation, secondary flow etc, are successfully explained by the classical theory, but it is inadequate to explain the rheological properties of certain materials like paints, slurries, ceramics, melts, poly-iso-butylene solutions in mineral oils or in tetralin, poly-methylmethacrylate solution etc. Certain phenomenon like anamolous viscosity, the Weissenberg effect, Merrington effect and the spinnability effect observed in these fluids could not be explained by the solution of Navier-Stokes equations and therefore a basic search into the foundations of fluid dynamics had to be undertaken.

In classical fluid dynamics any mathematical approach to a physical situation is based on these equations, viz.

1. Equation of Continuity

$$\frac{\partial \rho}{\partial t} + \left(\rho u^{i}\right)_{,i} = 0 \qquad \text{--- (1.3)}$$

where u^i and ρ are respectively the velocity vector and density of the fluid. For incompressible fluids and for steady flow, this equation reduces to

$$u^{i}_{,i} = 0 \qquad \text{--- (1.4)}$$

2. Equation of Motion:

$$\rho\left(\frac{\partial u_i}{\partial t} + u^m u_{i,m}\right) = F_{,i} + \tau^{m}_{i,m} \qquad \text{--- (1.5)}$$

where F is the impressed force per unit mass of the fluid and τ^{m}_{i} the stress tensor. The momentum equation for no extraneous force is simply

$$\rho\left(\frac{\partial u_i}{\partial t} + u^m u_{i,m}\right) = \tau^{m}_{i,m} \qquad \text{--- (1.6)}$$

3. Rheological Equation of State or Constitutive Equation.

- The constitutive equation for a non- Newtonian second- order fluid is

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij}\ , \qquad \text{--- (1.7)}$$

where

$$d_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),$$

$$e_{ij} = \frac{1}{2}\left(a_{i,j} + a_{j,i}\right) + u^{m}{}_{,i}\, u_{m,j}$$

$$c_{ij} = d_i^m d_{mj}$$

where p is an indeterminate hydrostatic pressure, μ_1, μ_2 and μ_3 are the coefficient of Newtonian viscosity, elastico-viscosity and cross-viscosity respectively.

- The constitutive equation of Walters liquid (Model B′) is

$$\begin{aligned} \sigma^{ik} &= -pg_{ik} + \sigma_{ik}' \\ \sigma_{ik} &= 2\eta_0 e^{ik} - 2k_0 e'^{ik}, \end{aligned} \qquad \text{--- (1.8)}$$

where σ_{ik} is the stress tensor, p is the isotropic pressure, g_{ik} the metric

tensor of fixed coordinate system x^i, v^i the velocity vector, e'^{ik} in the contravariant form is

$$\sigma'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^j e_{,j}^{ik} - v_{,j}^{k} e^{ij} - v_{,j}^{i} e^{ik}$$

It is the convected derivative of the deformation rate tensor e^{ik} defined

by $\quad e_{ik} = \frac{1}{2}\left(v_{i,k} + v_{k,i}\right)$

Here, η_0 is the limiting viscosity at small rate of shear that is given by

$$\eta_0 = \int_0^\infty N(\tau)\,d\tau$$

$$K_0 = \int_0^\infty \tau N(\tau)\, d\tau$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters. This idealized model is a valid approximation of Walters liquid (Model) B' taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau) d\tau, \; n \geq 2$$

have been neglected.

The equation (1.3) expresses a definite physical fact based on the hypothesis of mechanics of continuous media and can be given up only if we are prepared to do away with the great simplifying idealization of the continuous fluid. Similarly the equations of the motion (1.5) and (1.6) are based on Newton's laws of motions which continue to be the basis of all continuum mechanics except the relativistic mechanics. The constitutive equation (1.7) and (1.8) expressing linear relationship between stress and rate-of-strain tensors is a mere postulate and the main effect of such assumption is that the mathematics of problem is greatly simplified.

On the basis of large amount of data which could not be explained on the basis of linear assumption, it is felt that linearity is too drastic an assumption and one should explore mathematically the consequences of a more general functional relationship then that for a Newtonian Fluid. The availability of high speed computers have rendered possible the solution of non-linear equations involved. This development has led to the growth of subject matter of non-linear mechanism.

Non-linearity in the equation (1.1) has been attempted in a number of ways. The first generalization consisted in taking λ and μ is occurring in this equation as functions of three invariant of strain-rate tensor d_{ij}.. Non- linearity in stress-strain-rate law is thus introduced through the material constants. Other generalizations were obtained by including terms corresponding to elastic and plastic properties of the materials and micro-rotational inertia and micro-relational effects. These different generalizations gave rise to the study of fluids called plastics, pseudo-plastic, Bingham plastics, dilatant materials, non-Newtonian fluids including the rheopectic and thixotropic fluids. All these fluids along with their constitutive equations are treated in details and given in references Seth (1954), Eirich (1956), Prager (1961), Seegar (1965) Bhatnagar (1961, 1967), Eringen (1962, 1966], Fredrickson (1964), and Harris (1977).

The investigation is an attempt to study certain flow problems of visco-elastic fluids such as second-order fluid and Walters liquid B using computer oriented numerical techniques.

The section-4.1 is concerned with the visco-elastic fluid characterized by Walters liquid (Model B) in the annulus of two porous coaxial circular cylinders when both the boundaries are rotating with different angular velocities at a high injection in inner cylinder and high suction in outer cylinder. The governing differential equations and boundary conditions are replaced by difference equations in unknown variables by using finite-difference approximations for the derivatives. The numerical solution of the flow of visco-elastic fluid has been obtained by using Gauss-Elimination technique. The velocity function for different values of the parameter has been tabulated and shown graphically.

The section-4.2 deals with the study of non-Newtonian incompressible second- order fluid between two infinite porous rotating discs by finite difference scheme for small and large values of Reynolds numbers. It is assumed that the rate of suction of fluid at one disc is different from the rate of injection of fluid at the other disc. The governing differential equations and boundary conditions are replaced by difference equations in unknown variables by using finite difference approximations. The difference equations thus obtained are solved by iterative methods. The velocity components in transverse and axial direction have been investigated in detail and shown graphically.

The section-4.3 covers the study of unsteady flow of a viscous incompressible fluid filling the space between two parallel infinitely long rectangular (two dimensional) and two parallel circular plates (axisymmetric), which is of principal interest of many scientific and engineering applications. The numerical solution of unsteady squeezing of viscous fluid between two plates occurs is obtained by using finite difference scheme.

The section-4.4 is devoted to the study of the numerical solution of the flow of a visco-elastic fluid between coaxial rotating porous disks with uniform suction or uniform injection for small as well as large values of Reynolds numbers. The difference equations are solved by using Newton-Raphson iterative method. The behavior of velocity components have been investigated in detail in regions of recirculation and no-circulation for the cases of radial outflow and inflow, shown graphically and discussed in detail.

REVIEW OF LITERATURE

Chapter 2 REVIEW OF LITERATURE

The fluid flow through the porous boundaries is of great importance both in technological as well as biophysical fields, example of which is soil mechanics, hydrology, petroleum industry, transpiration cooling and gaseous diffusion technology, cooling of rocket, food preservation, cosmetic industry, blood flow and artificial dialysis. In resent years the problems of fluid flow past porous media or in channels with mass transfer, heat transfer have gained more importance because of varied applications, e.g. I. V. fluid containers made of PVC are commonly used these days. Water from inside permeates out thus increasing the concentration of drug inside and sometimes becomes hazardous to life. Thus the study relating to suction or injection is very important. The early researchers considered the blood to be a Newtonian fluid but being the suspension of cells, it behaves as a non- Newtonian fluid at low shear rates in small arteries.

A large number of theoretical investigations dealing with the study of incompressible laminar flow with either suction or injection have appeared during the last few decades.

Rivlin (1956), Coleman and Noll (1960) and Markovitz (1957) have studied elementary flow problems (steady as well as unsteady in nature) for non- Newtonian fluids. Parallel- plate torsional flows have been investigated by Coleman and Markovitz (1964), Coleman and Noll (1960) and Markovitz (1957) and cone and plate torsional motion have been considered by Markovitz and Williamson (1957) and Markovitz and Brown (1962). Some evidences favouring the Weissenberg effects etc. are given by Roberts (1954) and Jobling and Roberts (1958).

Oldroyd, Strawbridge and Toms (1951) have shown that the rheological behaviour of dilute solution of highly polymerized methyl-methacrylate in an organic liquid in a simple shearing motion can be represented, to a first approximation, at sufficiently small rates of shear by the stress-strain law proposed by Jeffreys (1926).

Coleman and Noll (1961), Crimnale, Ericksen and Filbey (1958) and Markovitz (1957) considered that the most general type of fluid is characterized by three functions of the rate of shear.

Berman (1956), Seller (1955) and Yuan (1956) have studied the two dimensional steady state flow in a rectangular channel with porous walls.

Nanda (1957) has obtained exact solution of the Navier- Stokes equation and energy equation for the case of steady state flow of the fluid through an annulus with porous walls, the inner wall is moving with a constant velocity parallel to the axis while the other is at rest.

Berman (1958) considered the steady state laminar flow of incompressible fluid in an annulus with porous walls.

Ting (1963) has taken positive values of the coefficient of elastico-viscosity but later it was confirmed by Markovitz and Brown (1962) and Coleman and Markovitz (1964) that it should be taken negative.

Terrill (1965) has considered the slow laminar flow of viscous liquid with suction at one wall and blowing at the other wall. Terrill (1964, 1965) obtained series solutions for small Reynolds numbers, large positive Reynolds numbers corresponding to suction and large negative Reynolds numbers corresponding to large blowing. Terrill (1982) also found the exact solution for a flow in a porous pipe.

Sharma (1966) has reconsidered this problem for an incompressible second order fluid. Kumar (1987) studied the same problem by the use of shooting method.

Kumar (1987) used the finite element Galerkin's method as well as the finite difference method to discuss the flow of non- Newtonian second order fluid through a converging or diverging channels when there is suction at one wall and equal blowing at the other.

The viscous flows past a circular cylinder and like problems are studied by many researchers. Datta (1961) has solved the problem of steady motion of an idealized viscoelastic liquid through an annulus between coaxial circular cylinders and between two parallel boundaries and flow of a non-Newtonian fluid through an annulus with porous walls.

Kapur and Mallic (1960) and Sinha and Chaudhary (1966) have discussed the steady state laminar flow of a viscous incompressible fluid between two coaxial porous cylinders rotating with constant angular velocity.

Sharma (1966) has extended the problems considered by Misra (1963) and Datta (1961) for a second order fluid.

Gupta and Singh (1975) have considered the steady problems of porous cylinder where both the cylinders are rotating with different velocities about the common axis and the cylinders are in relative motion along the axis and the visco-elastic fluid which is a second- order fluid is allowed to flow in the annulus under constant axial pressure gradient in which the suction and injection are small.

Gupta and Gupta (1976) considered the unsteady flow of a fluid through the annular space between two porous coaxial cylinders.

Kumar (1987) investigated the steady motion of a second order fluid when it flows in the first instance past a porous circular cylinder with suction or injection and secondly through the annulus between two coaxial right circular cylinders with suction and injection considered by Sharma (1966).

Kumar (1987) investigated the effect of suction and injection for the flow of a second-order fluid when both the discs rotate by applying shooting method. The boundary value problem is treated as initial value problem and Runge–Kutta method is applied to get the results at boundaries.

Sharma and Kumar (1986) and Sharma, Kumar and Arora (1986) studied the flow of a second order fluid past a circular cylinder with suction or injection on its walls by using Finite Element method.

Sharma (1992) deals with the steady flow of the effects of a transverse magnetic field on the flow of a slightly elastico-viscous liquid contained between two co-axial cylinders in relative rotation. It is assumed that there is suction on one of the cylinders accompanied by injection on the other. The analysis brings out the effects of magnetic field and its interaction with suction/injection.

Choudhary and Das (2002) considered the steady flow of visco-elastic fluid in the annulus of two porous coaxial cylinders, rotating with different uniform angular velocities, together with the translatory motion of inner cylinder along the axis of rotation. The analytical expression for toroidal and axial component has been obtained using series solution for small values of suction parameter.

The problem of forced flow of a fluid between two rotating discs has importance in chemical and mechanical engineering. The study has immense practical utility especially when the fluid is non- Newtonian. Flows induced by rotating disks are of considerable fundamental interest because of the richness of the physical phenomenon they encompass. These flows have technical applications in many areas, such as rotating machinery, lubrication, viscometry and crystal growth processes. The flow of a classical viscous fluid between a pair of infinite coaxial rotating disks has been studied by several authors due to its both theoretical and practical interest and that the problem offers the possibility on exact solution to the Navier- Stokes equations.

Karman (1921) was the first to give the solution to the Navier- Stokes equation for the flow of a Newtonian fluid in the vicinity of an infinitely large rotating disk. A number of approaches have been tried to investigate the rheological behavior of the so-called viscoelastic fluids i.e. fluids which when sheared demonstrate elastic properties besides viscous ones. The computation of flow of viscoelastic fluids has been on the leading edge of research in non-Newtonian fluid mechanics during last few years. The constitutive equation of even the simplest of viscoelastic fluids , such as the second order fluids are such that the momentum equations give rise to boundary value problems in which the order of the system of differential equations is greater than the number of boundary conditions. A large number of theoretical investigations dealing with the steady incompressible laminar flow with either injection or suction have appeared during the last few decades.

Von Karman (1921) has discussed the flow of viscous incompressible fluid under the influence of rotating disc which was later investigated by Cochran (1930).

Following Von Karman (1921), Batchelor (1951) studied the problem of steady flow of Newtonian fluid between two coaxial parallel infinite rotating disks, later it was solved by Stewartson (1953).

Stuart (1954) and Mithal (1961) have considered the effects of uniform high suction on the steady flow of viscous Rienier – Rivlin and second order fluids respectively due to rotating disc. They obtained the solutions by expanding the velocity – components in descending power of suction parameter.

Nanda (1960) has studied the effects of uniform suction on the revolving flow of a viscous liquid over the stationary disc and obtained that the presence of suction introduces an axial inflow at infinity and the same increases with an increase in suction.

Several different numerical procedures have been proposed in the literature during last years. Srivastava (1961) considered the study flow of a particular type of non- Newtonian fluid between a pair of rotating discs for small value of Reynolds numbers using regular perturbation procedure.

Bhatnagar (1961) studied the flow between torsrionally oscillating infinite plane disks in the presence of uniform magnetic field normal to the disks. Bhatnagar (1962) studied a particular type of non-Newtonian fluid for the unsteady flow only for small value of Reynolds numbers.

Lance and Rogers (1962) have discussed the flow between two rotating discs.

Bhatnagar and Rajeswari (1962) discussed the secondary flow induced in a particular class of non- Newtonian fluid between two parallel infinite oscillating planes again only for small value of Reynolds numbers.

Srivastava (196⁴) has studied the flow and also the heat transfer in the flow of a second order fluid between two disks one rotating and other at rest.

Pasha and Gupta (1967) have considered the effects of injection at the stationary disc (without heat transfer) in the problem solved by Srivastava (1964).

Sarma (1967) has considered the flow of a second order fluid for the case when both the disks rotate.

The problem of the flow between a rotating and a stationary disc has been independently solved by Mellor et. al. (1968) under the assumption of similarity solutions.

Narayan and Rudraiah (1972) and Wilson (1978) studied the same problem but they applied suction either on the stationary disc or on the rotating disc.

Gaur (1972) also considered the flow of a viscous incompressible fluid between two infinite porous rotating discs under the assumption that the rate of injection of the fluid at one disc is equal to the rate of suction of the fluid at the other.

Nguyen, Ribault and Florent (1975) obtained the multiple solution for the flow between coaxial disks. Roberts and Shipman (1975) have computed the flow between rotating and the stationary disks. Kubicek, Holodniok and Hlavacek (1976) calculated the flow between two coaxial disks by differentiation with respect to an actual parameter.

Hossain and Rahman (1984) studied the problem of the flow of a viscous incompressible fluid between two infinite porous rotating discs under the assumption that the rate of injection of the fluid at one disc is equal to the rate of suction of the fluid at the other in presence of transverse magnetic field.

The flow of a viscous incompressible fluid contained between two parallel disks which at time t are spaced a distance $h(1-\alpha t)^{1/2}$ apart has been studied by Hamza and MacDonald (1984), where h and α^{-1} denote a representative length and time.

El-Mistikawy (1991) investigated the flow of fluid between the two rotating disks in the presence of weak magnetic field.

Chaudhary and Das (1997) studied the flow and heat transfer of an incompressible second order fluid between two infinite porous rotating discs of infinite radius where the suction Reynolds number is assumed small.

The unsteady MHD flow near a rotating porous disk with uniform suction or injection has been studied by Attia (1988).

Ibrahim (2004) considered the steady flow of viscoelastic fluid between the two rotating disks for small value of Reynolds numbers by using regular perturbation method. The problem under consideration presents several difficulties from the viewpoint of application of numerical analysis, especially for high value of Reynolds numbers.

The flow of a viscous incompressible fluid between two parallel infinitely long rectangular (two dimensional) and two parallel circular plates (axisymmetric) is of principal interest of many scientific and engineering applications. Such flow situations have recently received special attention as a result of increasing practical interest in the design of thrust bearing, radial diffusers and lubrications. In unsteady loading the problem of unsteady squeezing of viscous fluid between two plates occurs. Chandrashekharan and Ramanaiah (1983) and Jackson and Symmons (1965) investigated the effect of inertia on the bearing characteristics.

By the use of mathematical modeling, large class of nonlinear ordinary differential equations are listed systematically by Sachdev (1991, 1997) and valuable hints for their analysis have been given by him.

MATERIALS AND METHODS

Chapter 3 MATERIALS AND METHODS

There are a lot of ways in which the word 'fluid' has been used in science. Fluid includes pretty much everything that's not a solid i.e. it includes both liquids and gases. Fluids are categorized into two types: ideal and real. The ideal fluids have zero compressibility and zero viscosity and are characterized by the assumption that the stress-tensor is a linear function of rate-of-strain and is normal to the surface. But real fluids have the property of viscosity. A fluid is called Newtonian fluid if it follows the Newton's law of viscosity:

$$\tau = \mu \left(\frac{dv}{dy} \right), \text{ where} \qquad \text{--- (3.1)}$$

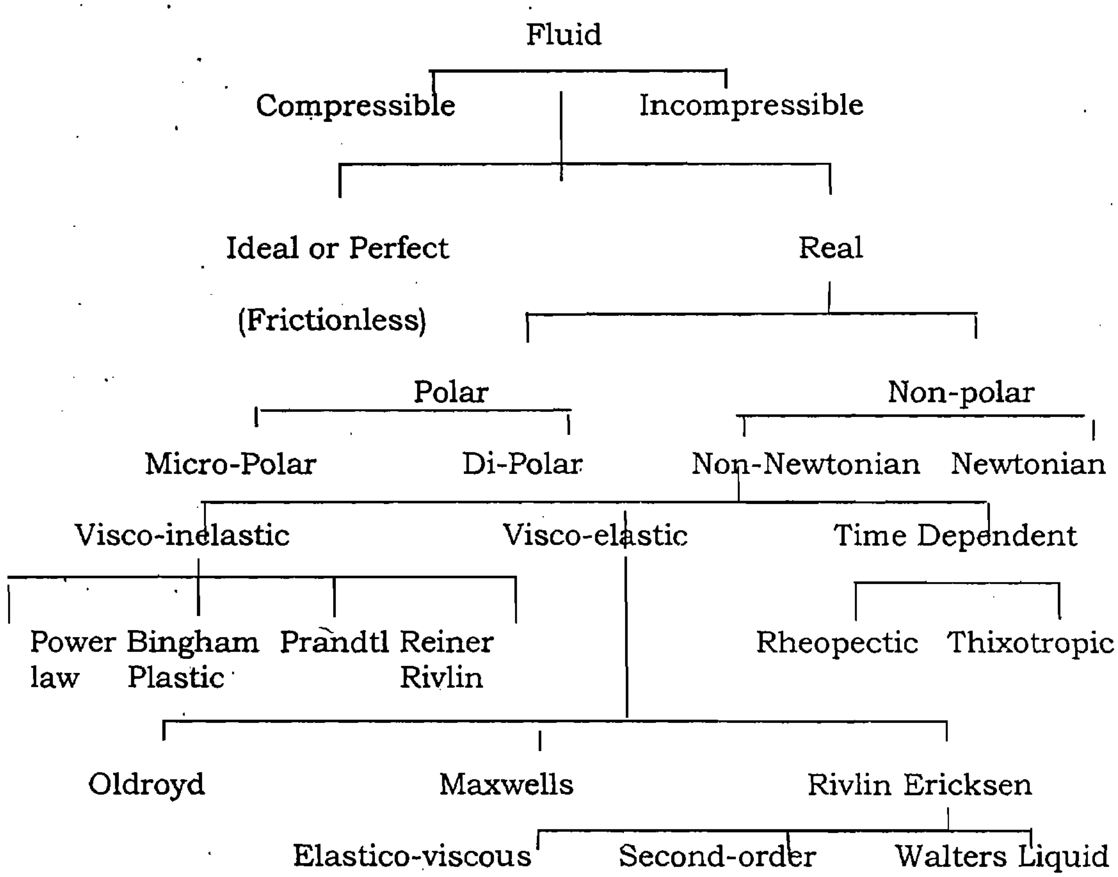
τ is shear stress, μ is viscosity of fluid and $\frac{dv}{dy}$ is shear rate or velocity gradient.

The fluids for which the relation (3.1) does not hold, are called non-Newtonian fluids. These are complex mixtures e.g. slurries, pastes, gels and polymer solutions etc.

The fluids having elastic properties besides viscosity are called visco-elastic fluids.

In the present study we have taken two types of visco-elastic fluids namely, second-order fluids and Walters liquid (Model B') have been considered. These fluids are described in the sequel. These fluids occur with wide variety and have importance in chemical and mechanical industries as well as in biological system.

- Classification of Fluid:



3.1 Second-order Fluid:

An incompressible simple fluid is an incompressible simple material if it possesses the property that all local states with the mass density are intrinsically equivalent in response. For a given history $g(s)$, a retarded history $g_c(s)$ can be defined as:

$$g_c(s) = g(s), 0 < s < \infty \qquad \text{--- (3.2)}$$

where c is retardation factor $0 < c < 1$. Taking into consideration, this definition of retarded history and assuming that the stress is more sensitive to recent deformation than to deformations which occurred in the distant past, Coleman and Noll (1960) proved that the theory of the simple fluids yields the theory of perfect fluids for $c \to 0$ and yields the theory of Newtonian fluids as the next approximation.

The theory of Newtonian fluids gives a correction to the theory of perfect fluids which is complete within terms of order one in c. If we neglect all the terms of order greater than two in c, then the simple fluid is called an incompressible second-order fluid. The constitutive equation is given by relation (1.7).

3.2 Walters liquid (Model B′):

The constitutive equation of Walters liquid is given by the relation (1.8).

Navier-Stokes Equations of Motion for viscous compressible fluid in Cartesian co-ordinates are:

$$\rho\frac{Du}{Dt} = \rho B_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu\left\{2\frac{\partial u}{\partial x} - \frac{2}{3}(\nabla.q)\right\}\right] + \frac{\partial}{\partial y}\left[\mu\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}\right] + \frac{\partial}{\partial z}\left[\mu\left\{\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right\}\right]$$

$$\rho\frac{Dv}{Dt} = \rho B_y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left[\mu\left\{2\frac{\partial v}{\partial y} - \frac{2}{3}(\nabla.q)\right\}\right] + \frac{\partial}{\partial z}\left[\mu\left\{\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right\}\right] + \frac{\partial}{\partial x}\left[\mu\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}\right]$$

$$\rho\frac{Dw}{Dt} = \rho B_z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[\mu\left\{2\frac{\partial w}{\partial z} - \frac{2}{3}(\nabla.q)\right\}\right] + \frac{\partial}{\partial x}\left[\mu\left\{\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right\}\right] + \frac{\partial}{\partial y}\left[\mu\left\{\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right\}\right]$$

--- (3.3)

where $q = ui + vj + zk$ and $B = B_x i + B_y j + B_z k$ are the velocity of the fluid at $P(x,y,z)$ at any time t and external body force at P per unit mass respectively. ρ is the density and μ is coefficient of viscosity of the fluid.

The equation of motion for a Walters's B' fluid in vector form:

$$\rho\left(\frac{\partial \underline{V}}{\partial t}+(\underline{V}.\underline{\nabla})\underline{V}\right)=-\underline{\nabla}P+\eta_0\nabla^2\underline{V}-K_0\left[\frac{\partial}{\partial t}\nabla^2\underline{V}+2(\underline{V}.\underline{\nabla})\nabla^2\underline{V}-\nabla^2(\underline{V}.\underline{\nabla})\underline{V}\right]$$

$$(\underline{\nabla}.\underline{V})=0 \qquad \text{--- (3.4)}$$

where V being the velocity.

3.3 Numerical Methods:

The numerical techniques which are used to solve the highly non-linear differential equations have been described as follows:

3.3.1 Taylor series for the function of several variables:

If f is the function of variables $x_1, x_2 .... x_n$ then expansion of the function f is given by

$$f(x_1+\Delta x_1, x_2+\Delta x_2+...........x_n+\Delta x_n)=f(x_1,x_2........x_n)$$
$$+\frac{\partial f}{\partial x_1}\Delta x_1+\frac{\partial f}{\partial x_2}\Delta x_2+........\frac{\partial f}{\partial x_n}\Delta x_n+\frac{1}{2}\left[\frac{\partial^2 f}{\partial x_1{}^2}(\Delta x_1)^2+...\frac{\partial^2 f}{\partial x_n{}^2}(\Delta x_n)^2\right.$$
$$\left.2\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2+....+2\frac{\partial^2 f}{\partial x_{n-1}\partial x_n}\Delta x_{n-1}\Delta x_n+.....\right]$$

--- (3.5)

3.3.2 Finite-Difference Method:

The finite-difference method for the solution of a two point boundary value problem consists in replacing the derivatives occurring in the differential equation by means of their finite difference approximations and then solving the resulting linear system of equations by a standard procedure.

Expanding $y(x+h)$ in Taylor series, we have

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + \text{--------} \qquad \text{--- (3.6)}$$

$$y'(x) = \frac{y(x+h) - y(x)}{h} - \frac{h}{2}y''(x)$$

$$y'(x) = \frac{y(x+h) - y(x)}{h} + O(h) \qquad \text{--- (3.7)}$$

which is forward difference approximation for $y'(x)$.

Similarly,

$$y(x-h) = y(x) - hy'(x) + \frac{h^2}{2}y''(x) - \frac{h^3}{6}y'''(x) + \text{--------} \qquad \text{--- (3.8)}$$

$$y'(x) = \frac{y(x) - y(x-h)}{h} + O(h) \qquad \text{--- (3.9)}$$

which is the backward difference approximation for $y'(x)$.

A central difference approximation for $y'(x)$ can be given by subtracting equation (3.8) by equation (3.6).

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h} + O(h^2)$$

It is a better approximation to $y'(x)$ than either equation (3.7) or equation (3.9) as the order of error involved in computation of higher order.

Again adding equation (3.6) and equation (3.8), we get approximation for $y''(x)$.

$$y''(x) = \frac{y(x-h) - 2y(x) + y(x+h)}{h^2} + O(h^2) \qquad \text{--- (3.10)}$$

In a similar manner, it is possible to derive finite difference approximation to higher derivative as

$$y'''(x) = \frac{y(x-2h) - 3y(x-h) + 3y(x) - y(x+h)}{h^2} + O\left(h^2\right) \quad \text{--- (3.11)}$$

$$y^{IV}(x) = \frac{y(x-2h) - 4y(x-h) + 6y(x) - 4y(x+h) + y(x+2h)}{h^2} + O\left(h^2\right) \quad \text{--- (3.12)}$$

$$y^{V}(x) = \frac{y(x-2h) - 5y(x-h) + 10y(x) - 10y(x+h) + 5y(x+2h) - y(x+3h)}{h^2} + O\left(h^3\right) \quad \text{--- (3.13)}$$

3.3.3 Discretization of Equation:

The two point boundary value problem is given by

$$y''(x) + f(x)\,y'(x) + g(x)y(x) = r(x)\ ,$$

with boundary condition $y(x_0) = a,\ y(x_n) = b$.

We divide the range $\left(x_0, x_n\right)$ into n equal subintervals of width h so that

$$x_i = x_0 + ih, \quad i = 1, 2, \ldots\ldots\ldots, n$$

The corresponding values of y at these points are denoted by

$$y(x_i) = y_i = y(x_0 + ih), \quad i = 0, 1 \ldots\ldots\ldots, n$$

Values of $y'(x)$ and $y''(x)$ at the points $x = x_i$ can be written as

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h} + O(h^2)$$

$$y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + O(h^2)$$

satisfying the differential equation at point $x = x_i$, we get

$$y_i'' + f_i y_i' + g_i y_i = r_i$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + f_i \frac{y_{i+1} - y_{i-1}}{2h} + g_i y_i = r_i, \; i = 1, \ldots\ldots\ldots\ldots, n-1$$

where $y_i = y(x_i)$, $g_i = g(x_i)$, etc.

Multiplying through by h^2 and simplifying, we obtain

$$\left(1 - \frac{h}{2} f_i\right) y_{i-1} + (-2 + g_i h^2) y_i + (1 + \frac{h}{2} f_i) y_i + 1 = r_i h^2 \qquad \text{--- (3.14)}$$
$$y_0 = a, \quad y_n = b.$$

To estimate the error in the numerical solution, we define the local truncation error, τ, by

$$\tau = \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - y_i'' \right) + f_i \left(\frac{y_{i+1} - y_{i-1}}{2h} - y_i' \right)$$

$$= \frac{h^2}{2} \left(y_i^{iv} + 2f_i y_i''' \right) + O(h^4) \qquad \text{--- (3.15)}$$

Thus, the finite difference approximation defined by equation (3.14) has second order accuracy for functions with continuous fourth derivatives on $\left(x_0, x_n\right)$. Further, it follows that $\tau \to 0$ as $h \to 0$, implying that greater accuracy in the result can be achieved by using a smaller value of h. In such a case more computational efforts would be required since the number of equations becomes larger.

3.3.4 Newton- Raphson Method:

Let a be an approximate root of $f(x) = 0$ and let $a_1 = a + h$ be the correct root so that $f(a_1) = f(a + h) = 0$

Expanding $f(a + h)$ by Taylor's series and neglecting the second and higher derivatives,

$$f(a) + hf'(a) = 0$$

$$h = -\frac{f(a)}{f'(a)}$$

A better approximation than a is therefore given by a_1, where

$$a_1 = a - \frac{f(a)}{f'(a)}$$

successive approximation are given by $a_2, a_3, a_4, \ldots\ldots\ldots a_{n+1}$, where

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)} \qquad \text{--- (3.16)}$$

which is the Newton-Raphson Formula.

The inherent error in the Newton- Raphson method:

If a is an approximate value of a root of $f(x) = 0$ and h is the necessary correction, so that $f(a + h) = 0$

$$f(a) + hf'(a) + \frac{h^2}{2} f''(a + \theta h) = 0, \quad 0 < \theta < 1 \qquad \text{--- (3.17)}$$

In Newton-Raphson method we neglect the term involving h^2 and got an approximate value h_1 from the equation

$$f(a) + h_1 f'(a) = 0, \qquad \text{--- (3.18)}$$

Subtracting equation (3.17) from equation (3.18)

$$(h - h_1) f'(a) + \frac{h^2}{2} f''(a + \theta h) = 0,$$

$$(h - h_1) = -h^2 \frac{f''(a + \theta h)}{2f'(a)} \qquad \text{--- (3.19)}$$

Now since h is the true value, it is plain that $h-h_1$ is the error in h_1.

3.3.5 Gauss- Elimination Method:

In the Gauss method of solving simultaneous linear equations, the unknown are eliminated successively by solving some equation for one unknown in terms of all the others; then substituting this value for the same unknown in all the remaining equations, thereby eliminating the unknown from the set. The process is repeated on the new set of equations, thus eliminating another unknown; and so on until the system is reduced to a single equation in one unknown.

The equation which expresses one unknown explicitly in terms of all the others called pivotal equations. After one unknown has been found, the remaining unknowns are found by back substitution into the pivotal equations.

Errors in the solution when the coefficients and constant terms are subject to error:

In system of linear equations occurring in applied mathematics the coefficient and the constant terms are often subject to errors due to rounding or to uncertainties in experimental data. The solutions obtained from such systems will therefore be inaccurate to some extent.

3.3.6 Factorization method:

This method is based on the fact that a square matrix A can be factorized into LU-decomposition where L is unit lower triangular matrix and U is upper triangular matrix, if all the principal minors of A are non-singular, i.e. if

$$a_{11} \neq 0, \quad \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0, \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0 \quad \text{etc.}$$

We consider, for definition, the linear system

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_3 = b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots a_{nn}x_3 = b_n$$

which can be written in the form

$$AX=B \qquad \text{--- (3.20)}$$

Let $\quad A = LU, \qquad$ --- (3.21)

where

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ - & - & - & 0 \\ l_{n1} & - & l_{n,n-1} & 1 \end{bmatrix} \qquad \text{--- (3.22)}$$

and

$$U = \begin{bmatrix} u_{11} & u_{12} & - & u_{1n} \\ 0 & u_{22} & 0 & u_{2n} \\ - & - & - & 0 \\ 0 & - & 0 & u_{nn} \end{bmatrix} \qquad \text{--- (3.23)}$$

Hence, (3.20) becomes $\quad LUX=B, \qquad$ --- (3.24)

If we set $\quad UX=Y, \qquad$ --- (3.25)

then (3.24) may be written as $\quad LY=B \qquad$ --- (3.26) which is equivalent to the system

$$
\begin{aligned}
&y_1 = b_1 \\
&l_{21}y_1 + y_2 = b_2 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&l_{n1}y_1 + l_{n2}y_2 + \cdots\cdots + l_{nn}y_n = b_n
\end{aligned}
$$

and can therefore be solved for $y_1, y_2, \ldots\ldots y_n$ by forward substitution.

When Y is known, the system (3.25) becomes,

$$
\begin{aligned}
u_{11}x_1 + u_{12}x_2 + \cdots\cdots + u_{1n}x_n &= y_1 \\
u_{22}x_2 + \cdots\cdots + u_{2n}x_n &= y_2 \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
u_{nn}x_3 &= y_n
\end{aligned}
$$

which can be solved for $x_1, x_2, \ldots\ldots x_n$ by backward substitution.

RESULTS AND DISCUSSION

Chapter 4 RESULTS AND DISCUSSION

The fluid flows through the porous boundaries are of considerable fundamental interest because of richness of the physical phenomenon they encompass. In the recent years the problems of non-Newtonian fluid flows, steady and unsteady, past through porous media or in channels with mass transfer have gained more importance because of varied applications in technological as well as biophysical fields.

Present investigations deals with the flows of classical viscous fluids such as second-order fluid and Walters liquid B′ between a pair of infinite coaxial rotating disks and in the annulus of coaxial cylinders due to its both theoretical and practical interest. These flows have technical applications in many areas, such as rotating machinery, lubrication viscometry and crystal growth process. We have investigated these types of problems for higher suction parameters and higher Reynolds numbers, which have immense practical utility in chemical industry. We have used four problems, these are:

Flow of Walters liquid B' through annulus of coaxial porous circular cylinders for high suction parameters.

Second-order fluid flow between two rotating discs of different transpiration for high Reynolds numbers.

Flow of incompressible fluid between two rectangular and circular plates.

Numerical solution of steady flows of a visco-elastic fluid between coaxial rotating porous disks with uniform suction or injection for high Reynolds numbers.

4.1 Flow of Walters liquid B′ through annulus of coaxial porous circular cylinders for high suction parameter:

The present investigation is concerned with the visco-elastic fluid characterized by Walters liquid B′ in the annulus of two coaxial circular cylinders when both the boundaries are rotating with different angular velocities at a high injection in inner cylinder and high suction in outer cylinder. The constitutive equation of Walters liquid B′ has already been described by the relation (1.8).

4.1.1 Formulation of problem:

Let us assume that the fluid is flowing in the annulus of coaxial circular cylinders whose radii are $a_1, b_1, (a_1 < b_1)$. Both inner and outer cylinders are rotating about the common axis with angular velocities w_1 and w_2 respectively while the inner cylinder is also rotating with uniform velocity W^* along its axis. There is suction in one cylinder and injection in another. The formulation of the problem has been done using the cylindrical polar coordinates (r, θ, z), where z-axis is considered as the common axis. The velocity components depend only on the radial distance r due to symmetry about the axis. Hence

$$\bar{u} = \bar{u}(r), \quad \bar{v} = \bar{v}(r), \quad \bar{w} = \bar{w}(r), \qquad \text{--- (4.1.1)}$$

where v and w are angular and axial velocities respectively. The boundary conditions are

$$\bar{u} = U_{a_1}, \bar{v} = a_1 w_1, \bar{w} = W^* \quad \text{at} \quad r = a_1$$

$$\bar{u} = U_{b_1}, \bar{v} = b_1 w_1, \bar{w} = 0 \quad \text{at} \quad r = b_1 \qquad \text{--- (4.1.2)}$$

where w_1, w_2 are the angular velocities U_{a_1} and U_{b_1} are the uniform injection and suction velocities.

Now introducing the non-dimensional quantities:

$$u = \frac{\bar{u}}{U_0},\ v = \frac{\bar{v}}{U_0},\ w = \frac{\bar{w}}{U_0},\quad P = \frac{\bar{P}}{\rho U_0^2},\ \alpha = \frac{K_0 U_0}{\eta_0 L},$$

$$R = \frac{\rho U_0 L}{\eta_0},\ r = \frac{\bar{r}}{L},\ z = \frac{\bar{z}}{L},\ a = \frac{a_1}{L},\quad b = \frac{b_1}{L},\quad \Omega_1 = \frac{w_1 L}{U_0},$$

$$\Omega_2 = \frac{w_2 L}{U_0}, H = \frac{W^*}{U_0}, U_a = \frac{U_{a_1}}{U_0}, U_b = \frac{U_{b_1}}{U_0} \qquad \text{--- (4.1.3)}$$

where L and U_0 are the characteristic length and characteristic velocity respectively, α is the non-dimensional visco-elastic parameter.

Under these considerations the governing equations in dimensionless form are:

$$\frac{du}{dr} + \frac{u}{r} = 0 \qquad \text{--- (4.1.4)}$$

$$-R\left[\frac{K^2}{r^3} + \frac{v^2}{r}\right] = -R\frac{\partial p}{\partial r} - 2\alpha\left[\frac{v}{r}\frac{d^2v}{dr^2} - \frac{3v}{r^2}\frac{dv}{dr} + \frac{1}{r}\frac{dv}{dr} + \frac{1}{r}\left(\frac{dv}{dr}\right)^2 + \frac{v^2}{r^3} + 4\frac{K^2}{r^5}\right]$$

---(4.1.5)

$$\frac{RK}{r}\left[\frac{dv}{dr} + \frac{v}{r}\right] = \frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2} - \frac{\alpha K}{r}\left[\frac{d^3v}{dr^3} + \frac{2}{r}\frac{d^2v}{dr^2} + \frac{3}{r^2}\frac{dv}{dr} - \frac{3v}{r^3}\right]$$

--- (4.1.6)

$$\frac{RK}{r}\frac{dw}{dr}=-R\frac{\partial P}{\partial z}+\frac{1}{r}\frac{dw}{dr}+\frac{d^2w}{dr^2}-\frac{\alpha K}{r}\left[\frac{d^3w}{dr^3}+\frac{3}{r}\frac{d^2w}{dr^2}-\frac{3}{r^2}\frac{dw}{dr}\right] \qquad \text{---(4.1.7)}$$

4.1.2 Numerical Solution:

Equation (4.1.4) on integration yields

$$u=\frac{K}{r},$$

where K is the non-dimensional constant related to injection and suction velocities as

$$K=U_a.a=U_b.b, \qquad \text{--- (4.1.8)}$$

where K is positive for injection on the inner cylinder and suction on the outer cylinder and negative for the reverse order.

From above equations we infer that the pressure should be a function of r and z only, which is the form

$$-P=\lambda z+g(r) \qquad \text{--- (4.1.9)}$$

where λ is an absolute constant and $g(r)$ is an arbitrary function of r.

Equations (4.1.6) and (4.1.7) are linear homogeneous in v and w respectively. The finite-difference approximations scheme for the first, second and third order derivatives have been used to discretize these differential equations.

Using these approximations in equation (4.1.6) and (4.1.7), we obtain

$$v_{i-1}\left[\frac{-\alpha K}{rh^3}+\frac{1}{h^2}\left(\frac{2\alpha K}{r^2}-1\right)-\frac{1}{2h}\left(\frac{RK}{r}-\frac{1}{r}+\frac{3\alpha K}{r^3}\right)\right]+v_i\left[\frac{3\alpha K}{rh^3}-\frac{2}{h^2}\left(\frac{2\alpha K}{r^2}-1\right)+\left(\frac{RK}{r^2}+\frac{1}{r}-\frac{\alpha K}{r^3}\right)\right]+$$
$$v_{i+1}\left[\frac{-3\alpha K}{rh^3}+\frac{1}{h^2}\left[\left(\frac{2\alpha K}{r^2}-1\right)\right]+\frac{1}{2h}\left(\frac{RK}{r}-\frac{1}{r}+\frac{3\alpha K}{r^3}\right)\right]+v_{i+2}\left[\frac{\alpha K}{h^3}\right]=0$$

$$\text{--- (4.1.10)}$$

$$w_{i-1}\left[\frac{-\alpha K}{rh^3}+\frac{1}{h^2}\left(\frac{3\alpha K}{r^2}-1\right)-\frac{1}{h^2}\left(\frac{RK}{r}-\frac{1}{r}-\frac{3\alpha K}{r^3}\right)\right]+w_i\left[\frac{3}{h^3}\left(\frac{\alpha K}{r}\right)-\frac{2}{h^2}\left(\frac{3\alpha K}{r^2}-1\right)\right]$$

$$w_{i+1}\left[\frac{-3\alpha K}{r^2}+\frac{1}{h^2}\left(\frac{3\alpha k}{r^2}-1\right)-\frac{1}{2h}\left(\frac{RK}{r}-\frac{1}{r}-\frac{3\alpha K}{r^3}\right)\right]+w_{i+2}\left[\frac{\alpha K}{rh^3}\right]-R\lambda=0$$

--- (4.2.11)

All v_i's and w_i's are functions of r. we divide r [1, 2] into hundred equal parts each of length 0.001.The boundary conditions can be written as:

$$u=U_a,\quad v_1=a\Omega_1,\quad w_1=H\quad \text{at}\quad r=1,$$

$$u=U_b,\quad v_{101}=\upsilon\Omega_2,\quad w_{101}=0\quad \text{at}\quad r=2. \qquad \text{--- (4.1.12)}$$

The equations (4.1.10) and (4.1.11) have been represented in the matrix form independently to solve them numerically using Gauss-eliminating technique.

4.1.3 Results and Discussion:

The Tables 4.1.1-4.1.6 represent the variation of angular velocity v with radial distance r for fixed value of suction parameter K. These tables have been depicted through figures 4.1.1-4.1.6. When Reynolds number is small, even for large value of suction parameter $K>0$, the angular velocity v increases with the increase in visco-elastic parameter α. For large value of Reynolds number, angular velocity increases with the increase in visco-elastic parameter but it becomes negative after increment in Reynolds number≥ 10. The results have also been obtained for the cases considered by Choudhury and Das (2002).

The study for Reynolds number and fixed visco-elastic-parameter α has also been made and it is shown through figure 4.1.7 and tabulated in table 4.1.7. It has been noticed that the velocity increases with the decrease in suction parameter.

For fixed value of Reynolds number R, the angular velocity v increases for all K<0 and K>0, with the increase in visco-elastic parameter α have also been depicted through the graphical presentation in figures 4.1.1, 4.1.6, 4.1.8-4.1.15 and tables 4.1.1, 4.1.6, 4.1.8-4.1.15

The corresponding results for Newtonian fluid can be deduced from the above results by setting $\alpha=0$ and it is worth mentioning here that these results agree well with that of Choudhury and Das. Tables 4.1.16-4.1.21 represent the variation of axial velocity w with the change of the visco-elastic parameter α for fixed K=0.9 and R=1, R=5, R=9, R=13, R=17, R=21 and the behavior is represented in figures 4.1.16-4.1.21.

It has been observed and concluded that for a fixed K, for a particular value of R, the axial velocity of the fluid increases with the increase in α. If K becomes very large (very small) and R is also very large, the axial velocity has a negative sign, i.e. the liquid comes out of the cylinder. The professionals from the chemical industry may be interested to find this limit of K and R for the fluid they are using these values depend on the characteristics of the fluid.

Tables 4.1.24-4.1.29 show that for small values of suction parameter K>0, the axial velocity increases with the increase in the value of α, whatever is the value of R and can be depicted from figures 4.1.22-4.1.27.

For $K<0$, the results have been tabulated in tables 4.1.28-4.1.33. The behavior of axial velocity with the increase in visco-elastic parameter α is shown in the figures 4.1.28-4.1.33.

Tables 4.1.36-4.1.39 represent the behavior of axial velocity with radial distance r for fixed values of R. From figure 4.1.34, fixing $R=1$ and $\alpha=0.0$, axial velocity decreases with decreasing values of K. From figure 4.1.35, fixing $\alpha=0.2$ and $k=0.1$ axial velocity w increases with the increasing value of R. Similar results can be obtained by fixing $\alpha=0.0$ and $K=0.9$ (figure 4.1.36). But as soon as the visco-elastic parameter α is increased (figure 4.1.37 ($\alpha=0.2$, $k=0.9$)), the axial velocity becomes suddenly very much negative at $R=13$ then becomes less negative on further increase in the value of R. Hence, one can't take the fluids whose Reynolds number as well as suction parameter is very large.

A study has also been made for different values of λ with the velocity component in tables 4.1.16, 4.1.38-4.1.39. The graphs in figures 4.1.16, 4.1.38-4.1.39 have been plotted and noticed that the axial velocity increases with the increase in values of λ.

r	V,R=1, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.109651	1.135066	1.177459
1.1	1.223125	1.266851	1.338178
1.15	1.340595	1.397217	1.487716
1.2	1.462193	1.527521	1.630005
1.25	1.588018	1.658764	1.767872
1.3	1.718144	1.791696	1.903382
1.35	1.852622	1.926888	2.038062
1.4	1.991488	2.064773	2.173057
1.45	2.134763	2.205685	2.309234
1.5	2.282458	2.349883	2.447253
1.55	2.434573	2.497566	2.587623
1.6	2.591102	2.648889	2.730738
1.65	2.752034	2.80397	2.876904
1.7	2.917352	2.962899	3.026357
1.75	3.087035	3.125744	3.179283
1.8	3.261059	3.292555	3.335825
1.85	3.439396	3.463368	3.496094
1.9	3.62202	3.638205	3.660173
1.95	3.808898	3.817081	3.828126
2	4	4	4

Table 4.1.1 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=0.9.

r	V,R=5, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.348207	1.372235	1.39815
1.1	1.65772	1.699356	1.744058
1.15	1.933567	1.987776	2.04575
1.2	2.17998	2.242782	2.309712
1.25	2.400555	2.468773	2.541253
1.3	2.598364	2.669445	2.744763
1.35	2.776053	2.847933	2.923913
1.4	2.935912	3.006912	3.0818
1.45	3.079935	3.148683	3.221056
1.5	3.209867	3.275239	3.343937
1.55	3.327242	3.388313	3.45239
1.6	3.433416	3.489422	3.548102
1.65	3.529589	3.579902	3.632547
1.7	3.616833	3.66093	3.707017
1.75	3.696103	3.733554	3.772653
1.8	3.768254	3.798704	3.830462
1.85	3.834058	3.857214	3.881342
1.9	3.894208	3.909829	3.926093
1.95	3.949332	3.957223	3.965431
2	4	4	4

Table 4.1.2 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=5and suction parameter k=0.9.

r	V,R=9, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.364264	1.648215	1.949154
1.1	1.534825	2.029242	2.551267
1.15	1.560825	2.207448	2.887955
1.2	1.483486	2.235704	3.025085
1.25	1.337626	2.157841	3.016426
1.3	1.152827	2.010496	2.906305
1.35	9.54E-01	1.824518	2.731591
1.4	7.64E-01	1.626052	2.523187
1.45	6.00E-01	1.437377	2.307172
1.5	4.79E-01	1.27758	2.105688
1.55	4.16E-01	1.163086	1.937636
1.6	4.21E-01	1.108081	1.819226
1.65	5.06E-01	1.124864	1.764418
1.7	6.81E-01	1.224132	1.78528
1.75	9.53E-01	1.415206	1.892274
1.8	1.329744	1.706233	2.094499
1.85	1.817567	2.104346	2.399883
1.9	2.422029	2.615797	2.81535
1.95	3.148058	3.246074	3.346955
2	4	4	4

Table 4.1.3 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=9and suction parameter k=0.9.

r	V,R=13, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	2.037351	2.450579	2.880775
1.1	2.599386	3.319155	4.066514
1.15	2.795459	3.73709	4.712586
1.2	2.717377	3.813046	4.945861
1.25	2.44274	3.637696	4.871004
1.3	2.037549	3.287282	4.575138
1.35	1.558249	2.826357	4.131364
1.4	1.05335	2.309905	3.601452
1.45	5.65E-01	1.78502	3.037925
1.5	1.29E-01	1.292229	2.485692
1.55	-2.23E-01	8.67E-01	1.983343
1.6	-4.63E-01	5.38E-01	1.564184
1.65	-5.68E-01	3.34E-01	1.257074
1.7	-5.15E-01	2.77E-01	1.087103
1.75	-2.87E-01	3.87E-01	1.076153
1.8	1.34E-01	6.83E-01	1.243357
1.85	7.60E-01	1.178551	1.605483
1.9	1.606363	1.888919	2.177255
1.95	2.682905	2.825834	2.971623
2	4	4	4

Table 4.1.4 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=13and suction parameter k=0.9.

r	V,R=17, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	3.074294	3.616778	4.176257
1.1	4.269755	5.214856	6.187577
1.15	4.779775	6.016392	7.286906
1.2	4.766765	6.205865	7.682144
1.25	4.368063	5.937738	7.545802
1.3	3.700528	5.342303	7.022238
1.35	2.86415	4.530187	6.233163
1.4	1.944919	3.595879	5.281871
1.45	1.017118	2.620506	4.256555
1.5	1.45E-01	1.674054	3.232908
1.55	-6.15E-01	8.17E-01	2.276204
1.6	-1.21383	1.03E-01	1.442973
1.65	-1.60898	-4.24E-01	7.82E-01
1.7	-1.76256	-7.22E-01	3.37E-01
1.75	-1.64137	-7.56E-01	1.45E-01
1.8	-1.21615	-4.95E-01	2.39E-01
1.85	-4.61E-01	8.85E-02	6.47E-01
1.9	6.47E-01	1.018234	1.395388
1.95	2.127927	2.315744	2.506477
2	4	4	4

Table 4.1.5 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=17and suction parameter k=0.9.

r	V,R=21, k=0.9,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	4.47509	5.146883	5.835616
1.1	6.54593	7.716418	8.914467
1.15	7.51377	9.045431	10.61092
1.2	7.631648	9.414239	11.23394
1.25	7.113592	9.058048	11.04082
1.3	6.141758	8.17564	10.2476
1.35	4.872061	6.936094	9.036984
1.4	3.438625	5.484059	7.564439
1.45	1.957361	3.943923	5.963055
1.5	5.29E-01	2.423147	4.347327
1.55	-7.59E-01	1.014957	2.816211
1.6	-1.83052	-1.99E-01	1.455582
1.65	-2.61772	-1.14921	3.40E-01
1.7	-3.06192	-1.77212	-4.64E-01
1.75	-3.11115	-2.01359	-9.01E-01
1.8	-2.71957	-1.82555	-9.20E-01
1.85	-1.8466	-1.16557	-4.76E-01
1.9	-4.56E-01	3.85E-03	4.70E-01
1.95	1.483117	1.71592	1.951506
2	4	4	4

Table 4.1.6 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=0.9.

R	V, R=1, α=0.0				
	k=0.9	k=0.5	K=0.1	k=-0.3	k=-0.7
1	1	1	1	1	1
1.05	1.109651	1.138181	1.171202	1.208715	1.25072
1.1	1.223125	1.27686	1.338074	1.406767	1.482939
1.15	1.340595	1.416285	1.501231	1.595432	1.698888
1.2	1.462193	1.556654	1.661184	1.775781	1.900446
1.25	1.588018	1.698126	1.81836	1.948719	2.089205
1.3	1.718144	1.840827	1.973118	2.115018	2.266525
1.35	1.852622	1.984861	2.125765	2.275337	2.433576
1.4	1.991488	2.130308	2.276561	2.430248	2.591369
1.45	2.134763	2.277235	2.425729	2.580246	2.740786
1.5	2.282458	2.425693	2.573461	2.725762	2.882597
1.55	2.434573	2.575723	2.719925	2.867177	3.017481
1.6	2.591102	2.727358	2.865265	3.004825	3.146036
1.65	2.752034	2.88062	3.009609	3.139	3.268794
1.7	2.917352	3.03553	3.153068	3.269968	3.386229
1.75	3.087035	3.192098	3.29574	3.397962	3.498763
1.8	3.261059	3.350333	3.437711	3.523192	3.606776
1.85	3.439396	3.510241	3.579058	3.645849	3.710612
1.9	3.62202	3.671822	3.71985	3.766102	3.810579
1.95	3.808898	3.835076	3.860145	3.884106	3.906957
2	4	4	4	4	4

Table 4.1.7 variation of angular velocity v with radial distance r for different values suction parameter k and fixed value of Reynolds numbers R=1 and alpha=0.0.

r	V,R=1, k=0.5,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.138181	1.142224	1.151507
1.1	1.27686	1.283834	1.299327
1.15	1.416285	1.42535	1.44487
1.2	1.556654	1.56716	1.589134
1.25	1.698126	1.709561	1.732836
1.3	1.840827	1.85278	1.876503
1.35	1.984861	1.996998	2.020527
1.4	2.130308	2.142353	2.165201
1.45	2.277235	2.288958	2.310751
1.5	2.425693	2.436901	2.457351
1.55	2.575723	2.586252	2.605134
1.6	2.727358	2.737068	2.754205
1.65	2.88062	2.889393	2.904646
1.7	3.03553	3.043261	3.056521
1.75	3.192098	3.1987	3.20988
1.8	3.350333	3.35573	3.36476
1.85	3.510241	3.514366	3.521193
1.9	3.671822	3.674619	3.679201
1.95	3.835076	3.836496	3.838799
2	4	4	4

Table 4.1.8 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=0.5.

r	V,R=1, k=0.1,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.171202	1.169995	1.168998
1.1	1.338074	1.336005	1.334277
1.15	1.501231	1.498565	1.496318
1.2	1.661184	1.658127	1.65553
1.25	1.81836	1.815073	1.81226
1.3	1.973118	1.969727	1.966807
1.35	2.125765	2.122369	2.119427
1.4	2.276561	2.273236	2.270344
1.45	2.425729	2.422538	2.41975
1.5	2.573461	2.570452	2.567814
1.55	2.719925	2.717137	2.714683
1.6	2.865265	2.862729	2.860489
1.65	3.009609	3.007348	3.005345
1.7	3.153068	3.1511	3.149353
1.75	3.29574	3.29408	3.292603
1.8	3.437711	3.43637	3.435175
1.85	3.579058	3.578045	3.57714
1.9	3.71985	3.71917	3.718562
1.95	3.860145	3.859804	3.859498
2	4	4	4

Table 4.1.9 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=0.1.

r	V,R=1, k=-0.3,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.208715	1.21838	1.229932
1.1	1.406767	1.423364	1.443027
1.15	1.595432	1.616863	1.642059
1.2	1.775781	1.800422	1.829192
1.25	1.948719	1.9753	2.006143
1.3	2.115018	2.142537	2.174292
1.35	2.275337	2.303	2.334765
1.4	2.430248	2.457423	2.488486
1.45	2.580246	2.606425	2.636229
1.5	2.725762	2.750533	2.778641
1.55	2.867177	2.890221	2.916271
1.6	3.004825	3.02587	3.04959
1.65	3.139	3.157834	3.179
1.7	3.269968	3.286415	3.304853
1.75	3.397962	3.411884	3.427453
1.8	3.523192	3.534475	3.547068
1.85	3.645849	3.654404	3.663933
1.9	3.766102	3.771857	3.778256
1.95	3.884106	3.887005	3.890223
2	4	4	4

Table 4.1.10 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=-0.3.

r	V,R=1, k=-0.7,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.25072	1.287379	1.334309
1.1	1.482939	1.54591	1.625578
1.15	1.698888	1.780244	1.882093
1.2	1.900446	1.994045	2.110121
1.25	2.089205	2.190242	2.314486
1.3	2.266525	2.371208	2.49896
1.35	2.433576	2.538893	2.666538
1.4	2.591369	2.694912	2.819628
1.45	2.740786	2.840619	2.960189
1.5	2.882597	2.977157	3.089832
1.55	3.017481	3.105503	3.209897
1.6	3.146036	3.226493	3.321507
1.65	3.268794	3.340852	3.425612
1.7	3.386229	3.449207	3.523022
1.75	3.498763	3.552111	3.614431
1.8	3.606776	3.650047	3.70044
1.85	3.710612	3.743444	3.781572
1.9	3.810579	3.832682	3.858283
1.95	3.906957	3.918099	3.930974
2	4	4	4

Table 4.1.11 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=-0.7.

r	V,R=21, k=0.5,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.772561	1.976104	2.184875
1.1	2.177221	2.531962	2.895208
1.15	2.300618	2.764941	3.239707
1.2	2.21547	2.755995	3.307974
1.25	1.983236	2.572977	3.174543
1.3	1.656171	2.273164	2.901911
1.35	1.278953	1.905216	2.542854
1.4	8.90E-01	1.510705	2.142224
1.45	5.22E-01	1.125326	1.738348
1.5	2.05E-01	7.80E-01	1.364147
1.55	-3.77E-02	5.01E-01	1.048021
1.6	-1.83E-01	3.12E-01	8.15E-01
1.65	-2.13E-01	2.33E-01	6.85E-01
1.7	-1.11E-01	2.81E-01	6.78E-01
1.75	1.39E-01	4.73E-01	8.11E-01
1.8	5.49E-01	8.21E-01	1.096225
1.85	1.131014	1.337935	1.547534
1.9	1.89423	2.034061	2.175654
1.95	2.847939	2.918685	2.990291
2	4	4	4

Table 4.1.12. variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=0.5.

r	V,R=21, k=0.1,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.051016	1.05779	1.064772
1.1	1.106795	1.118637	1.130819
1.15	1.169055	1.1846	1.200563
1.2	1.239215	1.257359	1.275962
1.25	1.318451	1.338296	1.358615
1.3	1.407743	1.428553	1.449834
1.35	1.507909	1.529077	1.550701
1.4	1.619633	1.640656	1.662111
1.45	1.743488	1.763947	1.784808
1.5	1.879953	1.899498	1.919413
1.55	2.029428	2.047767	2.066439
1.6	2.192248	2.209135	2.226317
1.65	2.368691	2.383918	2.399403
1.7	2.558986	2.572379	2.585992
1.75	2.763323	2.774735	2.786329
1.8	2.981853	2.99116	3.000612
1.85	3.214701	3.221799	3.229005
1.9	3.461962	3.466764	3.471636
1.95	3.72371	3.726142	3.728608
2	4	4	4

Table 4.1.13 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=0.1.

r	V,R=21, k=-0.3,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	2.310456	2.391941	2.475309
1.1	3.334653	3.476446	3.621301
1.15	4.119083	4.304408	4.493493
1.2	4.702883	4.918331	5.137903
1.25	5.119238	5.354008	5.593037
1.3	5.396476	5.64181	5.891375
1.35	5.55893	5.807679	6.060523
1.4	5.627612	5.873915	6.1241
1.45	5.62076	5.859788	6.102436
1.5	5.55428	5.782042	6.013124
1.55	5.442099	5.655283	5.871467
1.6	5.296463	5.492316	5.690836
1.65	5.128173	5.304404	5.482961
1.7	4.946792	5.10149	5.258171
1.75	4.760809	4.892381	5.025594
1.8	4.577782	4.684902	4.793321
1.85	4.404458	4.486021	4.568547
1.9	4.246871	4.30196	4.357683
1.95	4.110432	4.13829	4.166458
2	4	4	4

Table 4.1.14 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=-0.3.

r	V,R=21, k=-0.7,		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	5.550881	5.978558	6.416484
1.1	8.860794	9.605387	10.36665
1.15	11.1507	12.12437	13.1185
1.2	12.60647	13.73891	14.8938
1.25	13.3856	14.62011	15.87781
1.3	13.62237	14.91293	16.22653
1.35	13.43202	14.74102	16.07232
1.4	12.91391	14.21048	15.52819
1.45	12.15417	13.41285	14.69123
1.5	11.22779	12.4275	13.64528
1.55	10.20031	11.32354	12.4631
1.6	9.129229	10.16142	11.20812
1.65	8.065171	8.99417	9.935833
1.7	7.052832	7.868508	8.694977
1.75	6.131793	6.825682	7.5285
1.8	5.33719	5.902232	6.474353
1.85	4.700287	5.130599	5.566162
1.9	4.248958	4.539651	4.833796
1.95	4.008104	4.15513	4.30384
2	4	4	4

Table 4.1.15 variation of angular velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=-0.7.

r	W,R=1, k=0.9, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	9.72E-01	9.74E-01	9.76E-01
1.1	9.42E-01	9.44E-01	9.49E-01
1.15	9.10E-01	9.12E-01	9.19E-01
1.2	8.75E-01	8.78E-01	8.85E-01
1.25	8.38E-01	8.41E-01	8.49E-01
1.3	7.98E-01	8.02E-01	8.11E-01
1.35	7.56E-01	7.60E-01	7.69E-01
1.4	7.12E-01	7.15E-01	7.25E-01
1.45	6.66E-01	6.69E-01	6.79E-01
1.5	6.17E-01	6.20E-01	6.29E-01
1.55	5.66E-01	5.68E-01	5.78E-01
1.6	5.12E-01	5.15E-01	5.23E-01
1.65	4.56E-01	4.59E-01	4.67E-01
1.7	3.98E-01	4.00E-01	4.07E-01
1.75	3.37E-01	3.39E-01	3.46E-01
1.8	2.75E-01	2.76E-01	2.82E-01
1.85	2.09E-01	2.11E-01	2.15E-01
1.9	1.42E-01	1.43E-01	1.46E-01
1.95	7.21E-02	7.27E-02	7.41E-02
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.16 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=0.9.

r	W,R=5, k=0.9, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.058746179	1.126872264	3.0580289
1.1	1.114249911	1.251212694	5.150075859
1.15	1.165335318	1.371029182	7.241634183
1.2	1.21082411	1.484179122	9.296498658
1.25	1.24951415	1.58836463	11.27661021
1.3	1.280163085	1.68112818	13.14195115
1.35	1.301475798	1.759848599	14.85047861
1.4	1.312094774	1.821737411	16.35808743
1.45	1.310592657	1.863835556	17.61859642
1.5	1.295466479	1.883010532	18.58375399
1.55	1.265133165	1.875954099	19.20326119
1.6	1.217925985	1.839180696	19.42481197
1.65	1.152091689	1.769026875	19.19415194
1.7	1.065788029	1.661652161	18.45515858
1.75	9.57E-01	1.513041999	17.14994738
1.8	8.24E-01	1.319013773	15.21901003
1.85	6.64E-01	1.075227388	12.60139297
1.9	4.75E-01	7.77E-01	9.234591095
1.95	2.54E-01	4.20E-01	5.055381931
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.17 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=5and suction parameter k=0.9.

r	W,R=9, k=0.9, λ=1		
	α=0.0	α =0.2	α=0.4
1	1	1	1
1.05	1.043373273	1.030800546	1.228494025
1.1	1.08300165	1.090545498	1.418424929
1.15	1.1203181	1.185948962	1.564062815
1.2	1.156008217	1.322752826	1.660877303
1.25	1.19011255	1.505132065	1.705731025
1.3	1.222122128	1.735335356	1.697070667
1.35	1.251064659	2.012515923	1.635110263
1.4	1.275578901	2.332510286	1.522001464
1.45	1.293974404	2.686825992	1.361985322
1.5	1.304273045	3.061903686	1.161519944
1.55	1.304227099	3.438324339	9.29E-01
1.6	1.291305669	3.790039315	6.77E-01
1.65	1.262636503	4.083659923	4.17E-01
1.7	1.214883096	4.277854687	1.66E-01
1.75	1.144027243	4.322917914	-5.73E-02
1.8	1.044924392	4.160593654	-2.34E-01
1.85	8.88E-01	3.724266928	-3.40E-01
1.9	6.69E-01	2.93967224	-3.54E-01
1.95	3.77E-01	1.726336448	-2.49E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.18 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=9 and suction parameter k=0.9.

r	W,R=13, k=0.9, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.247441276	1.261490809	1.274530019
1.1	1.46577808	1.491639392	1.515365432
1.15	1.654526953	1.690212255	1.722559254
1.2	1.813242948	1.856966184	1.896105099
1.25	1.941513469	1.991653093	2.035952991
1.3	2.038954078	2.09402345	2.14202077
1.35	2.105205876	2.163828666	2.214202417
1.4	2.139933947	2.200822676	2.252374176
1.45	2.142826302	2.204762878	2.256399028
1.5	2.113592747	2.175410567	2.226129893
1.55	2.051963122	2.112530924	2.161411829
1.6	1.957684485	2.015892634	2.062083396
1.65	1.830516975	1.885267159	1.927977333
1.7	1.670228193	1.72042768	1.758920634
1.75	1.476586084	1.521147737	1.554734113
1.8	1.24935024	1.287199538	1.315231533
1.85	9.89E-01	1.018351953	1.040218494
1.9	6.94E-01	7.14E-01	7.29E-01
1.95	3.64E-01	3.75E-01	3.83E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.19. variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=13 and suction parameter k=0.9.

r	W,R=17, k=0.9, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.066159676	1.014142546	1.162973634
1.1	1.135629638	1.075201984	1.41622688
1.15	1.208199201	1.195342921	1.746977153
1.2	1.283662313	1.380113471	2.135804717
1.25	1.361782591	1.625887013	2.557511696
1.3	1.442233196	1.917897601	2.982415329
1.35	1.524506656	2.22928236	3.37805346
1.4	1.607786558	2.521549957	3.711255064
1.45	1.690767489	2.746864822	3.950502578
1.5	1.771401391	2.852449781	4.068486941
1.55	1.846537489	2.787271558	4.044731526
1.6	1.911409196	2.51096408	3.868138571
1.65	1.958904834	2.004666222	3.539292254
1.7	1.978539528	1.283099868	3.072337577
1.75	1.955022631	4.07E-01	2.496245304
1.8	1.866287516	-5.07E-01	1.855271994
1.85	1.680558445	-1.277510712	1.208432246
1.9	1.353252324	-1.658215121	6.28E-01
1.95	8.22E-01	-1.345764519	1.96E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.20 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=17 and suction parameter k=0.9.

r	W,R=21, k=0.9, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.064158675	9.93E-01	9.54E-01
1.1	1.131535442	9.91E-01	1.002354859
1.15	1.201976628	1.009557687	1.153892961
1.2	1.275364378	1.067045642	1.406162719
1.25	1.351604766	1.178207936	1.746555425
1.3	1.43059427	1.35119713	2.152210625
1.35	1.512162024	1.583643481	2.591172665
1.4	1.59598155	1.859605845	3.024476183
1.45	1.68143647	2.148087327	3.40911299
1.5	1.76740876	2.40392701	3.701759581
1.55	1.851934448	2.571781092	3.863068598
1.6	1.931639	2.593665693	3.862253397
1.65	2.000820768	2.420129043	3.681627507
1.7	2.049991016	2.024545126	3.320705578
1.75	2.0635956	1.419284778	2.799435636
1.8	2.016524508	6.72E-01	2.160119954
1.85	1.868218771	-8.34E-02	1.467599255
1.9	1.556034068	-6.38E-01	8.07E-01
1.95	9.83E-01	-7.16E-01	2.81E-01
2	0	0.00E+00	0.00E+00

Table 4.1.21 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=0.9.

r	W,R=1, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	9.58E-01	9.58E-01	9.57E-01
1.1	9.15E-01	9.15E-01	9.14E-01
1.15	8.72E-01	8.71E-01	8.70E-01
1.2	8.27E-01	8.27E-01	8.26E-01
1.25	7.82E-01	7.81E-01	7.81E-01
1.3	7.37E-01	7.36E-01	7.35E-01
1.35	6.90E-01	6.89E-01	6.88E-01
1.4	6.43E-01	6.42E-01	6.41E-01
1.45	5.95E-01	5.93E-01	5.93E-01
1.5	5.46E-01	5.44E-01	5.44E-01
1.55	4.96E-01	4.94E-01	4.94E-01
1.6	4.45E-01	4.43E-01	4.43E-01
1.65	3.93E-01	3.92E-01	3.91E-01
1.7	3.40E-01	3.39E-01	3.38E-01
1.75	2.86E-01	2.85E-01	2.85E-01
1.8	2.31E-01	2.30E-01	2.30E-01
1.85	1.75E-01	1.74E-01	1.74E-01
1.9	1.18E-01	1.17E-01	1.17E-01
1.95	5.94E-02	5.92E-02	5.91E-02
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.22 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=0.1.

r	W,R=5, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.065856721	1.069993623	1.073722673
1.1	1.117876468	1.125289494	1.131918387
1.15	1.156484701	1.166433423	1.175273643
1.2	1.182052454	1.193897244	1.20437166
1.25	1.194904172	1.20809182	1.219712869
1.3	1.195324801	1.209377244	1.221730209
1.35	1.183566062	1.198070921	1.210800867
1.4	1.159851867	1.17445401	1.187255448
1.45	1.124382827	1.13877661	1.151385223
1.5	1.077339871	1.091261918	1.103447887
1.55	1.018887026	1.032109589	1.043672152
1.6	9.49E-01	9.61E-01	9.72E-01
1.65	8.68E-01	8.80E-01	8.89E-01
1.7	7.76E-01	7.87E-01	7.95E-01
1.75	6.74E-01	6.82E-01	6.90E-01
1.8	5.60E-01	5.67E-01	5.74E-01
1.85	4.36E-01	4.42E-01	4.46E-01
1.9	3.01E-01	3.05E-01	3.08E-01
1.95	1.56E-01	1.58E-01	1.59E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.23 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=5and suction parameter k=0.1.

r	W,R=9, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.162248384	1.171320774	1.179582306
1.1	1.301361055	1.317846078	1.3327031
1.15	1.417546201	1.439990746	1.460020943
1.2	1.51098679	1.538107702	1.562086439
1.25	1.58184228	1.612499505	1.63936455
1.3	1.630251287	1.66342732	1.692251221
1.35	1.656334778	1.691117938	1.721086059
1.4	1.660199373	1.69576928	1.726162114
1.45	1.641940298	1.677554725	1.707733506
1.5	1.601643691	1.636626481	1.666021365
1.55	1.539388045	1.573118192	1.601218461
1.6	1.455244727	1.487146896	1.51349277
1.65	1.349277614	1.378814444	1.402990167
1.7	1.221541964	1.248208447	1.269836406
1.75	1.072082656	1.095402803	1.114138497
1.8	9.01E-01	9.20E-01	9.36E-01
1.85	7.08E-01	7.23E-01	7.35E-01
1.9	4.94E-01	5.04E-01	5.13E-01
1.95	2.58E-01	2.63E-01	2.67E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.24 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=9 and suction parameter k=0.1.

r	W,R=13, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.247441276	1.261490809	1.274530019
1.1	1.46577808	1.491639392	1.515365432
1.15	1.654526953	1.690212255	1.722559254
1.2	1.813242948	1.856966184	1.896105099
1.25	1.941513469	1.991653093	2.035952991
1.3	2.038954078	2.09402345	2.14202077
1.35	2.105205876	2.163828666	2.214202417
1.4	2.139933947	2.200822676	2.252374176
1.45	2.142826302	2.204762878	2.256399028
1.5	2.113592747	2.175410567	2.226129893
1.55	2.051963122	2.112530924	2.161411829
1.6	1.957684485	2.015892634	2.062083396
1.65	1.830516975	1.885267159	1.927977333
1.7	1.670228193	1.72042768	1.758920634
1.75	1.476586084	1.521147737	1.554734113
1.8	1.24935024	1.287199538	1.315231533
1.85	9.89E-01	1.018351953	1.040218494
1.9	6.94E-01	7.14E-01	7.29E-01
1.95	3.64E-01	3.75E-01	3.83E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.25 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=13and suction parameter k=0.1.

r	W,R=17, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.321442106	1.340677074	1.35844852
1.1	1.610529294	1.646521916	1.679238743
1.15	1.865735271	1.91632188	1.961427179
1.2	2.085647044	2.148908752	2.204071326
1.25	2.26896098	2.343157625	2.406237816
1.3	2.414476953	2.497985344	2.567007657
1.35	2.521090953	2.612348663	2.685479452
1.4	2.587787626	2.685242187	2.760771225
1.45	2.613634297	2.715696143	2.792021211
1.5	2.597777045	2.702773996	2.778387869
1.55	2.539437928	2.645569929	2.719049243
1.6	2.437911087	2.543206165	2.613201784
1.65	2.292554691	2.394830124	2.460058668
1.7	2.102775671	2.19961139	2.258847666
1.75	1.868004689	1.956738442	2.008808574
1.8	1.587659118	1.665415112	1.709190251
1.85	1.262019591	1.324856704	1.359247418
1.9	8.90E-01	9.34E-01	9.58E-01
1.95	4.69E-01	4.93E-01	5.05E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.26 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=17 and suction parameter k=0.1.

r	W,R=21, k=0.1, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.389003296	1.409161237	1.431435979
1.1	1.745148978	1.782553291	1.824053808
1.15	2.065974507	2.117814828	2.175690854
1.2	2.349113267	2.412659479	2.484227632
1.25	2.59227264	2.664872095	2.747590845
1.3	2.793219403	2.872304999	2.96375459
1.35	2.949772004	3.03287428	3.130740267
1.4	3.059796457	3.144556123	3.246615627
1.45	3.121201305	3.205383145	3.309493198
1.5	3.131928099	3.21344072	3.317528224
1.55	3.089936685	3.166863224	3.268916191
1.6	2.993188021	3.063830141	3.161889943
1.65	2.83962982	2.902561965	2.994716409
1.7	2.627191211	2.681315797	2.765692885
1.75	2.353791903	2.398380586	2.473142885
1.8	2.017370008	2.052071899	2.115411524
1.85	1.615101475	1.640726154	1.690860576
1.9	1.145739073	1.162694114	1.197859755
1.95	6.08E-01	6.16E-01	6.35E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.27 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21and suction parameter k=0.1.

r	W,R=1, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	9.50E-01	1.217014897	-3.01E-01
1.1	9.01E-01	1.15010875	-7.49E-01
1.15	8.52E-01	1.082561304	-8.30E-01
1.2	8.03E-01	1.016034173	-7.66E-01
1.25	7.54E-01	9.50E-01	-6.55E-01
1.3	7.06E-01	8.85E-01	-5.36E-01
1.35	6.58E-01	8.21E-01	-4.26E-01
1.4	6.09E-01	7.57E-01	-3.30E-01
1.45	5.61E-01	6.94E-01	-2.47E-01
1.5	5.12E-01	6.31E-01	-1.80E-01
1.55	4.63E-01	5.68E-01	-1.24E-01
1.6	4.13E-01	5.05E-01	-8.24E-02
1.65	3.64E-01	4.42E-01	-4.86E-02
1.7	3.13E-01	3.80E-01	-2.65E-02
1.75	2.63E-01	3.17E-01	-1.24E-02
1.8	2.11E-01	2.54E-01	-1.32E-03
1.85	1.59E-01	1.91E-01	1.28E-03
1.9	1.07E-01	1.28E-01	3.96E-03
1.95	5.38E-02	6.40E-02	3.42E-03
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.28 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1and suction parameter k=-0.3.

r	W,R=5, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.049938672	2.324268335	-1.078499355
1.1	1.082513269	2.260524325	-1.815176056
1.15	1.099833626	2.149338502	-1.933179915
1.2	1.103612071	2.0379059	-1.789191842
1.25	1.095251789	1.925649993	-1.55200309
1.3	1.075912563	1.812271507	-1.297403769
1.35	1.046560485	1.697499657	-1.056830055
1.4	1.008005676	1.581098121	-8.42E-01
1.45	9.61E-01	1.462859005	-6.55E-01
1.5	9.06E-01	1.342598794	-4.96E-01
1.55	8.43E-01	1.220154764	-3.61E-01
1.6	7.74E-01	1.095381924	-2.51E-01
1.65	6.98E-01	9.68E-01	-1.61E-01
1.7	6.15E-01	8.38E-01	-9.21E-02
1.75	5.27E-01	7.06E-01	-4.06E-02
1.8	4.32E-01	5.71E-01	-3.44E-03
1.85	3.32E-01	4.32E-01	1.54E-02
1.9	2.27E-01	2.91E-01	2.26E-02
1.95	1.16E-01	1.47E-01	1.72E-02
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.29 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=5 and suction parameter k=-0.3.

r	W,R=9, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.172747202	2.827795742	-2.844237355
1.1	1.296900476	2.819226674	-4.197369047
1.15	1.381136177	2.689823418	-4.334096559
1.2	1.432132891	2.556587473	-3.937099493
1.25	1.455091483	2.421861385	-3.353476761
1.3	1.454105301	2.285053124	-2.74791936
1.35	1.432427396	2.145727864	-2.190573665
1.4	1.392665972	2.003526257	-1.705612067
1.45	1.336929112	1.858151812	-1.295871604
1.5	1.266933167	1.709357822	-9.57E-01
1.55	1.184084793	1.556937668	-6.80E-01
1.6	1.089543633	1.400717149	-4.59E-01
1.65	9.84E-01	1.24054841	-2.84E-01
1.7	8.69E-01	1.076305029	-1.52E-01
1.75	7.45E-01	9.08E-01	-5.72E-02
1.8	6.11E-01	7.35E-01	7.16E-03
1.85	4.70E-01	5.58E-01	4.03E-02
1.9	3.21E-01	3.77E-01	4.89E-02
1.95	1.64E-01	1.91E-01	3.49E-02
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.30 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=9 and suction parameter k=-0.3.

r	W,R=13, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.312775462	3.112350955	-5.737593352
1.1	1.531155115	3.18139109	-8.102570788
1.15	1.676638699	3.043346747	-8.214177532
1.2	1.764888248	2.893536 21	-7.309759748
1.25	1.807474572	2.742466666	-6.063566774
1.3	1.813056659	2.589302047	-4.810672664
1.35	1.788191224	2.433292903	-3.694794522
1.4	1.737896916	2.273875508	-2.760137207
1.45	1.666053176	2.110616214	-2.003932589
1.5	1.575685916	1.943177784	-1.406135425
1.55	1.469174665	1.771295179	-9.41E-01
1.6	1.348404449	1.594758186	-5.86E-01
1.65	1.214878322	1.413398657	-3.22E-01
1.7	1.069801556	1.227080968	-1.31E-01
1.75	9.14E-01	1.035694721	-3.44E-03
1.8	7.49E-01	8.39E-01	7.45E-02
1.85	5.74E-01	6.37E-01	1.06E-01
1.9	3.91E-01	4.30E-01	1.01E-01
1.95	1.99E-01	2.18E-01	6.48E-02
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.31 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=13 and suction parameter k=-0.3.

r	W,R=17, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.463596926	3.275098141	-9.75011496
1.1	1.772567098	3.436544244	-13.48800807
1.15	1.969094553	3.297337776	-13.43023877
1.2	2.082237381	3.132598531	-11.63111175
1.25	2.132347133	2.967461436	-9.285474416
1.3	2.133895828	2.801276666	-7.009071782
1.35	2.097318809	2.632662878	-5.06264473
1.4	2.03023728	2.460665332	-3.511827881
1.45	1.938282591	2.284608436	-2.329287385
1.5	1.825660418	2.104005082	-1.454582137
1.55	1.69554233	1.918498457	-8.23E-01
1.6	1.550341069	1.727823683	-3.78E-01
1.65	1.391906421	1.531781897	-7.52E-02
1.7	1.221666122	1.330222304	1.19E-01
1.75	1.040728283	1.123029454	2.27E-01
1.8	8.50E-01	9.10E-01	2.71E-01
1.85	6.50E-01	6.91E-01	2.60E-01
1.9	4.41E-01	4.67E-01	2.07E-01
1.95	2.25E-01	2.36E-01	1.18E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.31 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=17 and suction parameter k=-0.3.

r	W,R=21, k=-0.3, λ=1		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.619511459	3.367540103	-15.02039752
1.1	2.010901883	3.632009909	-20.49112182
1.15	2.245997074	3.496926129	-19.99407382
1.2	2.371548701	3.315885733	-16.73769626
1.25	2.418579195	3.136108327	-12.70296512
1.3	2.408039371	2.95795776	-8.937501126
1.35	2.354308204	2.778798648	-5.870223551
1.4	2.267401159	2.596881077	-3.575152637
1.45	2.154389401	2.411082836	-1.958231507
1.5	2.020327473	2.220660956	-8.72E-01
1.55	1.868869257	2.025111845	-1.73E-01
1.6	1.702682959	1.824087443	2.54E-01
1.65	1.523734491	1.617343344	4.92E-01
1.7	1.333483425	1.404705703	6.03E-01
1.75	1.133020072	1.18604951	6.20E-01
1.8	9.23E-01	9.61E-01	5.75E-01
1.85	7.05E-01	7.30E-01	4.79E-01
1.9	4.78E-01	4.93E-01	3.46E-01
1.95	2.43E-01	2.50E-01	1.85E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.33 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=21 and suction parameter k=-0.3.

r	W, R=1, α=0.0				
	K=0.9	K=0.5	K=0.1	K=-0.3	K=-0.7
1	1	1	1	1	1
1.05	9.72E-01	9.65E-01	9.58E-01	9.50E-01	9.45E-01
1.1	9.42E-01	9.29E-01	9.15E-01	9.01E-01	8.92E-01
1.15	9.10E-01	8.91E-01	8.72E-01	8.52E-01	8.39E-01
1.2	8.75E-01	8.51E-01	8.27E-01	8.03E-01	7.88E-01
1.25	8.38E-01	8.10E-01	7.82E-01	7.54E-01	7.37E-01
1.3	7.98E-01	7.67E-01	7.37E-01	7.06E-01	6.86E-01
1.35	7.56E-01	7.23E-01	6.90E-01	6.58E-01	6.36E-01
1.4	7.12E-01	6.77E-01	6.43E-01	6.09E-01	5.87E-01
1.45	6.66E-01	6.29E-01	5.95E-01	5.61E-01	5.38E-01
1.5	6.17E-01	5.80E-01	5.46E-01	5.12E-01	4.89E-01
1.55	5.66E-01	5.29E-01	4.96E-01	4.63E-01	4.40E-01
1.6	5.12E-01	4.77E-01	4.45E-01	4.13E-01	3.92E-01
1.65	4.56E-01	4.23E-01	3.93E-01	3.64E-01	3.43E-01
1.7	3.98E-01	3.68E-01	3.40E-01	3.13E-01	2.95E-01
1.75	3.37E-01	3.10E-01	2.86E-01	2.63E-01	2.46E-01
1.8	2.75E-01	2.52E-01	2.31E-01	2.11E-01	1.97E-01
1.85	2.09E-01	1.91E-01	1.75E-01	1.59E-01	1.48E-01
1.9	1.42E-01	1.29E-01	1.18E-01	1.07E-01	9.93E-02
1.95	7.21E-02	6.54E-02	5.94E-02	5.38E-02	4.98E-02
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4.1.34 variation of axial velocity with radial distance r for different values of Suction parameters k and for fixed value of Reynolds number R=1 and viscoelastic parameter α=0.0.

r	W, k=0.1, α=0.2					
	R=1	R=5	R=9	R=13	R=17	R=21
1	1	1	1	1	1	1
1.05	9.58E-01	1.069993623	1.171320774	1.261490809	1.340677074	1.409161237
1.1	9.15E-01	1.125289494	1.317846078	1.491639392	1.646521916	1.782553291
1.15	8.71E-01	1.166433423	1.439990746	1.690212255	1.91632188	2.117814828
1.2	8.27E-01	1.193897244	1.538107702	1.856966184	2.148908752	2.412659479
1.25	7.81E-01	1.20809182	1.612499505	1.991653093	2.343157625	2.664872095
1.3	7.36E-01	1.209377244	1.66342732	2.09402345	2.497985344	2.872304999
1.35	6.89E-01	1.198070921	1.691117938	2.163828666	2.612348663	3.03287428
1.4	6.42E-01	1.17445401	1.69576928	2.200822676	2.685242187	3.144556123
1.45	5.93E-01	1.13877661	1.677554725	2.204762878	2.715696143	3.205383145
1.5	5.44E-01	1.091261918	1.636626481	2.175410567	2.702773996	3.21344072
1.55	4.94E-01	1.032109589	1.573118192	2.112530924	2.645569929	3.166863224
1.6	4.43E-01	9.61E-01	1.487146896	2.015892634	2.543206165	3.063830141
1.65	3.92E-01	8.80E-01	1.378814444	1.885267159	2.394830124	2.902561965
1.7	3.39E-01	7.87E-01	1.248208447	1.72042768	2.19961139	2.681315797
1.75	2.85E-01	6.82E-01	1.095402803	1.521147737	1.956738442	2.398380586
1.8	2.30E-01	5.67E-01	9.20E-01	1.287199538	1.665415112	2.052071899
1.85	1.74E-01	4.42E-01	7.23E-01	1.018351953	1.324856704	1.640726154
1.9	1.17E-01	3.05E-01	5.04E-01	7.14E-01	9.34E-01	1.162694114
1.95	5.92E-02	1.58E-01	2.63E-01	3.75E-01	4.93E-01	6.16E-01
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4.1.35 variation of axial velocity with radial distance r for different values of Reynolds number and for fixed value of suction parameter k=0.1 and viscoelastic parameter α=0.2.

r	W, k=0.9, α=0.0					
	R=1	R=5	R=9	R=13	R=17	R=21
1	1	1	1	1	1	1
1.05	9.72E-01	1.058746179	1.043373273	1.07032343	1.06615968	1.064158675
1.1	9.42E-01	1.114249911	1.08300165	1.144133409	1.13562964	1.131535442
1.15	9.10E-01	1.165335318	1.1203181	1.220879755	1.2081992	1.201976628
1.2	8.75E-01	1.21082411	1.156008217	1.299976031	1.28366231	1.275364378
1.25	8.38E-01	1.24951415	1.19011255	1.380734957	1.36178259	1.351604766
1.3	7.98E-01	1.280163085	1.222122128	1.4622794	1.4422332	1.43059427
1.35	7.56E-01	1.301475798	1.251064659	1.543427421	1.52450666	1.512162024
1.4	7.12E-01	1.312094774	1.275578901	1.622548915	1.60778656	1.59598155
1.45	6.66E-01	1.310592657	1.293974404	1.697389849	1.69076749	1.68143647
1.5	6.17E-01	1.295466479	1.304273045	1.764857975	1.77140139	1.76740876
1.55	5.66E-01	1.265133165	1.304227099	1.820761163	1.84653749	1.851934448
1.6	5.12E-01	1.217925985	1.291305669	1.85948642	1.9114092	1.931639
1.65	4.56E-01	1.152091689	1.262636503	1.873604137	1.95890483	2.000820768
1.7	3.98E-01	1.065788029	1.214883096	1.853378388	1.97853953	2.049991016
1.75	3.37E-01	9.57E-01	1.144027243	1.786160216	1.95502263	2.0635956
1.8	2.75E-01	8.24E-01	1.044924392	1.655636563	1.86628752	2.016524508
1.85	2.09E-01	6.64E-01	8.88E-01	1.442279252	1.68055845	1.868218771
1.9	1.42E-01	4.75E-01	6.69E-01	1.119142415	1.35325232	1.556034068
1.95	7.21E-02	2.54E-01	3.77E-01	6.53E-01	8.22E-01	9.83E-01
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4.1.36 variation of axial velocity with radial distance r for different values of Reynolds number and for fixed value of suction parameter k=0.9 and viscoelastic parameter α=0.0.

r	W, k=0.9, α=0.2					
	R=1	R=5	R=9	R=13	R=17	R=21
1	1	1	1	1	1	1
1.05	9.74E-01	1.126872264	1.030800546	1.44491036	1.014142546	9.93E-01
1.1	9.44E-01	1.251212694	1.090545498	2.04002049	1.075201984	9.91E-01
1.15	9.12E-01	1.371029182	1.185948962	2.74341594	1.195342921	1.00955769
1.2	8.78E-01	1.484179122	1.322752826	3.49088111	1.380113471	1.06704564
1.25	8.41E-01	1.58836463	1.505182065	4.19742563	1.625887013	1.17820794
1.3	8.02E-01	1.68112818	1.735335356	4.76108916	1.917897661	1.35119713
1.35	7.60E-01	1.759848599	2.012515923	5.06933457	2.22928236	1.58364348
1.4	7.15E-01	1.821737411	2.332510286	5.00825157	2.521549957	1.85960584
1.45	6.69E-01	1.863835556	2.686825992	4.47466697	2.746864822	2.14808733
1.5	6.20E-01	1.883010532	3.061903686	3.39108734	2.852449781	2.40392701
1.55	5.68E-01	1.875954099	3.438324339	1.72318058	2.787271558	2.57178109
1.6	5.15E-01	1.839180696	3.790039315	-5.01E-01	2.51096408	2.59366569
1.65	4.59E-01	1.769026875	4.083659923	-3.1693819	2.004666222	2.42012904
1.7	4.00E-01	1.661652161	4.277854687	-6.0678853	1.283099868	2.02454513
1.75	3.39E-01	1.513041999	4.322917914	-8.8600038	4.07E-01	1.41928478
1.8	2.76E-01	1.319013773	4.160593654	-11.07423	-5.07E-01	6.72E-01
1.85	2.11E-01	1.075227388	3.724266928	-12.097651	-1.277510712	-8.34E-02
1.9	1.43E-01	7.77E-01	2.93967224	-11.18113	-1.658215121	-6.38E-01
1.95	7.27E-02	4.20E-01	1.726336448	-7.4605674	-1.345764519	-7.16E-01
2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 4.1.37 variation of axial velocity with radial distance r for different values of Reynolds number and for fixed value of suction parameter k=0.9 and viscoelastic parameter α=0.2.

r	W,R=1, k=0.9, λ=3		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.019741529	1.039827027	1.05959886
1.1	1.031867278	1.067905305	1.103156539
1.15	1.036429556	1.084940635	1.132096427
1.2	1.033509968	1.091506766	1.147557285
1.25	1.023192802	1.088074845	1.150460532
1.3	1.005543227	1.075035177	1.141559498
1.35	9.81E-01	1.052713523	1.121476022
1.4	9.48E-01	1.021383445	1.090727963
1.45	9.09E-01	9.81E-01	1.049750134
1.5	8.62E-01	9.33E-01	9.99E-01
1.55	8.08E-01	8.75E-01	9.39E-01
1.6	7.47E-01	8.10E-01	8.69E-01
1.65	6.79E-01	7.37E-01	7.90E-01
1.7	6.03E-01	6.55E-01	7.03E-01
1.75	5.21E-01	5.65E-01	6.06E-01
1.8	4.31E-01	4.68E-01	5.02E-01
1.85	3.34E-01	3.63E-01	3.88E-01
1.9	2.30E-01	2.49E-01	2.67E-01
1.95	1.19E-01	1.29E-01	1.38E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.38 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1 and suction parameter k=0.9 for λ=3.

r	W,R=1, k=0.9, λ=5		
	α=0.0	α=0.2	α=0.4
1	1	1	1
1.05	1.067724089	1.106116217	1.143190178
1.1	1.122606831	1.191471059	1.257498298
1.15	1.164736399	1.257390259	1.345614089
1.2	1.194235784	1.30495177	1.409692011
1.25	1.211231023	1.335041335	1.451477615
1.3	1.215823832	1.348393548	1.472400089
1.35	1.208079552	1.34562262	1.473641028
1.4	1.188033938	1.327245723	1.456186217
1.45	1.155712787	1.293700894	1.42086507
1.5	1.111151912	1.245360907	1.368381003
1.55	1.054405582	1.182544108	1.299335076
1.6	9.86E-01	1.105522963	1.214244604
1.65	9.05E-01	1.014530867	1.113558018
1.7	8.12E-01	9.10E-01	9.98E-01
1.75	7.07E-01	7.91E-01	8.67E-01
1.8	5.90E-01	6.60E-01	7.22E-01
1.85	4.60E-01	5.14E-01	5.62E-01
1.9	3.19E-01	3.56E-01	3.88E-01
1.95	1.66E-01	1.85E-01	2.01E-01
2	0.00E+00	0.00E+00	0.00E+00

Table 4.1.31 variation of axial velocity with radial distance r for different values of viscoelastic parameter α=0.0, 0.2 and 0.4 and for fixed value of Reynolds number R=1 and suction parameter k=0.9 λ=5.

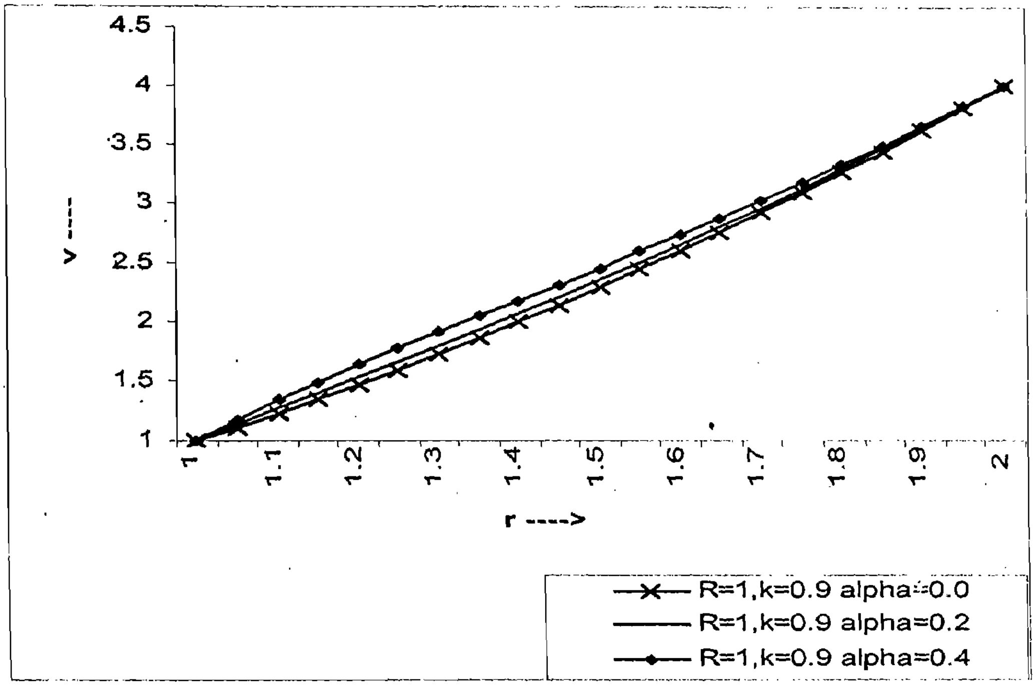


FIG. 4.1.1 variation of v with r for different values of α and fixed value of R=1 and K=0.9.

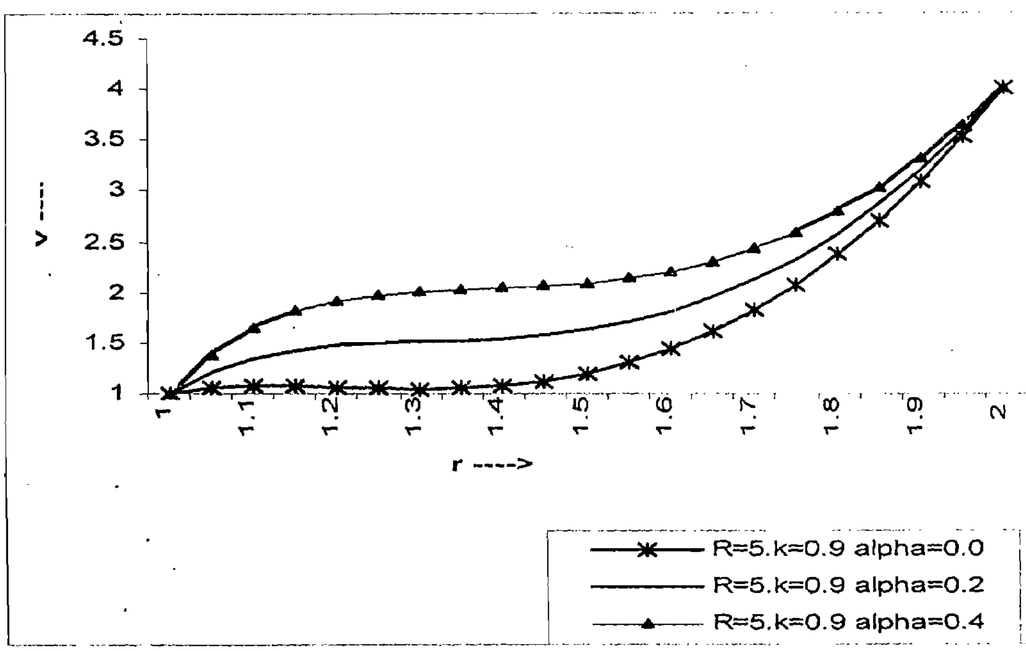


FIG. 4.1.2 variation of v with r for different values of α and fixed value of R=5 and K=0.9.

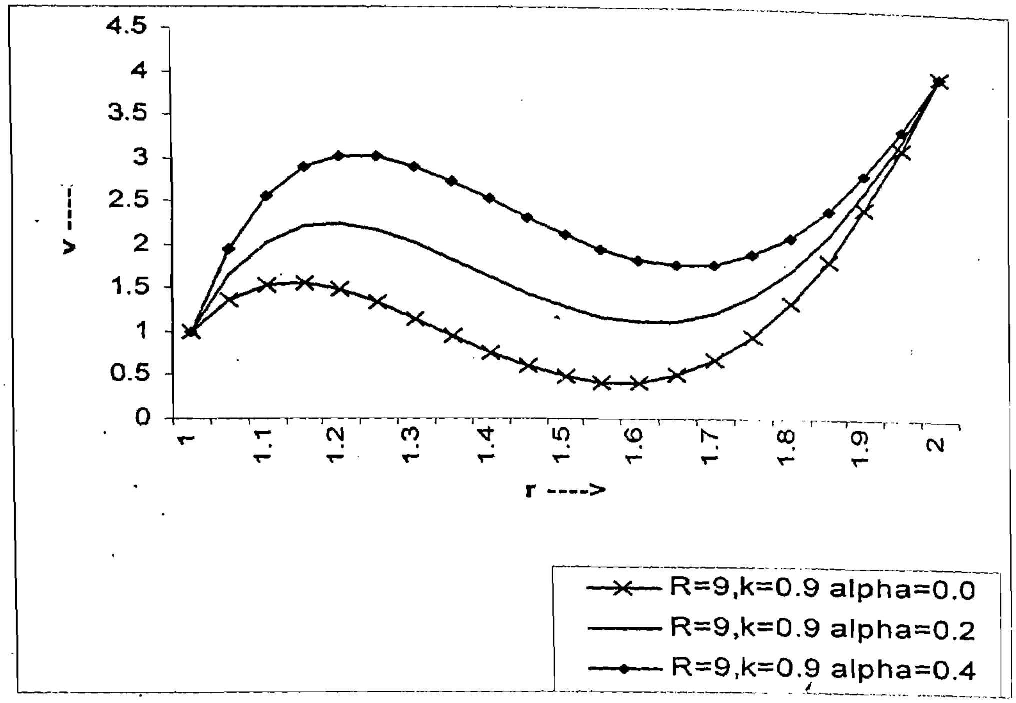


FIG. 4.1.3 variation of v with r for different values of α and fixed value of R=9 and K=0.9.

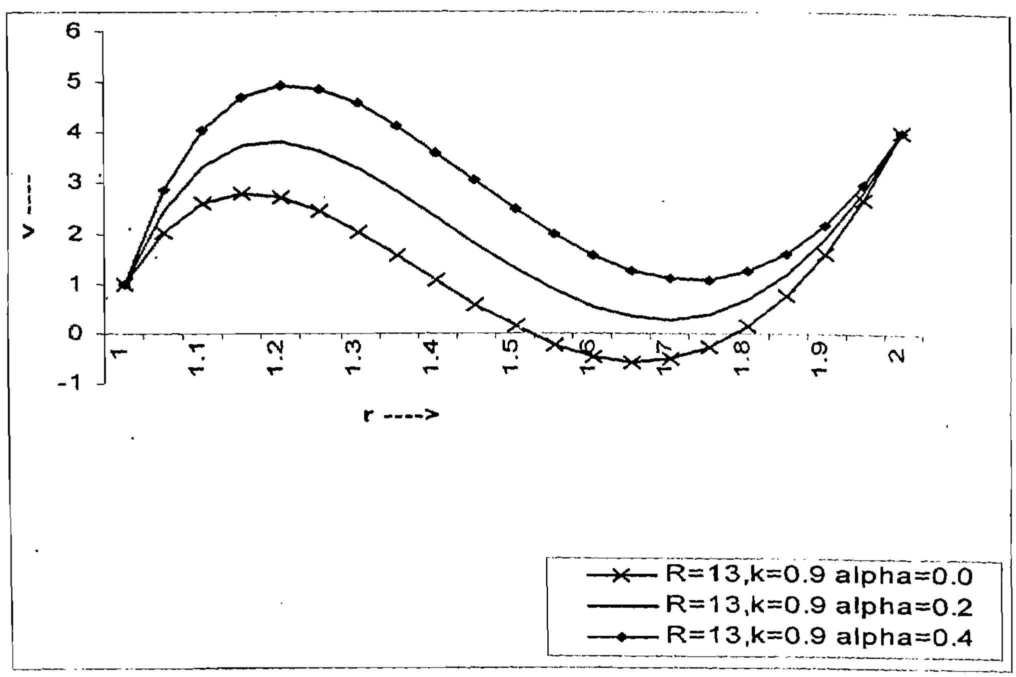


FIG. 4.1.4 variation of v with r for different values of α and fixed value of R=13 and K=0.9.

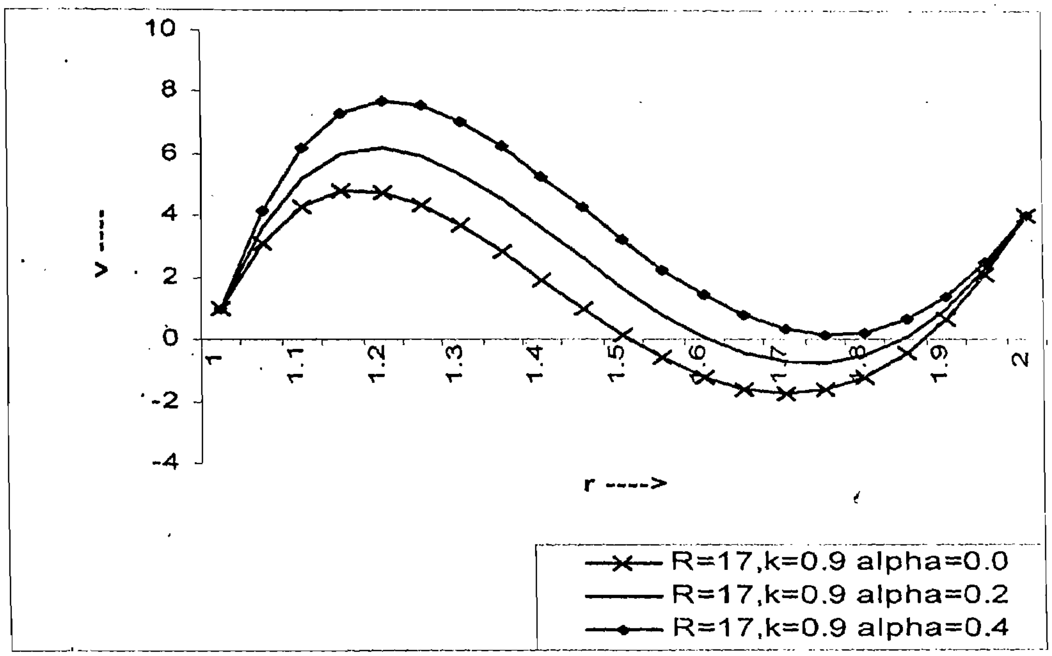


FIG. 4.1.5 variation of v with r for different values of α and fixed value of R=17 and K=0.9.

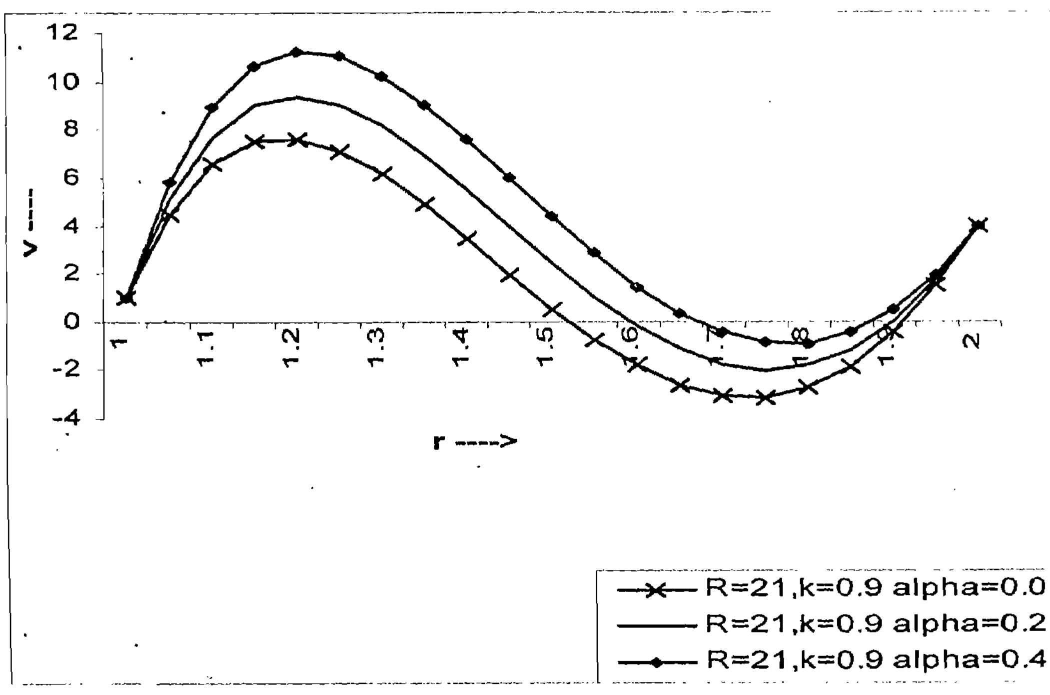


FIG. 4.1.6 variation of v with r for different values of α and fixed value of R=21 and K=0.9.

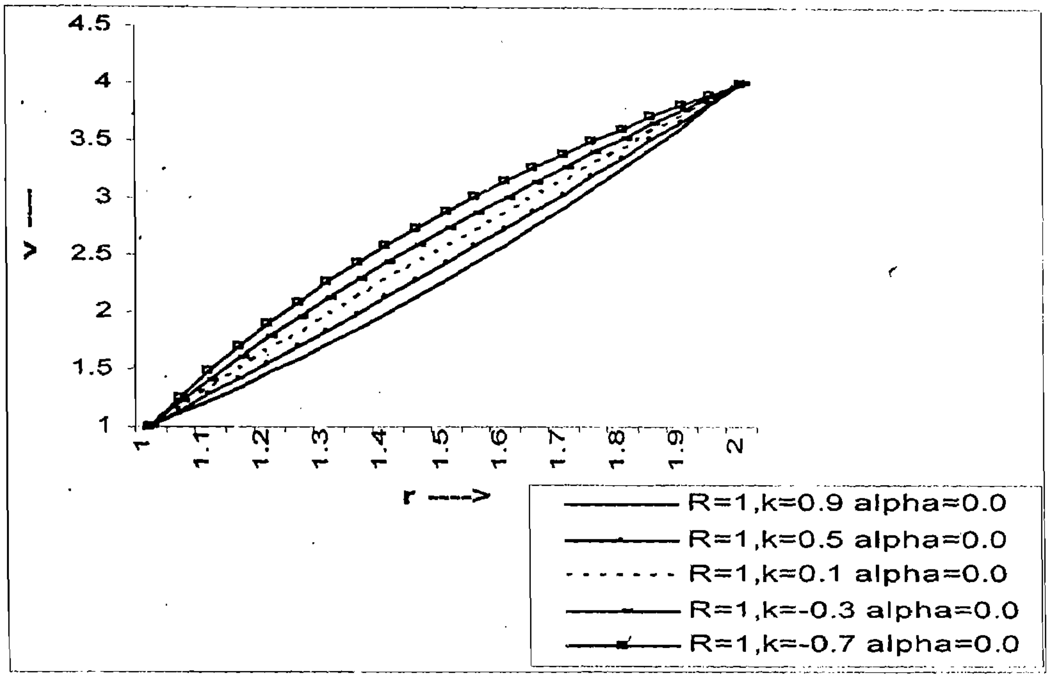


FIG 4.1.7 variation of angular velocity v with radial distance r for different values K and fixed value of R=1 and alpha=0.0

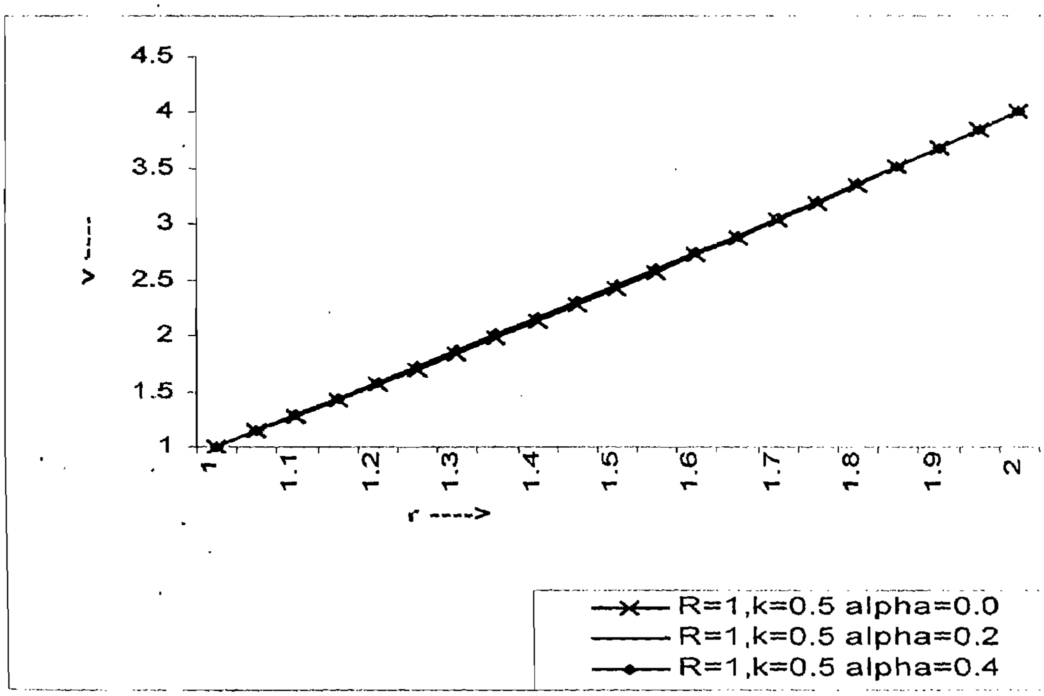


FIG. 4.1.8 variation of v with r for different values of α and fixed value of R=1 and K=0.5.

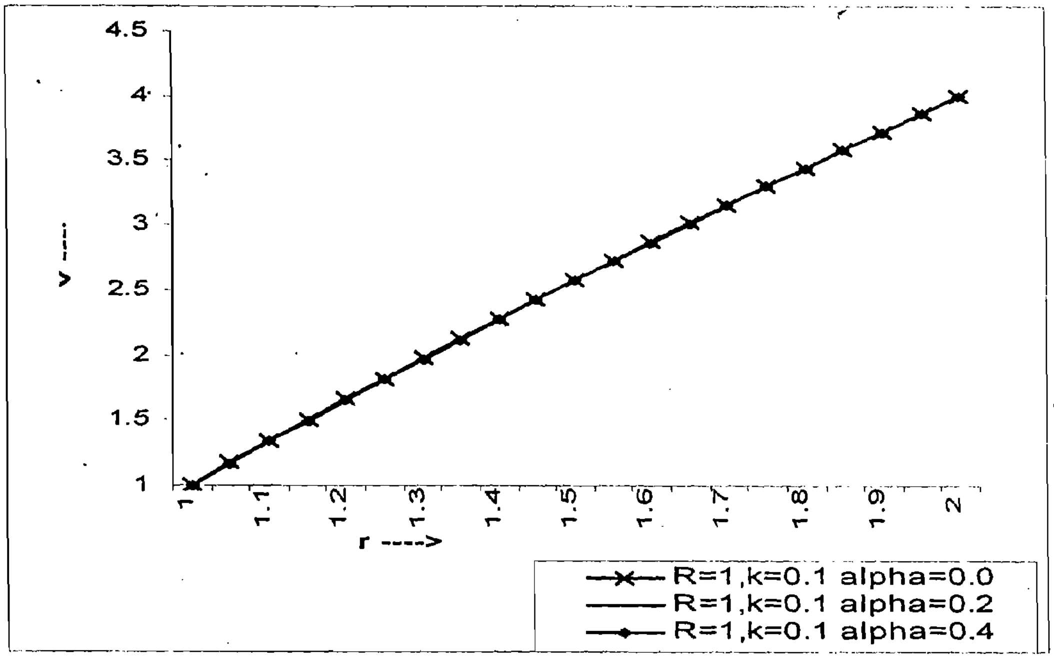


FIG. 4.1.9 variation of v with r for different values of α and fixed value of R=1 and K=0.1.

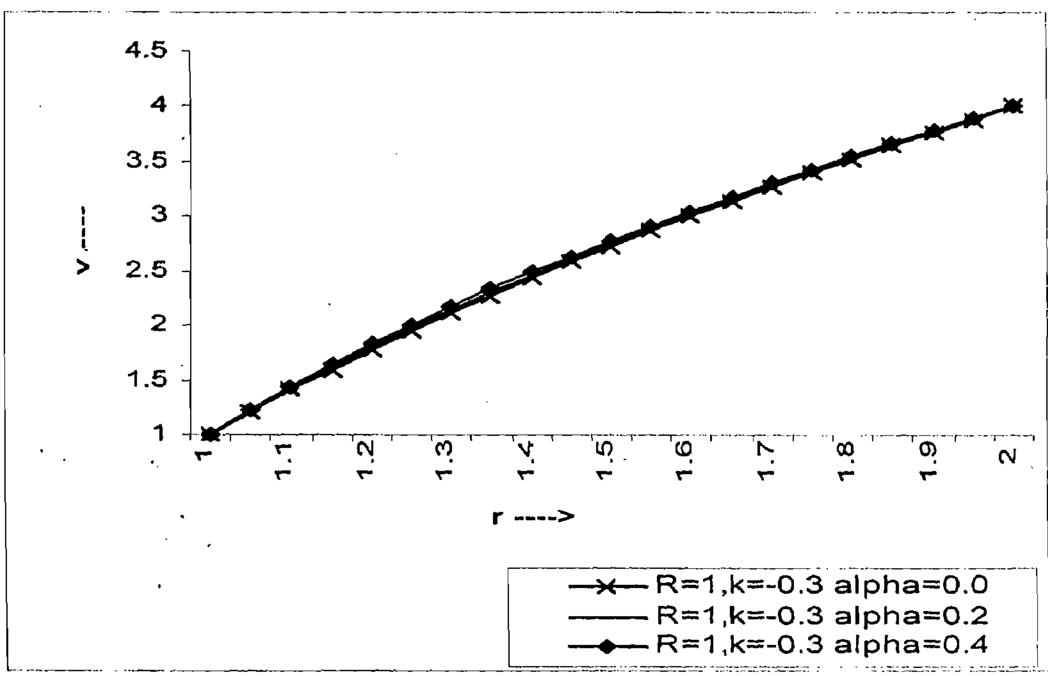


FIG. 4.1.10 variation of v with r for different values of α and fixed value of R=1 and K=-0.3.

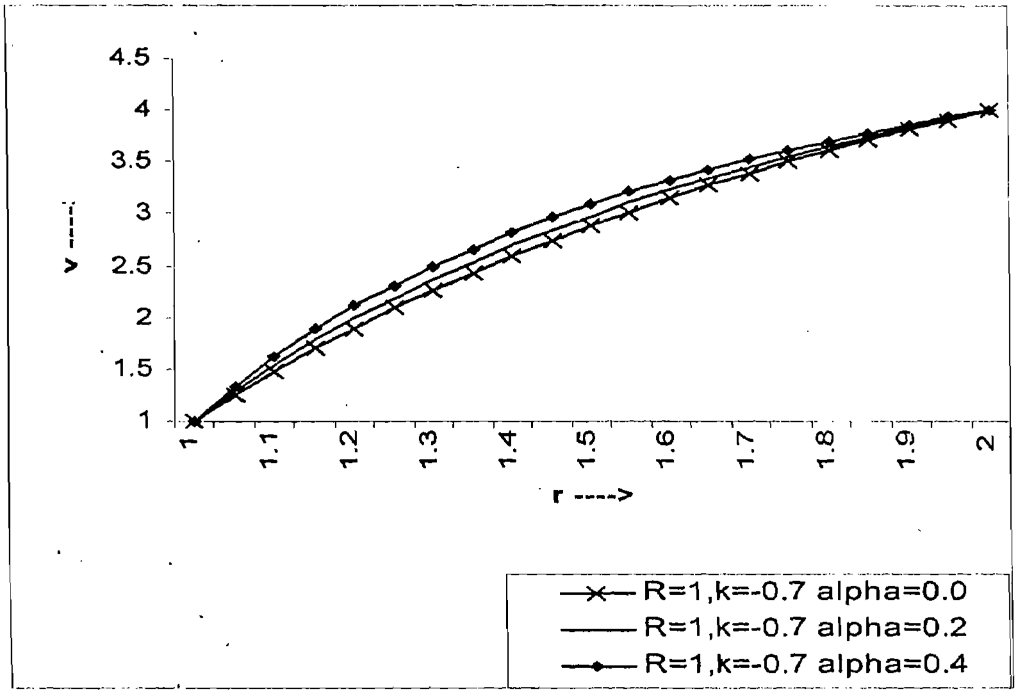


FIG. 4.1.11 variation of v with r for different values of α and fixed value of R=1 and K=-0.7.

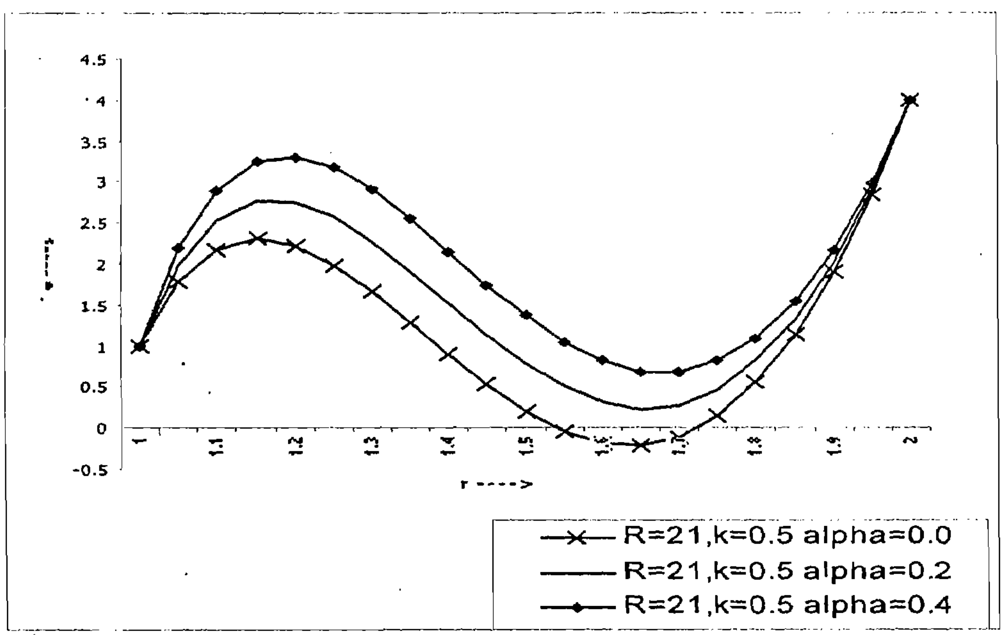


FIG. 4.1.12 variation of v with r for different values of α and fixed value of R=21 and K=0.5.

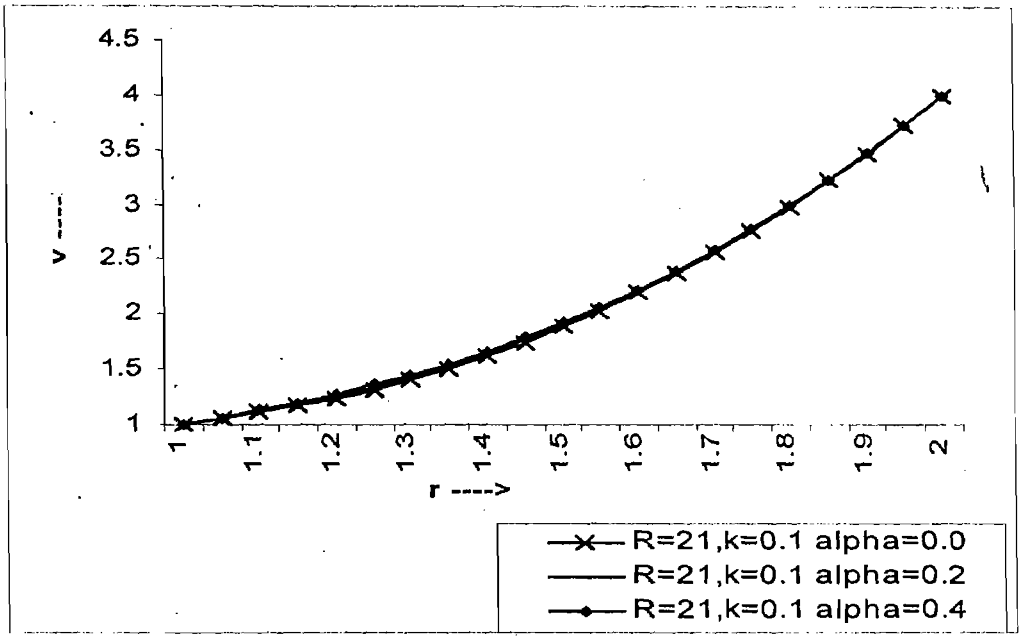


FIG. 4.1.13 variation of v with r for different values of α and fixed value of R=21 and K=0.1.

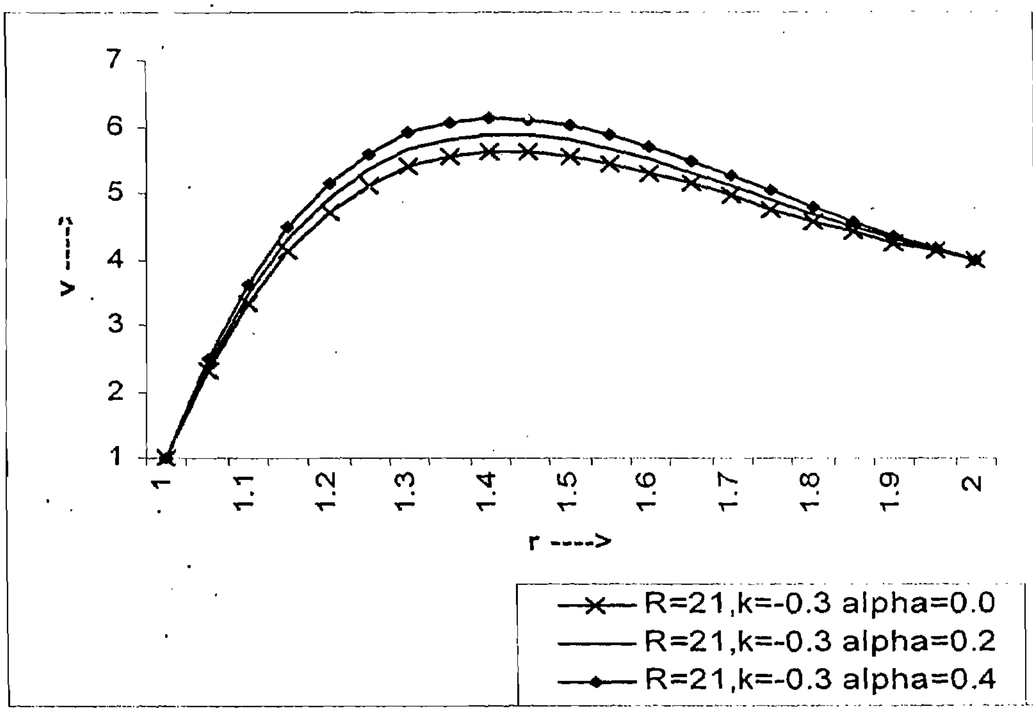


FIG. 4.1.14 variation of v with r for different values of α and fixed value of R=21 and K=-0.3.

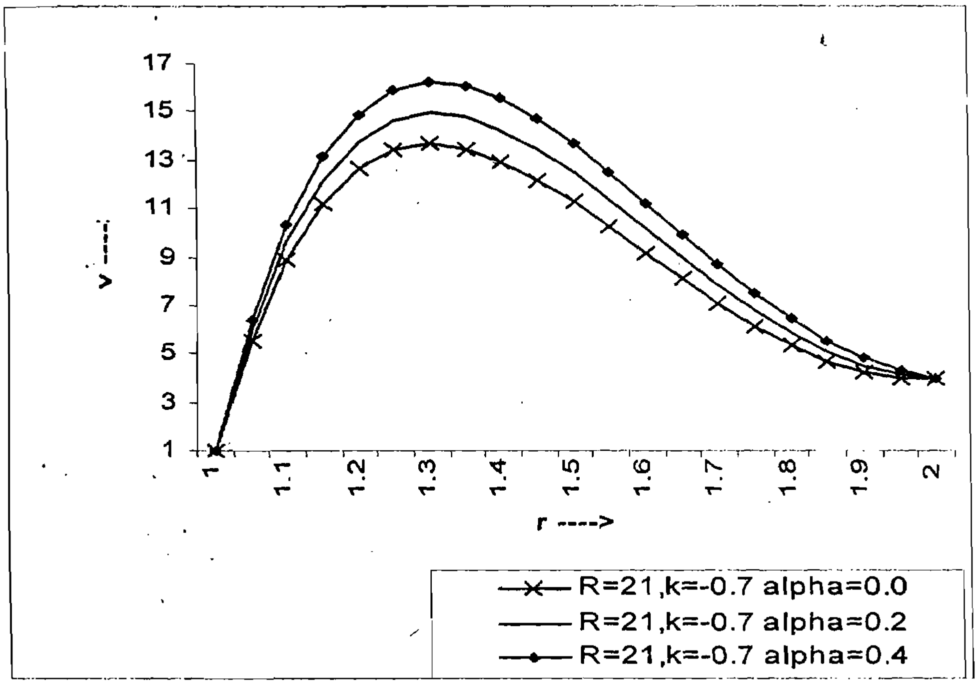


FIG. 4.1.15 variation of v with r for different values of α and fixed value of R=21 and K=-0.7.

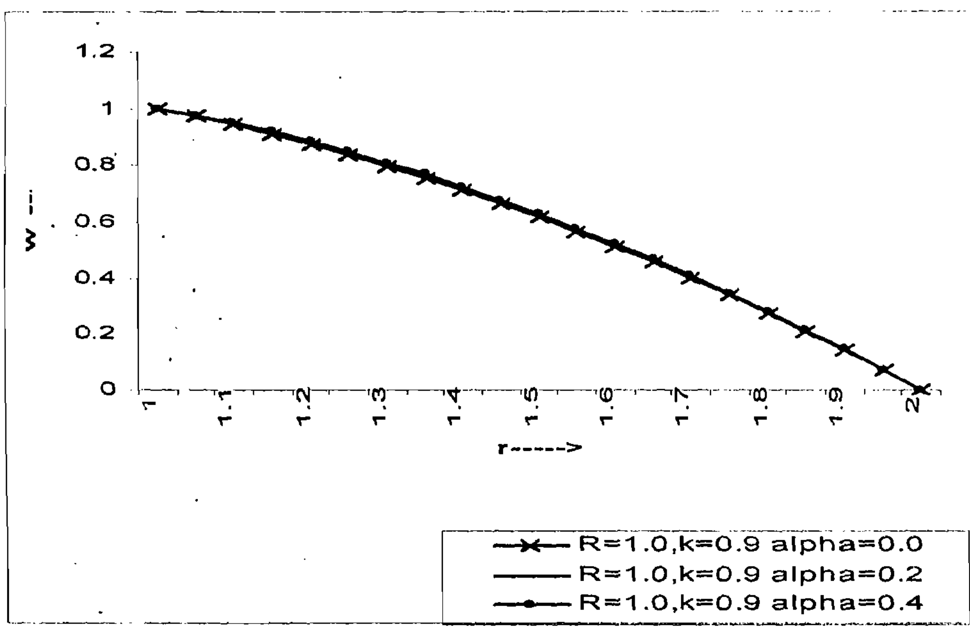


FIG. 4.1.16 variation of w with r for different values of α and fixed value of R=1 and K=0.9.

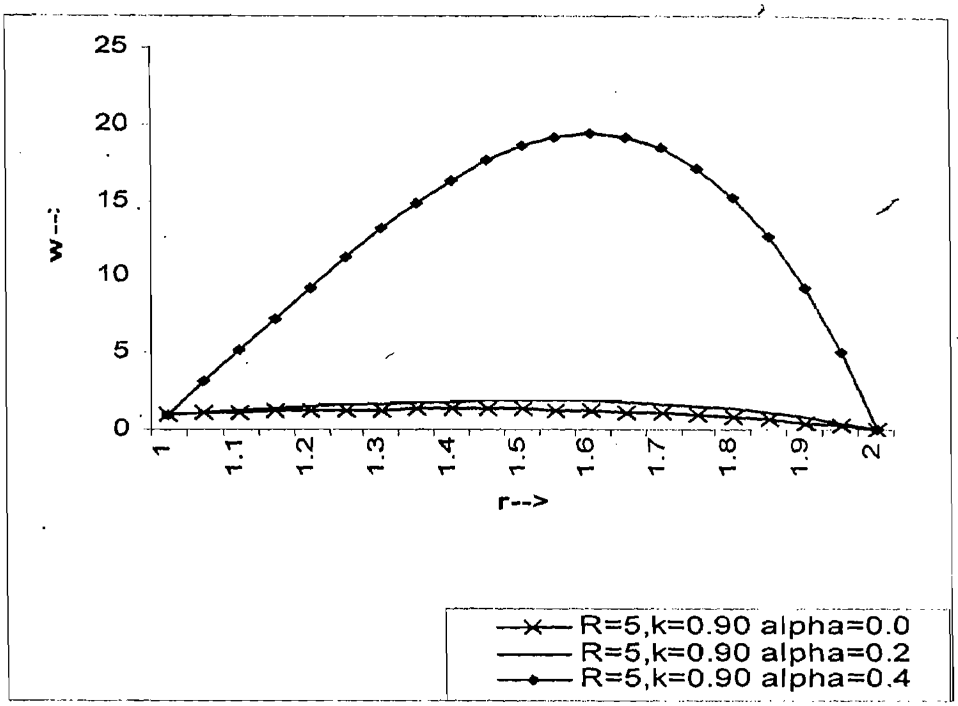


FIG. 4.1.17 variation of w with r for different values of α and fixed value of R=5 and K=0.9.

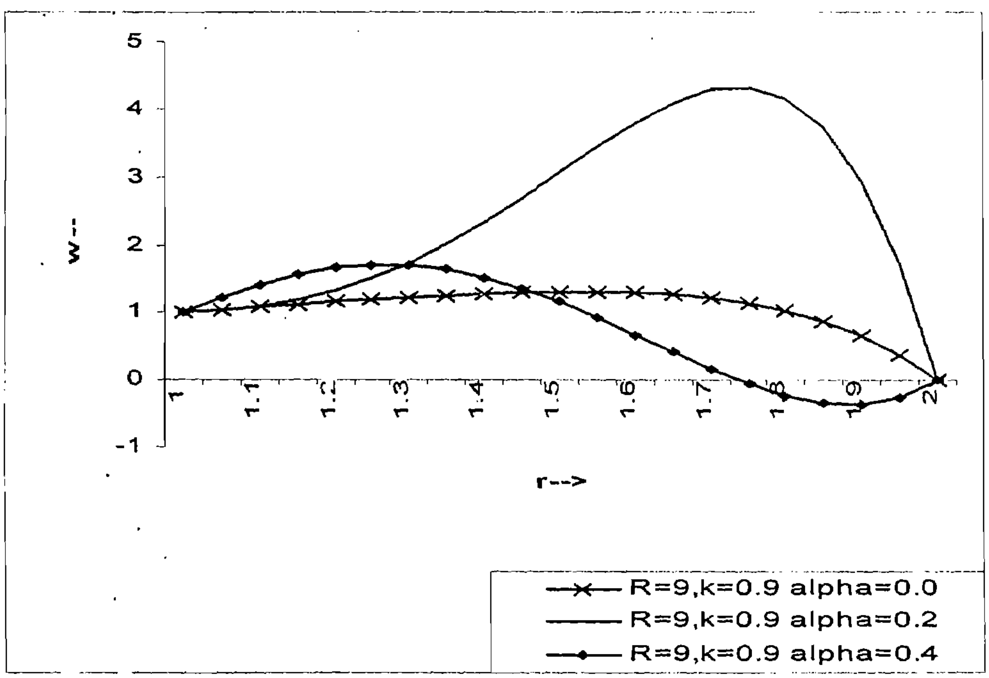


FIG. 4.1.18 variation of w with r for different values of α and fixed value of R=9 and K=0.9.

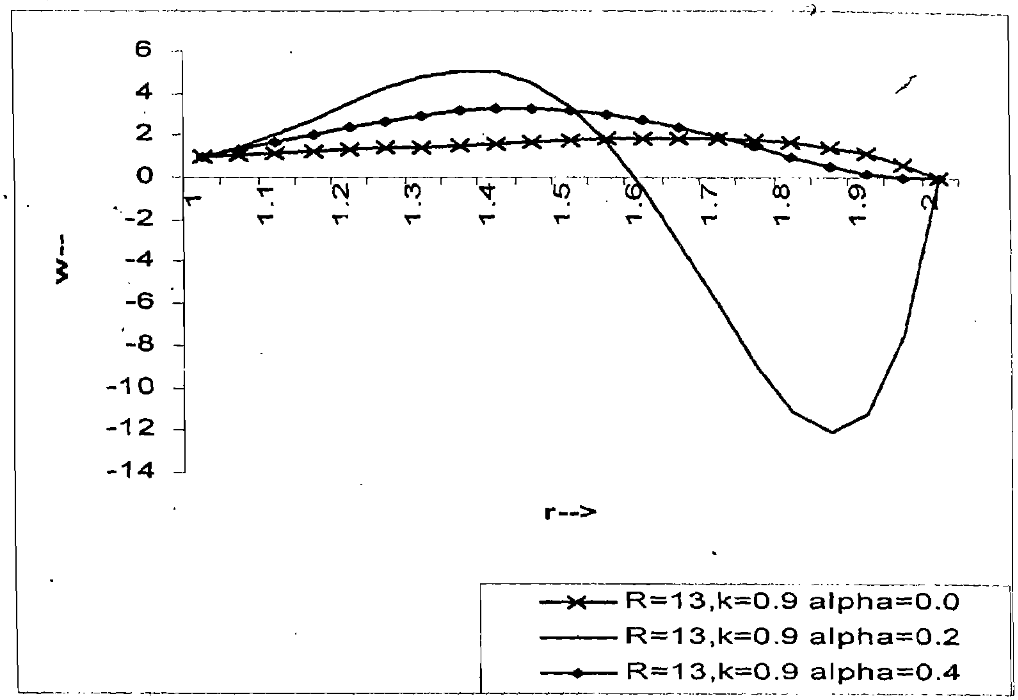


FIG. 4.1.19 variation of w with r for different values of α and fixed value of R=13 and K=0.9.

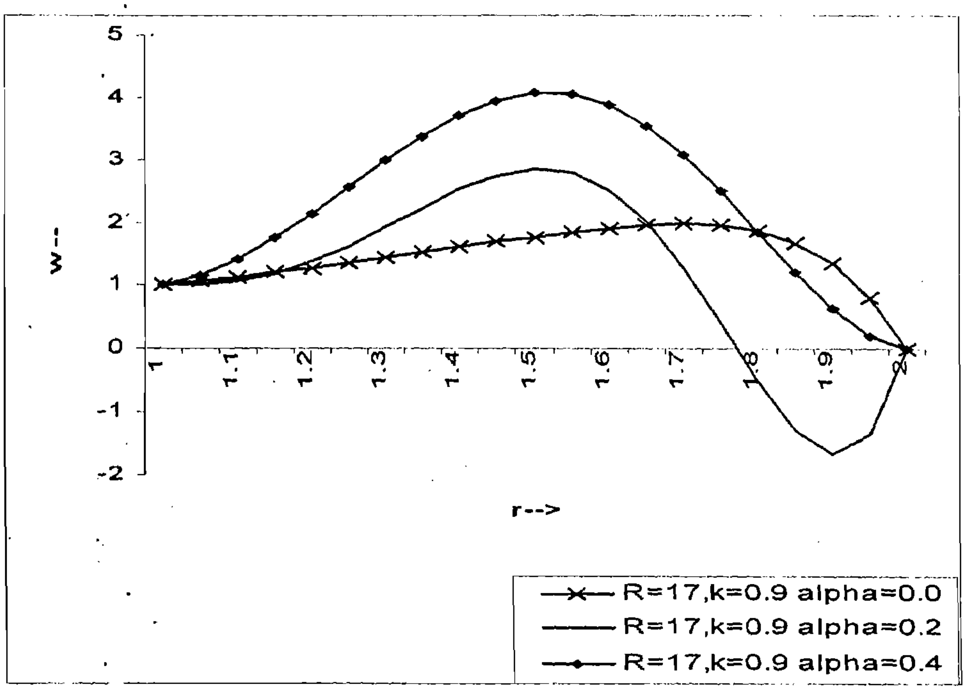


FIG. 4.1.20 variation of w with r for different values of α and fixed value of R=17 and K=0.9.

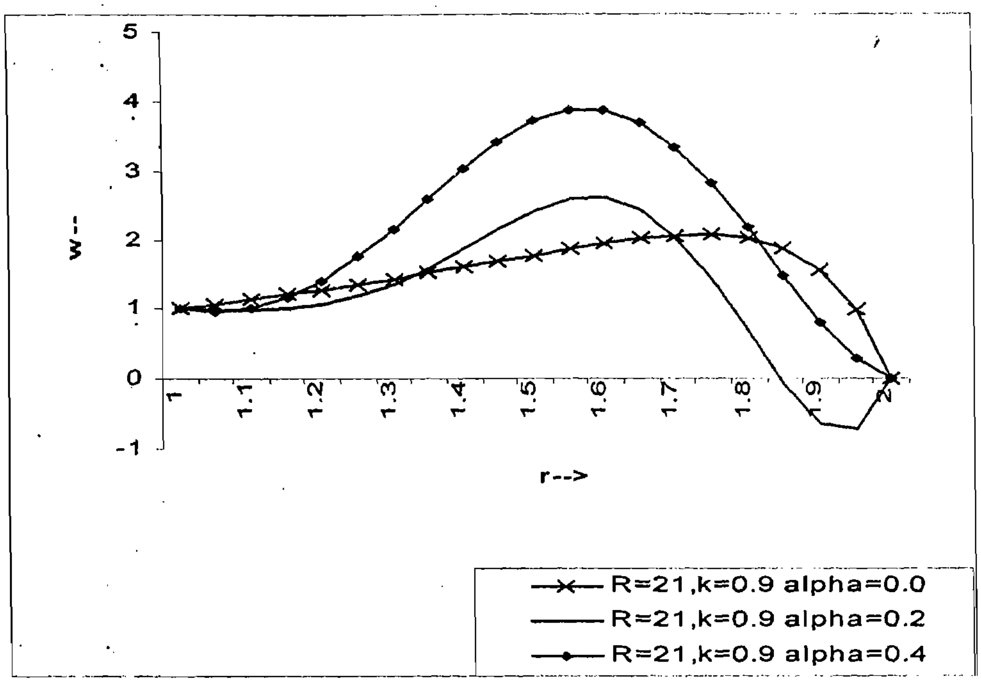


FIG. 4.1.21 variation of w with r for different values of α and fixed value of R=21 and K=0.9.

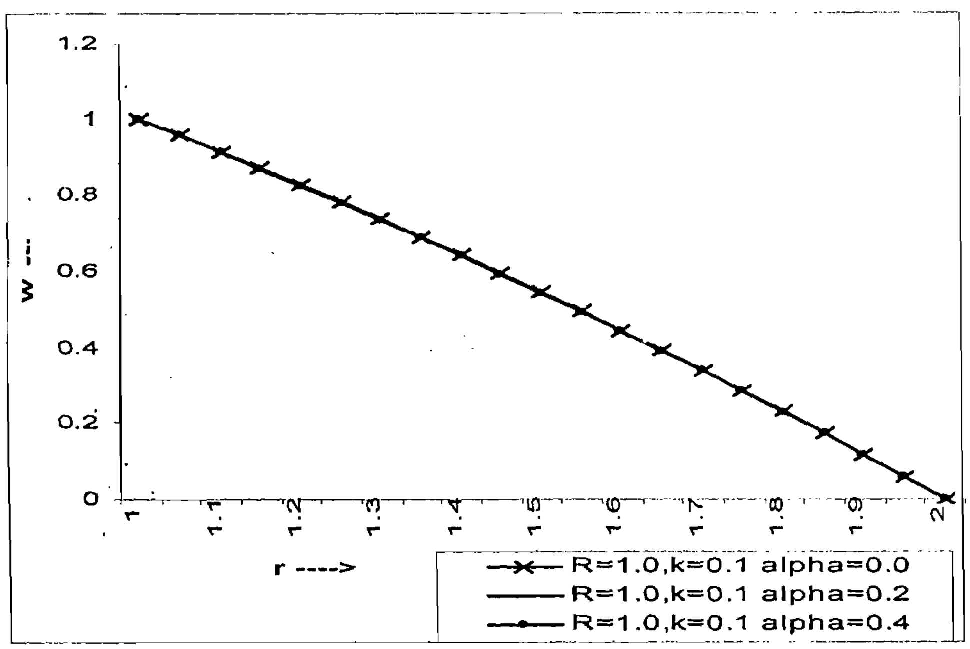


FIG. 4.1.22 variation of w with r for different values of α and fixed value of R=1 and K=0.1.

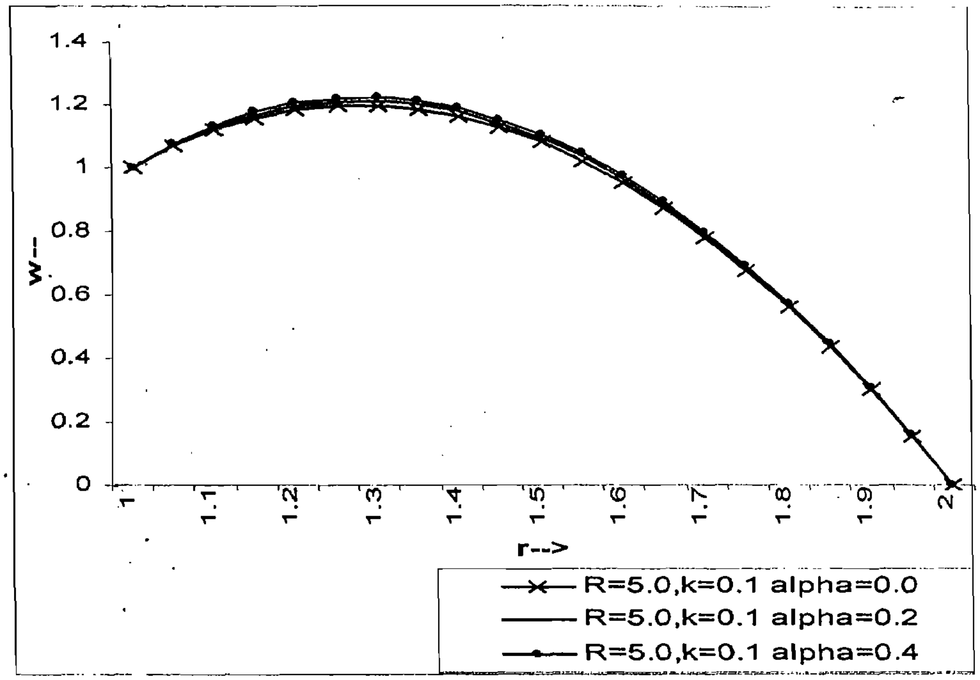


FIG. 4.1.23 variation of w with r for different values of α and fixed value of R=5 and K=0.1.

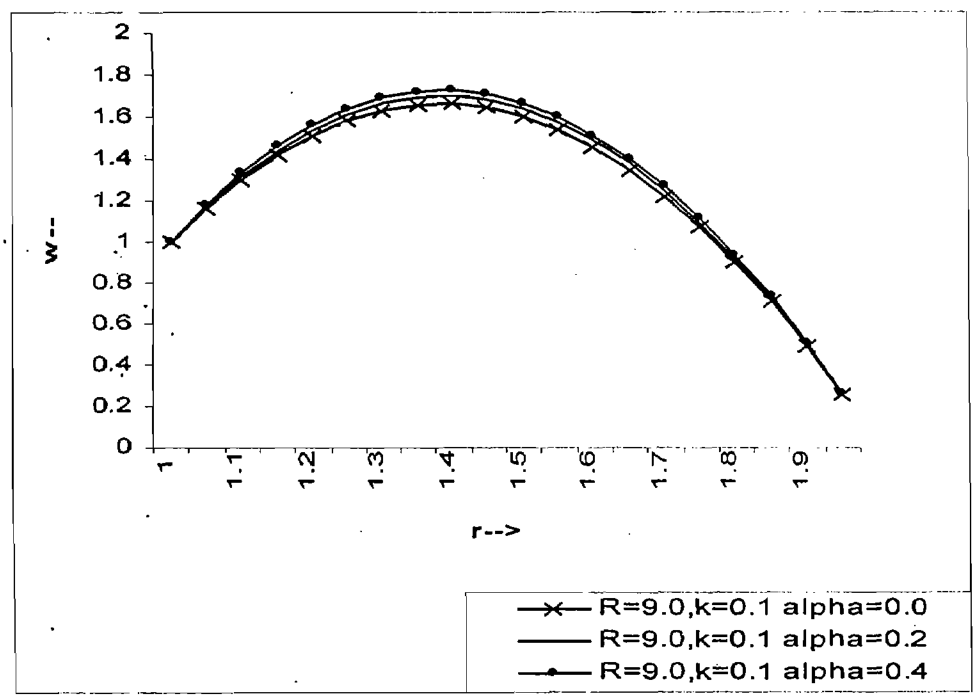


FIG. 4.1.24 variation of w with r for different values of α and fixed value of R=9 and K=0.1.

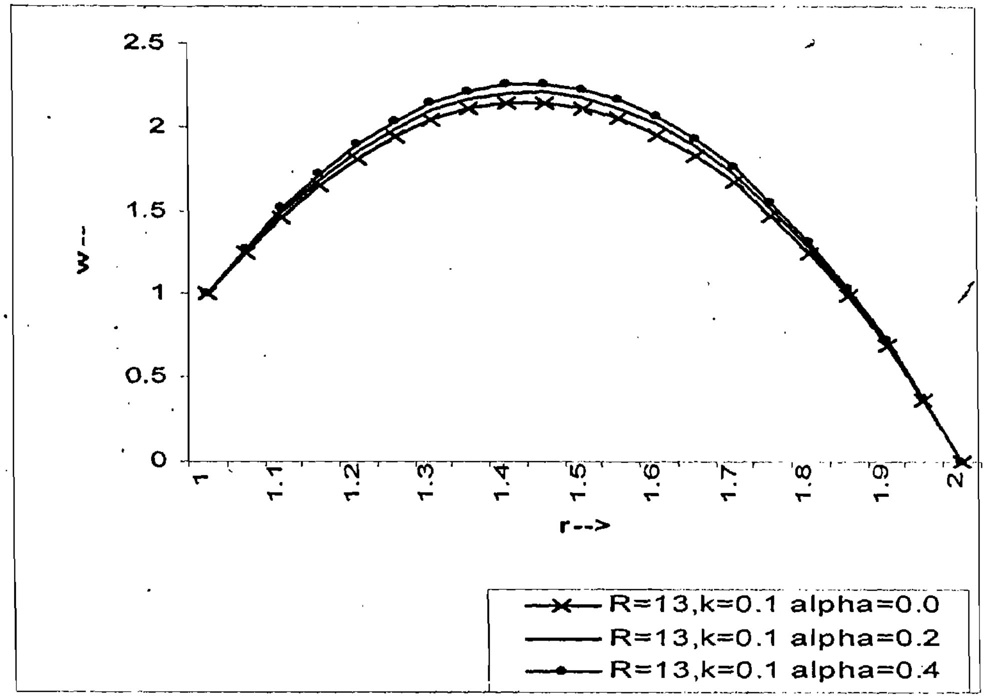


FIG. 4.1.25 variation of w with r for different values of α and fixed value of R=13 and K=0.1.

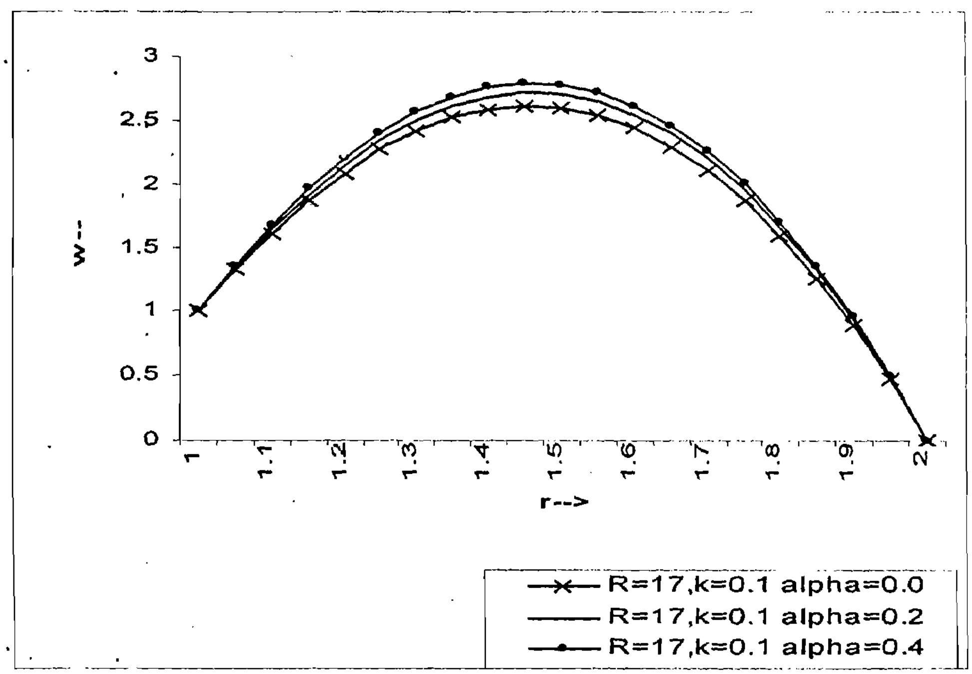


FIG. 4.1.26. variation of w with r for different values of α and fixed value of R=17 and K=0.1.

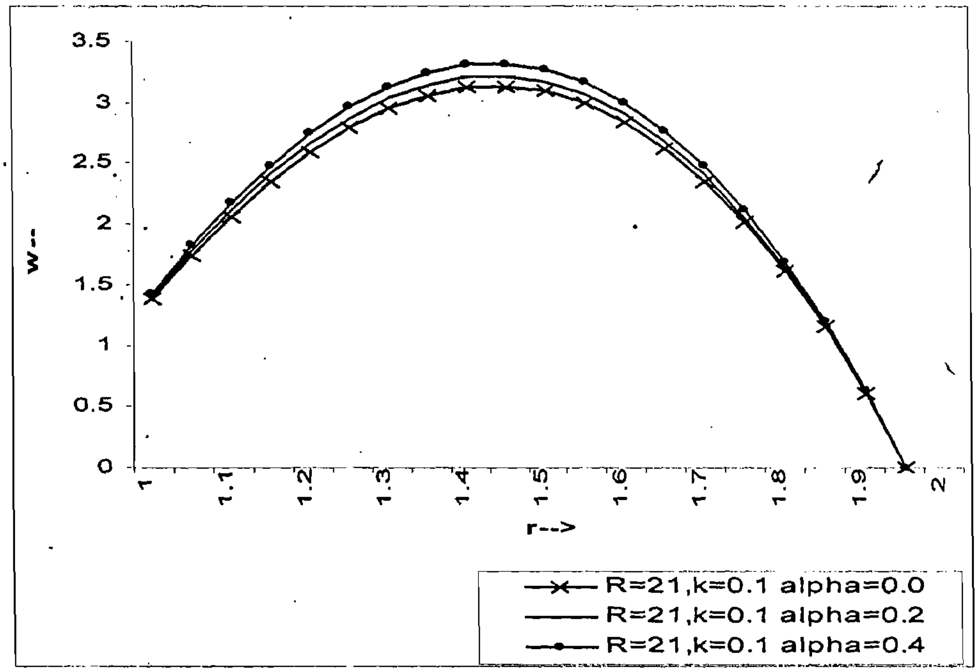


FIG. 4.1.27 variation of w with r for different values of α and fixed value of R=1 and K=0.1.

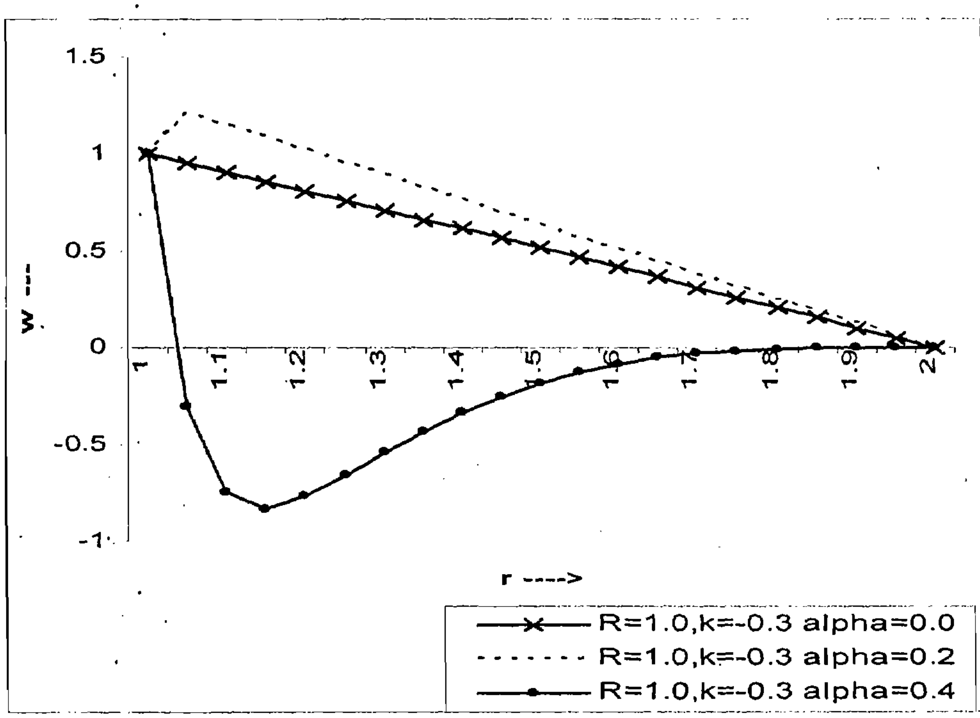


FIG. 4.1.28 variation of w with r for different values of α and fixed value of R=1 and K=-0.3.

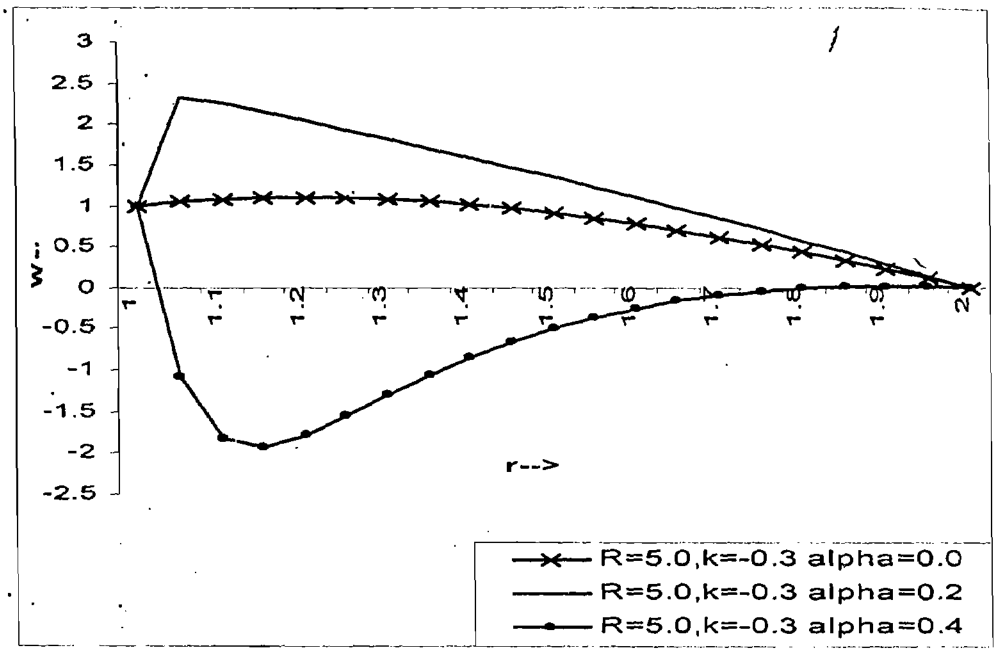


FIG. 4.1.29 variation of w with r for different values of α and fixed value of R=5 and K=-0.3.

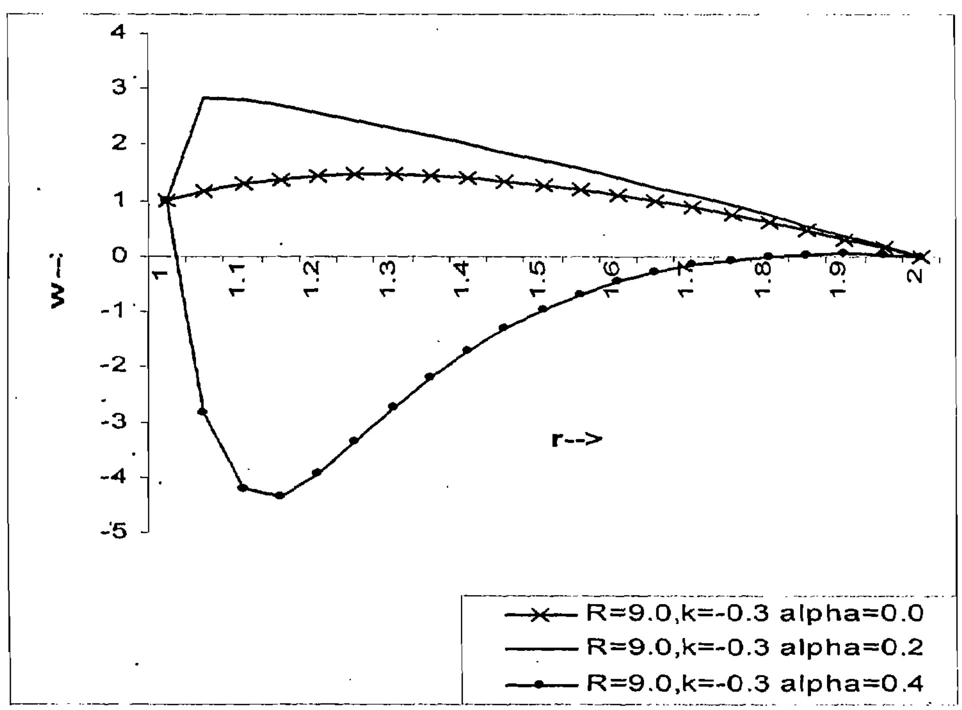


FIG. 4.1.30 variation of w with r for different values of α and fixed value of R=9 and K=-0.3.

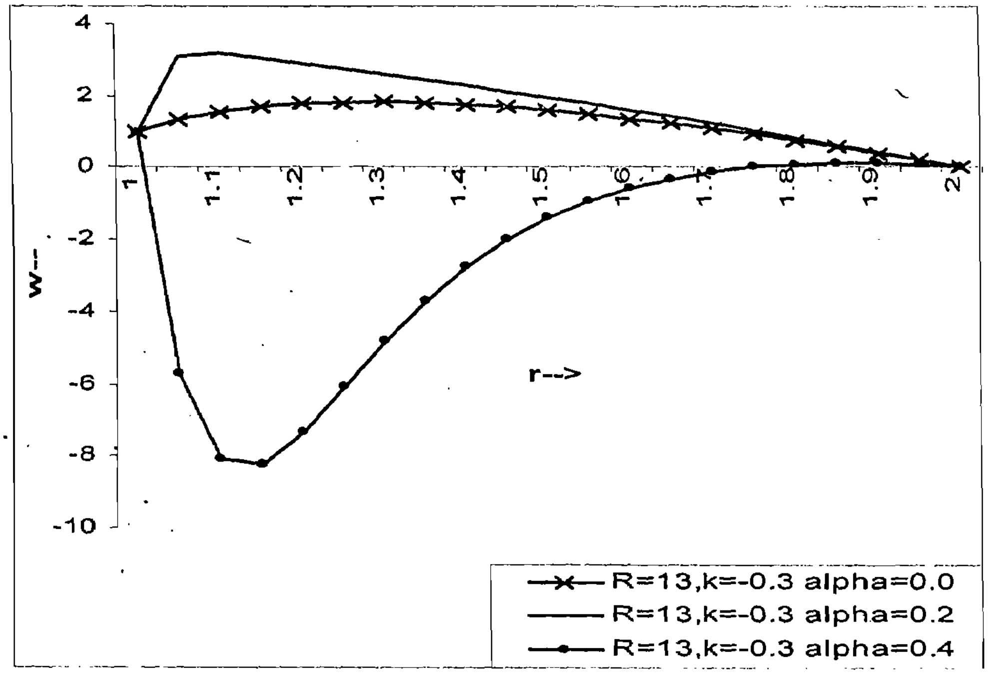


FIG. 4.1.31 variation of w with r for different values of α and fixed value of R=13 and K=0.1.

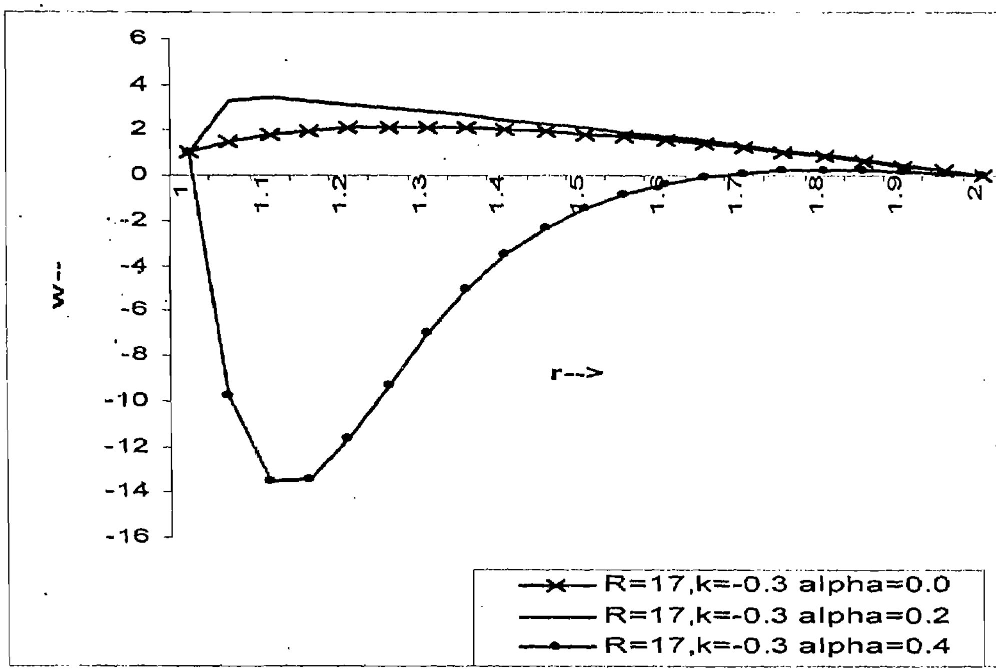


FIG. 4.1.32 variation of w with r for different values of α and fixed value of R=17 and K=-0.3.

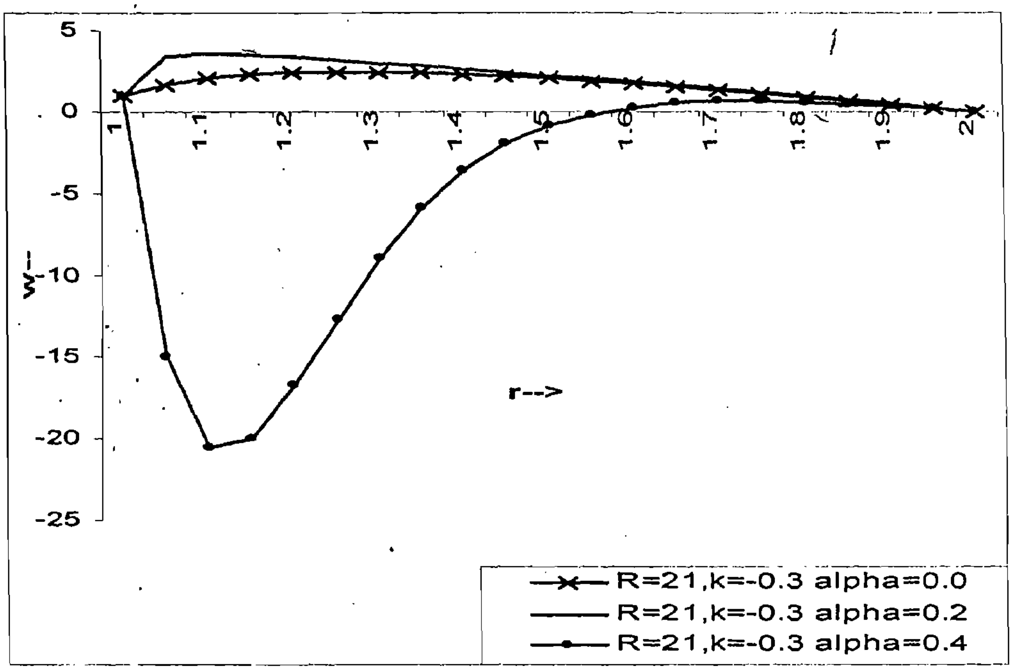


FIG. 4.1.33 variation of w with r for different values of α and fixed value of R=21 and K=0.1.

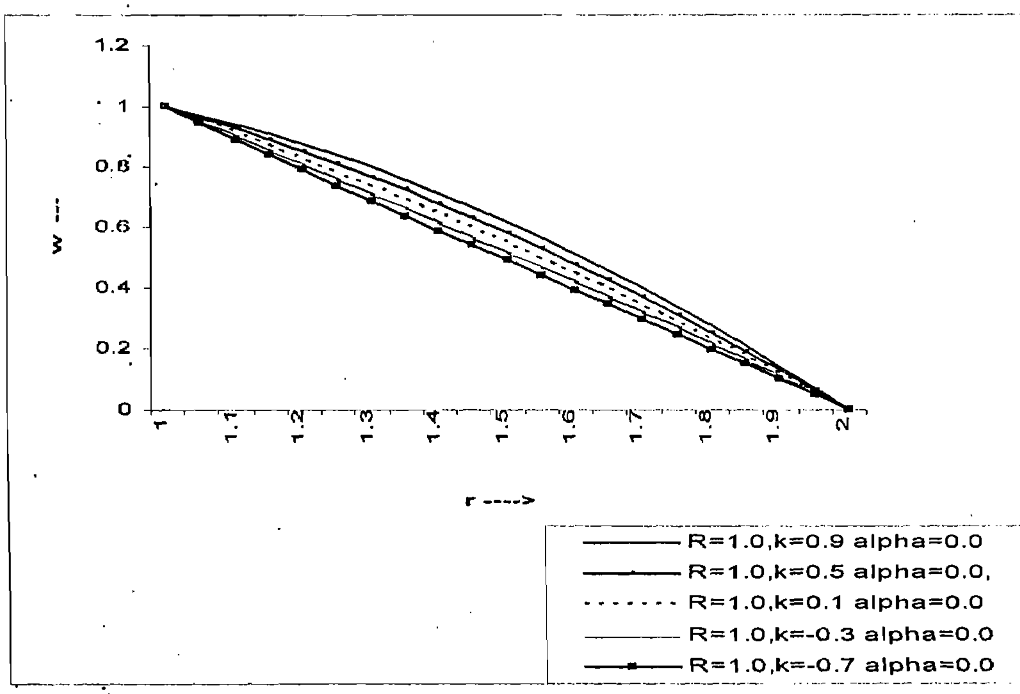


FIG 4.1.34 variation of w with radial distance r for different values K and fixed value of R=1 and alpha=0.0

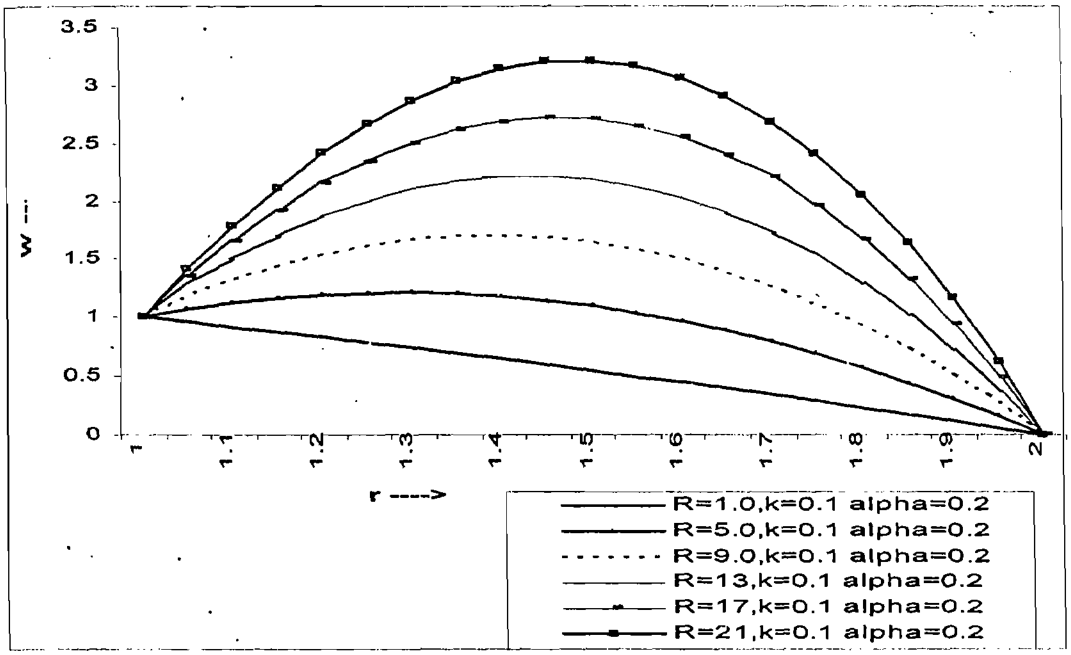


FIG 4.1.35 variation of w with radial distance r for different values R and fixed value of K=0.1 and alpha=0.2

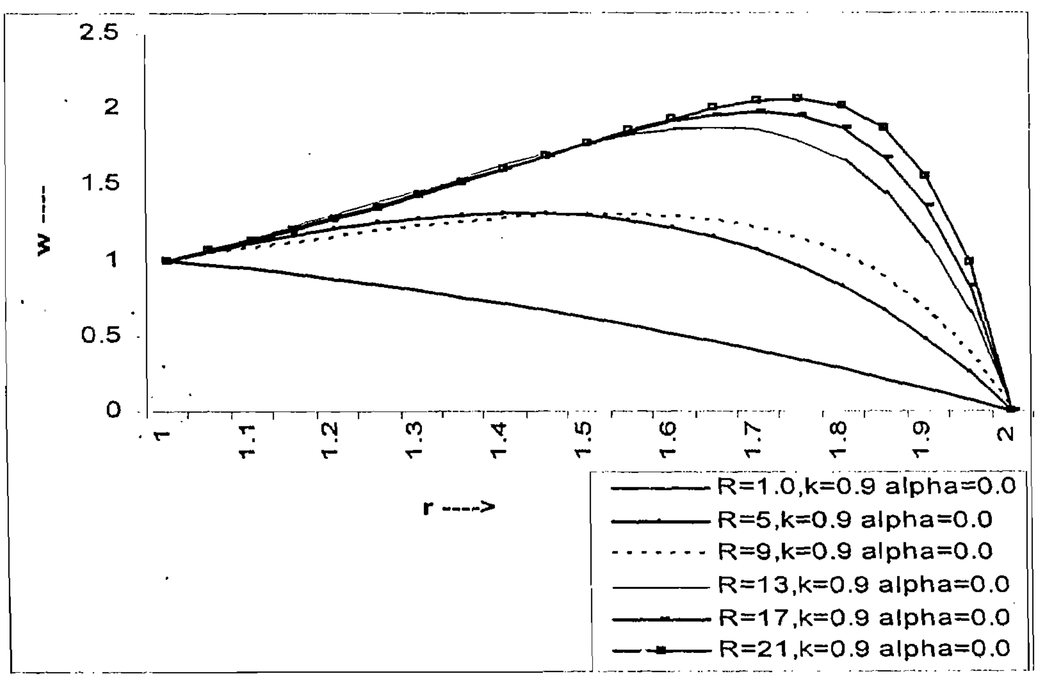


FIG 4.1.36 variation of w with radial distance r for different values R and fixed value of K=0.9 and alpha=0.0

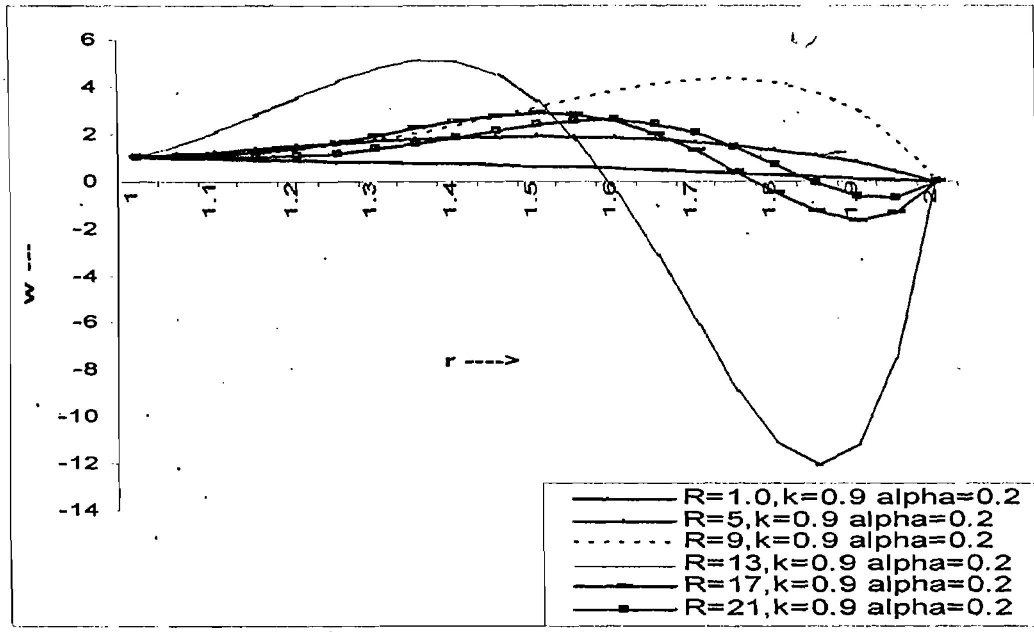


FIG 4.1.37 variation of w with radial distance r for different values R and fixed value of K=0.9 and alpha=0.2

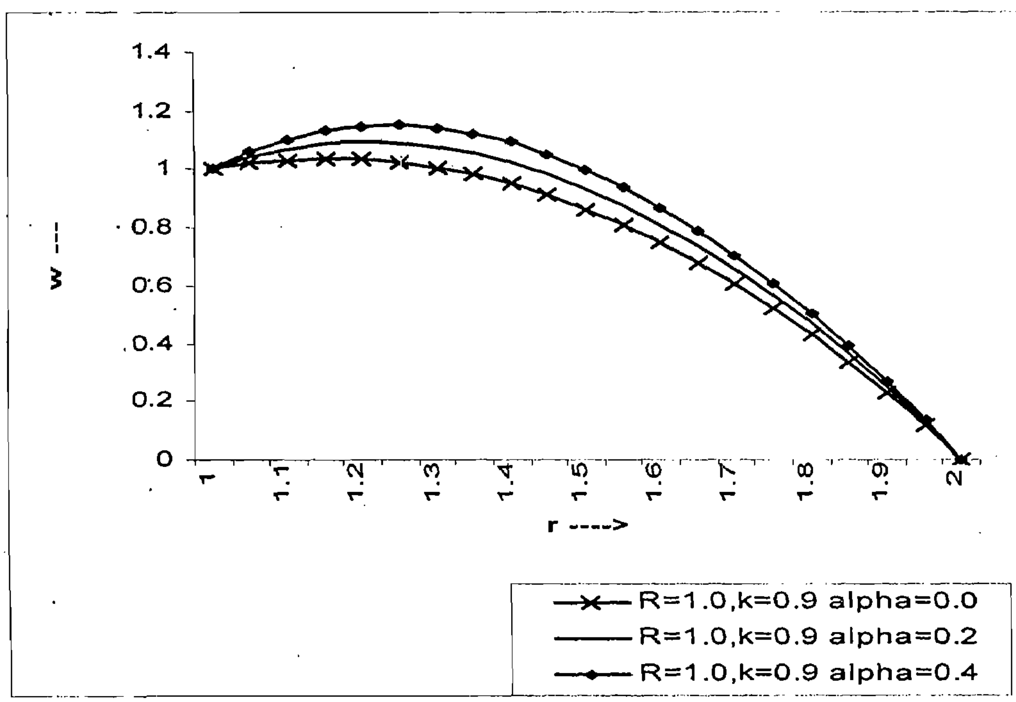


FIG 4.1.38 variation of w with r for different values alpha and fixed value of R=1,K=0.9, λ=3

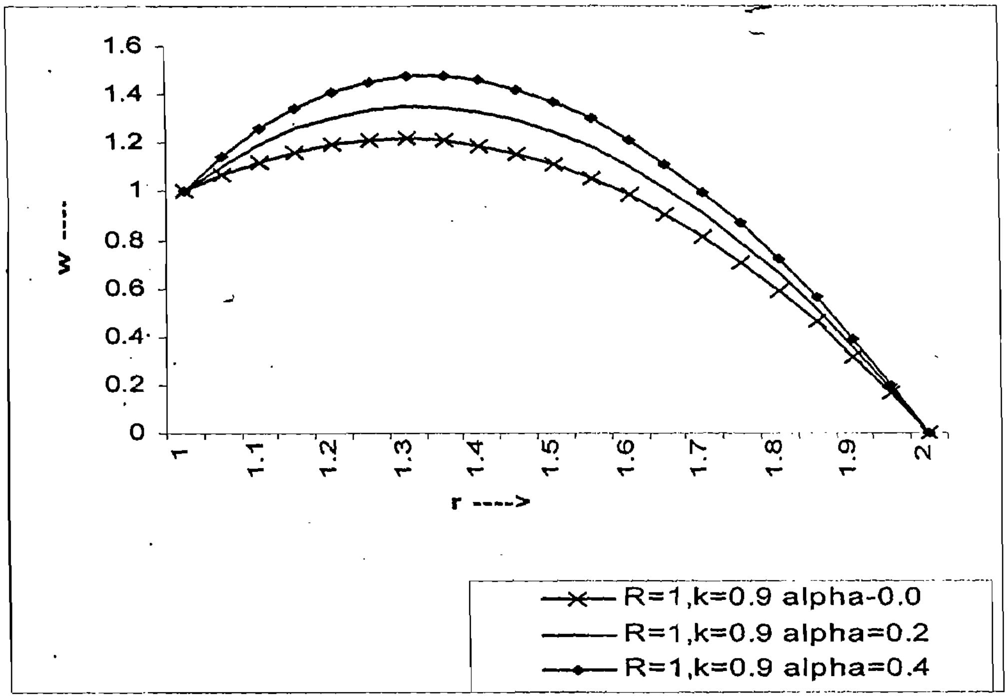


FIG 4.1.39 variation of w with r for different values alpha and fixed value of R=1,K=0.9, λ=5

4.2 Second-order fluid flow between two rotating discs of different transpiration for high Reynolds numbers:

The present investigation deals with the study of non-Newtonian incompressible second-order fluid between two infinite porous rotating discs by finite-difference scheme for small and large values of Reynolds numbers. The constitutive equation of the second order fluid has already been described in equation (1.7).

4.2.1 Formulation of the problem:

In a 3-dimensional cylindrical set of co-ordinates $(\bar{r}, \bar{\theta}, \bar{z})$, the system consists of two porous discs of infinite radius coinciding with the planes $z=0$ and $z=d$ respectively. Both the discs are rotating with different angular velocities. The lower disc $z=0$ is rotating with constant angular velocity $mR\Omega\bar{r}$ while the upper disc $(z=d)$ is rotating with constant angular velocity $R\Omega\bar{r}$ about the z-axis. The rate of suction at the upper discs is taken different from the injection rate at the lower disc. A uniform injection nW is applied on the lower disc while the uniform suction W is applied on the upper disc. The space between the discs is occupied by a homogeneous incompressible second-order fluid. By symmetry all the variables will be independent of each other. Now introducing the following non-dimensional quantities.

$$r = \frac{\bar{r}}{d},\quad z = \frac{\bar{z}}{d},\quad u = \frac{\bar{u}\,d}{v_1},\quad v = \frac{\overline{v_1}\,d}{v_1},\quad w = \frac{\bar{w}\,d}{v_1},$$

$$p = \frac{\bar{p}}{\rho\left(v_1/d\right)^2}$$

By assuming u, v and w as the radial, transverse and axial components of velocities respectively, the relevant boundary conditions of the problem are

$$\bar{z} = 0,\ \bar{u} = 0,\ \bar{v} = mR\ \Omega\ \bar{r},\ \bar{w} = nW$$

$$\bar{z} = 0,\ \bar{u} = 0,\ \bar{v} = R\,\Omega\,\bar{r},\ \bar{w} = W$$

we get the non-dimensional equations as

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \qquad \text{--- (4.2.1)}$$

$$\begin{aligned}
u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r} &= -\frac{\partial p}{\partial r}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}+\alpha\Bigg[u\left(\frac{\partial^3 u}{\partial r^3}+\frac{\partial^3 u}{\partial r\partial z^2}\right)\\
&+w\left(\frac{\partial^3 u}{\partial r^2\partial z}+\frac{\partial^3 u}{\partial z^3}+\frac{1}{r}\frac{\partial^2 u}{\partial r\partial z}\right)+\frac{\partial^2 u}{\partial r^2}\left(9\frac{\partial u}{\partial r}+\frac{u}{r}\right)+\frac{\partial u}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{3}{r}\frac{\partial u}{\partial r}+\frac{u}{r^2}\right)+\\
&\frac{\partial u}{\partial z}\left(3\frac{\partial^2 w}{\partial r^2}-\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+3\frac{\partial^2 u}{\partial r\partial z}-\frac{1}{r}\frac{\partial u}{\partial z}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial z^2}-\frac{2u}{r^2}\right)+2\frac{\partial v}{\partial r}\\
&\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}-\frac{1}{r}\frac{\partial v}{\partial r}+\frac{2v}{r^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\frac{\partial v}{\partial z}-\frac{2v}{r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{v}{r^2}\right)\\
&+2\frac{\partial w}{\partial r}\left(\frac{\partial^2 w}{\partial r\partial z}+2\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+4\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial r\partial z}\Bigg]+\beta\Bigg[\frac{\partial u}{\partial r}\left(6\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{2u}{r^2}\right)+\\
&\frac{\partial u}{\partial z}\left(2\frac{\partial^2 u}{\partial r\partial z}-\frac{1}{r}\frac{\partial u}{\partial z}+2\frac{\partial^2 w}{\partial r^2}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{2u}{r^2}\right)+\frac{\partial v}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}-\frac{2}{r}\frac{\partial v}{\partial r}+\frac{4v}{r^2}\right)\\
&+\frac{\partial v}{\partial z}\left(\frac{\partial^2 u}{\partial r\partial z}-\frac{2}{r}\frac{\partial v}{\partial z}\right)-\frac{v}{r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{2v}{r^2}\right)+\frac{\partial w}{\partial r}\left(2\frac{\partial^2 u}{\partial r\partial z}+2\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)\\
&+2\frac{\partial^2 w}{\partial r\partial z}\frac{\partial w}{\partial z}\Bigg] \qquad ---(4.2.2)
\end{aligned}$$

$$\begin{aligned}
u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}+\frac{uv}{r} &= \frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}+\alpha\Bigg[u\frac{\partial^3 u}{\partial r^3}+w\left(\frac{\partial^3 v}{\partial r^2\partial z}+\frac{1}{r}\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^3 v}{\partial z^3}\right)\\
&+\frac{\partial^2 u}{\partial r^2}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+\frac{\partial u}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{3}{r}\frac{\partial v}{\partial r}-2\frac{\partial^2 u}{\partial z^2}-\frac{3v}{r^2}\right)+\frac{\partial^2 u}{\partial z^2}\left(\frac{\partial v}{\partial z}-\frac{v}{r}\right)+2\frac{\partial u}{\partial z}\left(\frac{\partial^2 v}{\partial r\partial z}+\frac{1}{r}\frac{\partial v}{\partial z}\right)\\
&+\frac{\partial v}{\partial z}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+\frac{\partial^2 w}{\partial z^2}\right)+2\frac{\partial w}{\partial r}\frac{\partial^2 v}{\partial r\partial z}+\frac{u}{r}\left(4\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}\right)\Bigg]+\beta\Bigg[\frac{\partial^2 u}{\partial r^2}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+\\
&\frac{\partial u}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{3}{r}\frac{\partial v}{\partial r}-\frac{3v}{r^2}\right)+2\frac{\partial u}{\partial z}\left(\frac{\partial^2 v}{\partial r\partial z}+\frac{1}{r}\frac{\partial v}{\partial z}\right)+\frac{2u}{r}\frac{\partial^2 v}{\partial r^2}+\frac{\partial v}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r}\right)+\\
&2\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial w}{\partial z}+\frac{u}{r}\right)+\frac{\partial v}{\partial z}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\frac{\partial w}{\partial z}-\frac{v}{r}\left(\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)\Bigg] \qquad ---(4.2.3)
\end{aligned}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}+\alpha\left[u\left(\frac{\partial^3 u}{\partial r^3}+\frac{\partial^3 w}{\partial r\partial z^2}\right)\right.$$

$$+w\left(\frac{\partial^3 w}{\partial z^3}+\frac{\partial^3 w}{\partial r^2\partial z}+\frac{1}{r}\frac{\partial^2 w}{\partial r\partial z}\right)+\frac{\partial u}{\partial r}\left(3\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial w}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+2\frac{\partial u}{\partial z}$$

$$\left(2\frac{\partial^2 u}{\partial z^2}+\frac{2u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}\right)+2\frac{\partial v}{\partial z}\left(\frac{\partial^2 v}{\partial r^2}+2\frac{\partial^2 v}{\partial z^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+$$

$$\left.\frac{\partial w}{\partial r}\left(\frac{\partial^2 u}{\partial r^2}+3\frac{\partial^2 u}{\partial z^2}+4\frac{\partial^2 w}{\partial r\partial z}+\frac{3}{r}\frac{\partial w}{\partial z}\right)+\frac{\partial w}{\partial z}\left(\frac{\partial^2 w}{\partial r^2}+10\frac{\partial^2 w}{\partial z^2}\right)-\frac{u}{r}\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^2 w}{\partial r^2}\right)\right]$$

$$+\beta\left[2\frac{\partial u}{\partial z}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}+\frac{1}{r}\frac{\partial w}{\partial z}\right)+\frac{\partial v}{\partial z}\left(\frac{\partial^2 v}{\partial r^2}+2\frac{\partial^2 v}{\partial z^2}\right)+\frac{\partial^2 v}{\partial r\partial z}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+\right.$$

$$\left.2\frac{\partial w}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}\right)+2\frac{\partial w}{\partial z}\left(4\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^2 w}{\partial r^2}\right)\right]$$

--- (4.2.4)

subject to the boundary conditions

$$z = 0; \; , u = 0, \quad v = mR, \quad w = nR$$

$$z = 1; \quad u = 0, \quad v = \lambda r R, \quad w = R \qquad \text{--- (4.2.5)}$$

where $\lambda = \dfrac{d^2\Omega}{v_1}$, is a dimensionless rotational parameter,

$R = \dfrac{Wd}{v_1}$, is a suction Reynolds number,

$$\alpha = \frac{\mu_2}{\rho d^2}, \quad \beta = \frac{\mu_3}{\rho d^2}$$

α and β are dimensionless parameters representing elastico-viscous and cross-viscous effects. α is negative since $\mu_2 < 0$ for thermodynamic considerations. Further the constitutive relation (1.7) is valid for flow at low shear rates such that $|\alpha| << 1$.

4.2.2 Numerical solution:

To find the solution of equations (4.2.1) to (4.2.4), we introduce the following variables

$$u = -\left(\frac{rR}{2}\right)F'(z) ,$$

$$v = rR\lambda\, G(z),$$

$$w = RF(z) ,$$

$$P = p(z) + \frac{A}{2}\lambda^2 r^2 \qquad \text{--- (4.2.6)}$$

where $F(z)$ and $G(z)$ are non-dimensional functions of the variable z and primes denote differentiation with respect to z.

Then the equation of continuity (4.2.1) is identically satisfied with the equations (4.2.2) to (4.2.4) are transformed to

$$2RF''' - 2R^2FF'' + R^2F'^2 - 4R^2\lambda^2G^2 + 4A\lambda^2 + \alpha\left[2R^2FF^{IV} - 2R^2F''^2\right]$$
$$+\beta\left[2R^2F'F''' - R^2F''^2 + 4R^2\lambda^2G^2\right] = 0$$

--- (4.2.7)

$$G'' - RFG' + RF'G + \alpha\left[RFG''' - RF''G'\right] + \beta\left[RF'G'' - RF''G\right] = 0$$

--- (4.2.8)

$$RF'' - R^2FF' - P' + \alpha\left[11R^2F'F'' + R^2FF''' + r^2R^2F''F''' + 4r^2R^2\lambda^2 G'G''\right]$$
$$+\beta\left[7R^2F'F'' + \frac{1}{2}r^2R^2F''F''' + 2r^2R^2G'G''\right] = 0$$

--- (4.2.9)

The corresponding boundary conditions are

$$z = 0; \quad F' = 0, \quad G = m, \quad F = n, \qquad \text{--- (4.2.10)}$$

$$z = 1; \quad F' = 0, \quad G = 1, \quad F = 1, \qquad \text{--- (4.2.11)}$$

Equations (4.2.7) and (4.2.8) are highly non-linear in F and G which are functions of z. In finite-difference approximations, the first, second, third, fourth and fifth order derivatives of a function F and G as stated in relations (3.9)-(3.13),

The equations (4.2.7) and (4.2.8), we obtain

$$H_{1,i}\left(G_{i-1}, G_i, G_{i+1}, F_{i-1}, F_i, F_{i+1}, F_{i+2}\right)$$
$$= 2R\,\frac{F_{i+2} - 3F_{i+1} + 3F_i - F_{i-1}}{h^3} - 2RF_i\,\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2} + R$$
$$\left(\frac{F_{i+1} - F_{i-1}}{2h}\right)^2 - 4R^2\lambda^2 G_i^{\,2} + 4A\lambda^2 + \alpha\left[2RF_i\,\frac{F_{i+3} - 4F_{i+2} + 6F_{i+1} - 4F_i + F_{i-1}}{h^4} -\right.$$
$$\left. 2R^2\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right)^2\right] + \beta\left[2R\frac{F_{i+1} - F_{i-1}}{2h}\,\frac{F_{i+2} - F_{i+1} + F_i - F_{i-1}}{h^3} -\right.$$
$$\left. R^2\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right)^2 + 4R\lambda\left(\frac{G_{i+1} - G_{i-1}}{2h}\right)^2\right] = 0$$

--- (4.2.12)

$$H_{2,i}\left(G_{i+2}, G_{i+1}, G_i, G_{i-1}, F_{i+1}, F_i, F_{i-1}\right) =$$
$$\frac{G_{i+1} - G_i + G_{i+1}}{h^2} + R\frac{F_{i+1} - F_{i-1}}{2h}G_i - RF_i\,\frac{G_{i+1} - G_{i-1}}{2h} +$$
$$\alpha\left[RF_i\,\frac{G_{i+2} - 3G_{i+1} + 3G_i - G_{i-1}}{h^3} - R\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\,\frac{G_{i+1} - G_{i-1}}{2h}\right] +$$
$$\beta\left[R\frac{F_{i+1} - F_{i-1}}{2h}\,\frac{G_{i+1} - 2G_i + G_{i-1}}{h^2} - RG_i\,\frac{F_{i+1} - F_i + F_{i-1}}{h^2}\right] = 0$$

--- (4.2.13)

All F_i's and G_i's are the functions of z. divide z [0, 1] into twenty equal parts each of length 0.05 with mesh points 2 to 22 and 1 and 23 as the fictitious points. The boundary conditions (4.2.10) and (4.2.11) can be rewritten as:

$$G(0) = m, \qquad G(1) = 1,$$
$$F(0) = n, \qquad F(1) = 1,$$
$$F'(0) = n, \qquad F'(1) = 1,$$
$$G_2 = m, \qquad G_{22} = 1,$$
$$F_2 = m, \qquad F_{22} = 1$$
$$F_{23} = m, \qquad F_{21} = 1,$$

The differences of approximation Φ_i with exact values $\overline{\Phi}_i$ i.e. ΔF_i and ΔG_i can be calculated by

$$\overline{\Phi}_i = \Phi_i + \Delta\Phi_i, \ \Phi_i \text{ can be } F_i \text{ and } G_i.$$

Equations (4.2.12) and (4.2.13) involve only F_i's and G_i's. Expanding $H_{1,i}$ and $H_{2,i}$ by means of Taylor series expansion and neglecting the second and higher powers of small quantities.

$$\overline{H}_{1,j} = H_{1,j} + \left(\frac{\partial H_{1,j}}{\partial G_{i-1}} \Delta G_{i-1} + \frac{\partial H_{1,j}}{\partial G_i} \Delta G_i + \frac{\partial H_{1,j}}{\partial G_{i+1}} \Delta G_{i+1} + \frac{\partial H_{1,j}}{\partial F_{i-1}} \Delta F_{i-1} + \right.$$
$$\frac{\partial H_{1,j}}{\partial F_i} \Delta F_i + \frac{\partial H_{1,j}}{\partial F_{i+1}} \Delta F_{i+1} + \frac{\partial H_{1,j}}{\partial F_{i+2}} \Delta F_{i+2} = 0$$

--- (4.2.14)

$$\overline{H}_{2,j} = H_{2,j} + \left(\frac{\partial H_{2,j}}{\partial G_{i-1}} \Delta G_{i-1} + \frac{\partial H_{2,j}}{\partial G_i} \Delta G_i + \frac{\partial H_{2,j}}{\partial G_{i+1}} \Delta G_{i+1} + \frac{\partial H_{2,j}}{\partial G_{i+2}} \Delta G_{i+2} + \right.$$
$$\left. \frac{\partial H_{2,j}}{\partial F_{i-1}} \Delta F_{i-1} + \frac{\partial H_{2,j}}{\partial F_i} \Delta F_i + \frac{\partial H_{1,j}}{\partial F_{i+1}} \Delta F_{i+1} \right) = 0$$

--- (4.2.15)

equations (4.2.14) and (4.2.15) can be represented as

$$J \begin{bmatrix} F_3 \\ F_4 \\ - \\ - \\ F_{20} \\ F_{21} \\ G_3 \\ G_4 \\ - \\ - \\ G_{20} \\ G_{21} \end{bmatrix} + \begin{bmatrix} H_{1,3} \\ H_{1,4} \\ - \\ - \\ H_{1,20} \\ H_{1,21} \\ H_{2,3} \\ H_{2,4} \\ - \\ - \\ H_{2,20} \\ H_{2,21} \end{bmatrix} = 0$$

Where J is the jacobian matrix of order 38. Applying Newton-Raphson method (3.9) solving this system, a better approximation for F and G is found. The procedure is repeated till the desired accuracy is achieved. The number of intervals have been increased for the smooth flow function study.

4.2.3 Results and Discussion:

Tables 4.2.1-4.2.6 represent the variation of axial velocity F with z for different values of visco-elastic parameters α , β for fixed Reynolds number and these tables have been depicted through the figures 4.2.1-4.2.6. It has been observed and concluded that for larger values of Reynolds numbers, the axial velocity has negative sign i.e. liquid comes out of disc. The study is of great importance that the chemical industry may be interested to find the limit of Reynolds number for the fluid being used for such set-ups. These values depend on the characteristics of the fluid. The results corresponding to Newtonian fluid can be deduced from the above results by setting $\alpha = \beta = 0$.

Figures 4.2.1-4.2.2 depict that for $\alpha = \beta = 0$ and $\alpha = -0.01$, $\beta = 0.01$ the results are in good agreement with the results of Chaudhary and Das (1997), but for $\alpha = -0.05$, $\beta = 0.05$ recirculation occurs at the end points in the lower and upper discs. From point $z = 0.4$ to 0.9 the set of values shows the normal study of circulation and attains maximum value at $z = 0.7$. Hence, it is concluded that for non-Newtonian fluid large recirculation occurs at lower disc in comparison to the recirculation at upper disc. The behavior of the fluid is turbulent as higher value of Reynolds number is considered. It is also noticed that the behavior of the fluid in the practical purpose is to be seen by the technocrats and engineers in the industry.

Figure 4.2.3 represents that for $\alpha = -0.05$, $\beta = 0.05$ recirculation occurs at end point in lower disc and attains maximum value at $z = 0.05$. From $z = 0.15$ to $z = 0.1$ the set of values depicts normal behavior of circulation and attains the maximum value at point $z = 0.9$. For $\alpha = -0.01$, $\beta = 0.01$ the curve depicts normal behavior of circulation which has maxima at point $z = 0.8$ and minima at point $z = 0.05$

Figure 4.2.4 depicts that the axial velocity F with z shows normal circulation curve and attains maximum value at point $z = 0.25$ excepts from point $z = 0.5$ to 0.7, where recirculation occurs.

Figure 4.2.5 depicts recirculation for $\alpha = -0.05$, $\beta = 0.05$ at end point of lower disc and attains maximum value at point $z = 0.5$ and minimum value at $z = 0.95$. It is almost symmetrical about $z = 0.5$ except end points in the lower and upper discs.

Figure 4.2.6 shows that for $\alpha = -0.05$, $\beta = 0.05$ at the end point of the lower disc recirculation occurs and is maximum at $z = 0.35$, but for almost other values of α and β, the set of values represents normal behavior of circulation.

The comparison of different values of Reynolds number R has been tabulated in tables 4.2.7 and shown graphically in figure 4.2.7.

Figures 4.2.8-4.2.11 represent the behavior of transverse velocity G with z for different values of m, n and λ and visco-elastic parameters α, β for fixed Reynolds numbers R=1 and R=3 and given in detail in tables 4.2.8-4.2.11. It is observed that recirculation takes place for different values of z at for different parameters. Hence it is concluded that the distribution of transverse velocity is not symmetrical and for non- Newtonian fluid large recirculation occurs at upper disc in comparison to the recirculation at lower disc on increasing the value of Reynolds number.

The comparison of different values of Reynolds number has been represented by table 4.2.12 and has been depicted through graphs in figure 4.2.12. The present study could be of much interest to bio-engineers involved in design and construction of artificial organs, e.g. artificial dialysis etc.

z	F, m=2, n=2, λ=1, R=1		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	2	2	2
0.05	1.994123192	7.59E-01	-26.28936329
0.1	1.977542093	1.267115971	-60.79172157
0.15	1.951902772	1.565934293	-42.61009202
0.2	1.918944059	1.759896249	-6.748039627
0.25	1.880492583	1.926147198	-236.1655396
0.3	1.838457052	2.128035384	-104.9080007
0.35	1.794821915	2.422115487	-33.59782789
0.4	1.751640254	2.859077688	38.22107807
0.45	1.711026565	3.479597314	133.1253809
0.5	1.67514881	4.30529333	307.5344064
0.55	1.646220612	5.323149597	532.5361978
0.6	1.626493069	6.459995572	762.7643671
0.65	1.618246694	7.546113463	935.8063614
0.7	1.623783414	8.281345109	986.4666994
0.75	1.64541888	8.248719753	873.5249336
0.8	1.685475005	7.056792011	610.0880807
0.85	1.792	4.680600693	278.9769884
0.9	1.830126919	1.915317056	10.09140848
0.95	1.939338251	4.27E-01	-88.58554396
1	1	1	1

Table 4.2.1:variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=1, R=1.

z	F, m=2, n=2, λ=1, R=3		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	2	2	2
0.05	-1099529.535	-2.38E-01	-970.1294366
0.1	-137.5019638	-7.098131302	-409.4647002
0.15	-1052457.899	5.257978119	-451.8929952
0.2	-1181010.245	15.92430841	-233.9182545
0.25	-169517.8263	24.80386934	98.03040942
0.3	-4051.044821	33.1074951	335.4171111
0.35	-80313.5553	38.30451501	485.3890974
0.4	-85930.80962	35.36274624	561.4493394
0.45	-11565.83174	21.81511763	572.6262589
0.5	-1894.928198	3.944445991	523.9969443
0.55	446.7272841	-4.122060983	423.4005443
0.6	389.3839237	-5.48621967	288.5510826
0.65	-793.6766093	15.22571663	148.8542768
0.7	-1297.347564	34.28164181	38.34108518
0.75	-730.6642616	48.68535226	-18.13857454
0.8	217.129104	49.38372572	-14.0333611
0.85	-737.0173229	31.42473325	62.58659074
0.9	9.255582612	3.882235566	161.7635342
0.95	-1422.926233	-9.312949588	202.4965019
1	1	1	1

Table 4.2.2:variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=1, R=3.

z	F, m=2, n=2, λ=0.5, R=1		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	2	2	2
0.05	1.99462477	1.433878121	-6.16397871
0.1	1.979567593	1.667860335	-3.546575057
0.15	1.956533132	1.783456382	-1.418909061
0.2	1.927360059	1.825856187	1.79E-01
0.25	1.894017973	1.820871114	1.243172782
0.3	1.858603553	1.785040248	1.863037459
0.35	1.823336085	1.732314862	2.144401404
0.4	1.790552254	1.67844733	2.19052629
0.45	1.762700763	1.643248638	2.099651504
0.5	1.742336307	1.650287594	1.962117528
0.55	1.732113598	1.723374473	1.854089612
0.6	1.734781076	1.879338971	1.82975453
0.65	1.753174691	2.117267207	1.915143215
0.7	1.7902119	2.405663819	2.105055287
0.75	1.84888567	2.671296794	2.362521611
0.8	1.932259008	2.797604667	2.62092719
0.85	2.043459856	2.648898749	2.791897299
0.9	2.18567638	2.156928604	2.783841515
0.95	2.362153832	1.564441548	2.533762541
1	1	1	1

Table 4.2.3:variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=0.5 and R=1.

z	F, m=0.5, n=2, λ=1, R=3		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	2	2	2
0.05	1.99462477	1.433878121	-6.16397871
0.1	1.979567593	1.667860335	-3.546575057
0.15	1.956533132	1.783456382	-1.418909061
0.2	1.927360059	1.825856187	1.79E-01
0.25	1.894017973	1.820871114	1.243172782
0.3	1.858603553	1.785040248	1.863037459
0.35	1.823336085	1.732314862	2.144401404
0.4	1.790552254	1.67844733	2.19052629
0.45	1.762700763	1.643248638	2.099651504
0.5	1.742336307	1.650287594	1.962117528
0.55	1.732113598	1.723374473	1.854089612
0.6	1.734781076	1.879338971	1.82975453
0.65	1.753174691	2.117267207	1.915143216
0.7	1.7902119	2.405663819	2.105055287
0.75	1.84888567	2.671296794	2.362521611
0.8	1.932259008	2.797604667	2.62092719
0.85	2.043459856	2.648898749	2.791897299
0.9	2.18567638	2.156928604	2.783841515
0.95	2.362153832	1.564441548	2.533762541
1	1	1	1

Table 4.2.4:variation of F with z for different values of parameters α and β and fixed values of m=0.5, n=2, λ=1, R=3.

z	F, m=0, n=0, λ=1, R=1		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	0.00E+00	0.00E+00	0.00E+00
0.05	6.95E-03	6.87E-03	-9.07E-01
0.1	2.68E-02	2.65E-02	1.391674348
0.15	5.81E-02	5.74E-02	3.399313516
0.2	9.9	9.81E-02	5.154629018
0.25	1.49E-01	1.47E-01	6.720661981
0.3	2.05E-01	2.03E-01	8.116188475
0.35	2.67E-01	2.64E-01	9.289136725
0.4	3.33E-01	3.29E-01	10.12101808
0.45	4.00E-01	3.96E-01	10.45835609
0.5	4.69E-01	4.64E-01	10.16688996
0.55	5.36E-01	5.31E-01	9.198226052
0.6	6.01E-01	5.96E-01	7.648726097
0.65	6.62E-01	6.56E-01	5.781069423
0.7	7.16E-01	7.11E-01	3.976729357
0.75	7.63E-01	7.57E-01	2.604524175
0.8	8.00E-01	7.95E-01	1.82820429
0.85	8.25E-01	8.20E-01	1.468921383
0.9	8.36E-01	8.33E-01	1.094045319
0.95	8.32E-01	8.30E-01	5.69E-01
1	1	1	1

Table 4.2.5:variation of F with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=1.

z	F, m=0, n=0, λ=1, R=3		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	0.00E+00	0.00E+00	0.00E+00
0.05	6.36E-03	6.11E-03	-169.9479337
0.1	2.44E-02	2.35E-02	-163.9590596
0.15	5.27E-02	5.07E-02	-5.773794339
0.2	8.97E-02	8.62E-02	129.5855752
0.25	1.34E-01	1.29E-01	204.0011967
0.3	1.84E-01	1.77E-01	231.5703055
0.35	2.38E-01	2.29E-01	226.3814389
0.4	2.94E-01	2.83E-01	208.0428067
0.45	3.51E-01	3.38E-01	192.904701
0.5	4.07E-01	3.93E-01	188.4850578
0.55	4.60E-01	4.44E-01	191.9056708
0.6	5.08E-01	4.91E-01	190.5043239
0.65	5.49E-01	5.32E-01	169.3631373
0.7	5.81E-01	5.64E-01	123.1852037
0.75	6.01E-01	5.85E-01	63.11559941
0.8	6.07F-01	5.92E-01	12.43817763
0.85	5.96E-01	5.83E-01	-8.566840223
0.9	5.64E-01	5.55E-01	3.686160087
0.95	5.10E-01	5.04E-01	-3.11076689
1	1	1	1

Table 4.2.6:variation of F with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=3.

z	m=2, n=2 , λ=1 α=-0.01, β=0.01				
	R=1	R=3	R=5	R=7	R=9
0	2	2	2	2	2
0.05	7.59E-01	-2.38E-01	-295.670036	-29.864648	285.9172304
0.1	1.267115971	-7.0981313	-385.894013	-3.8857578	-941.020377
0.15	1.565934293	5.257978119	-244.520234	-4.27E-01	-482.0223566
0.2	1.759896249	15.92430841	6.45E-01	13.6017579	102.7515584
0.25	1.926147198	24.80386934	-260.206845	41.4670313	144.93206
0.3	2.128035384	33.1074951	-295.01866	32.2680872	-8.22E-01
0.35	2.422115487	38.30451501	-348.458222	14.2478716	-155.3726509
0.4	2.859077688	35.36274624	126.2757725	17.2501813	-63.90566168
0.45	3.479597314	21.81511763	235.8044862	16.3295582	77.43906403
0.5	4.30529333	3.944445991	29.3916735	8.35324298	73.44218932
0.55	5.323149597	-4.12206098	61.45978776	-4.9585971	-15.65655841
0.6	6.459995572	-5.48621967	24.93987021	-9.481939	-55.05674584
0.65	7.546113463	15.22571663	2.137426353	-11.511483	-10.38950591
0.7	8.281345109	34.28164181	-1.87155849	-16.298254	2.688198575
0.75	8.248719753	48.68535226	-4.45E-02	-19.54262	-6.953082215
0.8	7.056792011	49.38372572	-4.3999499	-16.279353	83.12016576
0.85	4.680600693	31.42473325	26.17377003	-5.1356317	91.35349135
0.9	1.915317056	3.882235566	7.53502198	7.00718672	13.71374467
0.95	4.27E-01	-9.31294959	-29.1004972	8.04621291	-59.73297944
1	1	1	1	1	1

Table 4.2.7:variation of F with z for different values R and fixed values of m=2, n=2, λ=1, α=-0.01 and β=0.01.

z	G, m=2, n=2, λ=0.5, R=1		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	2	2	2
0.05	1.950242249	2.180562211	-18.42772303
0.1	1.896876998	2.362520967	-48.6394384
0.15	1.841132368	2.54911777	-51.55819828
0.2	1.784063997	2.751348422	-52.93855280
0.25	1.7265625	2.977466953	-52.40308726
0.3	1.669360987	3.234158755	-49.94739222
0.35	1.613042632	3.526966937	-45.86522032
0.4	1.558047994	3.860232623	-40.59984204
0.45	1.504682763	4.236749582	-34.61151426
0.5	1.453125	4.657327273	-28.27980992
0.55	1.403432864	5.120471784	-21.85379307
0.6	1.355551978	5.622374173	-15.45959313
0.65	1.309323023	6.157252466	-9.160772626
0.7	1.264489012	6.717556015	-3.055112948
0.75	1.220703126	7.291765758	2.607158261
0.8	1.177535991	7.851746956	7.299974112
0.85	1.134483231	8.303257158	10.13260981
0.9	1.090973023	8.317411342	9.870923368
0.95	1.046373389	6.802165338	5.154604511
1	1	1	1

Table 4.2.8:variation of G with z for different values of parameters α and β and fixed values of m=2, n=2, λ=0.5, R=1.

z	G, m=0.5, n=2, λ=1, R=3		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	5.00E-01	5.00E-01	5.00E-01
0.05	4.13E-01	440631.4835	-431.4749495
0.1	3.34E-01	-965875.6999	-682.1874215
0.15	2.65E-01	-1037191.94	-658.4236486
0.2	2.06E-01	-903606.7258	-611.3514488
0.25	1.59E-01	-660255.1607	-606.6892638
0.3	1.24E-01	-430572.6929	-567.3778458
0.35	1.02E-01	-270634.4677	-537.0332264
0.4	9.33E-02	-192329.9973	-492.7235298
0.45	9.83E-02	-187339.3679	-525.5191434
0.5	1.17E-01	-203894.9344	-471.6358929
0.55	1.50E-01	-140833.2562	-335.07978
0.6	1.97E-01	-26788.80053	-180.1630424
0.65	2.58E-01	49665.41149	-83.16455018
0.7	3.32E-01	47893.9139	-36.67532286
0.75	4.18E-01	42843.68386	12.25997749
0.8	5.16E-01	53922.32801	71.84068089
0.85	6.25E-01	80491.62885	69.54955678
0.9	7.43E-01	87912.7567	54.91276678
0.95	8.68E-01	-142007.3091	10.69665534
1	1	1	1

Table 4.2.9:variation of G with z for different values of parameters α and β and fixed values of m=0.5, n=2, λ=1, R=3.

z	G, m=0, n=0, λ=1, R=1		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	0.00E+00	0.00E+00	0.00E+00
0.05	5.25E-02	5.29E-02	1.397592776
0.1	1.05E-01	1.06E-01	1.64E-01
0.15	1.57E-01	1.58E-01	-1.023602948
0.2	2.10E-01	2.11E-01	-2.165491709
0.25	2.62E-01	2.63E-01	-3.277374683
0.3	3.13E-01	3.14E-01	-4.351680125
0.35	3.65E-01	3.66E-01	-5.337242017
0.4	4.16E-01	4.17E-01	-6.128551735
0.45	4.66E-01	4.67E-01	-6.5646354
0.5	5.16E-01	5.16E-01	-6.443628595
0.55	5.65E-01	5.65E-01	-5.561522248
0.6	6.13E-01	6.14E-01	-3.781446584
0.65	6.61E-01	6.61E-01	-1.131352509
0.7	7.09E-01	7.09E-01	2.114910945
0.75	7.56E-01	7.56E-01	5.390153138
0.8	8.03E-01	8.03E-01	8.091581072
0.85	8.51E-01	8.51E-01	9.767612641
0.9	8.99E-01	8.99E-01	9.751210414
0.95	9.49E-01	9.49E-01	7.1418348
1	1	1	1

Table 4.2.10:variation of G with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=1.

z	G, m=0, n=0, λ=1, R=3		
	α=0.0,β=0.0	α=-0.01, β=0.01	α=-0.05, β=0.05
0	0.00E+00	0.00E+00	0.00E+00
0.05	5.75E-02	5.88E-02	1532.923556
0.1	1.15E-01	1.17E-01	2836.806444
0.15	1.72E-01	1.75E-01	3586.513432
0.2	2.29E-01	2.32E-01	3647.222266
0.25	2.85E-01	2.88E-01	3790.871827
0.3	3.40E-01	3.43E-01	3915.373344
0.35	3.94E-01	3.97E-01	3915.472476
0.4	4.47E-01	4.50E-01	3795.600629
0.45	4.98E-01	5.00E-01	3577.098787
0.5	5.47E-01	5.49E-01	3258.759723
0.55	5.94E-01	5.96E-01	2837.462382
0.6	6.39E-01	6.41E-01	2335.328516
0.65	6.83E-01	6.84E-01	1824.344475
0.7	7.26E-01	7.26E-01	1402.213669
0.75	7.68E-01	7.68E-01	1094.117985
0.8	8.09E-01	8.10E-01	825.8999602
0.85	8.52E-01	8.52E-01	661.2775542
0.9	8.97E-01	8.97E-01	1632.446706
0.95	9.46E-01	9.46E-01	-1330.006541
1	1	1	1

Table 4.2.11:variation of G with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=3.

z	G, m=2, n=2 , λ=1, α=-0.01, β=0.01				
	R=1	R=3	R=5	R=7	R=9
0	2	2	2	2	2
0.05	8.78148	325.617	3045.74	-610.46	72015.6
0.1	15.381	-275	5846.43	-888.89	42584.8
0.15	22.5575	-495.27	2747.06	-127.2	-15227
0.2	30.4159	-649.66	2821.89	421.952	-13974
0.25	39.0619	-716.99	-7202.5	146.973	-9303.3
0.3	48.5834	-699.61	-11754	168.073	6914.91
0.35	59.0289	-607.35	-11433	118.568	4748.53
0.4	70.3833	-466.45	-1262.5	60.3586	123.728
0.45	82.5431	-324.12	2873.57	14.826	-375.03
0.5	95.3056	-222.8	-3154.6	5.87659	-833.3
0.55	108.394	-120.72	-184.57	60.9603	-284.48
0.6	121.548	-17.102	249.46	64.9803	227.69
0.65	134.707	-65.377	356.884	-15.601	1439.74
0.7	148.253	-182.4	277.867	-86.688	-1450
0.75	163.118	-310.78	525.167	-139.41	859.562
0.8	180.211	-469.15	-726.62	-149.73	376.54
0.85	198.203	-626.29	-590.85	-102.69	140.442
0.9	208.086	-686.07	-795.46	-26.913	711.925
0.95	179.613	-543.45	-1256.5	45.541	1583.37
1	1	1	1	1	1

Table 4.2.12: variation of F with z for different values of R and fixed values of m=2, n=2, λ=1, α=-0.01and β=0.01.

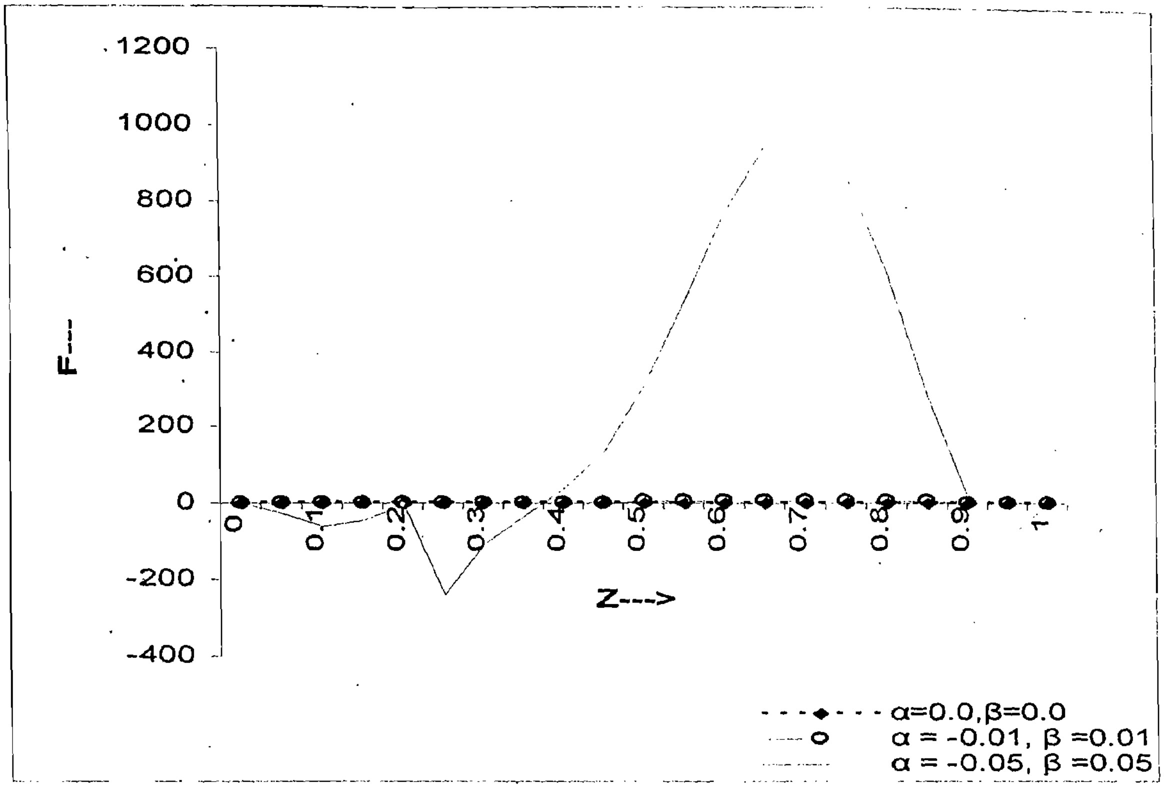


FIG. 4.2.1: variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=1, R=1.

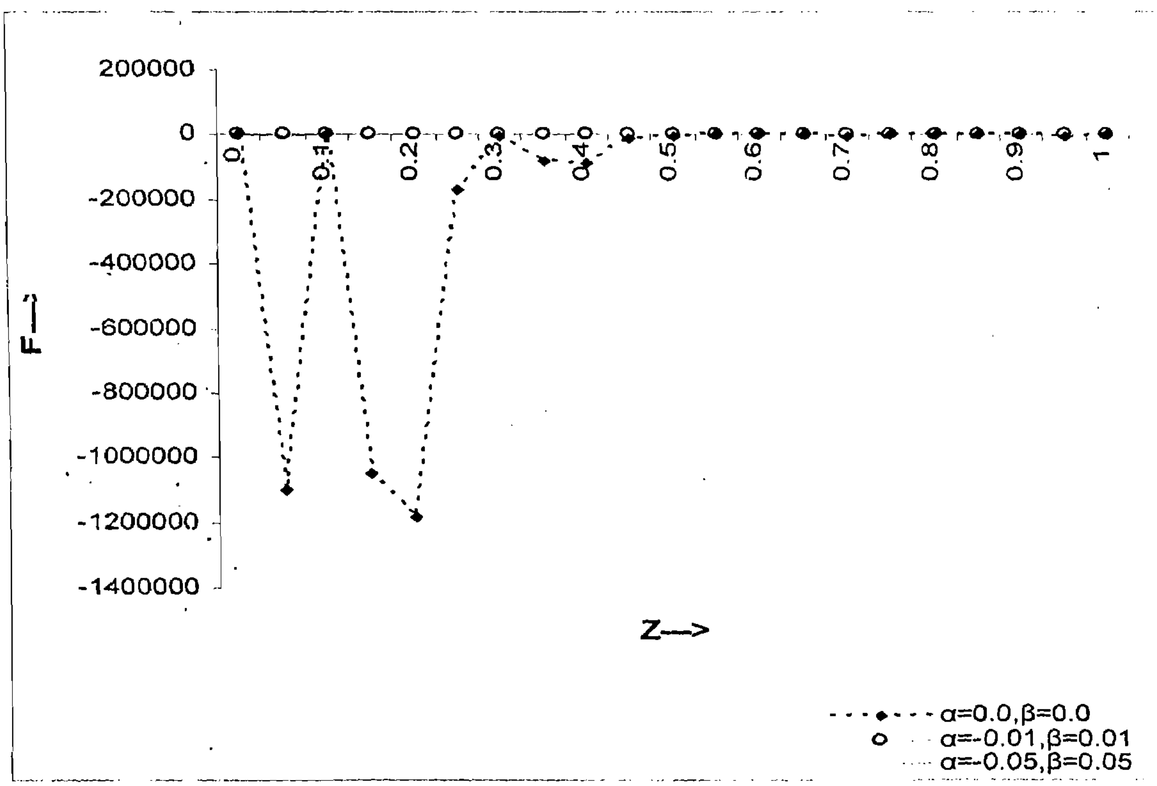


FIG. 4.2.2: variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=1, R=3.

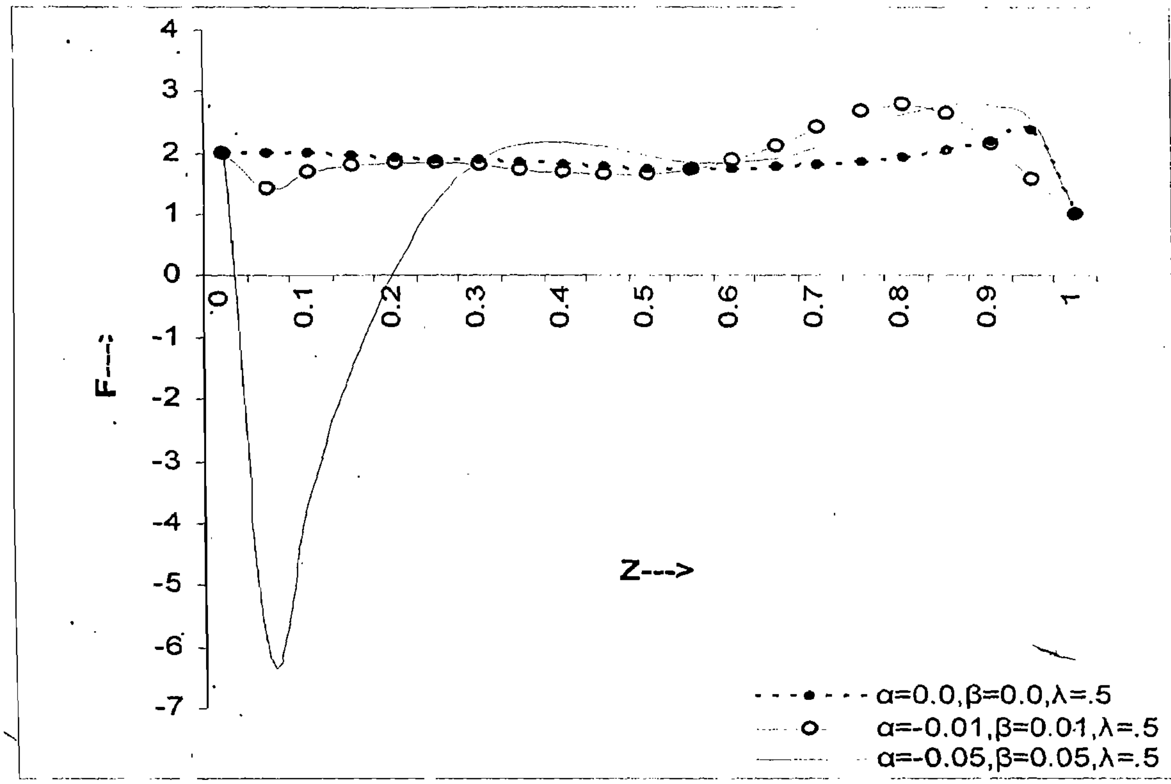


FIG. 4.2.3: variation of F with z for different values of parameters α and β and fixed values of m=2, n=2, λ=0.5 and R=1.

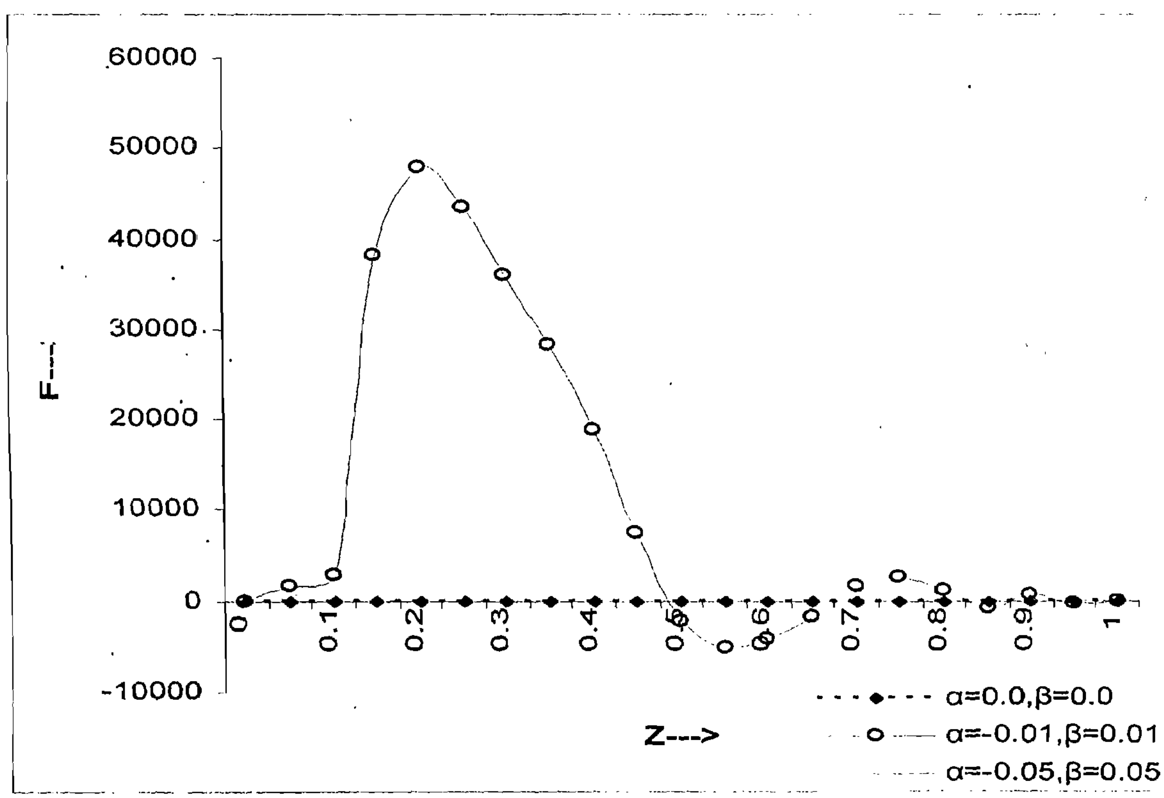


FIG. 4.2.4: variation of F with z for different values of parameters α and β and fixed values of m=0.5, n=2, λ=1, R=3.

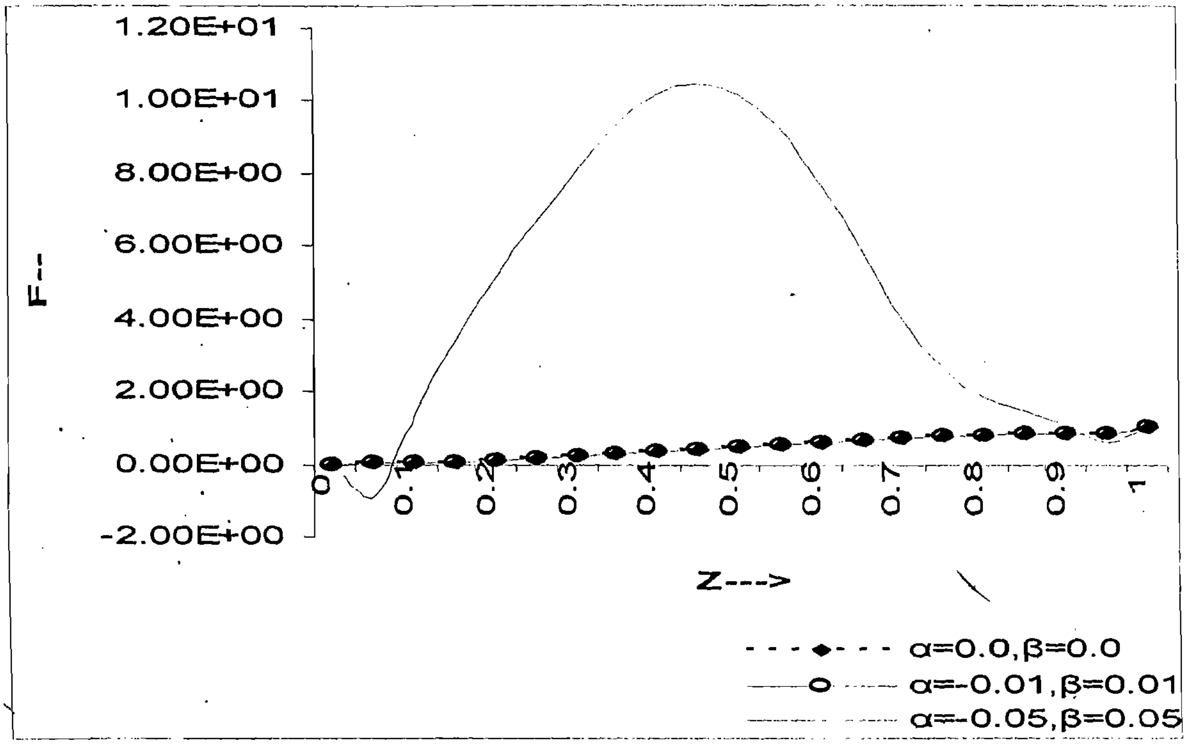


FIG. 4.2.5: variation of F with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=1.

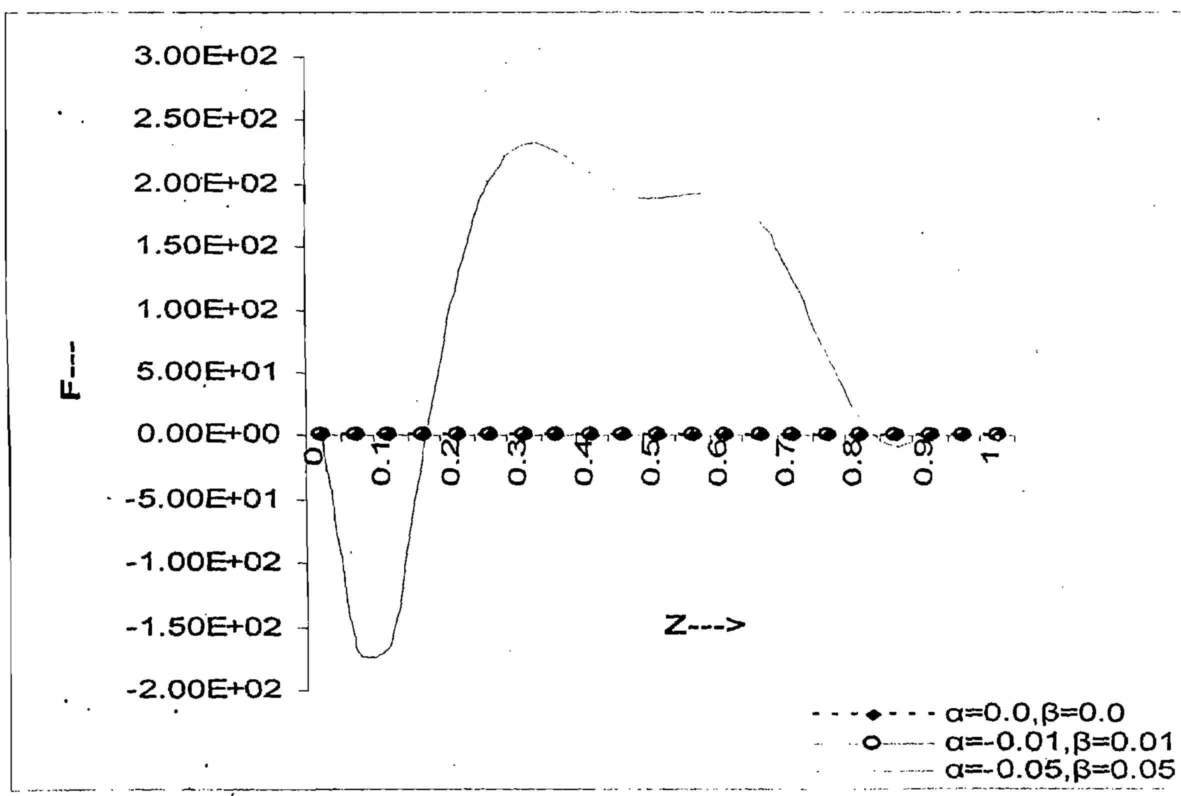


FIG. 4.2.6: variation of F with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=3.

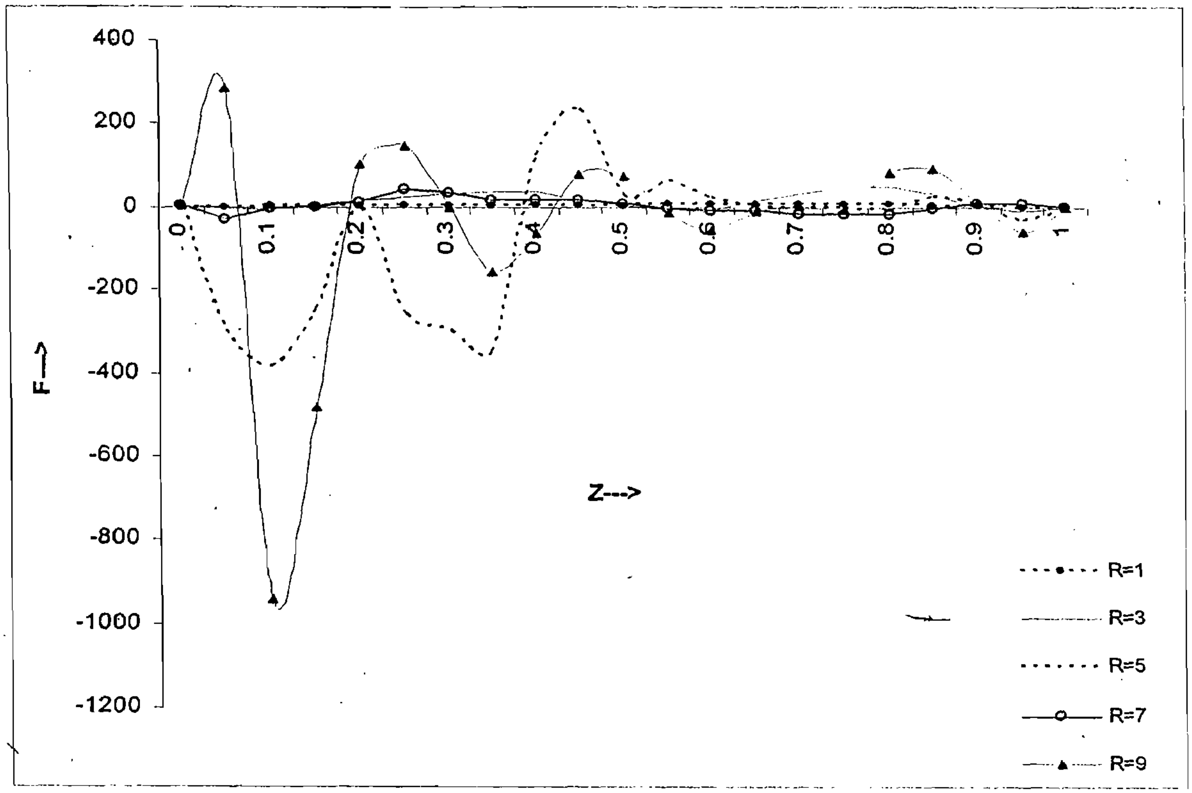


FIG. 4.2.7: variation of F with z for different values of R and fixed values of m=2, n=2, λ=1, α=-0.01 and β=0.01.

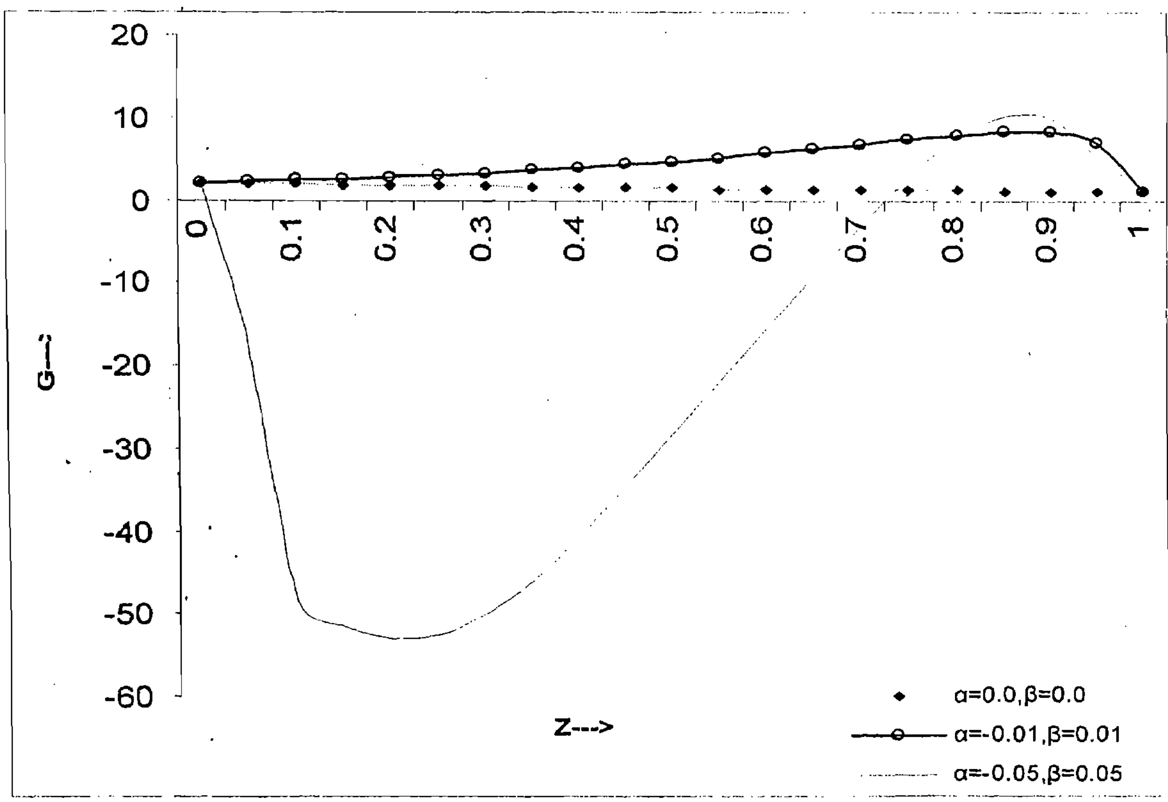


FIG. 4.2.8: variation of G with z for different values of parameters α and β and fixed values of m=2, n=2, λ=0.5, R=1.

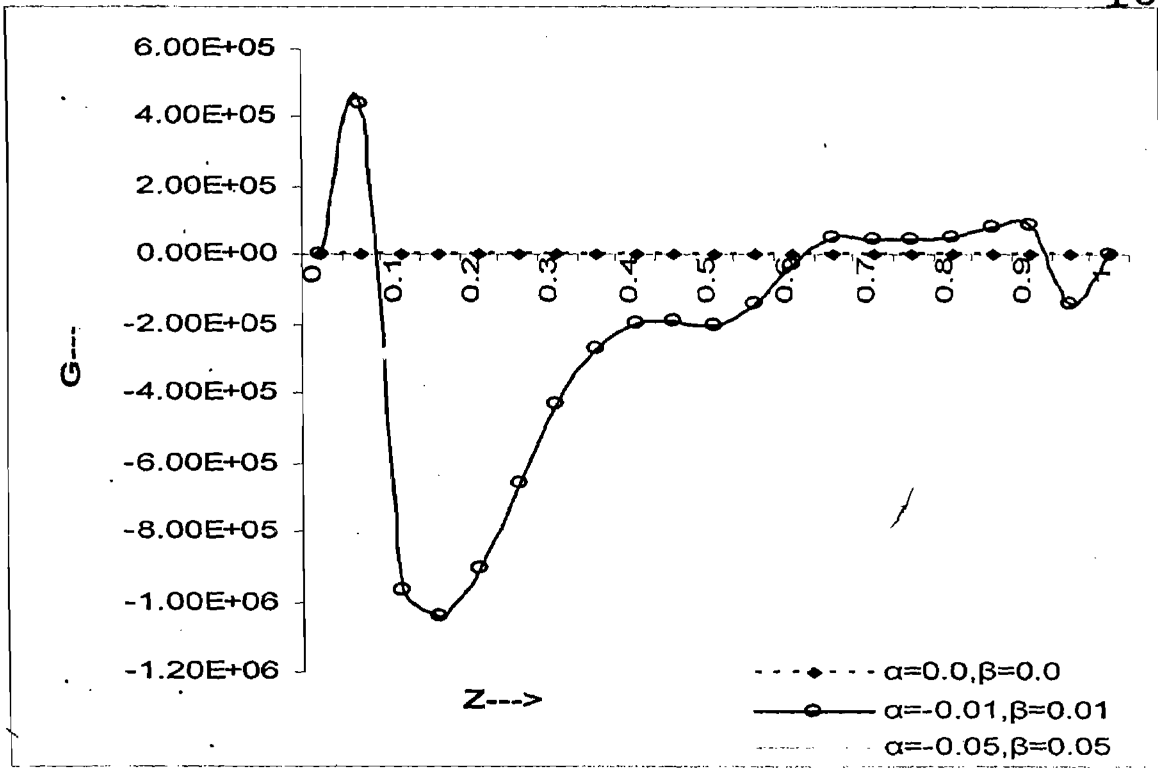


FIG. 4.2.9: variation of G with z for different values of parameters α and β and fixed values of m=0.5, n=2, λ=1, R=3.

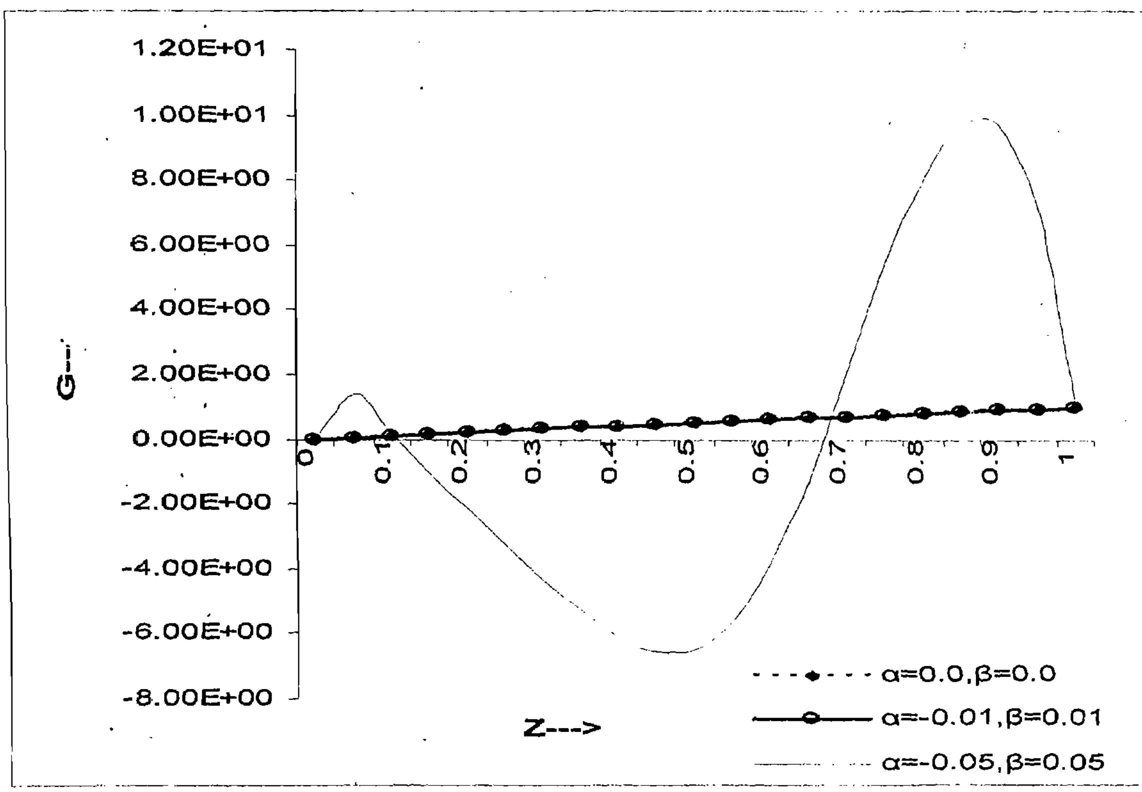


FIG. 4.2.10:variation of G with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=1.

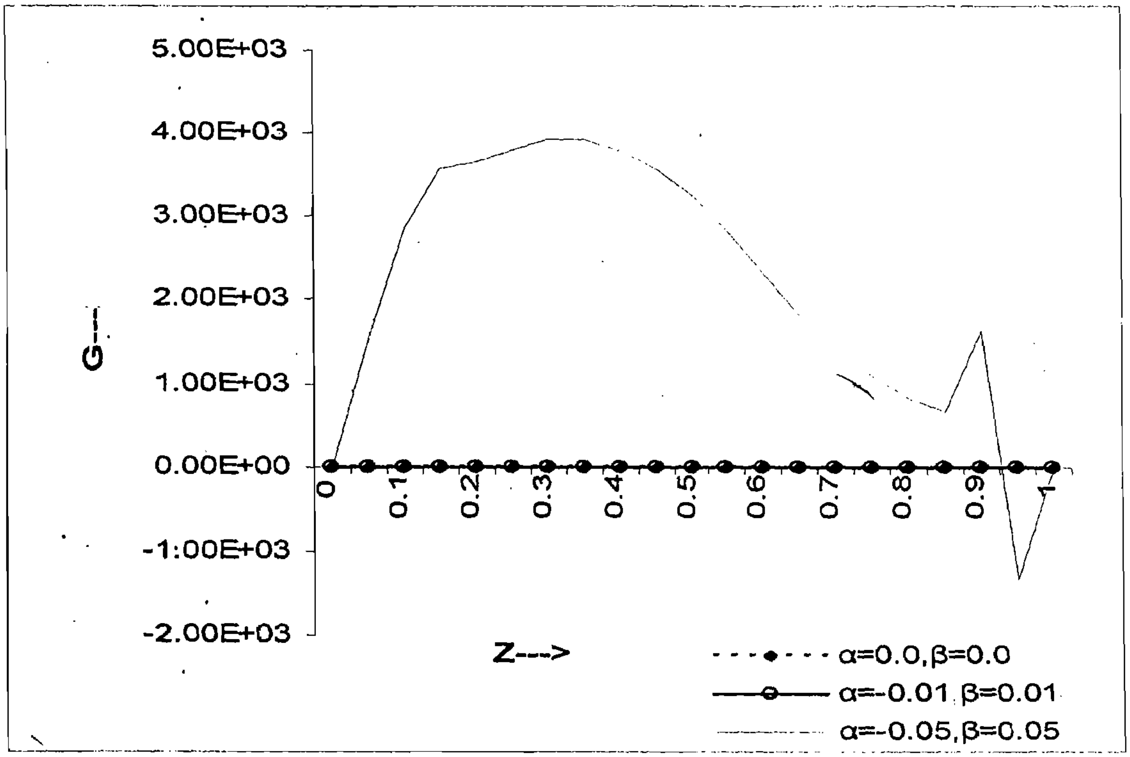


FIG. 4.2.11:variation of G with z for different values of parameters α and β and fixed values of m=0, n=0, λ=1, R=3.

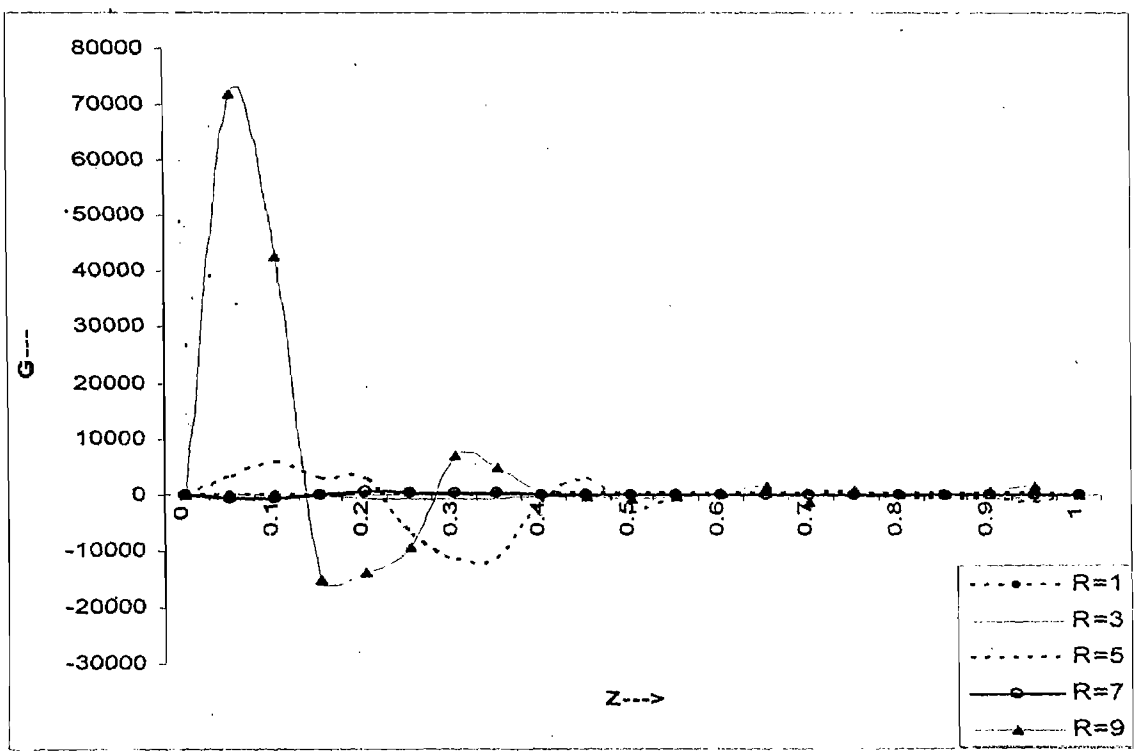


FIG. 4.2.12: variation of G with z for different values of R and fixed values of m=2, n=2, λ=1, α=-0.01and β=0.01.

4.3 Flow of Incompressible Fluid between two rectangular and circular plates:

In the present investigation, we study the problem of flow of incompressible fluid between two rectangular and circular plates for large value of Reynolds number α and present interesting results based on the numerical methods and represented graphically.

4.3.1 Formulation of the Problem:

TWO DIMENSIONAL FLOW:

The geometry of viscous incompressible fluid between two parallel infinitely long rectangular plates of width 2a separated by a small distance h (t) is shown in figure 4.3.1. The lower plate remains fixed and the upper plate is assumed to move downwards.

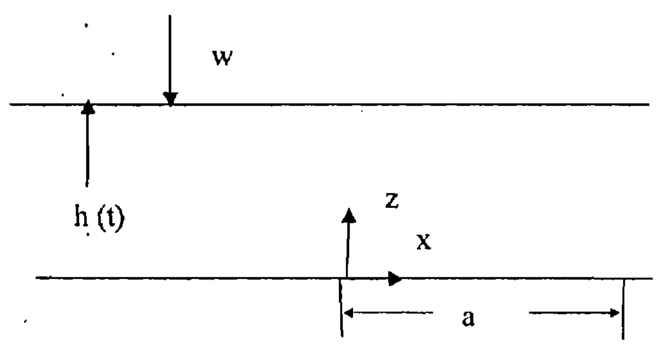


Figure 4.3.1 Geometry of flow between parallel rectangular plates and circular plates.

For the above two dimensional flow the fluid has velocity vector (u, 0, w) and the equation of continuity and of motion of fluid referred to Cartesian coordinates (Batchelor (1967)) are

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+v\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right) \qquad \text{--- (4.3.1)}$$

and the boundary conditions are

$u=w=0$ at $z=0$

$u=0$ and $w=\dot{h}$ at $z=h$ --- (4.3.2)

For two dimensional flow equations, we introduce the following transformations (Chandrashekeran and Ramnaih (1983))

Let $w=-\frac{v}{h}F(\eta)$

Where $\eta=\frac{z}{h(t)}$ so that $u=\frac{xvF'}{h^2}$ --- (4.3.3)

Where the prime denotes differentiation with respect to η and $\dot{h}=\frac{dh}{dt}$

Substituting equations (4.3.3) into equation (4.4.1) and eliminating pressure we obtain a non- linear ordinary differential equation

$$F''''+FF'''-\alpha\eta F'''-F'F''-3\alpha F''=0 \qquad \text{--- (4.4.4)}$$

with boundary conditions

$F=0,\quad F'=0$ at $\eta=0$

$F=\alpha,\quad F'=0$ at $\eta=1$ --- (4.4.5)

AXISYMMETRIC FLOW:

Figure 4.3.1 shows the cylindrical coordinates system used for the description of the problem of axisymmetric flow between two parallel circular plates of radius 'a' separated by a small distance h (t). In this case the upper plate moves towards the lower plate that is fixed. The u and w represent the velocity components in the radial and axial directions respectively. The axisymmetric unsteady flow is governed by the following equations of continuity and of motion

$$\frac{\partial}{\partial x}(xu)+\frac{\partial}{\partial z}(xw)=0$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{x}\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{x^2}\right)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+v\left(\frac{\partial^2 w}{\partial x^2}+\frac{1}{x}\frac{\partial w}{\partial x}+\frac{\partial^2 w}{\partial z^2}\right) \qquad \text{--- (4.3.6)}$$

The relevant boundary conditions are

$$u = w = 0 \qquad \text{at} \qquad z = 0$$

$$u=0 \text{ and } w=\dot{h} \qquad \text{at} \qquad z=h \qquad \text{--- (4.3.7)}$$

For axisymmetric flow, we introduce the transformations

Let $$w=-\frac{2v}{h}F(\eta)$$

Where $$\eta=\frac{z}{h(t)} \qquad \text{so that} \qquad u=\frac{xvF'}{h^2} \qquad \text{--- (4.3.8)}$$

Substituting equation (4.3.8) into (4.3.7), we get

$$F'''' + 2FF''' - \alpha\eta F''' - 3\alpha F'' = 0$$

with boundary conditions

$$F = 0, \quad F' = 0 \quad \text{at} \quad \eta = 0$$

$$F = \frac{\alpha}{2}, \quad F' = 0 \quad \text{at} \quad \eta = 1$$

These equations and boundary conditions for both the two dimensional and axisymmetric flows can be put in general form as

$$F'''' + AFF''' - \alpha\eta F''' - BF'F'' - 3\alpha F'' = 0 \qquad \text{--- (4.3.9)}$$

with boundary conditions

$$F = 0, \quad F' = 0 \quad \text{at} \quad \eta = 0$$

$$F = C\alpha \quad F' = 0 \quad \text{at} \quad \eta = 1 \qquad \text{--- (4.3.10)}$$

For two dimensional flow A=1, B=-1 and C=1 and

For axisymmetric flow A=2, B=0, C=1/2

4.3.2 Method of Solution:

Equation (4.4.6) is highly non- linear in F which is function of η. Divide η, [0,1] into hundred equal parts each of length 0.01. The finite difference approximation scheme for first, second, third and fourth order derivatives has been used to discretize the differential equation (4.4.6) to obtain,

$$\frac{F_{i+3} - 4F_{i+2} + 6F_{i+1} - 4F_i + F_{i-1}}{h^4} + \left(AF_i - \alpha\eta_i\right)\left(\frac{F_{i+2} - 3F_{i+1} + 3F_i - F_{i-1}}{h^3}\right)$$
$$- B\left(\frac{F_{i+1} - F_{i-1}}{2h}\right)\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right) - 3\alpha\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right) = 0$$

with boundary conditions

$$F_1 = 0, \quad F_1' = 0 \quad \text{at} \quad \eta = 1,$$

$$F_{101} = C\alpha \quad F_{101}' = 0 \quad \text{at} \quad \eta = 2.$$

Above equation can be solved by using iterative scheme.

4.3.3 Results and Discussion:

FOR TWO DIMENSIONAL FLOW:

Figure 4.3.2 represents the behavior of axial velocity w with η for different values of Reynolds numbers α. It has been observed and depicted that for two dimensional flow the axial velocity has turbulent flow at the lower discs from η=0.0 to η=0.15 and from η=0.15 to η =1.0 the fluid shows the normal behavior of circulation. But as soon as the Reynolds number increases the recirculation at the upper disc as well as at the lower disc increases. Though the computations have been made for higher value of Reynolds numbers, it has been noticed that the flow is turbulent in the study.

AXISYMMETRIC FLOW:

Figure 4.3.3 represents the graph between axial velocity and η for various values of Reynolds numbers α. Hence it is concluded that the distribution of axial velocity is symmetrical between the two discs only for small values of Reynolds number α and recirculation occurs at upper disc in comparison to the recirculation at lower disc on increasing the value of Reynolds number. Due to recirculation for larger values of Reynolds numbers, the axial velocity has negative sign i.e. liquid comes out of disc. The study is of great importance that the chemical industry may be interested to find the limit of Reynolds number for the fluid being used for such set-ups.

Figure 4.3.4 represents the behavior of axial velocity w with η for extremely high values of Reynolds numbers α. Hence it has been observed and concluded that for non- Newtonian fluids recirculation occurs at the lower disc as compared to recirculation at the upper disc. The recirculation occurs for η =0 to η =0.5 and after that the fluid shows the normal circulation. The behavior of the fluid is turbulent as higher value of Reynolds number is considered. It is also noticed that this behavior of the fluid in the practical purpose is to be seen by the technocrats and engineers in the industry.

η	Axial velocity w		
	α=0.5	α=1	α=2
0.0	0.00E+00	0.00E+00	0.00E+00
0.05	101.43426	114.09862	138.03267
0.10	79.049278	81.399545	86.699362
0.15	94.43755	94.276556	93.044341
0.20	115.3852	114.74268	112.01587
0.25	129.42872	128.35184	124.49096
0.30	136.94064	135.55535	130.81461
0.35	138.62251	137.04178	131.63867
0.40	135.2252	133.54728	127.67084
0.45	127.51926	125.82587	119.63583
0.50	116.28825	114.64331	108.27066
0.55	102.32869	100.7787	94.327913
0.60	86.454448	85.02662	78.586439
0.65	69.490698	68.212942	61.837022
0.70	52.332318	51.146976	44.996444
0.75	35.751338	34.904555	28.757911
0.80	21.052858	19.699668	14.640111
0.85	8.7370743	8.9408801	3.3399188
0.90	-3.71E-01	-1.94531	-5.530556
0.95	-1.45481	-2.088513	-3.103733
1.0	5.00E-01	1	2

Table 4.3.1. variation of axial velocity w with η for different values of α for two- dimensional flow.

η	Axial velocity w			
	α=1	α=3	α=5	α=10
0.0	0.00E+00	0.00E+00	0.00E+00	0.00E+00
0.05	16.81093989	22.84596846	28.8967283	2694.21976
0.10	50.91974849	67.09066636	87.9299017	771.627927
0.15	73.97101198	95.4131911	119.747543	312.618839
0.20	90.19872292	114.2084381	138.744819	-121.09434
0.25	100.3621201	124.811361	147.017869	-517.76703
0.30	105.2010638	128.4619363	146.49857	-847.23706
0.35	105.4358395	126.3148419	138.95952	-1086.6604
0.40	101.7708451	119.4533677	126.03571	-1235.6559
0.45	94.89798675	108.9001369	109.24217	-1302.7728
0.50	85.49931506	95.62558879	89.9899097	-1298.4347
0.55	74.24996853	80.55450178	69.6006414	-1231.5346
0.60	61.81996287	64.57248834	49.3230081	-1109.5804
0.65	48.87703053	48.53334198	30.351374	-941.27299
0.70	36.08690874	33.26597692	13.8332232	-739.38564
0.75	24.13351517	19.59380608	1.06507064	-521.94041
0.80	13.61286199	8.264560331	-8.0078654	-310.96435
0.85	5.628753863	4.79E-01	-6.1174393	-131.06316
0.90	-6.67E-01	-4.63877143	-7.8750893	9.97E-01
0.95	-5.35E-01	-8.63E-01	-1.0184486	10.8568876
1.0	5.00E-01	1.5	2.5	10

Table 4.3.2. variation of axial velocity w with η for different values of Reynolds numbers α for axisymmetric flow.

η	Axial velocity w	
	α=500	α=1000
0.0	0.00E+00	0.00E+00
0.05	-3857.26597	-16038.21464
0.10	-20204.6617	-154201.4436
0.15	-23752.2784	-185646.8045
0.20	-23654.6656	-189095.4403
0.25	-22082.5024	-179966.5236
0.30	-19148.983	-158950.4631
0.35	-14694.4345	-124871.0731
0.40	-8854.72898	-78940.10853
0.45	-2217.87997	-25939.2342
0.50	4192.170451	25956.5346
0.55	8995.329829	65935.30317
0.60	10326.86406	80675.31395
0.65	11732.49697	93829.21911
0.70	11644.37118	94990.68836
0.75	9800.300788	81206.7718
0.80	6518.471751	54803.18396
0.85	2754.155062	23541.9972
0.90	-8.41E-01	-2.089904806
0.95	-4.89E-03	-2.65E-05
1.0	250	500

Table 4.3.3 Variation of axial velocity w with η for very high values of α for axisymmetric flow.

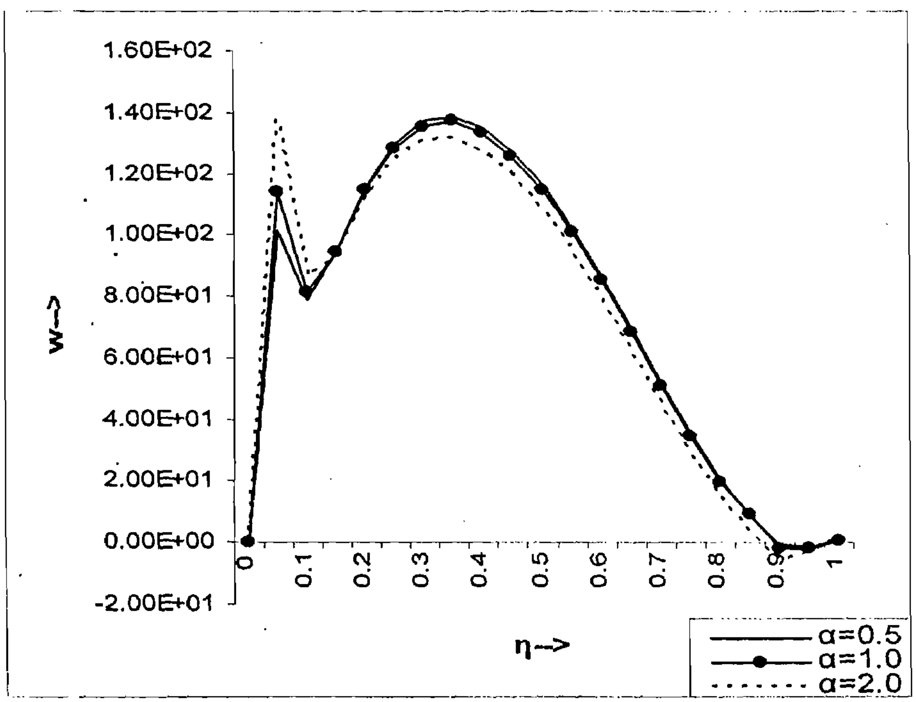


Fig. 4.3.2 Graph representing the variation of axial velocity w with η for different values of α for two- dimensional fluid flow.

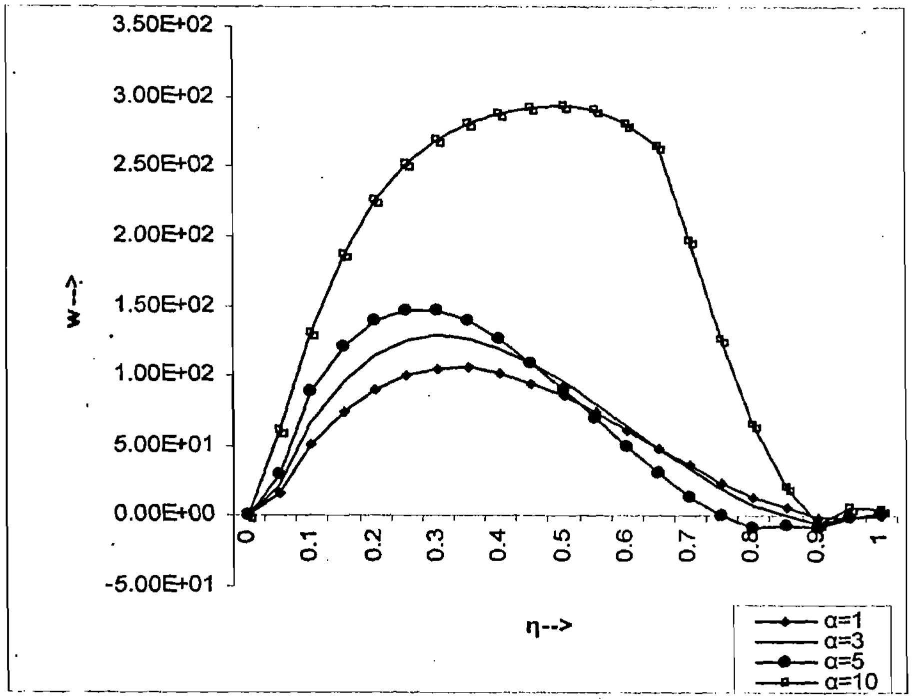


Fig.4.3.3 Graph representing variation of axial velocity w with η for different values of α for axisymmetric flow.

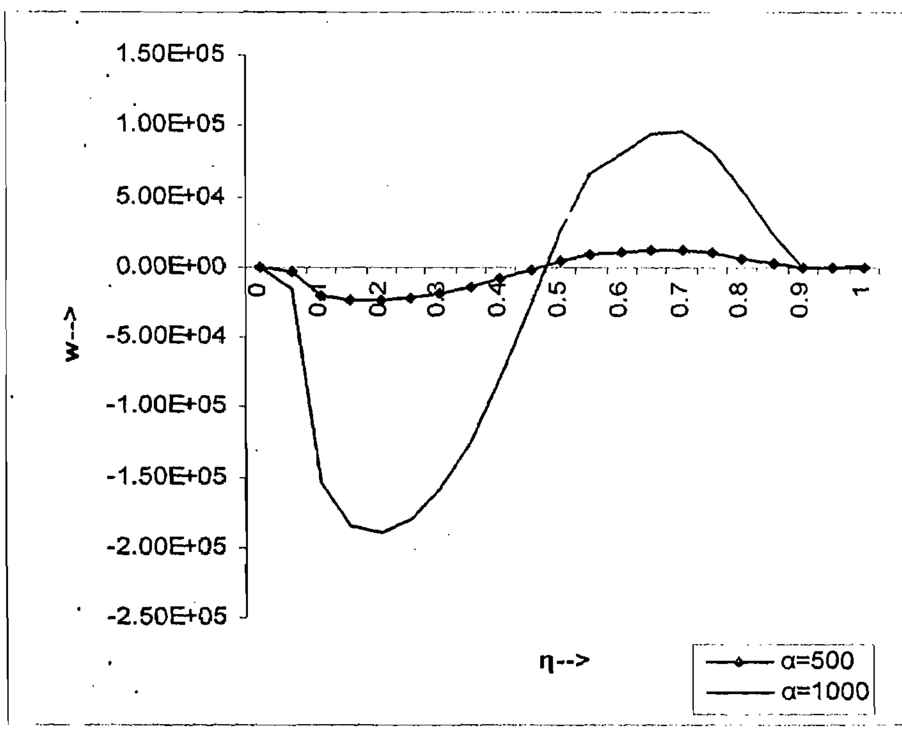


Fig.4.3.4 graph representing variation of axial velocity w with η for extremely high values of α for axisymmetric flow.

4.4 Numerical solution of steady flow of a viscoelastic fluid between coaxial rotating porous disks with uniform suction or injection for high Reynolds numbers:

Flows induced by rotating disks are of considerable fundamental interest because of the richness of the physical phenomenon they encompass. Thus present investigation is devoted to the computation of flow of visco-elastic fluids between a pair of coaxial porous rotating disks with uniform suction or injection for high value of Reynolds numbers.

4.4.1 Formulation of the Problem:

The computation of the flow of visco-elastic fluids is at best a complicated affair, therefore, in order to get some insight into their flow behavior, it is preferable to restrict to a model with minimum number of parameters in the constitutive equations. Accordingly, we have chosen the model of Walters liquid B' fluid in the present section as it involves only non-Newtonian parameters. The equation of motion for a Walters liquid B' is given in relation (3.4).

Let us consider the steady flow of a Walters B' fluid filling the space between two infinite parallel disks. Choosing a cylindrical polar coordinate system(r, θ, z), let the lower in the plane z=0 rotate with a constant angular velocity Ω while the upper disk at z=d has angular velocity Ω. Let u, v and w represent the velocity component in the increasing the direction of r, θ and z respectively.

The boundary conditions of the problem are:

$$u = 0, \quad v = rS\Omega, \quad w = w_0 \ \text{at} \ z = 0$$

$$u = 0, \quad v = r\Omega, \quad w = w_0 \ \text{at} \ z = d \qquad \text{--- (4.4.1)}$$

4.4.2 Numerical Solution:

In order to solve the problem under consideration, the equations (3.4) and (4.4.1) have to be written in cylindrical polar coordinate system. This leads to a set of three non- linear partial differential equations. In order to obtain their solution, we first transform them to a set of ordinary non-linear differential equations by making use of the transformation Bhatnagar (1961) as:

$$u = -\frac{1}{2} r\Omega H', \qquad v = r\Omega G(\eta), \quad w = d\Omega H(\eta) \qquad \text{--- (4.4.2)}$$

where $\eta = z/d$ and prime denotes differentiation with respect to η.

Substituting from (4.4.2) into the equation of motion (3.4), the equation in the θ-direction transforms to:

$$G'' + RH'G - RHG' + KR(2H'G - HG''' - H''G') = 0 \qquad \text{--- (4.4.3)}$$

while the elimination of the pressure between the equations in the radial direction r and axial direction z leads to:

$$H'''' + KR(H'H'''' + 2H''H''' + 8G'G'' - HH''''') - RHH''' - 4GG' = 0 \qquad \text{--- (4.4.4)}$$

where

1. $R = \dfrac{\Omega d^2}{v}$ (Reynolds number) and

2. $K = \dfrac{K_0}{\rho d^2}$ (viscoelastic parameter)

The transformations (4.4.2) applied to the boundary conditions (4.4.1) which give

$$G(0) = S, \quad G(1) = 1,$$
$$H(0) = a, \quad H(1) = a,$$
$$H'(0) = 0, \quad H'(1) = 0. \qquad \text{--- (4.4.5)}$$

In the above equations, S is the ratio of the angular velocities of the two disks, respectively located at z=0 (angular velocity ΩS) and z=d (angular velocity Ω), and "a" is the suction or injection parameter, which takes negative values for suction and positive values for injection, where

$$a = \frac{w_0}{\Omega d}. \qquad \text{--- (4.4.6)}$$

Equations 4.4.3 and 4.4.4 are highly non-linear in G and H, which are functions of η. The first, second, third, fourth and fifth order derivatives can be given by finite-difference approximation and the resulting equations are solved by Newton-Raphson iterative method.

4.4.3 Results and Discussion:

Tables 4.4.1, 4.4.2, 4.4.3 show the detailed values of the function $H(\eta)$ which characterizes the axial component of velocity, choosing the fluid to be visco-elastic and can be depicted from the figures 4.4.1, 4.4.2, 4.4.3.

The calculations show the magnitude of H is maximum for S=0 as shown in figure 4.4.1. Also H>0 near the lower disk means that the flow is directed towards the axis of the two disks and the corresponding value of H must be positive, i.e. the fluid flow towards the upper disk as shown in figure 4.4.1.

Figure 4.4.2 represents the variation of H with R for S=-1. It has been observed and concluded that at S=-1, a=0 and K=0.1 the axial velocity decreases first with an increase in Reynolds number and start increasing thereafter middle of the disks at η =0.5. The condition of recirculation occurs at the lower disk at point η =0 to η =0.5 and attains maximum at η =0.25 and from point η =0.5 to η =1.0 the set of values shows the normal study of circulation and attains maximum at η =0.75

The figure 4.4.3 depicts the variation of axial velocity H with K for a Newtonian fluid and a non-Newtonian fluid and tabulated in table 4.4.3. For Newtonian fluid the behavior of fluid between the two discs is normal but for non-Newtonian fluids the condition of recirculation takes place at the upper discs. With the increase in visco-elastic parameter the behavior of the fluid becomes turbulent near the upper discs.

Tables 4.4.4, 4.4.5, 4.4.6 show the detailed values of the function G(η), which characterize the transverse component of velocity and can be depicted from the figures 4.4.4, 4.4.5, 4.4.6.

Figure 4.4.4 shows that the transverse velocity decreases with the increase in Reynolds number.

From figure 4.4.5 it has been observed and concluded that for fixed value of S=0, R= 0.2 and a=1 transverse velocity G varies with visco-elastic parameter K and larger recirculation takes place at the lower disc as compared to the upper disc.

Figure 4.4.6 shows that for injection condition the flow of the visco-elastic the two discs is normal. But as soon as the suction parameter increases the behavior of the fluid between the two discs becomes turbulent and larger recirculation takes place near the upper disc as compared to the lower disc.

η	H, a=0, R=0.2, K=0.1		
	S=0.0	S=0.5	S=-1
0	0.00E+00	0.00E+00	0.00E+00
0.05	3.08E-05	2.65E-05	-2.71E-05
0.1	1.13E-04	9.58E-05	-8.64E-05
0.15	2.33E-04	1.94E-04	-1.52E-04
0.2	3.75E-04	3.07E-04	-2.05E-04
0.25	5.27E-04	4.25E-04	-2.34E-04
0.3	6.76E-04	5.37E-04	-2.35E-04
0.35	8.11E-04	6.34E-04	-2.07E-04
0.4	9.22E-04	7.10E-04	-1.54E-04
0.45	1.00E-03	7.61E-04	-8.16E-05
0.5	1.04E-03	7.81E-04	5.22E-08
0.55	1.04E 03	7.71E-04	8.17E-05
0.6	9.98E-04	7.30E-04	1.54E-04
0.65	9.14E-04	6.60E-04	2.07E-04
0.7	7.94E-04	5.66E-04	2.35E-04
0.75	6.45E-04	4.54E-04	2.34E-04
0.8	4.78E-04	3.33E-04	2.05E-04
0.85	3.09E-04	2.13E-04	1.52E-04
0.9	1.57E-04	1.07E-04	8.64E-05
0.95	4.44E-05	2.99E-05	2.71E-05
1	0.00E+00	0.00E+00	0.00E+00

Table 4.4.1: variation of H with η for different values of S and fixed values of a=0, R=0.2 and K=0.1

η	H, a=0, S=-1, K=0.1		
	Re=0.2	Re=0.4	Re=0.6
0	0.00E+00	0.00E+00	0.00E+00
0.05	-2.71E-05	-5.42E-05	-8.13E-05
0.1	-8.64E-05	-1.73E-04	-2.59E-04
0.15	-1.52E-04	-3.04E-04	-4.56E-04
0.2	-2.05E-04	-4.10E-04	-6.15E-04
0.25	-2.34E-04	-4.69E-04	-7.04E-04
0.3	-2.35E-04	-4.71E-04	-7.06E-04
0.35	-2.07E-04	-4.14E-04	-6.21E-04
0.4	-1.54E-04	-3.07E-04	-4.61E-04
0.45	-8.16E-05	-1.63E-04	-2.44E-04
0.5	5.22E-08	4.18E-07	1.41E-06
0.55	8.17E-05	1.64E-04	2.47E-04
0.6	1.54E-04	3.08E-04	4.63E-04
0.65	2.07E-04	4.15E-04	6.24E-04
0.7	2.35E-04	4.71E-04	7.08E-04
0.75	2.34E-04	4.69E-04	7.05E-04
0.8	2.05E-04	4.10E-04	6.16E-04
0.85	1.52E-04	3.04E-04	4.56E-04
0.9	8.64E-05	1.73E-04	2.60E-04
0.95	2.71E-05	5.42E-05	8.14E-05
1	0.00E+00	0.00E+00	0.00E+00

Table 4.4.2: variation of H with η for different values of R and for fixed value of a=0, S=-1 and K=0.1.

η	H, a=0.5, S=0.5, R=0.2		
	K=0	K=0.1	K=0.2
0	0.00E+00	0.00E+00	0.00E+00
0.05	5.00E-01	3.86E-01	4.47E-01
0.1	5.00E-01	4.42E-01	4.81E-01
0.15	5.00E-01	2.77E-01	4.98E-01
0.2	5.00E-01	1.45E-01	4.99E-01
0.25	5.00E-01	3.65E-01	5.01E-01
0.3	5.01E-01	7.32E-01	5.02E-01
0.35	5.01E-01	7.08E-01	5.02E-01
0.4	5.01E-01	4.84E-02	5.02E-01
0.45	5.01E-01	-3.40E-01	4.99E-01
0.5	5.01E-01	1.02361824	4.94E-01
0.55	5.01E-01	3.39829305	5.06E-01
0.6	5.01E-01	3.70694648	5.21E-01
0.65	5.01E-01	3.24E-01	4.76E-01
0.7	5.01E-01	-3.00E-01	4.17E-01
0.75	5.00E-01	2.19900275	5.96E-01
0.8	5.00E-01	1.96356699	7.86E-01
0.85	5.00E-01	-2.54084	-4.55E-02
0.9	5.00E-01	-7.0129167	-1.1319887
0.95	5.00E-01	5.38E-01	9.18E-01
1	5.00E-01	5.00E-01	5.00E-01

Table 4.4.3: variation of H with η for different values of K and fixed values of a=0.5, S=0.5 and R= 0.2.

η	G, a=1, S=0, K=0.1		
	R=0.2	R=0.4	R=0.6
0	0.00E+00	0.00E+00	0.00E+00
0.05	4.53E-02	4.07E-02	3.62E-02
0.1	9.10E-02	8.23E-02	7.38E-02
0.15	1.37E-01	1.25E-01	1.13E-01
0.2	1.84E-01	1.68E-01	1.53E-01
0.25	2.31E-01	2.13E-01	1.94E-01
0.3	2.79E-01	2.58E-01	2.37E-01
0.35	3.27E-01	3.05E-01	2.82E-01
0.4	3.76E-01	3.52E-01	3.28E-01
0.45	4.25E-01	4.00E-01	3.75E-01
0.5	4.75E-01	4.50E-01	4.24E-01
0.55	5.25E-01	5.00E-01	4.74E-01
0.6	5.76E-01	5.51E-01	5.26E-01
0.65	6.27E-01	6.04E-01	5.79E-01
0.7	6.79E-01	6.57E-01	6.34E-01
0.75	7.31E-01	7.12E-01	6.91E-01
0.8	7.84E-01	7.67E-01	7.49E-01
0.85	8.37E-01	8.24E-01	8.09E-01
0.9	8.91E-01	8.81E-01	8.71E-01
0.95	9.45E-01	9.40E-01	9.35E-01
1	1	1	1

Table 4.4.4: variation of G with η for different values of R and fixed values of a=1, S=0 and K=0.1

η	G, a=1, S=0, R=0.2		
	K=0.05	K=0.1	K=1.5
0	0.00E+00	0.00E+00	0.00E+00
0.05	-2.75E-01	4.53E-02	-1.95869
0.1	-2.63E-01	9.10E-02	-2.92909303
0.15	-1.18E-01	1.37E-01	-1.56604385
0.2	1.15E-01	1.84E-01	5.08E-01
0.25	3.64E-01	2.31E-01	1.811314532
0.3	5.52E-01	2.79E-01	1.598443098
0.35	6.30E-01	3.27E-01	4.69E-01
0.4	5.96E-01	3.76E-01	-4.44E-01
0.45	4.86E-01	4.25E-01	-4.82E-01
0.5	3.64E-01	4.75E-01	1.78E-01
0.55	2.91E-01	5.25E-01	8.88E-01
0.6	3.10E-01	5.76E-01	1.155133489
0.65	4.36E-01	6.27E-01	9.69E-01
0.7	6.38E-01	6.79E-01	6.82E-01
0.75	8.39E-01	7.31E-01	6.48E-01
0.8	9.61E-01	7.84E-01	8.07E-01
0.85	1.00965425	8.37E-01	8.66E-01
0.9	1.07150802	8.91E-01	7.04E-01
0.95	1.13116353	9.45E-01	6.84E-02
1	1	1	1

Table 4.4.5: variation of G with η for different values of K and fixed values of a=1, S=0 and R= 0.2.

H	G, K=0.15, S=0, R=0.2		
	a=0.0	a=0.5	a=1.5
0	0.00E+00	0.00E+00	0.00E+00
0.05	5.00E-02	-3.26402873	-14.3151004
0.1	1.00E-01	-5.80328882	-21.6568324
0.15	1.50E-01	-6.01923885	-8.83842647
0.2	2.00E-01	-3.5195133	3.963283822
0.25	2.50E-01	5.27E-01	10.63847898
0.3	3.00E-01	3.93555037	6.346511192
0.35	3.50E-01	5.139142158	-1.94975212
0.4	4.00E-01	3.843560974	-5.40710294
0.45	4.50E-01	1.116211803	-2.62153288
0.5	5.00E-01	-1.58474485	1.798318928
0.55	5.50E-01	-3.42977386	3.494467863
0.6	6.00E-01	-4.15909154	1.96723712
0.65	6.50E-01	-5.32470802	-3.35E-01
0.7	7.00E-01	-9.14909355	-1.66743345
0.75	7.50E-01	-9.27579126	-2.46615916
0.8	8.00E-01	-3.62726099	-5.18047713
0.85	8.50E-01	6.27E-01	-20.370823
0.9	9.00E-01	-3.58E-01	-25.8221322
0.95	9.50E-01	6.88E-01	-33.6434517
1	1	1	1

Table 4.4.6: variation of G with η for different values of a and fixed values of K=0.15, S=0 and R=0.2

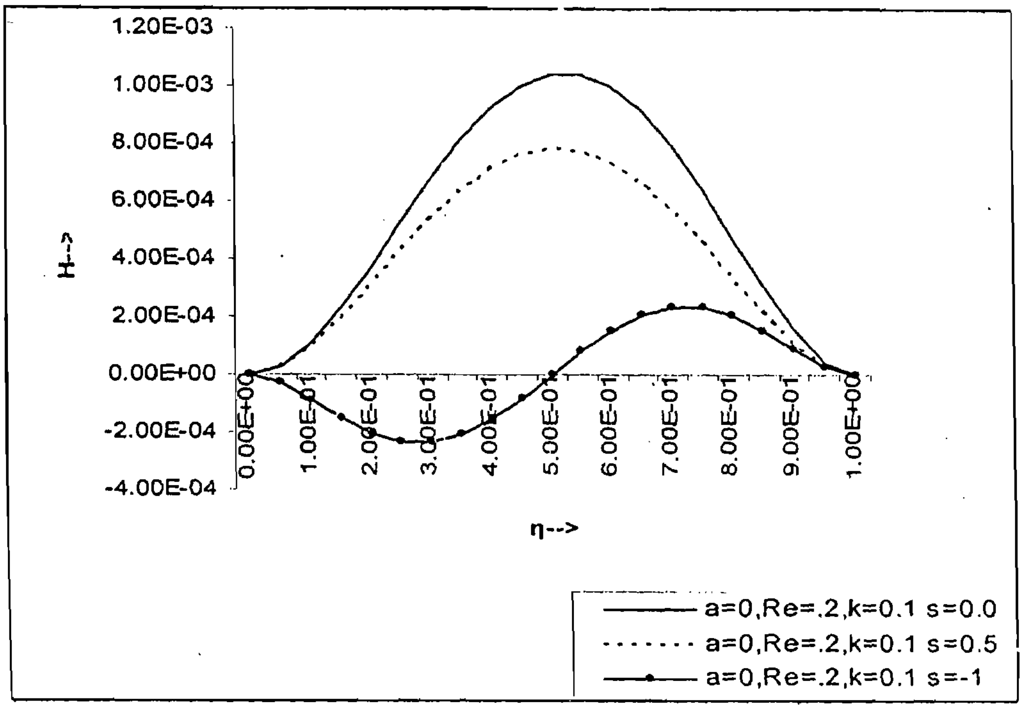


FIG. 4.4.1: variation of H with η for different values of S and fixed values of a=0, R=0.2 and K=0.1

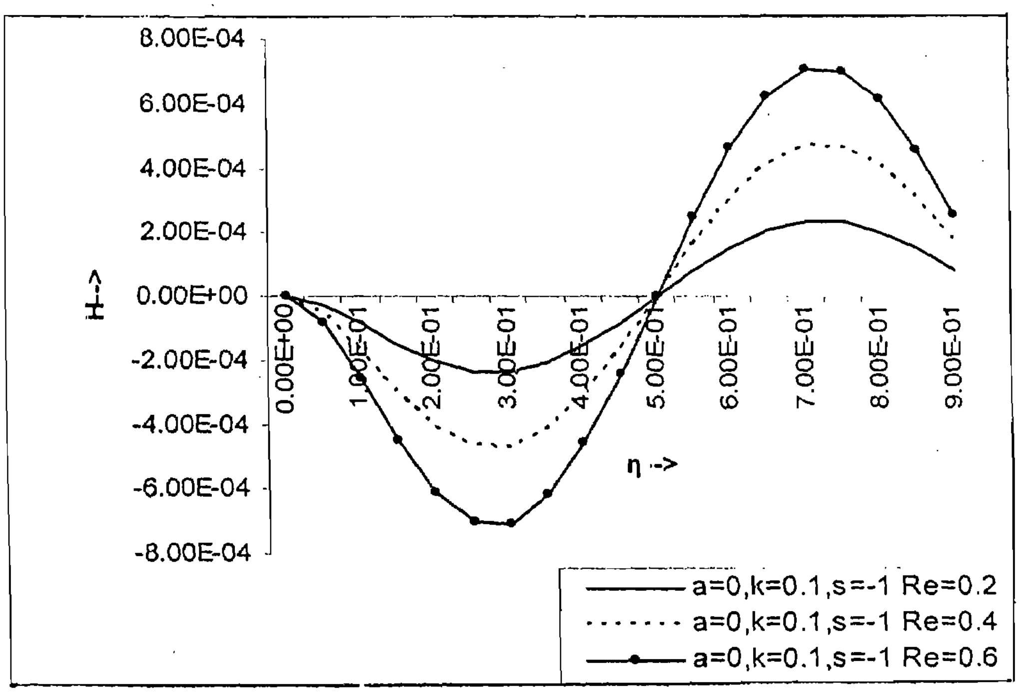


FIG.4.4.2: variation of H with η for different values of R and fixed value of a=0, K=0.1 and S=-1.

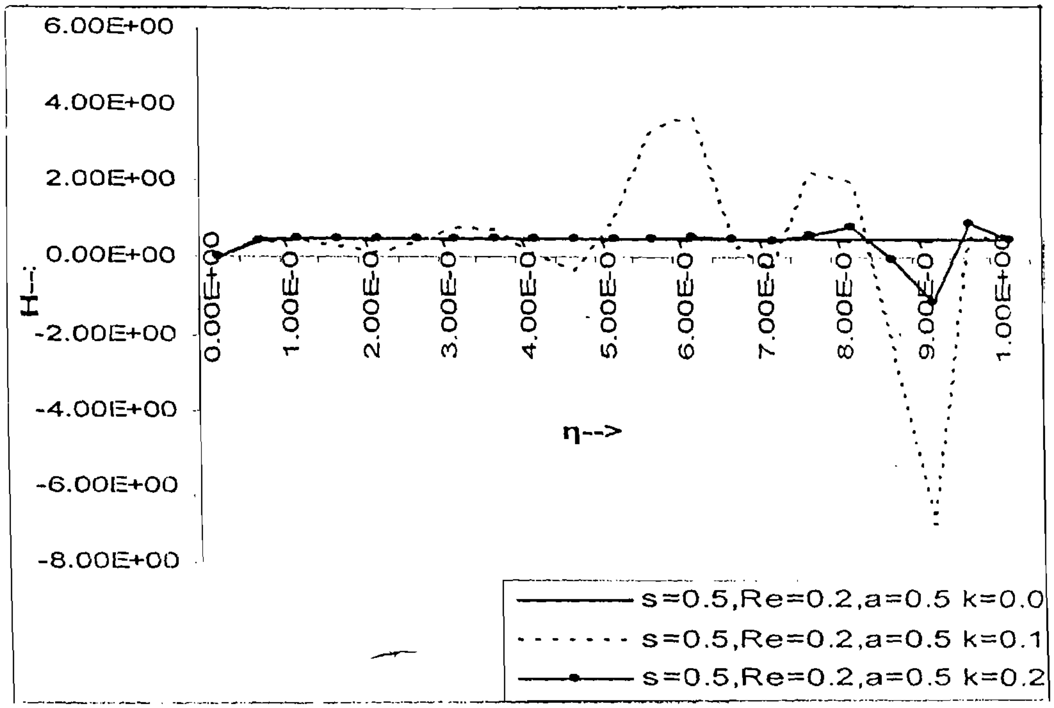


FIG. 4.4.3: variation of H with η for different values of K and fixed values of S=0.5, R=0.2 and a=0.5.

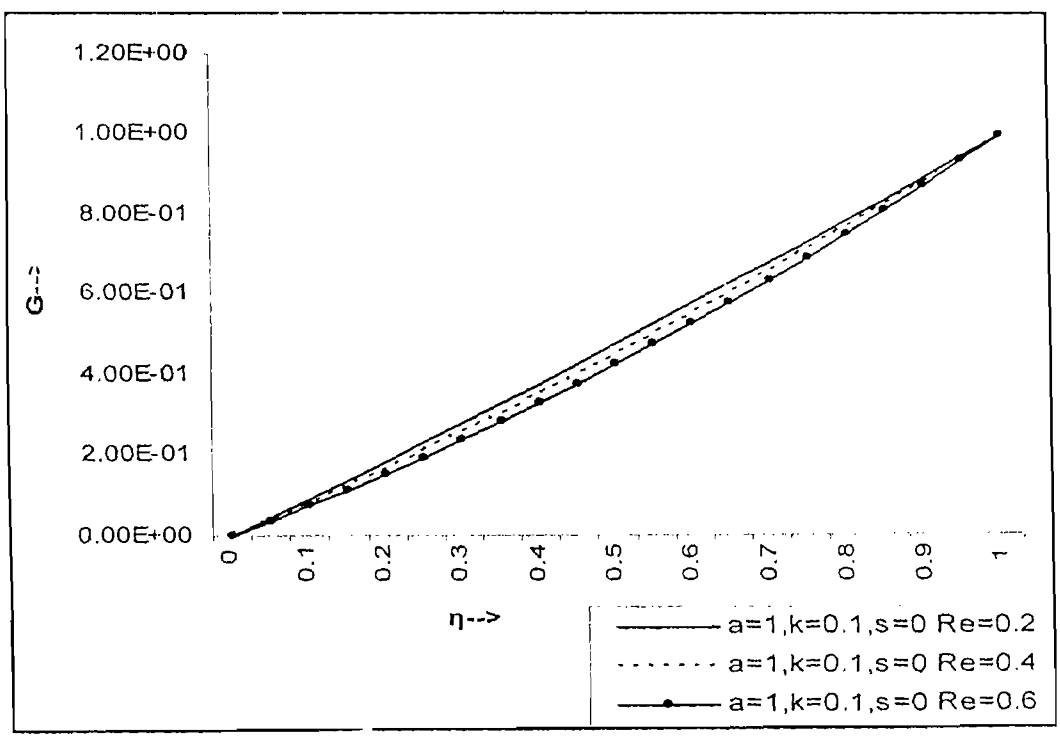


FIG. 4.4.4: variation of G with η for different values of R and fixed values of a=1, K=0.1 and S=0.

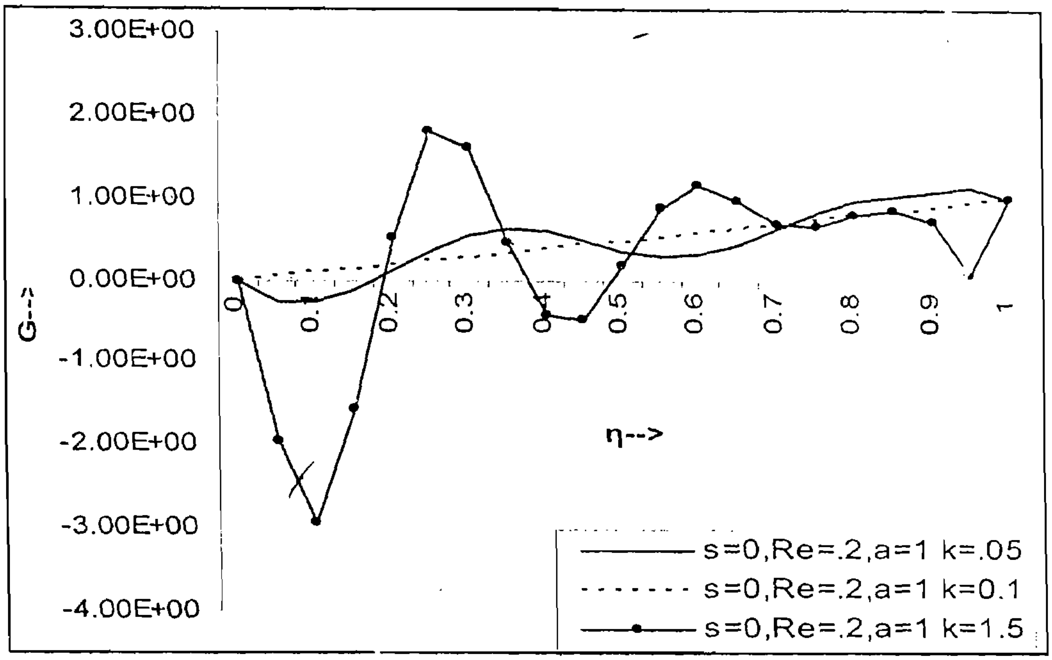


FIG. 4.4.5: variation of G with η for different values of K and fixed values of S=0, R= 0.2 and a=1.

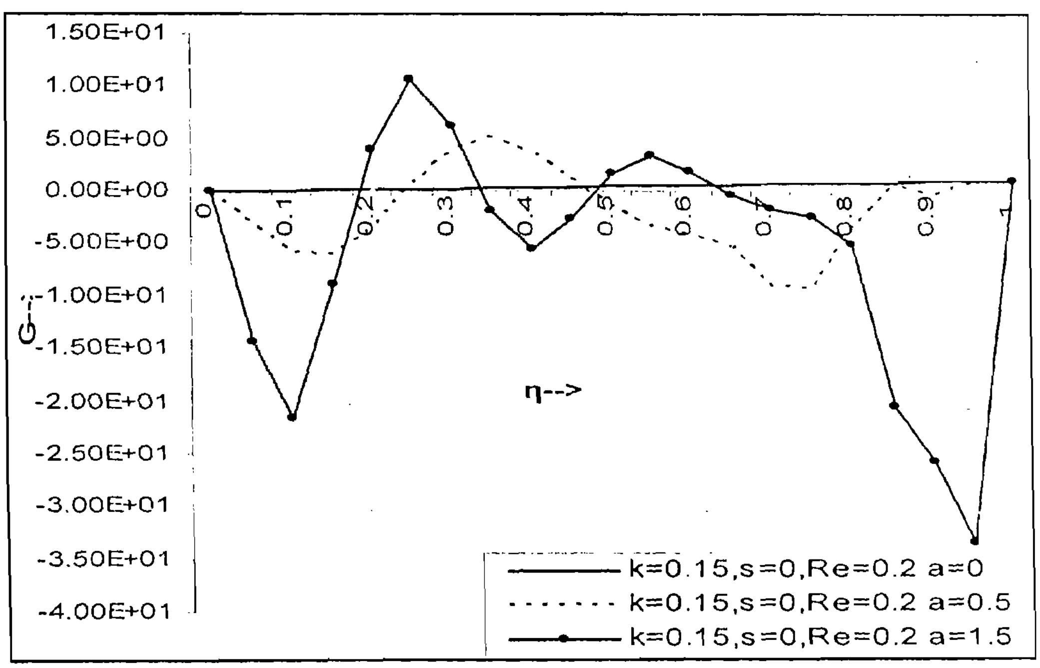


FIG. 4.4.6: variation of G with η for different values of a and fixed values of K=0.15, S=0 and R=0.2.

SUMMARY

Chapter 5 SUMMARY

The study of flow through porous boundaries is a vast area of research in the field of fluid mechanics. Due to its importance in various fields like soil physics, geo-hydrology, petrochemical industries, and filtration, it has remained the field of interest for numerous research workers. Tremendous work has been done in this area and is available in literature. It is hard to touch the field as a whole and hence the present work entitled "Numerical Solution of Some Flow Problems of non-Newtonian Fluids" is restricted to the study of flow of non-Newtonian visco-elastic incompressible fluid such as second-order fluid and Walters liquid B' through an annulus of two coaxial rotating cylinders, between the two rotating coaxial discs and parallel infinite rectangular and parallel circular plates and is restricted towards theoretical modeling of the phenomena.

The work has been divided into five major chapters. The first chapter is introductory in nature and covers the development of the fundamental concept of classical fluid mechanics so as to embrace generalization leading to the theories of second-order fluids and Walters liquid B'. It introduces the necessary foundations and fundamental equations of motion and constitutive equations governing the flow phenomena of these visco-elastic fluids. The Chapter second, Review of Literature, summarizes some relevant research work done in the past in the area to update the knowledge.

The third Chapter deals with the problem description, governing equations and various numerical techniques as Metarials and Methods, applied in solving the various algebraic equations involved during investigation. The Chapter fourth, Results and Discussion, has been divided into four sections. The section-4.1 is concerned with the visco-elastic fluid characterized by Walters liquid (Model B') in the annulus of two porous coaxial circular cylinders when both the boundaries are rotating with different angular velocities at a high injection in inner cylinder and high suction in outer cylinder. The governing differential equations and boundary conditions are replaced by difference equations in unknown variables by using finite-difference approximations for the derivatives. The section-4.2 deals with the study of non-Newtonian incompressible second-order fluid between two infinite porous rotating discs for small and large values of Reynolds numbers. It is assumed that the rate of suction of fluid at one disc is different from the rate of injection of fluid at the other disc. The governing differential equations and boundary conditions are replaced by algebraic equations in unknown variables by using finite-difference approximations. The section-4.3 covers the study of unsteady flow of a viscous incompressible fluid filling the space between two parallel infinitely long rectangular (two dimensional) and two parallel circular plates (axisymmetric), which is of principal interest of many scientific and engineering applications. The numerical solution of unsteady squeezing of viscous fluid occurring between two plates is obtained by using finite difference scheme. The section-4.4 is devoted to the study of the numerical solution of the flow of a visco-elastic fluid between coaxial rotating porous disks with uniform suction or uniform injection for small as well as large values of Reynolds numbers.

The numerical computations have been carried out and physical flow features, velocity components verses radial distance, have been detailed out through tabular representation for various parameters involved in the respective investigation and have been depicted graphically. Some new significant conclusions have been emerging from our study are of engineering applications especially for the chemical industry and mechanical industry perspective.

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Chapter 6 LITERATURE CITED

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APPENDICES

```
C     PROGRAM DETERMINING 'V'AND 'W'FOR SECTION 4.1
C     PROGRAM DETERMINING 'V'
      DIMENSION P(20,20),R(20),X(20),B(20),C(20),F(20),Q(20,20),
     1TEMP(20),DET(20),DELTAV(20),V(20)
      DOUBLE PRECISION P,Q,X,F,B,R,C,TEMP,DET,PROD,PRO,V
      CHARACTER *20 ARG,GETARG,FILNAM,FILO
C     FILO=GETARG()
C     FILNAM=GETARG()
      OPEN(UNIT=8,FILE='P709.OU',STATUS='NEW')
      WRITE(*,*)'GIVE ORDER OF EQUATION'
      READ(*,*)N
      ALPHA=1.0
      AK=0.4
      RE=1.0
      A=ALPHA*AK
      R(1)=1.0
      R(N+1)=2.0
      H=(R(N+1)-R(1))/(N+1)
      DO 10 I=2,N+1
      R(I)=R(1)+(I-1)*H
 10   CONTINUE
      DO 30 I=1,N+1
      X(I)=LOG(R(I))
 30   CONTINUE
      H2=H*H
      H3=H*H2
      V(1)=R(1)
      V(N+1)=R(N+1)
      DO 25 I=2,N
 25   V(I)=R(I)
      DO 20 I=2,20
      B(I)=-R(I)*R(I)
      C(I)=RE*AK*R(I)*R(I)
      C(I)=RE*AK*R(I)*R(I)
 20   CONTINUE
      DO 50 I=1,N
      P(I,I)=(3.0*A+2.0*(A+B(I+1))*H-(3.0*A-B(I+1)-C(I+1))*H3)/H3
 50   CONTINUE
      DO 60 I=2,N-1
      J=I-1
      P(I,J)=(-2.0*A-2.0*(A+B(I+1))*H-(3.0*A-
     1B(I+1)+C(I+1)*H))/(2.0*H3)
 60   CONTINUE
```

```
      DO 70 I=1,N-2
      J=I+1
      P(I,J)=(-6.0*A-2.0*(A+B(I+1))*H+(3.0*A-
     1B(I+1)+C(I+1))*H3)/(2.0*H3)
70    CONTINUE
      DO 80 I=1,N-3
      J=I+2
      P(I,J)=A/H3
80    CONTINUE
      CALL ALUD(P,Q,N)
      PROD=1.0
      DO 100 I=1,N
100   PROD=PROD*Q(I,I)
      PRO=PROD
      WRITE(8,*)PRO
114   CONTINUE
      DO 105 I=1,N
105   F(I)=V(I+1)
      DO 101 J=1,N
      DO 102 I=1,N
102   TEMP(I)=P(I,J)
      DO 103 I=1,N
103   P(I,J)=F(I)
      DO 104 I=1,N
      DO 104 K=1,N
104   Q(I,K)=0.0
      CALL ALUD(P,Q,N)
      PROD=1.0
      DO 200 L=1,N
200   PROD=PROD*Q(L,L)
      DET(J)=PROD
C     WRITE(8,*)'DETERMINENT VALUES'
      DO 1044 I=1,N
1044  P(I,J)=TEMP(I)
C     WRITE(8,*)DET(J)
101   CONTINUE
      DO 2222 I=1,N
2222  DELTAV(I)=DET(I)/PRO
      WRITE(8,*)'DELTAV'
      WRITE(8,*)(DELTAV(I),I=1,N)
      DO 111 I=2,N
      IF((DELTAV(I)).GT.0.001)GOTO 112
111   CONTINUE
      WRITE(8,*)'FINAL VALUE'
      WRITE(8,*)(V(K),K=1,N+1)
      STOP
112   DO 113 I=1,N
```

```
      V(I)=V(I)+DELTAV(I)
113   CONTINUE
      GOTO 114
      CLOSE(8)
      END
       SUBROUTINE ALUD(A,AL,N)
       Dimension a(20,20),au(20,20),al(20,20)
       DOUBLE PRECISION A,AU,AL
       do 110 i=1,n
       do 110 j=1,n
       al(i,j)=0.0
110    au(i,j)=0.0
       do 120 i=1,n
       au(i,i)=1.0
       al(i,1)=a(i,1)
120    au(1,i)=a(1,i)/al(1,1)
       do 130 j=2,n
       do 140 i=j,n
       sum=0
       do 150 k=1 ,j-1
150    sum=sum+al(i,k)*au(k,j)
       al(i,j)=a(i,j)-sum
140    continue
       if (j .lt. n) then
       do 160  jj=j+1,n
       sum=0
       do 170 kk=1,j-1
170    sum=sum+al(j,kk)*au(kk,jj)
       au(j,jj)=(a(j,jj)-sum)/al(j,j)
160    continue
       endif
130    continue
       RETURN
       End

c     PROGRAM DETERMINING 'W'
      dimension R(21),A(19,21),W(21),B(19),REN(10),AKS(19),ALP(6)
      double precision R,P,Q,A,W,AL,B,RE,AK,H3,AKAL,AKS,ALP
      N=19
      DO 15 K=1,5
      DO 15 P=1,10
      DO 15 Q=1,19
      DO 15 S=1,6
      AL=K
      RE=P
      AKS(Q)=1.0-Q/10.0
      AK=AKS(Q)
```

```
      ALP(S)=(S-1.0)/10.0
      ALPHA=ALP(S)
      WRITE(*,*)'AL=',AL
      WRITE(*,*)'ALPHA=',ALPHA
      WRITE(*,*)'AK=',AK
      WRITE(*,*)'RE=',P
      R(1)=1.0
      R(N+2)=2.0
      H=(R(N+2)-R(1))/(N+1)
      H2=H*H
      H3=H*H2
      DO 1 I=2,N+1
      R(I)=R(1)+(I-1)*H
1     continue
      AKAL=AK*ALPHA
      W(1)=1.0
      W(N+2)=0.0
      DO 2 I=1,N-1
      RR=R(I+1)
      A(I,I)=(3.0*AKAL*RR-2.0*H*(3.0*AKAL-RR*RR))/(RR*RR*H3)
2     CONTINUE
      DO 3 I=1,N-1
      J=I+1
      RR=R(I+1)
      A(I,J)=(-6.0*AKAL*RR*RR+2.0*H*RR*(3.0*AKAL-
     1RR*RR)+H2*(RE*AK*RR*RR
     1-RR*RR-3.0*AKAL))/(2.0*RR*RR*RR*H3)
3     CONTINUE
      DO 4 I=1,N-2
      J=I+2
      RR=R(I+1)
      A(I,J)=AKAL/(RR*H3)
4     CONTINUE
      DO 5 I=2,N-1
      J=I-1
      RR=R(I+1)
      A(I,J)=(-2.0*RR*RR*AKAL+2.0*H*RR*(3.0*AKAL-RR*RR)-
     1H2*(RE*AK*RR*RR
     1-RR*RR-3.0*AKAL))/(2.0*H3*RR*RR*RR)
5     CONTINUE
      A(N,N-2)=-AKAL/(R(N+1)*H3)
      A(N,N-1)=(6.0*AKAL*R(N+1)*R(N+1)+2.0*H*R(N+1)*(3.0*AKAL-
     1R(N+1)*R(N+1))-H2*(RE*AK*R(N+1)*R(N+1)-R(N+1)*R(N+1)-
     13.0*AKAL))/(2.0*H3*R(N+1)*R(N+1)*R(N+1))
      A(N,N)=(-3.0*AKAL*R(N+1)-2.0*H*(3.0*AKAL-R(N+1)*R(N+1)))/
     1(R(N+1)*R(N+1)*H3)
      DO 6 I=1,N
```

```
6       B(I)=RE*AL
        C=(-2.0*R(2)*R(2)*AKAL+2.0*H*R(2)*(3.0*AKAL-R(2)*R(2))-
       1H2*(RE*AK*R(2)*R(2)-R(2)*R(2)-
       13.0*AKAL))/(2.0*H3*R(2)*R(2)*R(2))
        B(1)=B(1)-W(1)*C
        CALL MINV(A,N)
        CALL MATMUL(A,B,W,N)
        WRITE(*,*)(W(I),I=1,N+2)
15      CONTINUE
        STOP
        END
         SUBROUTINE MATMUL(A,B,W,N)
         DIMENSION A(19,19),B(19),W(21)
         DOUBLE PRECISION A,B,W,SUM
         DO 10 I=1,N
         SUM=0.0
         DO 20 K=1;N
20       SUM=SUM+A(I,K)*B(K)
         W(I+1)=SUM
10       CONTINUE
         RETURN
         END
         subroutine minv(A,N)
c        program for solution of by matrix inversion
         dimension A(19,19),c(19,19),t(19)
         integer pivot(19)
         DOUBLE PRECISION A,C,T,TEMP
         do 300 j=1,n
300      pivot(j)=j
         do 410 i=1,n
         temp=A(i,i)
         it=i
         do 420 k=i,n
         if(abs(temp).lt.abs(A(k,i)))go to 430
         go to 420
430      temp=A(k,i)
         it=k
420      continue
         if(abs(temp).lt.1.0e-5)go to 440
         if(it.eq.i)go to 470
         it1=pivot(it)
         pivot(it)=pivot(i)
         pivot(i)=it1
         do 460 k=1,n
         temp=A(i,k)
         A(i,k)=A(it,k)
         A(it,k)=temp
```

```
460    continue
       go  to 470
440    write(*,901)
901    format(1x,'singular matrix')
       stop
470    do 480 j=1,n
       t(j)=A(j,i)
       A(j,i)=0.0
480    continue
       A(i,i)=1.0
       do 490 k=1,n
       A(i,k)=A(i,k)/t(i)
       do 500 j=1,n
       if(j.eq.i)go to 500
       A(j,k)=A(j,k)-A(i,k)*t(j)
500    continue
490    continue
410    continue
       do 510 i=1,n
       do 510 j=1,n
       isc=pivot(j)
510    c(i,isc)=A(i,j)
       DO 1 I=1,N
       DO 2 J=1,N
       A(I,J)=C(I,J)
2      CONTINUE
1      CONTINUE
       RETURN
       End

c      PROGRAM DETERMINING F AND G FOR SECTION 4.2.
       DIMENSION S(37,37),Q(37,37),TEMP(37),B(37),DET(37),
      1DELTA(37),Y(37)
       DOUBLE PRECISION F(22),G(22),Z(22),H,ALL,AM,AN,RE,S,H2,H3,H4
      1,H5,H6,Q,PROD,B,A2,C1,C2,C3,C4,C5,C6,C7,C8,PRO,DET,DELTA,
      1Y,TEMP
       OPEN(UNIT=8,FILE='P709.OU',STATUS='NEW')
       ALPHA=0.0
       BETA=0.0
       RE=1.0
       ALL=1.0
       AM=2.0
       AN=2.0
       F(2)=AN
       F(22)=1.0
       G(2)=AM
       G(22)=1.0
```

```
      Z(2)=0.0
      Z(22)=1.0
      H=(Z(22)-Z(2))/20.0
      H2=H*H
      H3=H2*H
      H4=H2*H2
      H5=H3*H2
      H6=H3*H3
      DO 1 I=3,21
      Z(I)=Z(2)+(I-2)*H
1     CONTINUE
C      WRITE(8,*)(Z(I),I=2,22)
      A2=1.0/10.0*(3.0*AM*AM+4.0*AM+3.0)-27.0/(35.0*ALL*ALL)+(1.0-
     1AN)*(1.0-AN)
      C1=2.0*ALL*ALL/15.0*(2.0+AM-3.0*AM*AM)-2.0/35.0*(1.0-
     1AN)*(22.0*AN+13.0)
      C3=1.0/6.0*AM*(1.0-AM)*ALL*ALL-1.0/2.0*AN*(1.0-AN)
      C4=1.0/2.0*ALL*ALL*(1.0-AM)-9.0/2.0*(1.0-AN)*(1.0-AN)
      C5=9.0/2.0*(1.0-AN)*(1.0-AN)-1.0/2.0*(1.0-AM)*(1.0-
     1AM)*ALL*ALL
      C6=1.0/20.0*(1.0+9.0*AM-11.0*AN+AM*AN)
      C7=1.0/4.0*(1.0-AN)*(3.0*AM-1.0)
      C8=(1.0-3.0*AM)*(1.0-AN)
      DO 10 I=3,20
      ZZ=Z(I)
      F(I)=AN+(1.0-AN)*(3.0-
     12.0*ZZ)*ZZ*ZZ+RE*ZZ*ZZ*(0.25*C1+C2*ZZ+C3*
     1ZZ*ZZ+1.0/30.0*(1.0*AM)*(1.0-AM)*ALL*ALL*ZZ*ZZ*ZZ-
     11.0/10.0*(1.0-AN)*(1.0-AN)*ZZ*ZZ*ZZ*ZZ+1.0/35.0*(1.0-
     1AN)*(1.0-AN)*ZZ*ZZ*ZZ*ZZ*ZZ-
     11.0/2.0*(1.0-AN)*ALPHA*(1.0-2.0*ZZ+ZZ*ZZ)+BETA*(C4+C5*ZZ))
10    CONTINUE
      DO 1101 I=3,21
      ZZ=Z(I)
      G(I)=(1.0-AM)*ZZ+AM+RE*(C6*ZZ+0.5*AN*(1.0-AM)*ZZ*ZZ-
     1AM*(1.0-1AN)
     1*(ZZ**3)+C7*(ZZ**4)+0.2*(1.0-AN)*(1.0-AM)*(ZZ**5)-ALPHA*ZZ
     1*(1.0-AN)*(1.0-AM)*(1.0-3.0*ZZ+2.0*ZZ*ZZ)-BETA*ZZ*(1.0-
     1AN)*((AM-3.0*AM*ZZ+(1.0-AM)*ZZ*ZZ*ZZ)-C8*ZZ*ZZ))
1101  CONTINUE
      F(1)=F(3)
      F(21)=F(22)
C     WRITE(8,*)(F(I),I=2,22)
      DO 12 I=1,17
      NX=I+2
      S(I,I)=-2.0*RE/H3-2.0*RE*RE*(F(NX+1)-2.0*F(NX)+F(NX-
     11))/H2+4.0*RE*RE/H2*F(NX)+2.0*RE*RE/H2*(F(NX+1)+RE*RE*(-
```

```
      1F(NX)+F(NX+2))/(2.0*H2)+ALPHA*(2.0*RE*RE*(F(NX+2)-
      14.0*F(NX+1)+6.0*F(NX)-4.0*
      1F(NX-1)+F(NX-2))/H4+12.0*RE*RE*F(NX)/H4+8.0*RE*RE*F(NX+1)/H4
      1-2.0*RE*RE*(6.0*F(NX)-4.0*F(NX-1)-
      12.0*F(NX+2))/H4)+BETA*(RE*RE
      1*(4.0*F(NX)-4.0*F(NX+2)+F(NX+3)-F(NX-1))/(2.0*H4)-RE*RE*(
      16.0*F(NX)-4.0*F(NX-1)-2.0*F(NX+2))/H4))
12     CONTINUE
       S(18,18)=6.0*RE/H3-2.0*RE*RE/H2*(F(21)-
      12.0*F(20)+F(19))+4.0*RE*RE
      1*F(20)+2.0*RE*RE/H2*F(21)+RE*RE*(6.0*F(20)-
      12.0*F(22)+2.0*F(21)-
      14.0*F(19))/(4.0*H2)+ALPHA*(2.0*RE*RE/H2*(12.0*F(20)+F(22)-
      110.0*F(21)+4.0*F(19)+F(18))-2.0*RE*RE/H2*(6.0*F(20)-
      12.0*F(22)+2.0*F(21)-4.0*F(19)))+BETA*(RE*RE/(2.0*H4)*(F(22)-
      13.0*F(21)+3.0*F(20)-F(19))-3.0*(F(22)-F(20))-
      1RE*RE/H4*(6.0*F(20)-2.0*F(22)+2.0*F(21)
      1-4.0*F(19)))
       DO 13 I=1,17
       J=I+1
       NX=I+2
       S(I,J)=-2.0*RE/H3-12.0*RE*RE*F(NX)/H2+2.0*RE*RE*(F(NX+1)-
      12.0*F(NX)+F(NX-1))/H2.0*RE*RE/H2*F(NX+1)+RE*RE/
      1(2.0*H2)*(F(NX+1))-RE*RE*F(NX-1)/(2.0*H2)+ALPHA*(-
      18.0*RE*RE*F(NX)/H4-2.0*RE*RE*(F(NX+3)-
      14.0*F(NX+2)+6.0*F(NX+1)-4.0*F(NX)+F(NX-1))/H4-
      12.0*RE*RE*F(NX+1)/H4-2.0*RE*RE*(-6.0*F(NX+1)+2.0*F(NX-
      11)+4.0*F(NX+2))/H4)+BETA*(RE*RE*(-4.0*F(NX+1)+F(NX+2)
      1+4.0*F(NX-1)-F(NX-2))/(2.0*H4)-RE*RE/H4
      1*(6.0*F(NX+1)-2.0*F(NX-1)+4.0*F(NX+2)))
13     CONTINUE
       S(18,17)=-4.0*RE/H3-2.0*RE*RE*F(20)+RE*RE/(2.0*H2)*(F(19)-
      12.0*F(20)+F(21))+ALPHA*(2.0*RE*RE/H4*(-4.0*F(20)+4.0*F(21))-
      12.0*RE*RE/H2*(2.0*F(21)-4.0*F(20)+2.0*F(20)))+BETA*
      1(RE*RE/(2.0*H4)*(-(F(22)
      1-2.0*F(21)+2.0*F(19)-F(18)))+2.0*(F(21)-F(19))+(F(22)-
      1F(20))-RE*RE/H4*(2.0*F(19)-4.0*F(20)+2.0*F(21)))
       DO 14 I=1,16
       J=I+2
       NX=I+2
       S(I,J)=3.0*RE/(2.0*H3)+2.0*RE*RE*F(NX+1)/H2-
      1RE*RE*F(NX+2)/(2.0*H2)+RE*RE*F(NX)/(2.0*H2)+ALPHA*
      1(2.0*RE*RE*F(NX)/H4+8.0*RE*RE*F(NX+1
      1)/H4+4.0*RE*RE*F(NX+2)/H4-8.0*RE*RE*F(NX+1)/H4+
      14.0*RE*F(NX)/H4)+BETA*(RE*RE*(4.0*F(NX+2)+F(NX+1)-F(NX+3)-
      14.0*F(NX)+2.0*F(NX-1))/(2.0*H4)-RE*RE*(-2.0*F(NX+2)
      1+4.0*F(NX+1)-2.0*F(NX))/H4)
```

```
14    CONTINUE
      S(18,16)=RE/H3+ALPHA*2.0*RE*RE/H4*(F(20)-F(21))-BETA*RE*RE/
     1(2.0*H4)*(F(21)-F(19))
      DO 15 I=2,15
      J=I+3
      NX=I+2
      S(I,J)=-RE/H3-2.0*ALPHA*RE*RE*F(NX+1)/H4+BETA**RE*RE*(-
     1F(NX+2)+F(NX))/(2.0*H4)
15    CONTINUE
      DO 16 I=2,17
      J=I-1
      NX=I+2
      S(I,J)=3.0*RE/H3-2.0*RE*RE*F(NX)/H2+RE*RE*(F(NX-1)-F(NX+1))/
     1(2.0*H2)+ALPHA*(-8.0*RE*RE*F(NX)/H4-12.0*RE*RE*H4*F(NX+1)-
     12.0*RE*RE/H4*(2.0*F(NX-1)-4.0*F(NX)+2.0*F(NX+1)+4.0*
     1F(NX+2)))+BETA*(RE*RE*(-4.0*F(NX-1)+4.0*F(NX+1)+F(NX-2)+
     12.0*F(NX+3)-F(NX))/(2.0*H4)-RE*RE/H4*(2.0*F(NX-1)-
     14.0*F(NX)+F(NX+1)))
16    CONTINUE
      DO 17 I=1,17
      J=I-2
      NX=I+2
      S(I,J)=-RE/H3+2.0*RE*RE/H4*F(NX)+BETA*RE*RE/(2.0*H4)*
     1(-F(NX+1)+F(NX-1))
17    CONTINUE
      DO 18 I=1,17
      J=I+18
      NX=I+2
      S(I,J)=-8.0*RE*RE*ALL*ALL*G(NX)
18    CONTINUE
      DO 19 I=1,16
      J=I+1+18
      NX=I+2
      S(I,J)=8.0*RE*RE*ALL*ALL*G(NX+1)
19    CONTINUE
      S(18,37)=-8.0*RE*RE*ALL*ALL*G(20)+BETA*RE*RE*ALL*ALL*G(20)
      S(18,36)=8.0*RE*RE*ALL*ALL*G(21)-BETA*RE*RE*ALL*ALL*G(21)
      DO 21 I=19,20
      J=I-18
      NX=I+2-18
      S(I,J)=-RE/(2.0*H)*(G(NX+1)-G(NX-1))+ALPHA*RE*(G(NX+2)-2.0*
     1G(NX+1)+2.0*G(NX-1)-G(NX-2))/(2.0*H3)-ALPHA*RE/
     1(2.0*H3)*(G(NX+1)-G(NX-1))+BETA*RE/(2.0*H3)*(G(NX+1)-
     12.0*G(NX)+G(NX-1))-BETA*RE*G(NX)/H2
21    continue
      S(19,19)=-2.0/H2+RE*(F(5)-F(3))/(2.0*H)+3.0*RE*
```

```
     1ALPHA*F(4)/H3+BETA*(-RE/H3*(F(5)-F(3))-RE/H2*(F(5)-
     12.0*F(4)+F(3)))
      S(19,20)=1.0/H2-RE*F(4)/(2.0*H)-ALPHA*(3.0*RE*F(4)/H3+RE/
     1(2.0*H3)*(F(5)-2.0*F(4)+F(3)))
      S(19,21)=ALPHA*RE*F(5)/H3
      DO 22 I=20,36
      NX=I+2-18
      S(I,I)=-2.0/H2+RE*(F(NX+1)-F(NX-1))/(2.0*H)+BETA*(-
     1RE/H3*(F(NX+1)
     1-F(NX-1))-RE/H2*(F(NX+1)-2.0*F(NX)+F(NX-1)))
22    CONTINUE
      DO 23 I=20,36
      J=I+1
      NX=I+2-18
      S(I,J)=RE*G(NX)/(2.0*H)-ALPHA*RE*(G(NX+1)-G(NX-
     11))/(2.0*H3)+BETA*(RE/(2.0*H3)*(G(NX+1)-2.0*G(NX)
     1+G(NX-1))-RE*G(NX)/H2)
23    CONTINUE
      S(37,18)=-RE*(G(22)-G(20))/(2.0*H)+ALPHA*RE/H3*(G(22)-
     13.0*G(21)+3.0*G(20)-G(19))+RE*ALPHA/H3*(G(22)-
     1G(20))+BETA*2.0*RE*G(21)/H2
      S(37,17)=-RE*G(21)/(2.0*H)-ALPHA*RE/(2.0*H3)*(G(22)-G(20))-
     1BETA*(RE/(2.0*H3)*(G(22)-2.0*G(21)+G(20))+RE/H2*G(21))
      S(37,37)=-2.0/H2+RE*(F(22)-F(20))/(2.0*H)-
     13.0*RE*ALPHA*F(21)/H3+BETA*(-RE/H3*(F(22)-F(20))-
     1RE/H2*(F(22)-2.0*F(21)+F(20)))
      S(37,36)=1.0/H2+RE*F(21)/(2.0*H)+3.0*ALPHA*RE*F(21)+ALPHA*R
     1*(F(22)-2.0*F(21)+F(20))/(2.0*H3)+BETA*RE*(F(22)-
     1F(20))/(2.0*H3)
      S(37,35)=-ALPHA*RE*F(21)/H3
      N=37
      CALL ALUD(S,Q,N)
      PROD=1.0
      DO 100 I=1,N
100   PROD=PROD*Q(I,I)
      PRO=PROD
C     WRITE(8,*)PRO
114   CONTINUE
      DO 105 I=1,37
      B(I)=5.0
      B(I)=-B(I)
      WRITE(8,*)B(I)
105   CONTINUE
      DO 101 J=1,N
      DO 102 I=1,N
102   TEMP(I)=S(I,J)
      DO 103 I=1,N
```

```
103    S(I,J)=B(I)
       DO 104 I=1,N
       DO 104 K=1,N
104    Q(I,K)=0.0
       CALL ALUD(S,Q,N)
       PROD=1.0
       DO 200 L=1,N
200    PROD=PROD*Q(L,L)
       DET(J)=PROD
C      WRITE(8,*)'DETERMINENT VALUES'
       DO 1044 I=1,N
1044   S(I,J)=TEMP(I)
C      WRITE(8,*)DET(J)
101    CONTINUE
       DO 2222 I=2,22
2222   DELTA(I)=DET(I)/PRO
C      WRITE(8,*)'DELTA'
C      WRITE(8,*)(DELTA(I),I=2,22)
       DO 111 I=2,22
       IF((DELTA(I)).GT.0.00001)GOTO 112
111    CONTINUE
112    DO 113 I=2,22
       Y(I)=F(I)+DELTA(I)
113    CONTINUE
C      WRITE(8,*)'INITIAL VALUE,FINAL VALUES'
       DO 987 I=2,22
987    WRITE(8,*)F(I)
       DO 97 I=2,22
97     WRITE(8,*)Y(I)
       STOP
       GOTO 114
       CLOSE(8)
       END
        SUBROUTINE ALUD(S,AL,N)
        dimension S(37,37),au(37,37),al(37,37)
        DOUBLE PRECISION S,AU,AL
        do 110 i=1,n
        do 110 j=1,n
        al(i,j)=0.0
110     au(i,j)=0.0
        do 120 i=1,n
        au(i,i)=1.0
        al(i,1)=S(i,1)
120     au(1,i)=S(1,i)/al(1,1)
        do 130 j=2,n
        do 140 i=j,n
        sum=0
```

```
      do 150 k=1 ,j-1
150   sum=sum+al(i,k)*au(k,j)
      al(i,j)=S(i,j)-sum
140   continue
      if (j .lt. n) then
      do 160  jj=j+1,n
      sum=0
      do 170 kk=1,j-1
170   sum=sum+al(j,kk)*au(kk,jj)
      au(j,jj)=(S(j,jj)-sum)/al(j,j)
160   continue
      endif
130   continue
      RETURN
      End

C     PROGRAM DETERMINING F FOR SECTION 4.3.
      Dimension R(21),P(19,19),F(21),S(19,19),Q(19),DET(19),
     1DELTAF(19);TEMP(19)
      Double precision R,A,B,F,P,RR,C,D,Q,S,PRO,PROD,DET,DELTAF
     1,TEMP,H2,H4,ALPHA,H,E,H3,X
      OPEN(UNIT=8,FILE='709.OU',STATUS='NEW')
      N=19
      A=1.0
      B=-1.0
      C=1.0
      ALPHA=10.0
      F(1)=0.01
      F(N+2)=ALPHA*C
      R(1)=0.0
      R(N+2)=1.0
      H=(R(N+2)-R(1))/(N+1)
      H2=H*H
      H3=H*H2
      H4=H3*H
      DO 1 I=2,N+1
      R(I)=R(1)+(I-1)*H
1     continue
      DO 12 I=2,N+1
      RR=R(I)
      F(I)=ALPHA*(-2.0*RR**3+3.0*RR**2)+ALPHA*ALPHA*(2.0/35.0
     1*RR**7-1.0/5.0*RR**6-0.1*RR**5+3.0/4.0*RR**4-
     124.0/35.0*RR**3+5.0/28.0*RR**2)+ALPHA**3*(4.0/5775.0*RR**11-
     12.0/525.0*RR**10+11.0/630.0*RR**9-1.0/20.0*RR**8+
     1227.0/4900.0*RR**7+187.0/4200.0*RR**6-71.0/700.0
     1*RR**5+5.0/112.0*RR**4+283.0/38808.0*RR**3-1229.0/
     11215600.0*RR**2)
```

```
12      CONTINUE
C       WRITE(8,*)(F(I),I=1,N+1)
114     DO 2 I=1,N-2
        RR=R(I+1)
        P(I,I)=-4.0/H4+3.0*A*F(I)/H3+A*(F(I+2)-3.0*F(I+1)+3.0*F(I)-
       1F(I-1))/H3-3.0*ALPHA*RR/H3+6.0*ALPHA/H2-B*(F(I+1)-F(I-1))/H3
2       CONTINUE
        DO 3 I=1,N-2
        J=I+1
        RR=R(I+1)
        P(I,J)=6.0/H4-3.0*A*F(I)/H3+3.0*ALPHA*RR/H3-3.0*ALPHA/H2+B/
       1(2.0*H3)*(F(I+1)-2.0*F(I)+F(I-1))+B*(F(I+1)-F(I-1))/(2.0*H3)
3       CONTINUE
        DO 4 I=1,N-2
        J=I+2
        RR=R(I+1)
        P(I,J)=-4.0/H4+A*F(I)/H3-ALPHA*RR/H3
4       CONTINUE
        DO 5 I=1,N-3
        J=I+3
        RR=R(I+1)
        P(I,J)=1.0/H4
5       CONTINUE
        DO 6 I=2,N-2
        J=I-1
        RR=R(I+1)
        P(I,J)=1.0/H4-A*F(I)/H3+ALPHA*RR/H3-3.0*ALPHA/H2
6       CONTINUE
        P(N-1,N-3)=1.0/H4
        P(N-1,N-2)=-4.0/H4-A*F(19)/H3+ALPHA*R(20)-3.0*ALPHA/H2-
       1B/(2.0*H3)*(F(20)-2.0*F(19)+F(18))+B/(2.0*H3)*(F(20)-F(18))
        P(N-1,N-1)=6.0/H4+3.0*A*F(19)/H3+A*(F(21)-3.0*F(20)+3.0*
       1F(19)-F(18))/H3-3.0*ALPHA*R(20)/H3+6.0*ALPHA/H2-B*(F(19)-
       1F(17))/H3
        P(N-1,N)=-4.0/H4-3.0*A*F(19)/H3+3.0*ALPHA*R(20)/H3-
       13.0*ALPHA/H2+B/(2.0*H3)*(F(20)-2.0*F(19)+F(18))
       1+B/(2.0*H3)*(F(20)-F(18))
        P(N,N-3)=1.0/H3
        P(N,N-2)=-4.0/H3-A*F(I)/H3+ALPHA*R(21)/H3
        P(N,N-1)=6.0/H3+3.0*A*F(20)/H3+3.0*ALPHA*R(21)/H3-
       13.0*ALPHA/H2-B/(2.0*H3)*(F(21)-2.0*F(20)+F(19))+B/
       1(2.0*H3)*(F(21)-F(19))
        P(N,N)=-4.0/H3-3.0*A*F(20)/H3+A*(F(21)-3.0*F(20)+3.0*F(19)-
       1F(18))/H3+3.0*ALPHA*R(21)/H3+6.0*ALPHA/H2-2.0*B*(F(21)-
       1F(19))/(2.0*H3)
        DO 7 I=1,N
7       Q(I)=0.0
```

```
      X=1.0/H4-A*F(2)/H3+ALPHA*R(2)/H3-3.0*ALPHA/H2-B*(F(3)-
     12.0*F(2)+F(1))/(2.0*H3)+B*(F(3)-F(1))/(2.0*H3)
      Q(1)=Q(1)-F(1)*X
      D=1.0/H4+A*F(19)/H3-ALPHA*R(19)/H3
      Q(18)=Q(18)-F(18)*D
      E=1.0/H3+A*F(20)/H3-ALPHA*R(20)/H3-3.0*ALPHA/H2
      Q(19)=Q(19)-F(19)*E
      Q(I)=-Q(I)
      WRITE(8,*)'Q'
      WRITE(8,*)(Q(I),I=1,N)
15    CONTINUE
      CALL ALUD(P,S,N)
      PROD=1.0
      DO 100 I=1,N
100   PROD=PROD*S(I,I)
      PRO=PROD
      WRITE(8,*)PRO
      DO 101 J=1,N
      DO 102 I=1,N
102   TEMP(I)=P(I,J)
      DO 103 I=1,N
103   P(I,J)=Q(I)
      DO 104 I=1,N
      DO 104 K=1,N
104   S(I,K)=0.0
      CALL ALUD(P,S,N)
      PROD=1.0
      DO 200 L=1,N
200   PROD=PROD*S(L,L)
      DET(J)=PROD
      DO 1044 I=1,N
1044  P(I,J)=TEMP(I)
101   CONTINUE
      DO 2222 I=1,N
2222  DELTAF(I)=DET(I)/PRO
      DO 111 I=1,N
      IF(DELTAF(I).GT.0.01)
     1GOTO 112
111   CONTINUE
      WRITE(8,*)'FINAL VALUE'
      DO 234 K=1,N+2
234   WRITE(8,*)F(K)
333   CONTINUE
      STOP
112   DO 113 I=2,N+1
      F(I)=F(I)+(DELTAF(I-1))
113   CONTINUE
```

```
      GOTO 114
      CLOSE(8)
      END
      SUBROUTINE ALUD(P,AU,N)
      dimension P(19,19),AU(19,19),AL(19,19)
      DOUBLE PRECISION P,AU,AL
      DO 75 I=1,N
      DO 75 J=1,N
75    AU(I,J)=P(I,J)
      DO 45 I=1,N
45    AL(I,I)=1.0
      MP=2
      MS=1
      DO 30 K=1,N-1
      DO 20 I=MP,N
      AL(I,K)=AU(I,K)/AU(K,K)
      DO 20 J=MS,N
20    AU(I,J)=AU(I,J)-AU(K,J)*AL(I,K)
      MS=MS+1
      MP=MP+1
30    CONTINUE
      DO 40 I=1,N
      AU(I,I)=AU(I,I)
40    CONTINUE
      RETURN
      END

C     PROGRAM DETERMINING H AND G FOR SECTION 4.4.
      DIMENSION H(21),Z(21),G(21),R(38,38),Q(38,38),TEMP(38)
     1,DET(38),DELTAH(21),DELTAG(21),B(38)
      DOUBLE PRECISION F,G,Z,H,RE,R,F2,F3,F4,F5,F6,Q,TEMP,
     1DET,DELTAH,DELTAG,B,PRO,PROD,AK,NMK,SS,AA
      OPEN(UNIT=8,FILE='79.OU',STATUS='NEW')
      N=19
      AA=0.5
      SS=0.5
      RE=0.2
      AK=0.25
      NMK=2*N
      H(1)=0.0
      H(N+2)=AA
      G(1)=SS
      G(N+2)=1.0
      Z(1)=0.0
      Z(N+2)=1.0
      F=(Z(N+2)-Z(1))/(N+1)
      F2=F*F
```

```
 F3=F2*F
 F4=F2*F2
 F5=F3*F2
 F6=F3*F3
 DO 1 I=2,N+1
 Z(I)=Z(1)+(I-1)*F
 DO 10 I=2,N+1
 ZZ=Z(I)
 H(I)=AA+RE*(1.0/30.0*(-1.0+SS)*(-2.0+SS*(-3.0+ZZ)-ZZ)*(-1.0+
1ZZ)**2*ZZ*ZZ)+RE*RE*(1.0/360.0*AA*ZZ*ZZ*(-1.0+SS)*(-
11.0+ZZ)*(-1.0+ZZ)*(9.0-60.0*AK*(-1.0+SS)-4.0*ZZ-8.0*ZZ*ZZ
1+SS*(3.0-20.0*ZZ+8.0*ZZ*ZZ)))+RE*RE*RE*(-1.0/2268000.0*((-
11.0+SS)*(-1.0+ZZ)*(-1.0+ZZ)*ZZ*ZZ*(-332.0-239.0*SS+34.0
1*SS*SS-63.0*SS*SS*SS-85.0*ZZ+85.0*SS*ZZ-495.0*SS*SS*ZZ-705.0
1*SS*SS*SS*ZZ+162.0*ZZ*ZZ-71.0*SS*ZZ*ZZ-64.0*SS*SS*ZZ*
1ZZ+273.0*SS*SS*SS*ZZ*ZZ+217.0*ZZ*ZZ*ZZ+349.0*SS*ZZ*ZZ*ZZ
1+631.0*SS*SS*ZZ*ZZ*ZZ+603.0*SS*SS*SS*ZZ*ZZ*ZZ+20.0*ZZ
1**4+55.0*SS*ZZ**4-270.0*SS*SS*ZZ**4-705.0*SS*SS*SS*ZZ**4-
115.0*ZZ**5-125.0*SS*ZZ**5-205.0*SS*SS*ZZ**5+345.0*SS*
1SS*SS*ZZ**5-20.0*ZZ**6-50.0*SS*ZZ**6+160.0*SS*SS*ZZ**6-
190.0*SS*SS*SS*ZZ**6-10.0*ZZ**7+30.0*SS*ZZ**7-10.0*SS*
1SS*SS*ZZ**7+1440.0*AK*AK*(-1.0+SS)**2*(16.0+19.0*SS
1+33.0*ZZ+37.0*SS*ZZ-20.0*ZZ*ZZ-50.0*SS*ZZ*ZZ-10.0*ZZ
1**3+10.0*SS*ZZ**3)+8.0*AK*(78.0+576.0*SS-36.0*SS*SS-
1618.0*SS*SS*SS-676.0*ZZ-11452.0*SS*ZZ+432.0*SS*SS*ZZ
1+1696.0*SS*SS*SS*ZZ+460*ZZ*ZZ+1875.0*SS*ZZ*ZZ+270.0*
1SS*SS*ZZ*ZZ-2605.0*SS*SS*SS*ZZ*ZZ-105.0*ZZ**3-1035.0
1*SS*ZZ**3-1215.0*SS*ZZ**3+2355.0*SS*SS*SS*ZZ**3-250.0*ZZ**4-
1375.0*SS*SS*ZZ**5+125.0*SS*SS*SS*ZZ**5)+90.0*AA*AA*(33.0-
1103.0*SS-236.0*ZZ-184.0*SS*ZZ-120*ZZ*ZZ+540.0*SS*ZZ*ZZ1
1+220.0*ZZ**3-220.0*SS*ZZ**3+420.0)*AK*(11.0+9.0*SS-12.0*ZZ
1+2.0*SS*ZZ))))
 G(I)=SS+(1.0-SS)*ZZ+RE*(-0.5*AA*(-1.0+SS)*(-
11.0+ZZ)*ZZ)+RE*RE*RE*(1.0/6300.0*((-1.0+SS)*(-1.0+ZZ)*
1ZZ*(-525.0*AA*AA*(-1.0+2.0*ZZ)-2.0*SS*(-1.0+2.0*ZZ)*
1(8.0+24.0*ZZ+105*AK*ZZ-14*ZZ*ZZ-105*AK*ZZ*ZZ
1-20.0*ZZ**3+10.0*ZZ**4)+SS*SS*(-3.0+ZZ)*(-9.0-12.0*ZZ+210
1*AK*ZZ+57.0*ZZ*ZZ-210.0*AK*ZZ*ZZ-60.0*ZZ**3+20.0
1*ZZ**4)+(2.0+ZZ)*(-4.0-2.0*ZZ+210*AK*ZZ-3.0*ZZ*ZZ-
1210.0*AK*ZZ*ZZ-20.0*ZZ**3+20.0*ZZ**4))))+RE*RE*RE
1*RE*(1.0/50400.0*(AA*(-1.0+SS)*(-1.0+ZZ)*ZZ*(34.0+2.0
1*ZZ+2100.0*AA*AA*ZZ+2.0*ZZ*ZZ-2100.0*AA*AA*ZZ*ZZ+
1247.0*ZZ**3-369.0*ZZ**4-75.0*ZZ**5+165.0*ZZ**6-140.0*AK*(-
12.0+180.0*AA*AA+4.0*SS-12.0*SS*SS+19.0*ZZ+28.0*SS
1*ZZ+12.0*SS*SS*ZZ-13*ZZ*ZZ-52.0*SS*ZZ*ZZ+5.0*SS*SS*
1ZZ*ZZ-18.0*ZZ**3+48.0*SS*ZZ**3-30*SS*SS*ZZ**3+12.0*ZZ**4
1-24.0*SS*ZZ**4+12.0*SS*SS*ZZ**4)+SS*SS*(6.0+114.0*ZZ+
```

```
     1254.0*ZZ*ZZ-1321.0*ZZ**3+1731.0*ZZ**3-915.0*ZZ**5+165.0
     1*ZZ**6)-2.0*SS*(20.0-12.0*ZZ-222.0*ZZ*ZZ+303.0*ZZ**3
     1+261.0*ZZ**4-495.0*ZZ**5+165.0*ZZ**6)+3360.0
     1*AK*AK*(-1.0+SS)*(-2.0+5.0*ZZ+SS*(-3.0+5.0*ZZ)))))
10     CONTINUE
       WRITE(8,*)'H(I)'
       DO 2 I=1,N+2
2      WRITE(8,*)H(I)
       WRITE(8,*)'G(I)'
       DO 3 I=1,N+2
3      WRITE(8,*)G(I)
C      ---------------------------------------------------------------
114    DO 12 I=1,N-1
       NX=I+1
12     R(I,I)=-RE*(G(NX+1)-G(NX-1))/(2.0*F)+AK*RE/F3*(-G(NX+2)+4.0*
     1G(NX+1)-3.0*G(NX))
       DO 13 I=1,N-1
       J=I+1
       NX=I+1
13     R(I,J)=RE*G(NX)/(2.0*F)+AK*RE/(2.0*F3)*(G(NX+1)-
     14.0*G(NX)+3.0*G(NX-1))
       DO 15 I=2,N-1
       J=I-1
       NX=I+1
       R(I,J)=-RE*G(NX)/(2.0*F)-AK*RE*(3.0*G(NX+1)-4.0*G(NX)+G(NX-
     11))/(2.0*F3)
15     CONTINUE
       DO 16 I=1,N-1
       J=N+I
       NX=I+1
     16     R(I,J)=-2.0/F2+RE*(H(NX+1)-H(NX-1))/(2.0*F)-
     1AK*RE*(2.0*(H(NX+1)-2.0*H(NX-1))+3.0*H(NX))/F3
       DO 17 I=1,N-1
       J=N+I+1
       NX=I+1
17     R(I,J)=1.0/F2-RE*H(NX)/(2.0*F)+AK*RE*(H(NX+1)+8.0*H(NX)-3.0*
     1H(NX-1))/(2.0*F3)
       DO 18 I=1,N-2
       J=N+I+2
       NX=I+1
       R(I,J)=-RE*H(NX)*AK/F3
18     CONTINUE
       DO 19 I=2,N-1
       J=N+I-1
       NX=I+1
       R(I,J)=1.0/F2+RE*H(NX)/(2.0*F)+AK*RE*(3.0*H(NX+1)-H(NX-1))/
     1(2.0*F3)
```

```
19    CONTINUE
      R(19,19)=-RE/(2.0*F)*(G(21)-G(19))+AK*RE/F3*(3.0*G(20)-4.0*
     1G(19)+G(18))
      R(19,18)=-RE*G(20)/(2.0*F)-AK*RE/(2.0*F3)*(3.0*G(21)-
     14.0*G(20)+G(19))
      R(19,38)=-2.0/F2+RE/(2.0*F)*(H(21)-H(19))+AK*RE/F3*(-
     12.0*H(21)+2.0*H(19)+3.0*H(20))
      R(19,37)=1.0/F2+RE*H(20)/(2.0*F)+AK*RE/(2.0*F3)*(3.0*H(21)-
     18.0*H(20)-H(19))
      R(19,36)=AK*RE/F3*H(20)
C     ************************************************************
      DO 20 I=N+2,NMK-2
      J=I-N
      NX=I+1-N
      R(I,J)=-4.0/F4-RE/F3*(H(NX+2)-3.0*H(NX+1)+6.0*H(NX)-
     1H(NX-1))+AK*RE/F5*(-H(NX+3)+H(NX+2)+6.0*H(NX+1)-24.0*H(NX)
     1+7.0*H(NX-1)+H(NX-2))
      R(20,1)=-4.0/F4-RE/F3*(H(4)-3.0*H(3)+7.0*H(2)-H(1))+
     1AK*RE/F5*(-H(5)+H(4)+6.0*H(3)-23.0*H(2)+7.0*H(1))
      R(38,19)=7.0/F4-4.0/(2.0*F4)*(H(21)-H(19))+AK*RE/F5*(-
     129.0*H(20)+26.0*H(21)+17.0*H(19)-5.0*H(18)+H(17))+
     1AK*RE/(2.0*F5)*(5.0*H(21)-10.0*H(20)+3.0*H(19))
      R(38,18)=-4.0/F4+RE/(2.0*F4)*(3.0*H(21)+4.0*H(20)-
     15.0*H(19))+AK*RE/(2.0*F5)*(-32.0*H(21)+57.0*H(20)+16.0
     1*H(19)+H(18))
      R(38,17)=1.0/F4+AK*RE/(2.0*F5)*(H(21)-H(19)-10.0*H(20))
      R(38,16)=AK*RE*H(20)/F5
20    CONTINUE
      DO 21 I=N+1,NMK-2
      J=I+1-N
      NX=I+1-N
      R(I,J)=6.0/F4+3.0*RE*H(NX)/F3+AK*RE/(2.0*F5)*(H(NX+3)-
     124.0*H(NX+1)+12.0*H(NX)-9.0*H(NX-1))
21    CONTINUE
      DO 22 I=N+1,NMK-2
      J=I+2-N
      NX=I+1-N
      R(I,J)=-4.0/F4+AK*RE/F5*(H(NX)+4.0*H(NX-1))-RE*H(NX)/F3
22    CONTINUE
      DO 23 I=N+1,NMK-3
      J=I+3-N
      NX=I+1-N
      R(I,J)=1.0/F4+AK*RE/(2.0*F5)*(H(NX+1)-2.0*H(NX)-H(NX-1))
23    CONTINUE
      DO 24 I=N+2,NMK-2
      J=I-1-N
      NX=I+1-N
```

```
      R(I,J)=1.0/F4+RE*H(NX)/F3+AK*RE/(2.0*F5)*(-
     1H(NX+3)+8.0*H(NX+2)-21.0*H(NX+1)+14.0*H(NX)-10.0*H(NX-1))
24    CONTINUE
      DO 244 I=N+3,NMK-2
      J=I-2-N
      NX=I-N
244   R(I,J)=AK*RE*H(NX)/F5
      DO 25 I=N+1,NMK-1
      NX=I+1-N
      R(I,I)=-8.0*AK*RE/F3-2.0*RE/F*(G(NX+1)-G(NX-1))
25    CONTINUE
      DO 26 I=N+1,NMK-2
      J=I+1
      NX=I+1-N
      R(I,J)=8.0*AK*RE/F3*(G(NX+1)-G(NX))-2.0*RE*G(NX)/F
26    CONTINUE
      DO 27 I=N+2,NMK-2
      J=I-1
      NX=I+1-N
      R(I,J)=8.0*AK*RE/F3*(G(NX)-G(NX-1))+2.0*RE*G(NX)/F
27    CONTINUE
      R(37,19)=-4.0/F4-RE/F3*(H(20)-3.0*H(20)+6.0*H(19)-
     1H(18))+AK*RE/F5*(H(21)+5.0*H(20)-24.0*H(19)+7.0*H(18)+H(17))
      R(37,18)=1.0/F4+RE*H(20)/F3+AK*RE/(2.0*F5)*(8.0*H(21)-
     122.0*H(20)+14.0*H(19)-10.0*H(18))
      R(37,17)=AK*RE*H(20)/F5
      R(38,38)=-AK*RE/F3-2.0*RE/F*(G(21)-G(19))
      R(38,37)=2.0*RE*G(20)+8.0*AK*RE/F3*(G(20)-G(19))
      DO 1111 I=1,NMK
      B(I)=0.0
      B(1)=H(2)*(-RE*G(2)/(2.0*F)-AK*RE/(2.0*F3)*(3.0*G(3)-
     14.0*G(2)+G(1)))+G(2)*(1.0/F2+RE*H(2)/(2.0*F))+AK*RE/(2.0*F3)
     1*(3.0*H(3)-H(1))
      B(19)=H(20)*(RE*G(20)/(2.0*F)-AK*RE/(2.0*F3)*(G(21)-4.0*
     1G(20)+3.0*G(19)))+G(20)*(1.0*F2-RE*H(20)/(2.0*F)+
     1AK*RE/(2.0*F)*(H(21)-H(19)))
      B(20)=(1.0/F4+RE*H(2)/F3+AK*RE/(2.0*F5)*(-H(5)+8.0*H(4)
     1-21.0*H(3)+14.0*H(2)-10.0*H(1)))*H(2)
      B(21)=H(20)*(AK*RE*H(2)/F5)
      B(37)=H(19)*(-4.0/F4+RE/(2.0*F4)*(3.0*H(21)+4.0*H(20)-
     15.0*H(19)))
      B(38)=G(20)*(8.0*AK*RE/F3*(G(21)-G(19))-
     12.0*RE*G(20)/F)+H(20)*(-4.0/F4+RE/(2.0*F4)*(3.0*H(21)
     1+4.0*H(20)-5.0*H(19))+AK*RE/(2.0*F5)*(-32.0*H(21)+
     157.0*H(20)+16.0*H(19)+H(18)))
      B(I)=-B(I)
1111  CONTINUE
```

```
888   FORMAT(6f15.6)
      CALL ALUD(R,Q,NMK)
      PROD=1.0
      DO 100 I=1,NMK
100   PROD=PROD*Q(I,I)
C     write(8,*)'prod'
C     write(8,*)prod
      PRO=PROD
      DO 101 J=1,NMK
      DO 102 I=1,NMK
102   TEMP(I)=R(I,J)
      DO 103 I=1,NMK
103   R(I,J)=B(I)
      DO 104 I=1,NMK
      DO 104 K=1,NMK
104   Q(I,K)=0.0
      CALL ALUD(R,Q,NMK)
      PROD=1.0
      DO 200 L=1,NMK
200   PROD=PROD*Q(L,L)
      DET(J)=PROD
      DO 1044 I=1,NMK
1044  R(I,J)=TEMP(I)
C     write(8,*) det(j)
101   CONTINUE
      DO 2222 I=1,N
      DELTAG(I)=DET(N+I)/PRO
2222  DELTAH(I)=DET(I)/PRO
c     WRITE(8,*)'DELTAH'
c     WRITE(8,*)(DELTAH(I),I=1,N)
c     WRITE(8,*)'DELTAG'
c     WRITE(8,*)(DELTAG(I),I=1,N)
      DO 111 I=1,N
      IF((DELTAH(I)).GT. .0001 .AND. (DELTAG(I)).GT. .0001)
     1GOTO 112
111   CONTINUE
      WRITE(8,*)'FINAL VALUE'
      WRITE(8,*)'H(K)'
      DO 234 K=1,N+2
234   WRITE(8,*)H(K)
      WRITE(8,*)'G(K)'
      DO 235 K=1,N+2
235   WRITE(8,*)G(K)
      STOP
112   DO 113 I=2,N+1
      H(I)=H(I)+(DELTAH(I-1))
      G(I)=G(I)+(DELTAG(I-1))
```

```
113   CONTINUE
      GOTO 114
      CLOSE(8)
      END
       SUBROUTINE ALUD(R,AU,N)
       dimension R(38,38),AU(38,38),AL(38,38)
       DOUBLE PRECISION R,AU,AL,N
       DO 75 I=1,N
       DO 75 J=1,N
75     AU(I,J)=R(I,J)
       DO 45 I=1,N
45     AL(I,I)=1.0
       MP=2
       MS=1
       DO 30 K=1,N-1
       DO 20 I=MP,N
       AL(I,K)=AU(I,K)/AU(K,K)
       DO 20 J=MS,N
20     AU(I,J)=AU(I,J)-AU(K,J)*AL(I,K)
       MS=MS+1
       MP=MP+1
30     CONTINUE
       DO 40 I=1,N
       AU(I,I)=AU(I,I)
40     CONTINUE
       RETURN
       END
```

VITA

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Abstract

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Major:	Mathematics	Minor:	Computer Science
Department:	Mathematics, Stat and Computer Science		
Thesis Title:	Numerical Techniques of Some Flow Problems		
Advisor:	Dr. Manoj Kumar		

The solutions of problems of engineering interest in the flow of visco-elastic fluid require a good understanding of the behavior of such fluids under a variety of circumstances. The study has immense practical utility. A large number of theoretical investigations dealing with the study of incompressible laminar flow with either suction or injection have appeared during the last few decades. The problem of forced flow of a fluid between two rotating discs has importance in chemical and mechanical engineering. Flows induced by rotating disks are of considerable fundamental interest because of the richness of the physical phenomenon they encompass. These flows have technical applications in many areas, such as rotating machinery, lubrication, viscometry and crystal growth processes.

The present investigations are based on the steady and unsteady flow of visco-elastic incompressible fluids such as Second-order fluid and Walters liquid B', in the annulus of two porous coaxial circular cylinders, between the two parallel rectangular plates and parallel circular plates, and between the two coaxial infinite discs. The discs/cylinders are rotating with different angular velocities and the rate of suction of the fluid in one discs/cylinder is different from the rate of injection in the other discs/cylinder. In the unsteady squeezing of thin film of visco-elastic fluid in between the plates when the lower plate is fixed and the upper plate is moving towards the lower plate has been reinvestigated.

All these problems have been discretized by using finite-difference approximation scheme and later solved by using Numerical techniques such as Newton-Raphson technique, Factorization technique and Gauss-elimination technique. The results have been obtained for the small as well as large value of Reynolds numbers or suction parameters through graphical representations and it has been concluded that the present investigation is in the use of technical as well as biophysical fields.

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