ADMISSIBILITY OF POOLING IN A RANDOM MODEL ANALYSIS OF VARIANCE OF STRIP-PLOT EXPERIMENT

THESIS

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Dedicated to

My Beloved Parents

SMT. SUMITRA SHARMA

SHRI DAMODAR PRASAD SHARMA

CERTIFICATE-I

This is to certify that the thesis entitled 'ADMISSIBILITY OF POOLING IN A RANDOM MODEL ANALYSIS OF VARIANCE OF STRIP-PLOT EXPERIMENT', submitted in partial fulfilment of the requirements for the degree of **MASTER OF SCIENCE IN AGRICULTURAL STATISTICS** of the Jawaharlal Nehru Krishi Vishwa Vidyalaya, Jabalpur; is a record of the bonafied research work carried out by **SHRI NEEL KANTH SHARMA** under my guidence and supervision. The subject of the thesis has been approved by the Student's Advisory Committee and Director of Instructions.

No part of the thesis has been submitted for any other degree or diploma (certificate awarded etc.) or has been published. All the assistance and help received during the course of investigations has been duly acknowledged by him.

Jabalpur
Dated 9th October, 1992.

(Kripa Shanker)
Chairman of the Advisory Committee

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CERTIFICATE-II

This is to certify that the thesis entitled 'ADMISSIBILITY OF POOLING IN A RANDOM MODEL ANALYSIS OF VARIANCE OF STRIP-PLOT EXPERIMENT', submitted by SHRI NEEL KANTH SHARMA to Jawaharlal Nehru Krishi Vishwa Vidyalaya, Jabalpur in partial fulfilment of the requirement for the degree of MASTER OF SCIENCE IN AGRICULTURAL STATISTICS in the Department of MATHEMATICS AND STATISTICS has after evaluation been approved by the external examiner and by the Student's Advisory Committee after an oral examination on the same.

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CHAPTER I

INTRODUCTION

INTRODUCTION

The analysis of variance and associated tests of significance were first developed by R.A. Fisher. In the original description of analysis of variance tests, Fisher (1937) was of the opinion that for every well-designed experiment there can only be one correct analysis and the test(s) of significance are completely determined before the experimental results are available. According to him, the appropriate test of significance is determined by a specification of the population from which the experimental data were sampled. The problem of specification arise in the choice of the statistical model. Fisher suggested that an appropriate statistical model, to be used in describing an observation in an investigation, should be determined in advance by the investigator. The analysis of variance tests carried out by considering an appropriate statistical model leads to valid subsequent inferences.

Model specification :

If the model specification is fixed in advance, that is if no attempt is made to use the data in hand as an aid in determining the model specification to be used in subsequent inferences, Bancroft (1964) refer to the analysis as being determined by a 'completely specified model'. This is referred to as the case of unconditional specification by

Bancroft and Han (1977). However, in experimental designs, situations frequently arise in which the model is not completely specified.

The present study is concerned with an experiment conducted in strip-plot design with two factors. In specifying the model for an analysis of variance of the strip-plot design, there exists an uncertainty about the existence of component of variances involving interactions of the factors with replications and the interaction between two factors. In such cases, the available data from the experiment are used to perform preliminary tests of significance as an aid in determining an appropriate final model specification for subsequent inferences. This is referred to as the case of conditional specification by Bancroft and Han (1977). In these situations, the model is said to be incompletely specified.

General Structures for Incompletely Specified Models :

Incompletely specified models involving the use of preliminary test(s) of significance are explained by Bancroft (1964) as follows :

An incompletely specified model which may take completely specified form on the basis of one preliminary test of significance may be given as follows :

$$S^* = IS_1 + (1-I)S_2$$

An incompletely specified model which may take completely specified form on the basis of three preliminary tests of significance may be given as follows :

$$S^{***} = IJKS_1'' + IJ(1-K)S_2'' + I(1-J)K\, S_3'' + I(1-J)(1-K)\, S_4''$$
$$+ (1-I)JK\, S_5'' + (1-I)\, J\, (1-K)S_6'' + (1-I)(1-J)K\, S_7''$$
$$+ (1-I)(1-J)(1-K)\, S_8''$$

Where S_i'' ($i= 1,2, \ldots, 8$) are eight forms of the completely specified models which S^{***}, the incompletely specified model, will take depending on whether the three preliminary tests of significance T_p^*, T_p^{**} and T_p^{***} are significant or not, i.e.

$I = 1$, if $T_p^* \geq T^* (\alpha_1)$ and $I = 0$, if $T_p^* < T^* (\alpha_1)$;

$J = 1$, if $T_p^{**} \geq T^{**} (\alpha_2)$ and $J=0$, if $T_p^{**} < T^{**} (\alpha_2)$;

$K = 1$, if $T_p^{***} \geq T^{***}(\alpha_3)$ and $K = 0$, if $T_p^{***} < T^{***}(\alpha_3)$.

Where α_1, α_2 and α_3 are levels of significance of the preliminary tests of significance T_p^*, T_p^{**}, and T_p^{***} respectively.

Incompletely specified models which may take completely specified form on the basis of more than three preliminary tests of significance may be obtained in analogous manner.

Test Procedure :

A test procedure for a null hypothesis regarding treatment effects for an incompletely specified model when three preliminary tests of significance are used, consists eight mutually exclusive alternatives. The occurrence of any one of the alternatives would reject the null hypothesis.

Admissibility of the Test Procedure :

A test procedure is completely ruled out of consideration if it is inadmissible even though it may have sufficiently large power and controlled size. Therefore admissibility of a test procedure is also an important and desired property.

In the present study, for testing a hypothesis about one of the main effects in two-factor strip-plot design, two test procedures based on three preliminary tests of significance are developed using Satterthwaite approximate F statistics discussed by Anderson and Bancroft (1952) (Page 350). Necessary and sufficient conditions for admissibility of the two test procedures are derived.

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CHAPTER II
REVIEW OF LITERATURE

REVIEW OF LITERATURE

A brief resume of work done by various statisticians on the topic under study is as follows :

Cohen (1968) proved that the 'some times pooling' procedure is an admissible test procedure for the general linear hypothesis model. The proof follows from a well known invariance result and a theorem of Matthes and Truax (1967). The estimation procedures based on a preliminary test were found to be inadmissible for the squared error loss function.

Cohen (1974) studied three types of problems i.e. testing the normal mean, fixed effects models of the analysis of variance and random effects models. Necessary and sufficient conditions for admissibility of the test procedures based on preliminary test of significance were derived. According to him, admissibility is equivalent to the intuitive and practical condition that acceptance regions of the procedures have convex sections in certain variables, while other variables are fixed. Optimality properties of the pooling procedures is also discussed.

Agarwal and Gupta (1981a) developed three test procedures based on one preliminary test of significance considering Davenport and Webster (1973) approximate F - statistics for a mixed model ANOVA of a three factor experiment. They proved the admissibility of the test procedures, and derived necessary and sufficient conditions for admissibility of the test procedures.

Agarwal and Gupta (1981b) in a separate study derived the necessary and sufficient conditions for the three test procedures based on two preliminary tests of significance considering Davenport and Webster (1973) approximate F-statistics for a three factor factorial experiment under mixed model.

Gupta and Gupta (1985) developed two test procedures based on two preliminary tests of significance using approximate F - statistics for a mixed model ANOVA of group of experiments. Admissibility of the test procedures is proved with the derivation of the necessary and sufficient conditions for their admissibility.

Kripa Shanker (1990a) proved the admissibility of a some times-pool test procedure based on one preliminary test of significance in a two-level nested random model analysis of variance with unequal subclass numbers. Necessary and sufficient condition for admissibility of the test procedure is also derived.

Kripa Shanker (1990b) developed a test procedure based on two preliminary tests of significance for a mixed model ANOVA of three level nested classification with unequal subclass numbers. Admissibility of this test procedure is proved. A necessary and sufficient condition for admissibility of the test procedure is also derived.

Kripa Shanker (1991) developed a test procedure based on one preliminary test of significance for a mixed model analysis of variance of balanced incomplete block design. A necessary and sufficient condition for admissibility of the test procedure is also derived.

CHAPTER III

MATERIALS AND METHODS

MATERIALS AND METHODS

In agricultural field experiments, certain treatments such as irrigation, dates of transplanting, application of bulky manures, cultivation methods, etc. can be conveniently applied only to large plots. In a two-factor factorial experiment with such type of treatments if the effects of both factors are of relatively little interest compared to the effect of interaction between them, it is preferred to conduct the experiment in strip-plot design.

In the present investigation, an experiment conducted on paddy in strip-plot design which involves two factors i.e. dates of transplanting (D) and green manuring (G), are considered.

3.1 Statistical Model and ANOVA :

Each observation in the experiment conducted in strip-plot design is represented by the following linear statistical model :

$$Y_{ijk} = \mu + r_i + d_j + e_{ij} + g_k + e_{ik} + (dg)_{jk} + e_{ijk} \qquad (3.1.1)$$

Where

$i = 1, 2, \ldots , I$; $j = 1, 2, \ldots , J$; $k = 1, 2, \ldots , K$

Y_{ijk} = observation of the plot of the i^{th} replication in which transplanting was done on j^{th} date with k^{th} green manuring.

μ = the general mean,

r_i = effect of the i^{th} replication,

d_j = effect of the j^{th} date of transplanting,

g_k = effect of the k^{th} green manuring,

$(dg)_{jk}$ = effect of the interaction between j^{th} level of date of transplanting and k^{th} level of green manuring,

e_{ij} = random error, N.I.D. $(0, \sigma^2_{rd})$,

e_{ik} = random error, N.I.D. $(0, \sigma^2_{rg})$,

e_{ijk} = random error, N.I.D. $(0, \sigma^2_e)$.

The model (3.1.1.) is a random model, where r_i, d_j, g_k and $(dg)_{jk}$ are random effects. The assumptions are

r_i $(i = 1,2, \ldots, I)$ are N.I.D. $(0, \sigma^2_r)$,

d_j $(j = 1,2, \ldots, J)$ are N.I.D. $(0, \sigma^2_d)$,

g_k $(k = 1,2, \ldots, K)$ are N.I.D. $(0, \sigma^2_g)$,

$(dg)_{jk}$ $(i=1,2,\ldots, I; J=1,2,\ldots, J)$ are N.I.D. $(0, \sigma^2_{dg})$.

The skeleton of the analysis of variance with expected mean squares is given in Table 1. The expected mean squares in the Table are derived following Steel and Torrie (1980).

Table 1 : Random Model Analysis of Variance of Strip-Plot Experiment with Expected Mean Squares.

Source of Variation	Degrees of freedom	Mean Square Observed	Mean Square Expected
Replications (R)	$(I-1)$	-	-
Dates of transplanting (D)	$(J-1) = n_5$	V_5	$\sigma_e^2 + K\ \sigma_{rd}^2 + I\ \sigma_{dg}^2 + I\ K\ \sigma_d^2 = \sigma_5^2$
Error (a)	$(I-1)(J-1) = n_4$	V_4	$\sigma_e^2 + K\ \sigma_{rd}^2 = \sigma_4^2$
Green manuring (G)	$(K-1)$ -	-	$\sigma_e^2 + I\ \sigma_{dg}^2 + J\ \sigma_{rg}^2 + IJ\ \sigma_g^2 =$
Error (b)	$(I-1)(K-1) = n_3$	V_3	$\sigma_e^2 + J\ \sigma_{rg}^2 = \sigma_3^2$
DxG	$(J-1)(K-1) = n_2$	V_2	$\sigma_e^2 + I\ \sigma_{dg}^2 = \sigma_2^2$
Error (c)	$(I-1)(J-1)(k-1) = n_1$	V_1	$\sigma_e^2 = \sigma_1^2$

3.2 Model Specification :

If $\sigma^2_{dg} > 0$, $\sigma^2_{rg} > 0$, $\sigma^2_{rd} > 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + e_{ij} + g_k + e_{ik} + (dg)_{gk} + e_{ijk}, \qquad (3.2.1)$$

and model (3.1.1) is completely specified.

If $\sigma^2_{dg} > 0$, $\sigma^2_{rg} \leq 0$, $\sigma^2_{rd} > 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + e_{ij} + g_k + (dg)_{gk} + e_{ijk} \qquad (3.2.2)$$

and model (3.1.1) is completely specified.

If $\sigma^2_{dg} > 0$, $\sigma^2_{rg} > 0$, $\sigma^2_{rd} \leq 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + g_k + e_{ik} + (dg)_{jk} + e_{ijk} \qquad (3.2.3)$$

and the model (3.1.1) is completely specified.

If $\sigma^2_{dg} > 0$, $\sigma^2_{rg} \leq 0$, $\sigma^2_{rd} \leq 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + g_k + (dg)_{jk} + e_{ijk} \qquad (3.2.4)$$

and the model (3.1.1) is completely specified.

If $\sigma^2_{dg} \leq 0$, $\sigma^2_{rg} > 0$, $\sigma^2_{rd} > 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + e_{ij} + g_k + e_{ik} + e_{ijk} \qquad (3.2.5)$$

and the model (3.1.1) is completely specified.

If $\sigma^2_{dg} \leq 0$, $\sigma^2_{rg} \leq 0$, $\sigma^2_{rd} > 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + e_{ij} + g_k + e_{ijk} \tag{3.2.6}$$

and the model (3.1.1) is completely specified.

If $\sigma^2_{dg} \leq 0$, $\sigma^2_{rg} > 0$, $\sigma^2_{rd} \leq 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + g_k + e_{ik} + e_{ijk} \tag{3.2.7}$$

and the model (3.1.1) is completely specified.

If $\sigma^2_{dg} \leq 0$, $\sigma^2_{rg} \leq 0$, $\sigma^2_{rd} \leq 0$

then the appropriate model is

$$Y_{ijk} = \mu + r_i + d_j + g_k + e_{ijk} \tag{3.2.8}$$

and the model (3.1.1) is completely specified.

The abridged ANOVA for testing the hypothesis about the effect of dates of transplanting (D) i.e. $H_o : \sigma^2_d = 0$ vs $H_1 : \sigma^2_d > 0$, in case of above mentioned appropriate models (3.2.1) to (3.2.8) under different specifications is given in Table 2.

Table 2 : Abridged Random Model ANOVA for Strip-Plot Experiment with Expected Mean Squares under different specifications.

	Source of variation				
	D	Error (a)	Error (b)	DG	Error (c)
Degrees of Freedom (D.F.)	n_5	n_4	n_3	n_2	n_1
Observed Mean Squares	V_5	V_4	V_3	V_2	V_1
E.M.S. if (i) $\sigma^2_{dg}>0;\sigma^2_{rg}>0,\sigma^2_{rd}>0$	$\sigma^2_e+K\sigma^2_{rd}+I\sigma^2_{dg}+IK\sigma^2_d$	$\sigma^2_e+K\sigma^2_{rd}$	$\sigma^2_e+J\sigma^2_{rg}$	$\sigma^2_e+I\sigma^2_{dg}$	σ^2_e
(ii) $\sigma^2_{dg}>0;\sigma^2_{rg}\leq 0;\ \sigma^2_{rd}>0$	$\sigma^2_e+K\sigma^2_{rd}+I\sigma^2_{dg}+IK\sigma^2_d$	$\sigma^2_e+K\sigma^2_{rd}$	σ^2_e	$\sigma^2_e+I\sigma^2_{dg}$	σ^2_e
(iii) $\sigma^2_{dg}>0;\sigma^2_{rg}>0;\ \sigma^2_{rd}\leq 0$	$\sigma^2_e+I\sigma^2_{dg}+IK\sigma^2_d$	σ^2_e	$\sigma^2_e+J\sigma^2_{rg}$	$\sigma^2_e+I\sigma^2_{dg}$	σ^2_e
(iv) $\sigma^2_{dg}>0;\sigma^2_{rg}\leq 0;\ \sigma^2_{rd}\leq 0$	$\sigma^2_e+I\sigma^2_{dg}+IK\sigma^2_d$	σ^2_e	σ^2_e	$\sigma^2_e+I\sigma^2_{dg}$	σ^2_e
(v) $\sigma^2_{dg}\leq 0;\ \sigma^2_{rg}>0;\sigma^2_{rd}>0$	$\sigma^2_e+K\sigma^2_{rd}+IK\sigma^2_d$	$\sigma^2_e+K\sigma^2_{rd}$	$\sigma^2_e+J\sigma^2_{rg}$	σ^2_e	σ^2_e
(vi) $\sigma^2_{dg}\leq 0;\ \sigma^2_{rg}\leq 0,\ \sigma^2_{rd}>0$	$\sigma^2_e+K\sigma^2_{rd}+IK\sigma^2_d$	$\sigma^2_e+K\sigma^2_{rd}$	σ^2_e	σ^2_e	σ^2_e
(vii) $\sigma^2_{dg}\leq 0,\ \sigma^2_{rg}>0;\ \sigma^2_{rd}\leq 0$	$\sigma^2_e+IK\sigma^2_d$	σ^2_e	$\sigma^2_e+J\sigma^2_{rg}$	σ^2_e	σ^2_e
(viii) $\sigma^2_{dg}\leq 0,\ \sigma^2_{rg}\leq 0;\ \sigma^2_{rd}\leq 0$	$\sigma^2_e+IK\sigma^2_d$	σ^2_e	σ^2_e	σ^2_e	σ^2_e

E.M.S. = Expected Mean Squares.

3.3 Never pool Test :

From Table 2, it may be observed that there is no estimator of error variance which is an appropriate denominator for exact F-test to test H_o when the appropriate models are given by (3.2.1) and (3.2.2).

If the appropriate model is given by (3.2.1), Satterthwaite approximate F-statistics denoted by

$$F^* \left(= \frac{V_5}{V_4+V_2-V_1}\right) \text{ and } F^{**} \left(= \frac{V_5+V_1}{V_4+V_2}\right)$$

are usually considered for testing H_o. F^* retains a true Chi-square statistic in the numerator while F_1^* avoids negative coefficients in the linear functions. Both the statistics are discussed by Anderson and Bancroft (1952) (Page 350). If the computed value of F^* (or F^{**}) exceeds $F\ (n_5, \nu_1; \alpha_4)$ [or $F\ (\nu_1' \ \nu_2' \ ; \alpha_4')$], the null hypothesis H_o is rejected. This test is never pool test (NPT). Here $F\ (p, q; \alpha_t)$ refers to the upper 100 α_t% point of the central F-distribution with (p, q) degrees of freedom. The degrees of freedom ν_1, ν_1' and ν_2' of synthesised variances $(V_4+V_2-V_1)$, (V_5+V_1) and (V_4+V_2) respectively were obtained following Satterthwaite (1946) and given as follows :

$$\nu_1 = (V_4+V_2-V_1)^2 \,/\, (n_4^{-1} V_4^2 + n_2^{-1} V_2^2 + n_1^{-1} V_1^2)$$

$$\nu_1' = (V_5+V_1)^2 \,/\, (n_5^{-1} V_5^2 + n_1^{-1} V_1^2)$$

$$\nu_2' = (V_4+V_2)^2 \,/\, (n_4^{-1} V_4^2 + n_2^{-1} V_2^2)$$

If the appropriate model is given by (3.2.2), Satterthwaite approximate F-statistics denoted by $F_1^{*}\ (= \frac{V_5}{V_4+V_2-V_{13}})$ and $F_1^{**}(= \frac{V_5+V_{13}}{V_4+V_2})$ are considered for testing H_o. $V_{13} = (n_1V_1+n_3V_3)/(n_1+n_3)$. The degrees of freedom ν_2 and ν_3' of synthesised variances $(V_4+V_2-V_{13})$ and (V_5+V_{13}) respectively were obtained following Satterthwaite (1946) and given as follows :

$$\nu_2 = [n_{13}(V_4+V_2)-(n_1V_1+n_3V_3)]^2 \; / \; [n_{13}^2(n_4^{-1}V_4^2+n_2^{-1}V_2^2)+n_1V_1^2+n_3V_5^2\ ]$$

$$\nu_3' = (n_{13}V_5+n_1V_1+n_3V_3)^2 \; / \; (n_5^{-1}n_{13}^2V_5^2 + n_1V_1^2 + n_3V_3^2\ )$$

The estimated degrees of freedom ν_i (i=1,2), ν_j' (j=1, 2, 3) may be in fractions, therefore, $F\ (n_5\nu_1;\ \alpha_4)$, $F(n_5, \nu_2;\ \alpha_5)$, $F\ (\nu_1', \nu_2';\ \alpha_4')$ and $F\ (\nu_1', \nu_2';\ \alpha_5')$ can be interpolated as per procedure suggested by Laubscher (1965).

3.4 Always pool Test :

If the appropriate model is given by (3.2.8), then the estimator of error variance which is an appropriate denominator for exact F-test to test H_o will be V_{1234}, where $V_{1234} = (n_1V_1+n_2V_2+n_3V_3+n_4V_4)/n_{1234}$, $n_{1234} = n_1+n_2+n_3+n_4$. The null hypothesis H_o is rejected if, the computed value of the ratio V_5/V_{1234} exceeds $F(n_5, n_{1234};\ \alpha_9)$. This test is always pool test.

3.5 Some times pool Test :

However, incase of uncertainty about the existence of component of variances σ^2_{dg} and / or σ^2_{rg} and/or σ^2_{rd}, the model (3.1.1) is conditionally specified (Bancroft and Han, 1977). Using F-tests, the appropriate F-statistics for exact/approximate F-test for testing H_o depends on the outcome of the following three preliminary tests of significance carried out for testing the hypotheses :

$$\left.\begin{array}{llll} H_{10} : \sigma^2_{dg} = 0 & \text{vs} & H_{11} : \sigma^2_{dg} > 0, \\ H_{20} : \sigma^2_{rg} = 0 & \text{vs} & H_{21} : \sigma^2_{rg} > 0, \\ H_{30} : \sigma^2_{rd} = 0 & \text{vs} & H_{31} : \sigma^2_{rd} > 0. \end{array}\right\} \qquad (3.5.1)$$

On the basis of outcome of the above preliminary tests of significance, the model would be selected finally and then the appropriate F-statistic for testing H_o would be determined. This test of H_o is called some times pool test (SPT).

3.6 Formulation of the Test Procedure :

In order to test H_o vs H_1, using the F-statistics obtained on the basis of the outcome of the preliminary tests of significance (3.5.1), the following two test procedures, each of which consist of eight mutually exclusive alternatives, [A_{1i}, for test procedure I and A_{2i} for test procedure II, i=1,2,..., 8] are developed and given below. The occurrence of any one of the alternatives A_{1i} (or A_{2i}) $(i=1,2,\ldots, 8)$ would reject H_o.

Test Procedure I :

A_{11} : $V_2/V_1>F_1$, $V_3/V_1>F_2$, $V_4/V_1>F_3$, $V_5/(V_4+V_2-V_1)>F_4$, (3.6.1)

A_{12} : $V_2/V_1>F_1$, $V_3/V_1\leq F_2$, $V_4/V_1>F_3$, $V_5/(V_4+V_2-V_{13})>F_5$, (3.6.2)

A_{13} : $V_2/V_1>F_1$, $V_3/V_1>F_2$, $V_4/V_1\leq F_3$, $V_5/V_2>F_6$, (3.6.3)

A_{14} : $V_2/V_1>F_1$, $V_3/V_1\leq F_2$, $V_4/V_1\leq F_3$, $V_5/V_2>F_6$, (3.6.4)

A_{15} : $V_2/V_1\leq F_1$, $V_3/V_1>F_2$, $V_4/V_1>F_3$, $V_5/V_4>F_7$, (3.6.5)

A_{16} : $V_2/V_1\leq F_1$, $V_3/V_1\leq F_2$, $V_4/V_1>F_3$, $V_5/V_4>F_7$, (3.6.6)

A_{17} : $V_2/V_1\leq F_1$, $V_3/V_1>F_2$, $V_4/V_1\leq F_3$, $V_5/V_{124}>F_8$, (3.6.7)

A_{18} : $V_2/V_1\leq F_1$, $V_3/V_1\leq F_2$, $V_4/V_1\leq F_3$, $V_5/V_{1234}>F_9$ (3.6.8)

Test Procedure II :

A_{21} : $V_2/V_1>F_1$, $V_3/V_1>F_2$, $V_4/V_1>F_3$, $(V_5+V_1)/(V_4+V_2)>F_4'$, (3.6.9)

A_{22}: $V_2/V_1>F_1$, $V_3/V_1\leq F_2$, $V_4/V_1>F_3$, $(V_5+V_{13})/(V_4+V_2)>F_5'$, (3.6.10)

Alternatives A_{23} to A_{28} are same as that of A_{13} to A_{18} respectively.

In the above test procedures

$V_{124} = (n_1V_1+n_2V_2+n_4V_4)/n_{124}$, $n_{124} = n_1+n_2+n_4$,

$F_1 = F(n_2, n_1; \alpha_1)$, $F_2=F(n_3, n_1; \alpha_2)$, $F_3=F(n_4, n_1; \alpha_3)$,

$F_4= F(n_5, \nu_1; \alpha_4)$, $F_5=F(n_5, \nu_2; \alpha_5)$, $F_6=F(n_5,n_2; \alpha_6)$,

$F_7 = F(n_5, n_4; \alpha_7)$, $F_8 = F(n_5, n_{124}; \alpha_8)$,

$F_9 = F(n_5, n_{1234}; \alpha_9)$, $F_4' = F(\nu_1', \nu_2'; \alpha_4')$,

$F_5' = F(\nu_3', \nu_2'; \alpha_5')$.

The levels α_1, α_2 and α_3 are called preliminary levels of significance and, α_4, α_4', α_5, α_5', α_6, α_7, α_8 and α_9 are called final levels of significance.

3.7 Admissibility of Test Procedures :

Let the test procedure I (or II) be denoted by $\varphi(V)$, such that

$\varphi(V) = 1$, for $V \in A_{1i}$ (or A_{2i}) $(i=1, 2, \ldots, 8)$

$= 0$, otherwise.

The test procedure would be admissible if it is possible to determine the acceptance region of $\varphi(V)$, which has convex section in certain variables while other variables are fixed.

It may be noted that the mean squares $V_i (i=1,2,3,4,5)$ are distributed as $\chi_i^2 \sigma_i^2 / n_i$, where χ_i^2 is a central Chi-square statistic based on n_i degrees of freedom. The joint distribution of independent mean squares $V_1, V_2, V_3 V_4$ and V_5 which belongs to a multivariate exponential family, is given by

$$K \prod_{i=1}^{5} (V_i^{\frac{n_i}{2}-1}) \exp\left[-\frac{1}{2}\sum_{i=1}^{5}\frac{n_i V_i}{\sigma_i^2}\right] \prod_{i=1}^{5} dV_i \tag{3.7.1}$$

Where $K = \prod_{i=1}^{5}\left[\left\{n_i/(2\sigma_i^2)\right\} \Big/ \Gamma\left(n_i/2\right)\right]$

Let us consider the following orthogonal transformation

$$W = T\,V \tag{3.7.2}$$

Where $W' = (w_4, w_5, w_3, w_2, w_1)$

$V' = (V_1, V_2, V_3, V_4, V_5)$

$$T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} \\ 0 & \frac{-\sqrt{3}}{2} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ 0 & 0 & \frac{-\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

From (3.7.2) we have

$$\left.\begin{aligned} V_1 &= \frac{w_4}{\sqrt{5}} - \frac{2w_5}{\sqrt{5}} \\ V_2 &= \frac{w_4}{\sqrt{5}} + \frac{w_5}{2\sqrt{5}} - \frac{\sqrt{3}\,w_3}{2} \\ V_3 &= \frac{w_4}{\sqrt{5}} + \frac{w_5}{2\sqrt{5}} + \frac{w_3}{2\sqrt{3}} - \frac{\sqrt{2}\,w_2}{\sqrt{3}} \\ V_4 &= \frac{w_4}{\sqrt{5}} + \frac{w_5}{2\sqrt{5}} + \frac{w_3}{2\sqrt{3}} + \frac{w_2}{\sqrt{6}} - \frac{w_1}{\sqrt{2}} \\ V_5 &= \frac{w_4}{\sqrt{5}} + \frac{w_5}{2\sqrt{5}} + \frac{w_3}{2\sqrt{3}} + \frac{w_2}{\sqrt{6}} + \frac{w_1}{\sqrt{2}} \end{aligned}\right\} \tag{3.7.3}$$

The transformation (3.7.2) of variables in the distribution (3.7.1) gives an exponential distribution of W of the form :

$$dp\ (W;\ \Theta) = C(\Theta)\ e^{W'\Theta}\ d\lambda(w)$$

Where $C(\Theta) = K$, $\Theta = -\frac{1}{2}\ TN$, $N' = \left(\frac{n_1}{\sigma_1^2}, \frac{n_2}{\sigma_2^2}, \frac{n_3}{\sigma_3^2}, \frac{n_4}{\sigma_4^2}, \frac{n_5}{\sigma_5^2}\right)$

and $d\lambda(w)$ is the function of w's and differential terms.

It may be observed that the conditional distribution of w_5 given (w_1, w_2, w_3, w_4) belongs to one-dimensional family with parameters

$$\Theta_2 = \left[\frac{1}{4\sqrt{5}}\left(\frac{4n_1}{\sigma_1^2} - \sum_{i=2}^{5} \frac{n_i}{\sigma_i^2}\right)\right]$$

Under the transformation of variables. The original hypothesis $H_o : \sigma_d^2 = 0$ vs $N_1 : \sigma_d^2 > 0$ reduces to $H_o^* : \Theta_2 = 0$ vs $H_1^* : \Theta_2 > 0$.

The test procedure $\varphi(V)$, denoted by $\varphi(W)$ after transformation of variables in terms of w's would be admissible if the acceptance region of $\varphi(W)$ has convex section in w_5 while w_1, w_2, w_3, w_4 are fixed.

3.7.1 Admissibility of Test procedure I :

The tests under the test procedure I[(3.6.1) to (3.6.8)] after substitution of the values of V_i's from (3.7.3) would be

$V_2/V_1 > F_1 \Longrightarrow$

$$w_5 > [2(F_1-1)w_4+\sqrt{15}w_3]/(4F_1+1) \tag{3.7.1.1}$$

$V_3/V_1 > F_2 \Longrightarrow$

$$w_5 > [2(F_2-1)w_4-\sqrt{5}/\sqrt{3}\ w_3+2\sqrt{10}/\sqrt{3}\ w_2]/(4F_2+1) \tag{3.7.1.2}$$

$V_4/V_1 > F_3 \Longrightarrow$

$$w_5 > [2(F_3-1)w_4-\sqrt{5}/\sqrt{3}\ w_3-\sqrt{10}/\sqrt{3}\ w_2+\sqrt{10}w_1]/(4F_3+1) \tag{3.7.1.3}$$

$V_5/(V_4+V_2-V_1) > F_4 \Longrightarrow$

$$\begin{aligned} w_5 < [&-2(F_4-1)w_4+\sqrt{5}/\sqrt{3}(2F_4+1)w_3-\sqrt{10}/\sqrt{3}(F_4-1)w_2 \\ &+\sqrt{10}(F_4+1)w_1]/(6F_4-1) \end{aligned} \tag{3.7.1.4}$$

$V_5/(V_4+V_2-V_{13}) > F_5 \Longrightarrow$

$$\begin{aligned} w_5 > [&2n_{13}(3F_5-1)w_4 - \sqrt{5}/\sqrt{3}\left\{(2n_1+n_3)F_5+n_{13}\right\}w_3 \\ &+\sqrt{10}/\sqrt{3}\left\{(n_1-n_3)F_5-n_{13}\right\}w_2-\sqrt{10}n_{13}(F_5+1)w_1]/[(2n_1-3n_3)F_5+n_{13}] \end{aligned} \tag{3.7.1.5}$$

$V_5/V_2 > F_6 \Longrightarrow$

$$w_5 < [-2(F_6-1)w_4+\sqrt{5}/\sqrt{3}(3F_6+1)w_3+\sqrt{10}/\sqrt{3}w_2+\sqrt{10}w_1]/(F_6-1) \tag{3.7.1.6}$$

$V_5/V_4 > F_7 \Longrightarrow$

$$\begin{aligned} w_5 < [&-2(F_7-1)w_4-\sqrt{5}/\sqrt{3}(F_7-1)w_3-\sqrt{10}/\sqrt{3}(F_7-1)w_2 \\ &+\sqrt{10}(F_7+1)w_1]/(F_7-1) \end{aligned} \tag{3.7.1.7}$$

$V_5/V_{124} > F_8 \Longrightarrow$

$$\begin{aligned} w_5 > [&2n_{124}(F_8-1)w_4-\sqrt{5}/\sqrt{3}\left\{(3n_2-n_4)F_8+n_{124}\right\}w_3 \\ &+\sqrt{10}/\sqrt{3}(n_4F_8-n_{124})w_2-\sqrt{10}(n_4F_8+n_{124})w_1]/[(4n_1-n_{24})F_8+n_{124}] \end{aligned} \tag{3.7.1.8}$$

$V_5/V_{1234} > F_9 \Longrightarrow$

$$w_5 > [2n_{1234}(F_9-1)w_4+\sqrt{5}/\sqrt{3}\{(n_{34}-3n_2)F_9-n_{1234}\}w_3$$

$$+\sqrt{10}/\sqrt{3}\{(-2n_3+n_4)F_9-n_{1234}\}w_2$$

$$-\sqrt{10}\{n_4F_9+n_{1234}\}w_1]/[(2n_1-n_{234})F_9+n_{1234})] \quad (3.7.1.9)$$

Let the right-hand side expressions of the inequalities (3.7.1.1) to (3.7.1.9) be denoted by E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9 respectively. The acceptance region of $\varphi(w)$ will be the union of the following eight sets :

$$w_5 : w_5 \geq \max(E_1, E_2, E_3, E_4) \quad (3.7.1.10)$$

$$w_5 : w_5 < \min(E_2, E_5) \cap w_5 \geq \max(E_1, E_3) \quad (3.7.1.11)$$

$$w_5 : w_5 < E_3 \cap w_5 \geq \max(E_1, E_2, E_6) \quad (3.7.1.12)$$

$$w_5 : w_5 < \min(E_2, E_3) \cap w_5 \geq \max(E_1, E_6) \quad (3.7.1.13)$$

$$w_5 : w_5 < E_1 \cap w_5 \geq \max(E_2, E_3, E_7) \quad (3.7.1.14)$$

$$w_5 : w_5 < \min(E_1, E_2) \cap w_5 \geq (E_3, E_7) \quad (3.7.1.15)$$

$$w_5 : w_5 < \min(E_1, E_3, E_8) \cap w_5 \geq E_2 \quad (3.7.1.16)$$

$$w_5 : w_5 < \min(E_1, E_2, E_3, E_9) \quad (3.7.1.17)$$

The union of the eight sets given by (3.7.1.10) to (3.7.1.17) will be a convex set for a large number of ordered arrangements of E_1, E_2, ... , E_9, out of $9!$ maximum possible ordered arrangements. One of the ordered arrangements for which the union of the eight sets[(3.7.1.10) to (3.7.1.17)] will be convex set, is given by

$$E_2<E_9<E_4<E_6<E_1<E_7<E_3<E_5<E_8 \qquad (3.7.1.18)$$

As suggested by Cohen (1968), E_i's $(i=1,2,\ldots,9)$ may be represented by spheres centred at the origin as shown by the venn diagram in Fig. 1. It may be observed that the union of the eight sets [(3.7.1.10) to (3.7.1.17)] under the condition (3.7.1.18) is the set E_8, as depicted by shaded portion in Fig. 1, which is convex set. Hence the test procedure I [(3.6.1) to (3.6.8)] is admissible.

<u>Necessary and Sufficient Condition for Admissibility of the Test Procedure I</u> :

Using (3.7.1.1) to (3.7.1.9) the inequalities $E_2<E_9$, $E_9<E_4$, $E_4<E_6$, $E_6<E_1$, $E_1<E_3$, $E_3<E_8$, obtained from the condition (3.7.1.18) are given in terms of w_i $(i=1,2,3,4)$ as follows :

$$E_2<E_9 \Longrightarrow$$

$$\begin{aligned} w_4>\Big[&-\sqrt{5}/\sqrt{3}\left\{2(n_{34}-3n_2)F_2F_9+(n_1-2n_2)F_9-2n_{1234}F_2\right\}w_3 \\ &+\sqrt{5}/\sqrt{6}\left\{4(2n_3-n_4)F_2F_9+(4n_1-2n_2-3n_4)F_9+4n_{1234}F_2+3n_{1234}\right\}w_2 \\ &+\sqrt{5}\left\{4n_4F_2F_9+n_4F_9+4n_{1234}F_2+n_{1234}\right\}w_1\Big]/\Big[(5n_{234}+2n_1)F_2F_9 \\ &+3n_1F_9-5n_{1234}F_2\Big] \end{aligned} \qquad (3.7.1.19)$$

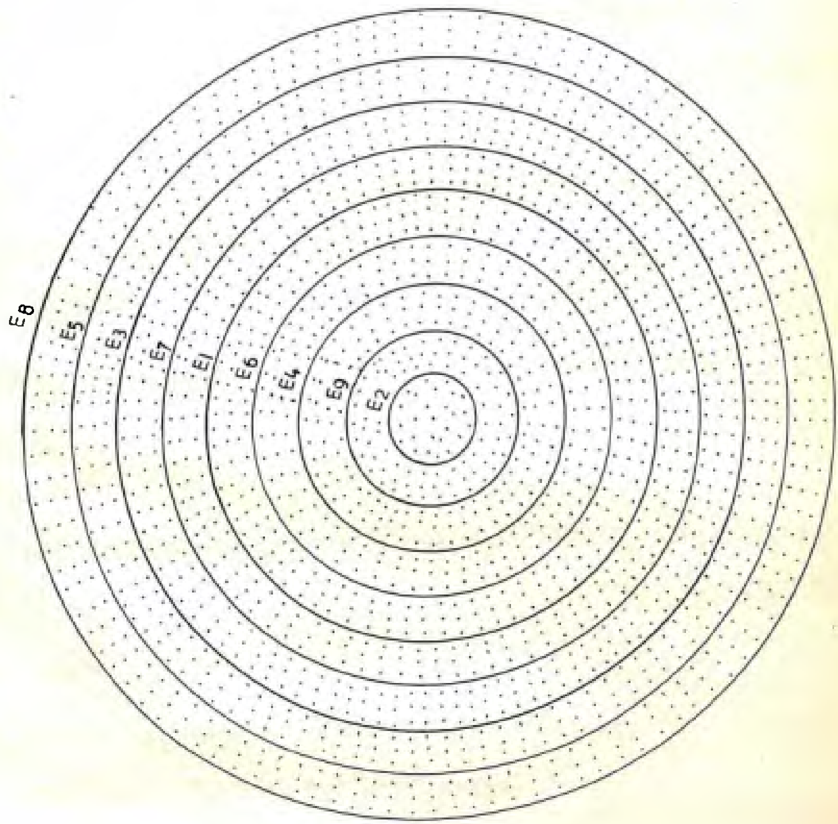

FIG.1

$E_9 < E_4 \Longrightarrow$

$$w_4 < [\sqrt{5}/\sqrt{3}\{(4n_1 - 8n_{34} + 16n_2)F_4F_9 + (8n_1 + 2n_2 + 6n_{34})F_9 + 2n_{1234}F_4\}w_3$$
$$- \sqrt{10}/\sqrt{3}[(2n_1 - n_2 + 11n_3 - 7n_4)F_4F_9 - (2n_1 - n_2 + n_3 - 2n_4)F_9 + 7n_{1234}F_4$$
$$- 2n_{1234}\}w_2 + \sqrt{10}\{(2n_1 - n_{23} + 5n_4)F_4F_9 + (2n_1 - n_{23} - 2n_4)F_9$$
$$+ 7n_{1234}F_4\}w_1 \quad]/[(9n_1 + 5n_{234})F_4F_9 - 4n_1F_9 - 5n_{1234}F_4]$$

(3.7.1.20)

$E_4 < E_6 \Longrightarrow$

$$w_4 < [2/\sqrt{15}(4F_4F_6 - F_6 + 2F_4)w_3 + 1/\sqrt{30}(F_4F_6 - F_6 + 5F_4) \quad w_2$$
$$-1/\sqrt{10}(F_4F_6 + F_6 - 7F_4) \quad w_1]/F_4(F_6 - 1) \qquad (3.7.1.21)$$

$E_6 < E_1 \Longrightarrow$

$$w_4 > [2/\sqrt{15}(3F_6F_1 + F_1 + 1)w_3 + 1/\sqrt{30}(4F_1 + 1)w_2 + 1/\sqrt{10}(4F_1 + 1)w_1]/F_1(F_6 - 1)$$

(3.7.1.22)

$E_1 < E_3 \Longrightarrow$

$$w_4 < [-2/\sqrt{15}(3F_3 + F_1 + 1)w_3 - 1/\sqrt{30}(4F_1 + 1)w_2 + 1/\sqrt{10}(4F_1 + 1)w_1]/(F_1 - F_3)$$

(3.7.1.23)

$E_3 < E_8 \Longrightarrow$

$$w_4 < [-\sqrt{5}/\sqrt{3}\{(6n_2 - 2n_4)F_3F_8 + (2n_1 + n_2 - n_4)F_8 + 2n_{124}F_3\}w_3$$
$$+ \sqrt{5/6}\{4n_4F_3F_8 + (4n_1 - n_2)F_8 - 4n_{1234}F_3\}w_2$$
$$- \sqrt{10}\{4n_4F_3F_8 + (4n_1 - n_2)F_8 + 2n_{124}F_3$$
$$+ 2n_{124}\}w_1 \quad]/[(4n_1 - n_{24} - 4)F_3F_8 - (4n_1 - n_{23})F_8 + (n_{124} + 4)F_3 - n_{124} + 1]$$

(3.7.1.24)

Eliminating w_4, w_3, w_2 and w_1 from the inequalities (3.7.1.19) to (3.7.1.24), the necessary and sufficient condition for admissibility of the test procedure I [(3.6.1) to (3.6.8)] is obtained and given as follows :

$$\beta_{10}\,[\beta_1(\beta_2\beta_3+\beta_4\,\beta_5)+\beta_6(\beta_7\beta_8-\beta_9\beta_3)\,] <$$
$$(\beta_{11}\beta_{12}+\beta_6\beta_{13})[\beta_8(\beta_{14}+\beta_{17})+\beta_3(2\beta_{15}+\beta_{16})] \qquad (3.7.1.25)$$

Where

$\beta_1 = 4F_1^2\,(F_6+2)+F_3(4F_1+1)+F_1(F_6-2)$,

$\beta_2 = 4(n_1-2n_{34}+4n_2)\,F_4F_9+2(4n_1+n_2+3n_{34})\,F_9+2n_{1234}F_4$,

$\beta_3 = (5n_{234}+2n_1)\,F_2F_9+3n_1F_9-5n_{1234}F_2$,

$\beta_4 = 4n_1F_4F_9+(5n_{234}+n_1)\,F_9-5n_{1234}$,

$\beta_5 = -2n_1F_2F_9+(n_1-2n_2)\,F_9$,

$\beta_6 = (F_1F_6-F_3)(4F_1+1)$,

$\beta_7 = 8n_4F_2F_9+2n_4F_9+8n_{1234}F_2+2n_{1234}$,

$\beta_8 = (5n_{234}+9n_1)\,F_4F_9-5n_{1234}F_4-4n_1F_9$,

$\beta_9 = 2(4n_1-n_{23}+5n_4)\,F_4F_9+2(2n_1-n_{23}-2n_4)\,F_9+28n_{1234}F_4$,

$\beta_{10} = (12n_4-18n_2-12)\,F_1F_3F_6F_8+(16n_1+10n_2-20n_4-20)\,F_1F_3F_8$
$-(15n_{124}-12)\,F_1F_3F_6-(18n_1+3n_{24}+3)\,F_1F_6F_8$
$+(19n_{124}+4n_3+20)\,F_1F_3+(50n_1-3n_{24}+3)\,F_1F_8$
$+(4n_1-2n_{24}-8)\,F_3F_8+(3n_{124}+3)\,F_1F_6+(5n_{124}+4n_3+5)\,F_1$
$+(n_{1234}+8)\,F_3+(8n_1+2n_{24}-2)\,F_8-n_3+2$,

$$\beta_{11} = (4F_1^2\ (F_6+2)+F_1(F_6-1)+(4F_1+1)\ ,$$

$$\beta_{12} = [(12n_1-18n_2+2n_4-12)F_1F_3F_6F_8-(22n_1+2n_2-3n_4+3)F_1F_6F_8$$
$$-(2n_{124}-12)F_1F_3F_6-(14n_1-6n_2-n_4+1)F_1F_8-5n_{124}F_1F_3$$
$$+(4n_1-n_{24}-4)F_1F_3F_8-(3n_1-n_{24}+3)F_1F_6+(n_{124}+4)F_3$$
$$-(4n_1-n_{24}+1)F_3F_8-(n_{124}-1)F_1+(n_{124}+4)F_3$$
$$-(4n_1-n_{24}+1)F_8-n_{124}+1\ ],$$

$$\beta_{13} = [40n_4F_1F_3F_6F_8+40n_{124}F_1F_3F_6+(40n_1-10n_2)F_1F_6F_8$$
$$+(16n_1-4n_2-44n_4-16)F_1F_3F_8-(56n_1-14n_2-4n_4+4)F_1F_8$$
$$-(36n_{124}-16)F_1F_3+(4n_1-n_{24})F_3F_8+20n_{124}F_1F_6$$
$$-(24n_{124}+4)F_1+(n_{124}+4)F_3-(4n_1-n_{24}+1)F_8-n_{124}+1],$$

$$\beta_{14} = 8(2n_3-n_4)F_2F_9+2(4n_1-2n_2-3n_4)F_9+8n_{1234}F_2+6n_{1234},$$

$$\beta_{15} = 2(2n_1-n_2+11n_3-7n_4)F_4F_9-2(2n_1-2n_2+n_3-n_4)F_9+14n_{1234}\ ,$$

$$\beta_{16} = 4(n_1-2n_3+4n_2)F_4F_9+2(4n_1+n_2+3n_{34})F_9+2n_{1234}F_4,$$

$$\beta_{17} = (2n_{34}-6n_2)F_2F_9+(n_1-2n_2)F_9-2n_{1234}F_2.$$

3.7.2 Admissibility of Test Procedure II :

The tests under test procedure II after substitution of the values of V_i's from (3.7.3) would be

$$(V_5+V_1)/(V_4+V_2) > F_4' \Longrightarrow$$

$$w_5<[-4(F_4'-1)w_4+\sqrt{5}/\sqrt{3}(2F_4'+1)w_3-\sqrt{10}/\sqrt{3}(F_4'-1)w_2 +\sqrt{10}(F_4'+1)w_1] \;/\; (2F_4'+3) \qquad (3.7.2.1)$$

$$(V_5+V_{13})/(V_4+V_2) > F_5' \Longrightarrow$$

$$w_5<[-4/\sqrt{5}n_{13}(F_5'-1)w_4+\sqrt{5}/\sqrt{3}(2n_{13}F_5' +2n_3+n_1)w_3 -\sqrt{10}/\sqrt{3}(n_{13}F_5'+n_3-n_1)w_2+\sqrt{10}n_{13}(F_5'+1)w_1]/[2n_{13}F_5'-2n_3+3n_1]$$

(3.7.2.2)

Let the right hand sides expression of the inequalities (3.7.2.1) and (3.7.2.2) be denoted by E_4' and E_5' respectively. The acceptance region of $\varphi(w)$ will be the union of the following eight sets :

$$w_5 \;:\; w_5 \geq \max\,(E_1,\ E_2,\ E_3,\ E_4') \qquad (3.7.2.3)$$

$$w_5 \;:\; w_5<E_2 \cap w_5 \geq \max\,(E_1,\ E_3,\ E_5') \qquad (3.7.2.4)$$

The remaining six sets will be same as (3.7.1.12) to (3.7.1.17).

One of the order arrangements of E_1, E_2, E_3, E_4', E_5', E_6, E_7, E_8, E_9 for which the union of the above eight sets will be convex sets, is given by

$$E_2<E_9<E_4'<E_6<E_1<E_3<E_5'<E_8. \tag{3.7.2.5}$$

It may be observed that the union of the eight sets [(3.7.2.3), (3.2.2.4) and (3.7.1.12) to (3.7.1.17)] under the condition (3.7.2.5) is the set E_8 as depicted by shaded portion in Fig. 2, which is convex set. Hence the test procedure II[(3.6.3) to (3.6.10)] is admissible.

<u>Necessary and Sufficient Condition for Admissibility of the Test Procedure II</u> :

Using (3.7.2.1), (3.7.1.6) and (3.7.1.9) the inequalities $E_9<E_4'$ and $E_4' < E_6$[out of the inequalities $E_2<E_9$, $E_9<E_4'$, $E_4'<E_6$, $E_6<E_1$, $E_1<E_3$, $E_3<E_8$ obtained from the condition (3.7.2.5)]are given in terms of $w_i (i=1,2,3,4)$ as follows :

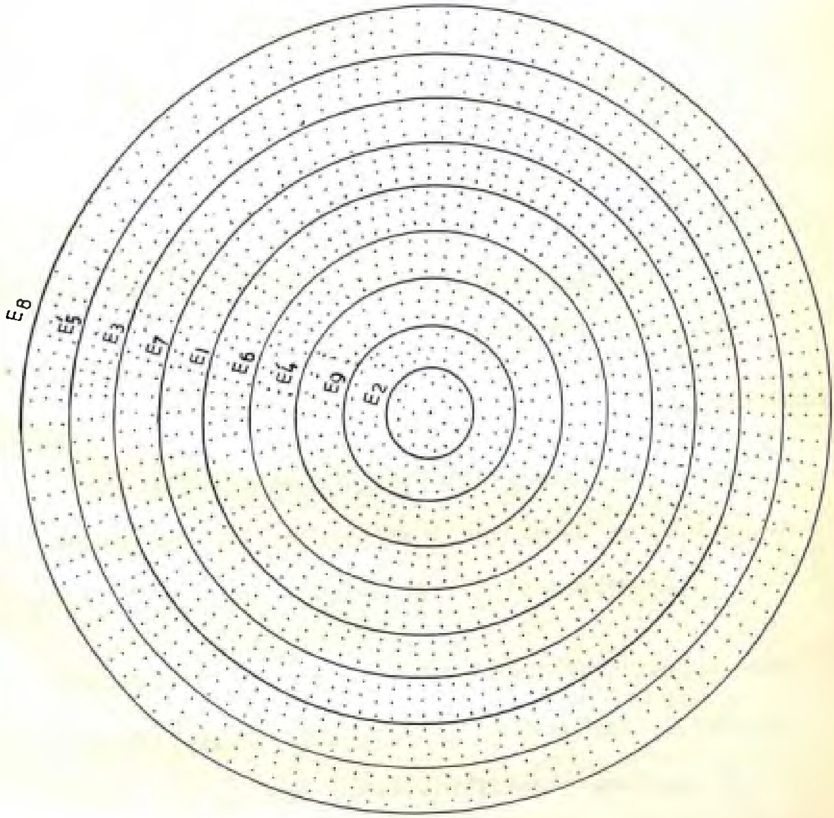

FIG. 2

$E_9 < E_4' \Longrightarrow$

$$w_4 < [\sqrt{5}\sqrt{3}\{2(n_{12}-n_{34})F_4'F_9+(n_1+4n_2-n_{34})F_9+2n_{1234}F_4'+2n_{1234}\}w_3$$
$$+\sqrt{5}/\sqrt{6}\{-(2n_1-n_2-5n_3+n_4)F_4'F_9+(2n_1-n_2+5n_3-4n_4)F_9+n_{1234}F_4'$$
$$+2n_{1234}\}w_2+\sqrt{5}/\sqrt{2}\{(2n_1-n_{23}-3n_4)F_4'F_9+(2n_1-n_{23}-4n_4)F_9$$
$$-2n_{1234}F_4''-4n_{1234}\}w_1] \;/\; [6n_1F_4'F_9+(5n_{234}-n_1)F_9-5n_{1234}]$$

(3.7.2.6)

$E_4' < E_6 \Longrightarrow$

$$w_4 < [2/\sqrt{15}\ (F_4'F_6+2F_6+F_4'+1)w_3+1/\sqrt{30}(F_4'F_6-F_6+F_4'+4)w_2$$
$$+\sqrt{10}(F_4'F_6-F_6+F_4'+4)w_1\ ]/(F_6-1)$$

(3.7.2.7)

Inequalities $E_2 < E_9$, $E_6 < E_1$; $E_1 < E_3$ and $E_3 < E_8$ have been obtained for test procedure I and given in (3.7.1.19) (3.7.1.22), (3.7.1.23) and (3.7.1.24) respectively.

Eliminating w_4, w_3, w_2 and w_1 from the inequalities (3.7.1.19), (3.7.1.22), (3.7.1.23), (3.7.1.24), (3.7.2.6) and (3.7.2.7) the necessary and sufficient condition for admissibility of the test procedure II[(3.6.1) to (3.6.8)] is obtained and given as follows :

$$\beta_{10} [\beta_1 (\delta_1\beta_3 + \delta_2\beta_{13}) + \beta_6 (\delta_3\beta_3 - \delta_2\beta_7)] <$$

$$(\beta_{11} \beta_{12} + \beta_6 \beta_{13}) [\delta_2(\beta_{14}+\beta_{17}) + \beta_3(\delta_1-\delta_4)] \qquad (3.7.2.8)$$

Where

$$\delta_1 = 2(n_{12}-n_{34}) F_4' F_9+(n_1+4n_2-2n_{34}) F_9+2n_{1234}F_4'+2n_{1234} ,$$

$$\delta_2 = 6n_1 F_4' F_9 + (5n_{234}-n_1) F_9-5n_{1234} ,$$

$$\delta_3 = (2n_1-n_{23}-3n_4) F_4' F_9+(4n_1-2n_{23}-8n_4) F_9-2n_{1234}F_4'-2n_{1234},$$

$$\delta_4 = (4n_1-2n_2-10n_3+2n_4) F_4' F_9+(4n_1-2n_2+10n_3-8n_4) F_9 +n_{1234}F_4'+8n_{1234}.$$

Out of the two admissible test procedures, the one is selected which has the largest power for the given value of the size.

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3.7.3. <u>Illustration</u> :

Consider an example given by Panse and Sukhatme (1978). In the example, the experiment was conducted in a strip-plot design with 3 dates of transplanting, 3 types of green manuring and 6 replications. The abridged analysis of variance is given in Table 3.

Table 3 : Abridged Analysis of variance of strip-plot design.

Source of variation	d.f.	S.S.	M.S.
Replications	5	-	-
Dates of transplanting	$2 = n_5$	-	$7985.58 = V_5$
Error (a)	$10 = n_4$	-	$61.33 = V_4$
Manuring	2	-	-
Error (b)	$10 = n_3$	-	$41.22 = V_3$
Dates x Manuring	$4 = n_2$	-	$3.82 = V_2$
Error (c)	$20 = n_1$	-	$9.21 = V_1$

Using formulae given in 3.3, the degrees of freedom ν_1, ν_1' and ν_2' are obtained by substituting the values of $n_i V_i$ (i=1,2,4,5) shown in Table 3 and given as follows :

$\nu_1 = 8$, $\nu_1' = 2$, $\nu_2' = 11$,

For $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_4' = \alpha_6 = \alpha_8 = \alpha_9 = 0.05$

we get

$F_1 = 2.87$, $F_2 = 2.35$, $F_3 = 2.35$, $F_4 = 4.46$, $F_4' = 3.89$,

$F_6 = 6.94$, $F_8 = 3.32$, $F_9 = 3.21$.

Substituting the values of n_1, n_2, n_3, n_4, n_5, F_1, F_2, F_3, F_4, F_6, F_8, F_9 in the inequality (3.7.1.25), the left hand side and right hand side of the inequality are obtained as 1.06 and 42.42 respectively, which establishes the admissibility of the test procedure I.

Substituting the values of n_1, n_2, n_3, n_4, n_5, F_1, F_2, F_3, F_4', F_6, F_8, F_9 in the inequality (3.7.2.8), the left hand side and right hand side of the inequality are obtained as -21.61 and 1.01 respectively, which establishes the admissibility of the test procedure II.

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CHAPTER IV

SUMMARY

SUMMARY

In the agricultural field experiments, the two factor strip-plot design is used if both factors require large plot size and the effects of both the factors are of relatively little interest compare to the interaction between the factors.

A two factor strip-plot design is considered in the present study. The effects of the factors are assumed to be random. The analysis of variance of the strip-plot design, a-part from the two main effects of the factors, consist error (a), error (b), error (c) and the interaction effect of the factors. Error (c) is the experimental error which may be called as true error, error (a) which may be called as doubtful error (1), is influenced by experimental error component of variance and component of variance due to interaction of replication and one of the factors, error (b) which may be called as doubtful error (2), is influenced by experimental error component of variance and component of variance due to interaction of replication and other factor, interaction of one factor with other which may be called as doubtful error (3), is influenced by experimental error component of variance and component of variance of the interaction between the two factors.

When the three components of variances i.e. interaction of replication with each of the two factors and interaction of one factor with other are present, it may be observed from the random model analysis of variance of strip plot design that there is no estimator of error variance which claims to be an appropriate denominator for exact/approximate F-test in testing a hypothesis about a main effect. In such a case, two Satterthwaite approximate F-statistics discussed by Anderson and Bancroft (1952) may be used to test the hypothesis about the effect of a factor.

In case of uncertainty about the existence of interaction components of variances, three preliminary tests of significance are carried out to know the presence of one or more interaction components of variances. Using the two Satterthwaite approximate F-statistics, two test procedures based on three preliminary test of significance are developed. Each test procedure consists eight mutually exclusive alternatives and occurrence of one of the alternatives would reject the null hypothesis about the main effect.

Following Cohen (1974), the admissibility of both the test procedures are proved. Necessary and sufficient conditions for admissibility of the test procedures are also derived.

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V I T A

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