MARKOV MODELS FOR RELIABILITY AND COST ANALYSIS OF REPAIRABLE SYSTEMS INCORPORATING MAINTENANCE AND INSPECTIONS

Thesis

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By

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Himani Pant

Pantnagar **(*Himani Pant*)**
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CERTIFICATE-I

This is to certify that the thesis entitled **"MARKOV MODELS FOR RELIABILITY AND COST ANALYSIS OF REPAIRABLE SYSTEMS INCORPORATING MAINTENANCE AND INSPECTIONS"**, submitted in partial fulfillment of the requirements for the degree of **Doctor of Philosophy** with major in **Mathematics** and minor in **Computer Science,** of the College of Post Graduate Studies, G. B. Pant University of Agriculture and Technology, Pantnagar, is a record of bonafide research carried out by **Ms. Himani Pant,** Id. No. **54239,** under my supervision and no part of the thesis has been submitted for any other degree or diploma.

The assistance and help received during the course of this investigation have been duly acknowledged.

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CERTIFICATE-II

We, the undersigned, members of the Advisory Committee of **Ms. Himani Pant,** Id. No. **54239**, a candidate for the degree of **Doctor of Philosophy in Mathematics** with major in **Mathematics** and minor in **Computer Science** agree that the thesis entitled **"MARKOV MODELS FOR RELIABILITY AND COST ANALYSIS OF REPAIRABLE SYSTEMS INCORPORATING MAINTENANCE AND INSPECTIONS"** may be submitted in partial fulfillment of the requirements for the degree.

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Chapter 1 INTRODUCTION

Reliability, as a human attribute, has long been applauded. The term reliability dates back to 1816. Prior to World War II the term meant repeatability, to be more precise, an experiment was treated reliable if the same result was obtained repeatedly. In the 1920s, the development of reliability engineering was in parallel with quality. In 1940s, the US military redefined reliability and it meant that a product would work when intended for fixed time. Now a days, industries are upsetting to offer more robotisation in their industrial mechanism so as to meet the accelerating needs of the society and accordingly the complexity of industrial products and industrial system are expanding day by day. Due to the increased competition, complexity and development of technology, and increasing sophistication in manufacturing processes, the question of reliability has become one of concerns.

For major engineering systems, design and reliability management is a challenging issue. If reliability is out of control, many complicated issues like manpower or maintainer's shortages, availability of spare part, lack of repair facilities, and others may arise. Hence, reliability must be improved considering the availability and the overall cost owing to maintenance hours, cost of spare parts, transport cost, storage cost, etc. Effective reliability engineering needs knowledge, experience and vast engineering expertise. Reliability engineers must comply with the requirements for variety of reliability tasks and documentation at various stages like system development, production, testing, and operation. The aim of reliability engineering is to perform an assessment on reliability and determine the areas of improvement. This includes improving not only the design of the equipment, but also the manner in which it is used and maintained.

1.1 Reliability engineering

Reliability engineering is a division of engineering that aims at annihilating failure cost owing to system downtime, maintenance needs, repair equipments and the cost of spares, among other things. It is critical to note that reliability engineering is a broad field to gain information while it is hard to combine its goals. However, some of the most important reliability engineering objectives are given below in order of preference:

- Using technical expertise and unique approaches to avoid or reduce failures.
- Detecting and repairing failure reasons that occur despite attempts to avoid them.
- To figure out how to deal with failures that does occur, if their causes aren't corrected.
- Applying the methodologies to assess the reliabilities of new layouts and analysing reliability data.

1.2 Reliability theory

Reliability engineering is dependent on reliability theory, which gives rise to all of the features of reliability engineering. Reliability engineers frequently have to handle systems with distinct configurations and determine their reliability characteristics. Reliability is the probability that a device will carry out its objective in an adequate manner for the period foreseen in the given operating conditions. Mathematically, if t is any time and T is the time of the system failure, then the reliability within the interval $(0, t)$ can be defined as:

$$R(t) = P\{T > t\}$$

Reliability can also be expressed in terms of cumulative distribution function $(F(t))$ as:

$$R(t) = 1 - F(t)$$

Reliability is basically partitioned into two parts:

- Component reliability
- System reliability

Component reliability refers to the reliability of a single unit, whereas system reliability refers to the overall reliability of all system components.

1.2.1 Objectives of reliability theory

Systems in the current times are turning more advanced and sophisticated day by day. The complexity of the systems is related to the huge number of subsystems/components required to construct them. The fundamental goal of reliability engineers is to design highly dependable systems under particular limitations such as system cost, environmental conditions, and so on. Reliability engineers can help in:

- The system's proper operation.
- Satisfactory performance for a predetermined time span.
- Minimizing the chances of system failure.
- Using numerous approaches to increase the system's reliability.
- Achieving great reliability at a minimal cost.

1.3 Fundamental concepts of reliability

Reliability is one large concept. It is employed whenever we anticipate that something will behave in a particular way. If users of a system are seldom confronted with a failure, the system is regarded as more reliable than a system that fails more frequently. The key concepts to the reliability discussion include:

1.3.1 Failure

Failure is the state or condition of not achieving a desirable or anticipated goal. A failure is reported to occur if the observable result of a program execution differs from the expected result.

1.3.2 Failure modes

The term failure modes refer to the various ways in which something might fail.

1.3.3 Fault

The suspected cause of the failure is referred to as fault. One fault may be the cause of several failures. Depending on whether the fault is going to turn into a failure; we have three kinds of faults:

- Never executed faults, so they prevent failures.
- Executed faults not converted to failure.
- Executed faults converted to failure.

1.3.4 Error

It may be defined as a wrong or missing human action which results in a system/component to fail or contain a fault. An example includes misinterpreting a user's requirements in a product specification.

1.3.5 Time

Time is an important concept in formulating reliability. If the interval between two consecutive failures is short, the system is considered less reliable. Two forms of time are:

(i) Execution time: It is the actual time spent in performing the desired functions of the system.

(ii) Calendar time: It is the time experienced in years, days, etc. It is very useful in order to correspond to the reliability of the system.

1.4 Reliability Metrics

Reliability parameters serve to quantify reliability. They assess the performance of the system quite well. The following reliability parameters are utilized to quantify a system's reliability:

1.4.1 Up-time

It is the entire amount of time required for a system to perform its task correctly under specified operating circumstances.

1.4.2 Downtime

It is the entire amount of time system did not perform its task correctly under specified operating circumstances.

1.4.3 Failure rate

The failure frequency of components is referred as hazard/failure rate and expressed by the Greek letter $\lambda(t)$. Mathematically, failure rate is the limit of the probability that a failure occurs per unit time interval Δt given that no failure has occurred before time t. It may also be defined in the two contexts listed below.

- **Failure rate in discrete environment**

  The failure rate $\lambda(t)$ is given by

$$\lambda(t) = \frac{R(t) - R(t + \Delta t)}{\Delta t. R(t)}$$

  where $R(t)$ is the system's reliability at time t.

- **Failure rate in continuous environment**

$$\lambda(t) = \frac{f(t)}{R(t)}$$

  where, $f(t)$ is the failure density function and $R(t)$ is the system's reliability.

1.4.4 Mean Time To Failure (MTTF)

The MTTF is the average time it is expected that a component will be up and running. It is the mean time between two successive failures. An MTTF of 200 implies that out of every 200-time units one failure can be expected.

1.4.5 Mean Time To Repair (MTTR)

It is a factor representing the mean corrective maintenance time required to restore a system to perform as desired. It includes replacement, restoration etc.

1.4.6 Mean Time Between Failures (MTBF)

It is the combination of MTTF and MTTR i.e., MTBF = MTTF + MTTR. Hence, MTBF of 600 hours implies that after the occurrence of first failure, the next failure is expected after 600 hours.

1.4.7 Probability of Failure on Demand (POFOD)

This is one of the very important measures, and it's used to analyze the likelihood of the system failing when a service request is made. It does not involve any measurements of time explicitly. A POFOD of 0.01 signifies that out of a hundred requests for services one of them might result in failure. It is anticipated to be as low as possible.

1.4.8 Maintainability

Maintainability is stated as the likelihood of completing a successful repair activity within a certain time frame. Otherwise stated, maintainability assesses how easily and quickly a system can be returned to operational state following a breakdown.

1.4.9 Availability

The availability is the probability of functioning of the system at any given instant. Availability can be classified as follows:

(i) Point availability

The system probability of being functional at any random time *t* is called point availability. Point availability $A(t)$ is defined as:

$$A(t) = P(X(t) = 1)$$

where $X(t)$ denotes the system status as:

$$X(t) = \begin{cases} 1, & \text{when system is operational} \\ 0, & \text{when system is down} \end{cases}$$

(ii) Limiting availability

The point availability function's limit as time *t* approaches infinite is called limiting availability. It is expressed as:

$$A = \lim_{t\to\infty} A(t)$$

(iii) Limiting average availability

The limiting average availability is systems critical performance index and is given by:

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t A(u)du$$

1.4.10 Sensitivity analysis

It tests the rate of change in a model outcome when a change is made to the model data. The model input is nothing but the parameters involved in the formulation of the model and the corresponding sensitivity analysis is known as parametric sensitivity analysis (PSA). It helps guide system optimization, assess reliability, and determine model parameters that could significantly impact modeling errors. If R and η are reliability and any parameter of the systems model respectively, then sensitivity corresponding to this parameter, denoted by S is given by:

$$S = \frac{\partial R}{\partial \eta}$$

1.5 Reliability Improvement

Reliability improvement is a process to enhance the availability and suppress the cost of the system. There are various ways to improve the system's reliability. Some of them are discussed under:

1.5.1 Maintenance

In repairable system, actions frequently used to restore or renew the system are called maintenance. Generally, it affects the system's reliability characteristics such as reliability, MTTF, availability etc. Maintenance can be classified into following two types:

(i) Preventive maintenance (PM)

Like by the name, it is clear that this maintenance prevents the system failure. It promotes the continuous system performance i.e., provides repair to the components before their failure. For the preventive maintenance one has to observe the system's past behavior, mechanism of components wear out and knowledge of vital components within the system. Therefore, with the application of preventive maintenance cost of the system or product is very much fluctuated.

(ii) Corrective maintenance (CM)

As from its name, it is clear that this maintenance corrects the system failure i.e., corrective maintenance is linked with replacing or repairing the failed system. This maintenance involves three steps:

- Deciding the component responsible for system failure.
- Replacing or repairing the failed component.
- Verifying the repair or replacement once it's done.

1.5.2 Inspection

An inspection means an organized or formal examination of a system. Inspection is used to search the hidden failure inside the system. Inspections can be divided on the basis of its frequency.

(i) Continuous monitoring

As the name suggests it means that the system is continuously/constantly monitored. In continuous monitoring, a continuous alarm system constantly monitors the system and triggers a warning whenever something goes wrong with it. It provides us the current condition of the system.

(ii) Periodic inspection

Periodic inspection means inspections are performed periodically.

There are two main types of periodic inspections:

- **Age-based inspection policy:** In age-based inspection policy, schedules inspections are carried out at fixed age intervals.
- **Calendar-based inspection policy:** In calendar-based inspection policy, schedules inspections at fixed calendar intervals are performed, say, for example, every Monday (once a week).

(iii)Non-periodic inspections

Non-periodic inspections means inspections are carried out non-periodically.

1.6 System

A system is a group of interactive or interdependent entities or components that make up a united whole. Generally systems are of two types: Binary state system (BSS) and Multi state system (MSS). A system with two possible states namely working and complete failure is known as BSS while a system with more than two states is called MSS. If component's reliability is known then system's reliability can be evaluated by applying the structure function associated with it. As a result, knowledge of the structure of the system becomes mandatory. Different types of system configurations are discussed below.

1.6.1 Series system

In regard to reliability, a system is referred a series system if all components operate to ensure system success and the single component failure is the cause of system failure.

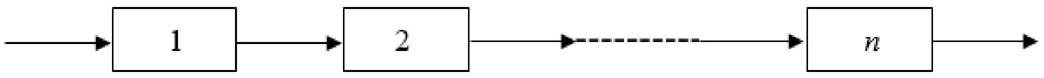


Figure 1.1: Series System

1.6.2 Parallel system

In regard to reliability, a system is referred a series system if only one component must operate to ensure system success and failure of all components is the cause of system failure.

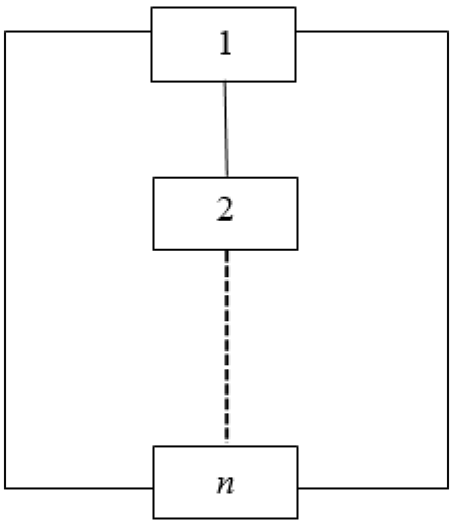


Figure 1.2: Parallel System

1.6.3 Series-parallel system

A series-parallel system comprises of serially-connected subsystems such that each subsystem contains units arranged in parallel. The system failure occurs once any of the subsystem fails.

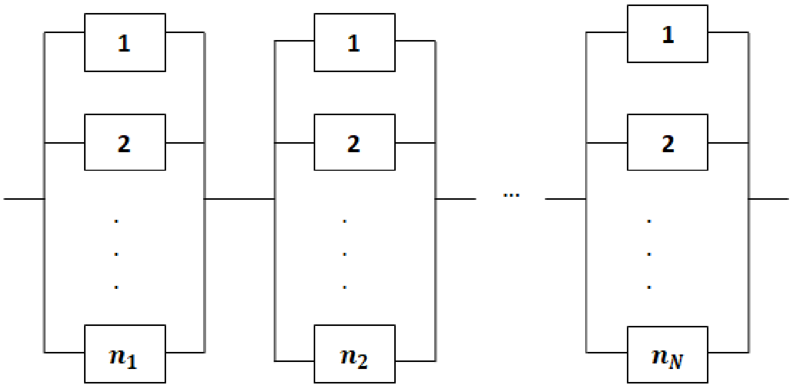


Figure 1.3: Series-parallel system

1.6.4 Parallel-series system

A parallel-series system comprises of parallel-connected subsystems such that each subsystem contains units arranged in series. The system failure occurs only if all the subsystem fails.

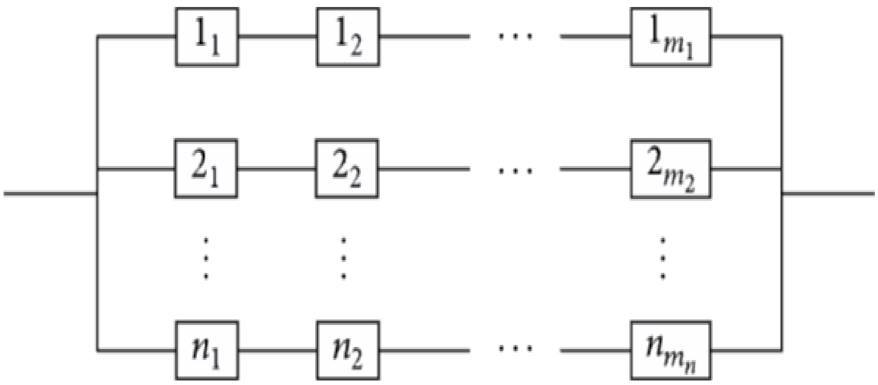


Figure 1.4: Parallel-series system

1.6.5 *k*-out-of-*n* system

This system is split into two kinds: k-out-of-n:G and k-out-of-n:F system. A k-out-of-n:G (F) system consists of n parallel components and works (fails) if and only if the total number of working (failed) components is at least k. This system is same as series (parallel) system if it is in the form *n*-out-of-*n*:G (1-out-of-*n*:G) or 1-out-of-*n*:F (*n*-out-of-*n*:F).

1.6.6 Linear (circular) consecutive *k*-out-of-*n* system

This system is an add-on to the *k*-out-of-*n* system. It is also partitioned into two types namely Linear (circular) consecutive *k*-out-of-*n*:G and Linear (circular) consecutive *k*-out-of-*n*:F system. Linear (circular) consecutive *k*-out-of-*n*:G (F) system comprises of *n* components ordered in linear (circular) manner and works (fails) if and only if overall number of successively working (failed) components is at least *k*.

1.6.7 Weighted *k*-out-of-*n* system

In some real-world systems, each component is important and this significance gives weight to the systems component. This concept added to the *k*-out-of-*n* system generates a new system named weighted *k*-out-of-*n* system. This system is also of two types: weighted *k*-out-of-*n*: G system and weighted *k*-out-of-*n*:F system. A weighted *k*-out-of-*n*:G (F) system contains of *n* components and works (fails) if and only if the overall weight of the working components is at least *k*.

1.7 Lifetime distributions for reliability modeling

Lifetime distributions are crucial statistical tools for reliability modeling. Some of the distributions which could be applied as lifetime distributions are given below.

1.7.1 Exponential distribution

It is a distribution that assumes a constant failure rate. If a random variable *T* is assumed to be exponentially distributed, then the PDF is as follows:

$$f\ (t) = \lambda e^{-\lambda t}, \text{for } t \geq 0$$

where λ is the distribution parameter and $\lambda > 0$.

The corresponding CDF and reliability function are obtained as

$$F(t) = P\{T \leq t\} = 1 - e^{-\lambda t}, R(t) = 1 - F(t) = e^{-\lambda t}.$$

1.7.2 Weibull distribution

It is perhaps the most widely used probabilistic model to analyze the failure time behavior of components, systems, or equipment in a reliability community. A random variable T is said to follow a Weibull distribution if it possesses the following PDF:

$$f\ (t)\ = \beta\theta(t\theta)^{(\beta-1)}e^{-(t\theta)^{\beta}}, \text{ for } t \geq 0$$

where θ and β are the scale and shape parameters, respectively.

In general, $\theta > 0$ and $0 < \beta < \infty$.

It's reliability function is expressed as $R(t) = e^{-(t\theta)^{\beta}}$.

1.7.3 Gamma Distribution

A continuous random variable is said to follow gamma distribution with parameters α and λ if its density function is given by

$$f(x) = \frac{\lambda^{\alpha}x^{\alpha-1}e^{-\lambda x}}{(\alpha-1)!}, \text{ for } x > 0$$

In general, $\lambda > 0, \alpha > 0$.

1.7.4 Log-normal distribution

It is frequently used to model products in which physical fatigue significantly results in primary failure. A random variable X is said to follow log-normal distribution with parameters μ and σ, then the PDF $f(x)$ is given by

$$f(u) = \frac{1}{u\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{lnu-\mu}{\sigma})^{2}}, \text{ for } \sigma, x > 0 \text{ and } \mu \in (-\infty, \infty).$$

1.7.5 Normal distribution

The PDF of normal distribution is given by:

$$f(u) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{u-\mu}{\sigma})^{2}}$$

where parameters μ and σ are the mean and standard deviation of the distribution respectively.

1.8 Stochastic process

A stochastic process is a mathematical concept that is often characterized as a set of random variables $\{S(t), t \in T\}$, indexed using a mathematical set. In a stochastic process, each random variable is uniquely connected with an element in the set. The set employed to index random variables (T) is named as index set. Each variable in the set gets its values from the state space, which is a mathematical space. As the stochastic process evolves from a random variable hence, it is influenced by them over time t.

Classification of stochastic process based on index set:

- **Discrete-time stochastic process:** If the index set have a finite or countable number of elements.
- **Continuous-time stochastic process:** If the index set is some interval of the real line.

Classification of stochastic process based on state space:

- **Discrete/integer-valued stochastic process:** If the state space is the integers or natural numbers.
- **Real-valued stochastic process:** If the state space is the real line.

A stochastic process can also be written $\{S(t, r), t \in T\}$as to reflect that it is actually a function of two variables, $t \in T$ and $r \in \Omega$, where Ω represents sample space i.e., the stochastic process varies on the outcome of a random trial over time.

1.9 Markov process

A Markov process is one in which all of the information required to make predictions about the result at some point is provided by the most recent observation. Its outcome and the period since then are all we need to give a probability to a new observation. Whatever is obtained before the recent observation has no bearing on the goal we aim to achieve next. Precisely, a Markov process is a stochastic process with a memoryless property.

1.10 Optimization

Optimization is the process of selecting the optimal element from a group of alternatives based on some criterion. All quantitative fields, from engineering and computer to economics and operations research, include optimization issues, and the discovery of solution techniques has been of concern in Mathematics for millennia.

In its most basic form, an optimization problem is methodically selecting input values from within an authorized set and computing the function's value to minimize or maximize a real function. More broadly, optimization entails determining the "best available" values of some objective function provided a specific domain, which can encompass a wide range of objective functions and domains.

As a result, there are three fundamental components to an optimization issue:

- **Variables:** These are elements of the optimization model that may be altered to generate new possibilities.
- **Objective functions:** The optimization's goal is defined by objective functions.
- **Constraints:** These are the variables and objective function's restrictions.

An optimization issue can be expressed as follows:

Given $f: A \to R$

Search $y \in A$ such that

$$f(\boldsymbol{y}) \le f(\boldsymbol{x})\ \forall \boldsymbol{x} \in A \text{ (minimization)}$$

or

$$f(\boldsymbol{y}) \ge f(\boldsymbol{x}) \forall \boldsymbol{x} \in A \text{ (maximization)}$$

Satisfying

$$g_i(x) \le 0, i = 1,2, \dots, n_1$$

$$h_i(x) = 0, i = 1,2, \dots, n_2$$

$$k_i(x) \ge 0, i = 1,2, \dots, n_3$$

1.11 Particle Swarm Optimization (PSO)

PSO is a computational approach in computer science that optimizes a problem by iteratively attempting to enhance a candidate solution in relation to a certain level of quality. It solves a problem by generating a population of possible solutions, nicknamed particles in this context, and moving these particles about in the search-space using a simple mathematical formula based on the particle's velocity and position. The movement of each particle is controlled by its local best known position, but it is also steered toward the best known positions in the search-space, which are updated when better places are discovered by other particles. This is supposed to direct the swarm's attention to the optimal solution.

Kennedy and Eberhart (1995) presented PSO. Sociobiologists think that a fish’s school or bird’s flock moving in a group "may benefit from the other group member’s experience". In other words, if a bird is flying about randomly looking for food, all of the birds in the flock may share their discoveries and assist the entire flock to have the greatest hunt.

A study of flock behavior is provided to illustrate how the PSO inspired the design of an optimization method to handle complicated mathematical problems. A bird’s flock flying over a location must locate a site to land, and determining where the entire swarm should land is a complicated task since it depends on numerous factors, including maximizing food supply and reducing the danger of predators' existence. In this perspective, the birds move synchronously for a time period until the optimal spot to land is determined and the entire flock lands at once. The described movement occurs only when all the members are able to communicate among themselves; otherwise, they are most likely to land at different spots and at different times.

The mentioned problem of determining the optimum landing spot defines an optimization problem. In order to maximize their survival circumstances, the flock must select the optimal place, such as longitude and latitude. To accomplish this, each bird flies about seeking for and assessing different spots while employing numerous survival criteria simultaneously. All of them have the benefit of knowing where the optimal placement point is until it is discovered by one of them. As a result, each member updates its individual and social knowledge.

In populations of species each individual seeks to attain the optimal solution within a multidimensional search space. Therefore, each particle designates a candidate solution. Individual particles by the virtue of their experience have the ability to modify their positions against their best positions. On constantly adjusting their directions, we can expect all particles to progressively arrive to their best positions. PSO's advantage is that it is easy to perceive, simple to operate, and fast to search. In typical PSO, the initial population is produced randomly. Meanwhile, the velocity and position factors describe the particle status in the space, as follows:

$$v_i^{a+1} = w * v_i^a + c_1 * r_1 * (pbest_i^a - x_i^a) + c_2 * r_2 * (gbest^a - x_i^a)$$

$$x_i^{a+1} = x_i^a + v_i^{a+1}$$

where, v_i^a / x_i^a is the velocity/position vector of the i^{th} particle at the a^{th} iteration.

v_i^{a+1}/ x_i^{a+1} denotes the velocity/position vector of the i^{th} particle at the $(a+1)^{th}$ iteration.

$pbest_i^a$ denotes the personal best of the i^{th} particle at the a^{th} iteration.

$gbest^a$ is the global best of the all the particles at the a^{th} iteration.

c_1 and c_2 are acceleration coefficients, which control movement of particles.

w is an inertia weight, which along with c_1 and c_2 controls the effect of prior velocities on the new one.

r_1 and r_2 are arbitary numbers between 0 and 1.

The velocity update equation's first term is a product of inertia weight (w) and the particle's prior velocity (v_i^a), which is why it indicates the particle's past motion into the current one. The second term of the equation also called the cognitive term is the difference between particle's individual best ($pbest_i^a$) and the current solution (x_i^a), drawing the particle to its best individual position. And, the third term also known as the social term is the difference between particle's best point ($gbest^a$) and the current solution (x_i^a), attracting particle to its global best spot. Figure 1.5 illustrates the update in particles position and velocity.

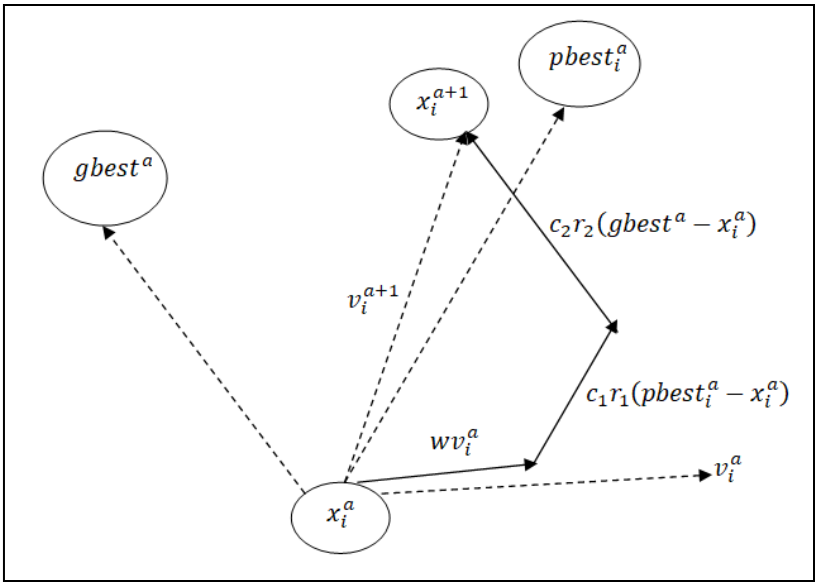


Figure 1.5: Particles position and velocity updates in PSO

1.12 Overview of proposed work

The basic needs of modern technology urge the engineers to design a methodological approach to investigate the problem of reliability. Engineers deal with the issue of designing and developing complex systems with greater reliability and profit. Reliability theory helps to compute several reliability characteristics like reliability, availability (point, limiting and limiting-average availability), MTTF, sensitivity analysis, expected cost rate of the various complex systems.

Reliability and availability are one of the major concerns for most of the complex systems. Different types of failure that system experiences during their life cycle cannot be ruled out. Also some systems need to be inspected so as to decrease their chances of failure and subsequently increasing the reliability, availability and profit of the system. So, keeping aforementioned facts in view the objectives of present research work are listed below:

- To develop mathematical models for multi-component systems.
- To derive methodical prepositions on point availability, limiting availability and maintenance cost rate for multi-component systems.
- To evaluate reliability indices of the proposed systems incorporating different types of failure and inspection policies.

Review of Literature

Chapter 2 REVIEW OF LITERATURE

The reliability analysis have significant links with function examination, requirements specification, designing of system, manufacturing, testing, maintenance, technical documentation and many more. Considering the importance of the subject, a large number of researchers have focused on system reliability. Reliability and availability models provides a mathematical means of predicting and estimating the relationships based on the failure rates between different components of the system. Various parameters such as MTTF, MTTR etc. can aid to provide input for such models. The most fundamental causes of failures can be identified and estimated with the help of engineering tools.

Some of the research works linked to the current study are briefly outlined and reviewed in this chapter. Based on the requirement, the chapter is further divided into two sections:

Section 2.1: Review of literature related to reliability assessment of systems

Section 2.2: Review of literature highlighting particle swarm optimization (PSO)

2.1 Review of literature related to reliability assessment of systems

Dhillon (1978) presented a repairable *k*-out-of-*n* three-state model with common-cause failures and developed the Laplace transforms of the state probability equations.

Kenyon and Newell (1983) presented a computer program and solution for limiting availability of a k-out-of-n:G system with single repair.

McGrady (1985) described a method to calculate the availability of a *k*-out-of-*n*:G network analytically, where each unit may have distinct availability. Author also gave an algorithm and a FORTRAN subroutine to determine this kind of availability.

Gupta and Gupta (1986) considered an electronic system comprising of two serially-connected subsystems. One subsystem had two parallel-joined identical units while another had only one unit. The system had three states: good, degraded and failed; also, it had two types of failures: failure due to major human error and unit failure. The repair was supposed to occur only in case of unit failure following a general distribution. Time dependent probabilities were evaluated to forecast the operational availability and expected profit, making the system more applicable to real-world problems.

Moustafa (1994) utilized Markov models to get closed form answers for the series-parallel systems reliability. The system was composed of two identical serially-connected pieces of equipment. There was only one unit in each piece of equipment. An additional component was connected in parallel with the existing one to increase equipments reliability. As a result, each piece of equipment was regarded to be two-component fault-tolerant. The unit had a Poisson failure and a constant repair rate. If at least one piece of equipment broke, the system failed.

Barlow and Proschan (1996) presented several mathematical models useful in solving reliability problems. Certain life distributions and their use in determining maintenance policies were discussed, and topics such as the theory of increasing (decreasing) failure rate distributions, the theory of coherent systems, and optimum maintenance policies were also covered.

Moustafa (1996a) presented Markov models for determining the availability of *K*-out-of-*N* systems undergoing *M* modes of failure. Author calculated the closed form solutions of the steady-state probabilities and consequently the availability of the system.

Moustafa (1996b) provided Markov models for two failure mode *K*-out-of-*N*:G systems, to present its transient reliability analysis with and without repair. In case of repairable systems, the simultaneous set of linear differential equations solution was utilized to calculate the reliability. Whereas, for non-repairable systems, closed form solutions of the transient probabilities were employed to determine reliability.

Moustafa (1997) offered a Markov model for analyzing the reliability of *K*-out-of-*N*: G systems with poor coverage due to dependent failures. The reliability and

MTTF were calculated using closed form probabilistic solutions. To demonstrate the outcomes, a numerical example was presented.

Moustafa (1998) represented Markov models for analysis of transient reliability for M failure mode K-out-of-$N:G$ system with and without repair. The reliability and the MTBF were calculated by virtue of simultaneous set of linear differential equations solution in case of repair. Although, to obtain the reliability and the MTBF of non-repairable system, closed form solutions of the transient probabilities were utilized.

Levitin and Lisnianski (1999) expanded the joint replacement and redundancy schedule optimization issue to a multistate system with a variety of performance levels for the units and the system. The system elements were picked from a list of market-available items, and the quantity of such elements for each system unit was calculated. Each unit was distinguished by its capacity, cost, and reliability. The lifespan distribution of a system element with the hazard rate, which grows with time, defined its reliability. The number of element replacements and ideal system structure were defined as those that offer the required degree of system reliability while incurring the least amount of maintenance, unsupplied demand, and capital investment due to failures. The reliability of a multistate system was assessed using a universal generating function approach. As an optimization approach, a genetic algorithm was utilized. Examples of how to determine the best system structure and replacement timetable were provided.

Pham and Wang (2000) studied the strategic maintenance of a k-out-of-n:G system undergoing imperfect PM and partial failures. Two (τ, T) opportunistic maintenance models were proposed in regards of reliability requirements. In the two models, only minimal repairs were conducted on failed units ahead time τ and the CM of all failed units were combined with PM of all functional but degraded units after τ; if the system persist till time T with no perfect maintenance, it underwent PM at time T. System cost rate and availability were deduced considering maintenance time.

Sarkar and Sarkar (2000) considered a periodically inspected system under perfect repair and determined its availability. They considered two different models, viz. Model A and Model B. In Model A an unfailed system was treated as new on each inspection and the failed system was immediately perfectly repaired while in Model B nothing was done to an unfailed system but the failed one was repaired at the next inspection.

Cui and Xie (2001) investigated the systems point availability undergoing periodic inspection using random walk models. Authors considered two cases; First case: the system underwent maintenance at periodic inspection and modified to be as good as new. Second case: the system did not undergo maintenance at periodic inspection. Few assumptions were made like: the failures were detected through inspection only, failed system was perfectly repaired but time taken to repair could be constant/ random length.

Moustafa (2001) derived the availability and the steady-state probabilities of K-out-of-N:G systems subject to general repairs and constant failure rate. An imbedded Markov chain at repair completion epochs was used. Results were illustrated considering the special case of constant repair time, Erlang-2 and Coxian-2 repair time.

Frostig and Levikson (2002) derived formulas for the availability and the expected up-time and down-time of the R out of N repairable system using Markov renewal processes under the assumption that either the components repair times follow general distribution and the lifetimes are exponential or vice-versa. Numerical examples for various life time and repair time distributions were given.

Klutke and Yang (2002) studied systems with hidden failures deteriorating owing to shocks as well as graceful degradation. Authors assumed that shocks occur based on Poisson process and deterioration occurred at a constant rate. Periodic inspections were performed. Expression for limiting average availability was derived, which helped in understanding the effect of system life distribution on availability, and suggested more effective inspection strategies.

Levitin (2002) proposed a method for determining the best series–parallel configuration for systems with units of nominal performance rates and varying reliability. These systems were multi-state because their output performance varied based on the combination of units available at any one time. A universal moment generating function (UMGF) for quick extraction of multi-state system reliability and a genetic algorithm (GA) for optimization were used in the technique devised to address this challenge. Fundamental UMGF method operators were created for two types of systems depending on processing time and transmitting capacity, respectively. Basic GA methods and parameters as well as solution encoding for GA implementation were determined for the given issue.

Moustafa (2002) considered a Markov model to calculate the limiting availability of a system adopting several stages of degradation and capable of random failures at each phase of degradation. Partial repairs and minimal maintenance restored the system to operational state prior to failure and the previous degraded state, respectively. After degradation failure, overhaul repair returned the system to "as good as new". The mean time to minimal maintenance was determined with respect to minimal unavailability.

Bérenguer *et al.* (2003) considered a continuously monitored system subject to gradual and stochastic deterioration. For triggering a PM operation, an alarm threshold is set on the degradation level of the system. Authors developed a mathematical model to find the optimal value of the alarm threshold that helps to minimize the systems unavailability.

Ramirez-Marquez and Coit (2004) posed a redundancy allocation issue with the goal of reducing design cost of a system displaying multi-state reliability behavior, given system-level performance limitations. This problem was looked using capacitated binary units that can give various levels of multi-state system performance. The multi-state character of the system was due to the various demand levels that must be met during the system's operational time. The heuristic allowed for faster and clearer analysis of the problem. Three distinct issues were solved, demonstrating the heuristic's simplicity and convenience of implementation without jeopardizing the intended optimization goals.

Cui and Xie (2005) examined the availability of a system bearing periodic inspections. If the system is found to be failed, then the system will either be replaced or perfectly repaired. They discussed two models: Model A and Model B. Model A treated a system as a new one after completing the inspection or repair while in Model B if during inspection the system was found to be in working state then no maintenance action was done.

Levitin *et al.* (2006) presented a technique incorporating the Markov chain and universal generating function technique for analyzing a series-parallel safety-critical system with two system states viz. failure-safe and failure-dangerous. The overall system safety function and state distributions were computed, considering periodic inspection and repair (perfect and imperfect) of system units. The proposed method could be applied for analyzing state distributions and decision-making in complex systems.

Liao *et al.* (2006) considered a condition-based maintenance model for constantly deteriorating systems undergoing continuous monitoring. Authors investigated a maintenance policy, which helped in achieving the maximum availability level. Search algorithm was used to find the optimum threshold of maintenance.

Wang and Pham (2006) examined the optimal maintenance, maintenance cost and availability of the series system with n constituting units under the common presumption that each unit was liable to correlated failure and repair, shut-off rule, imperfect repair and arbitrary distributions of times to failure and repair. Mean time between system failures, mean time between system repairs, and downtime of the system was also evaluated by them. They also studied the properties of maintenance cost rates and system availability. Optimum maintenance policies were discussed through a numerical example.

Zhang *et al.* (2006) investigated the k-out-of-(m+n):G warm standby system, which had two types of units. One group of units in the system was type 1 and the other was type 2. There were m type 1 and n type 2 units in total. Type 1 units had a reduced hazard rate, and if one failed, it was preferable to fix it. There were r-repair shops in the area. The system state transition process was easily explained using a Markov model and solutions for system availability/reliability was also derived using the model. To demonstrate the solutions for system availability/ reliability, an example of a power generation/transmission system was presented.

Zheng *et al.* (2006) presented a new model for a one-unit Markov repairable system in which repair durations were sufficiently low that the system does not fail; i.e., the repair period wasn't recorded in the downtime log. Authors began by assuming that the crucial repair time is constant. The model was then modified to allow for a non-negative random variable for the crucial repair time. As a measure of reliability, system availability for these new models was calculated. To illustrate the findings, several numerical examples were provided.

Zhou *et al.* (2006) developed a dynamic maintenance policy for a continuously monitored series system incorporating imperfect maintenance. The optimality was determined by maximizing the cost for the continuously monitored system.

Nourelfath and Ait-Kadi (2007) found the least cost design of a multi-state series–parallel system under reliability constraints subject to a particular maintenance policy. The number of repairable units was more than the number of maintenance teams, and a maintenance policy set out the preferences amongst the system units. Dependencies as a result of maintenance teams were reflected by coupling the universal generating function with a Markov model.

Chelbi *et al.* (2008) proposed and modeled a preventive maintenance and inspection policy for randomly failing systems having both revealed and non-revealed failure. The system was undergoing inspection on reaching age T. A general expression of the system limiting availability was being stated and conditions of existence and uniqueness of optimality were developed. Authors focused on determining the inspection age that maximizes the systems limiting availability.

Juang *et al.* (2008) presented a genetic algorithm-based optimization approach to enhance design efficiency. The goal was to find the most cost-effective component MTBF and MTTR. Authors also created a knowledge-based interactive decision support system to help designers build up and save component parameters during the repairable series-parallel system's entire design process.

Xu and Hu (2008) determined the optimal limiting availability of a system having six states. Both PM and CM were considered. Availability was derived using the method of strong continuous semi-group theory. Furthermore, the optimal time to perform PM was analyzed.

Peng *et al.* (2009) proposed a model codetermining inspection and preventive replacement policies for microengines prone to wear degradation. Optimal limits for inspection and replacement interval were demonstrated based on optimizing the quality and reliability of microengines. The proposed model was suitable for wider array of devices experiencing wear degradation.

Wang and Watada (2009) looked at improving the reliability of a series-parallel system with fuzzy random lifetimes. To maximize system reliability, a fuzzy random reliability model was created. To solve the problem, a fuzzy random simulation technique was developed first to calculate system reliability, and a theorem was established to assure the fuzzy random simulation's convergence. A hybrid binary PSO approach was also suggested, which incorporated the fuzzy random simulation. A numerical example of the suggested hybrid method was also presented.

Cui *et al.* (2010) built a periodically inspected maintenance model taking into account the real situation for storage products. Authors presented the limiting average and the point availability for the storage products using the virtual age concept and several lemmas. Finally, an example was presented.

Li *et al.* (2010) constructed the optimum model of a multi-state series–parallel system exposed to common cause failures in order to offer a desired level of reliability at a low cost. The universal generating function was used to assess the reliability of the system with components of various sorts mixed together, and a genetic algorithm was used to find the best model. To exemplify the suggested technique, a numerical example was provided. The findings demonstrate that the redundancy allocation approach differs due to common cause failures. Mixing components of various sorts to get the necessary degree of reliability at a low cost was a highly successful approach.

Ram and Singh (2010) discussed the availability, MTTF, and expected profit of a system comprised of serially-connected subsystems A (1-out-of-2: F) and B (1-out-of-*n*: F) with partial and catastrophic failures. The repair and failure time followed general and exponential distributions respectively. Analysis was being made under "preemptive-repeat repair discipline" prioritizing subsystem A.

Tian and Liao (2011) investigated multi-unit systems having dependency among units subject to condition monitoring. Authors proposed a policy based on proportional hazards model (PHM). An algorithm for the exact cost evaluation was developed. Real-world examples were also provided.

Cheng and Li (2012) studied a degrading simple repairable system experiencing inspections. Authors assumed that the system failures are discovered by inspections only, repair not being as good as new, and the consecutive working (repair) times formed a decreasing (increasing) geometric process. They presented a bivariate mixed policy (T, N), respectively, predicated on the time between two consecutive inspections and the failure-number of the system. Authors determined an optimal policy $(T, N)^*$ such that the the average cost rate is minimized. Expression of the average cost rate, and the corresponding optimal mixed policy was derived. Finally, numerical example was provided to pursue some discussions and sensitivity analysis.

Golmakani and Moakedi (2012) found the optimal periodic inspection interval for a two-component repairable system over a finite time horizon with failure interaction. First component has soft failure and the second has hard failure. First component's failure does not affect the second component; meanwhile, second component's failure increased the hazard rate of first component. First component increased the operating cost and was detected on inspection only. Repair/restoration of components was as good as new. Authors found the optimal inspection interval minimizing the expected total cost.

Hu *et al.* (2012) designed a repairable failure dependent series-parallel system. A dependency function for determining the failure probability of units in each subsystem, as well as a Markov model for determining the subsystem's state distribution was offered. Under the constraints of system availability, an optimum allocation issue was given with the goal of reducing the system cost, which included costs associated with the units and the repair teams. Because of its strong search capabilities and versatility in expressing discrete design variables, the genetic algorithm was utilized to discover the best allocation methods to solve the optimization issue. A numerical example was given to demonstrate how different dependencies lead to different allocation techniques.

Berrade *et al.* (2013) considered a system with three likely states viz. good, defective and failed. Faults were revealed on their occurrence; the defective state was only revealed by inspection and did not affect the systems task performing ability. Imperfect periodic inspections were incorporated for revealing the systems state. The model was illustrated employing paradigm of railways. The system lifetime was supposed heterogeneous. Thus, the time spent in the defective state was a random variable. The circumstance under which the cost of maintenance was closely linked to its effectiveness was determined.

El-Damcese and Shama (2013) investigated the reliability, MTTF and availability of a restoration system prone to degradation, following exponential distribution for failure and repair times. The system was repaired if it was in failed or degraded state. Several cases were analyzed to observe graphically the consequence of distinct system parameters on reliability characteristics. Sensitivity analysis for the system reliability was also investigated.

Liu *et al.* (2013) developed an optimum maintenance policy for constantly monitored deteriorating systems with multiple modes of failure. The degradation is demonstrated with the help of a stochastic process. A maintenance alarm signalizes on degradation reaching a threshold level. Multiple sudden failures are assumed to occur during degradation. This model was utilized to obtain the optimum maintenance threshold maximizing the limiting availability or minimizing the long-run cost.

Ram *et al.* (2013) investigated the reliability of a system comprising of a main unit and a standby unit incorporating waiting repair time. On the failure of main unit, the load is instantaneously transferred to the standby one by the means of a switchingover device. On its failure, the main unit had to wait for repair because of unavailability of repair facility. On failure of both the units, the system undergoes complete failure mode. The system was also considered to fail owing to systems incorrect start, as a result of an inexperienced/untrained operator. The repair of both the units followed general distribution, whereas repair in case of human error was obtained using Gumbel-Hougaard copula. The system was analyzed using Laplace transform and supplementary variable technique. Various measures like availability, MTTF and profit function were being evaluated and further justified using numerical examples.

Singh *et al.* (2013) dealt with analyzing the availability of a system, consisting of two subsystems, viz. subsystem-1 (*k*-out-of-*n*: G) and subsystem-2 (1-out-of-2: G) under the assumption that hazard rates are constant but repairs obey general and exponential distributions. Authors evaluated the state probabilities; availability, MTTF, reliability, asymptotic behavior, and the cost effectiveness of the system using supplementary variable technique, copula methodology and Laplace transformations. Numerical example and particular cases were used to describe the model.

Tang *et al.* (2013) evaluated the availability of a system prone to hidden failures, inspected periodically. Both the inspection and repair/replacement were considered to take non-negligible time. Two inspection policies were considered: Calendar-based and Age-based. Furthermore two assumptions were considered. Assumption A: Unfailed system being restored to as good as new on inspection. Assumption B: No intervention done on unfailed system. The point and limiting availability for all the cases were being derived.

Khatab *et al.* (2014) dealt with imperfect PM optimisation problem. A production system was considered, and was continuously monitored and assumed to be prone to stochastic degradation. The system underwent PM as its reliability reached a desired value, however CM was performed at system failure. On reaching a fixed number of maintenance, the system was replaced with a new one. Both CM and PM were considered imperfect, i.e. the system was restored between as bad as old state and as good as new state. A PM optimisation model on examining the optimal reliability threshold combined with optimal PM actions, maximizing the availability was proposed and an algorithm was provided.

Munjal and Singh (2014) dealt with the reliability estimation of a system consisting of two rectifiable parallel-connected subsystems L and M. Both L and M were 2-out-of-3: G systems having 3 type-A and 3 type-B units respectively in parallel. Also, a hot spare of type-A to subsystem L and of type-B to subsystem M was connected. Supplementary variable technique was used to mathematically establish the model and Gumbel-Hougaard copula for reliability and cost estimation. Various measures such as MTTF, availability, long-run probability and cost were analyzed. Some specific cases were considered to highlight different chances.

Li and Peng (2014) calculated the availability and the operation cost of multi-state series–parallel system incorporating Markov process to examine the changing dynamics of component state and system phase. Operation cost was calculated using Markov reward model, and availability using universal generating function. Genetic algorithm was employed to solve the optimization problem to minimize the overall cost with availability being more than a desired value. The proposed model was illustrated using maritime oil transportation system.

Aliyu *et al.* (2015) optimized the availability and the profit of a series-parallel system comprising of three subsystems A (linear consecutive k-out-of-n), B (single unit) and C (single unit) with A and B in cold standby. Authors maximized the limiting availability and profit. n = 2, 3, 4 and 5 was considered in order to solve the optimization problem. Definite expressions for limiting availability, repairmen busy period, and profit function were evaluated with the help of linear first order differential equations. The effects of system parameters on profit and availability were also analyzed graphically.

Negi and Singh (2015) studied a non-repairable complex system which consists of two serially connected subsystems namely P (weighted m-out-of-n: G) and Q (weighted u-out-of-v: G) having linear (u, f, g): G and circular (v, f, g): G components respectively. The reliability, mean time to failure and sensitivity of the considered system were evaluated with the help of universal generating function.

Ye *et al.* (2015) used a random-effects Wiener process with measurement errors to describe the degradation data to account for the heterogeneous degradation rate and non-negligible measurement errors. A filtering method that predicted the joint distribution of the deterioration rate and the present degradation levels in an iterative manner was developed. The distribution of the remaining usable life was projected in real-time based on the estimates. The approach was both storage and computationally efficient. Simulation and real-world data were used to illustrate its usefulness.

Zhao and Nakagawa (2015) optimized a random inspection policy as per random procedure times. They compared it with periodic scrutiny and computed checking cost to determine when to adopt such a random inspection. Three new inspection models named inspection first, last and overtime were also proposed, where inspections with deterministic rules were strategically timed, but their performance were regulated by operation process completion durations. The total expected downtime and inspection costs until failure detection were obtained, and optimal policies were derived minimizing costs. Furthermore, the inspection policies were compared with periodic inspection. Comparisons of first and last inspection models were done.

Babishin and Taghipour (2016) studied a k-out-of-n system with its units following a non-homogeneous Poisson process with power law intensity function for failure. The system was inspected periodically, and if the number of failed units was less than n-k+1, the failed units were detected and corrected only at the inspection. Meanwhile, the system failed if number of failed units were n-k+1, and instantly all the failed units were detected and fixed. A model to collectively obtain the optimal periodic inspection interval and maintenance actions resulting in minimal overall expected cost of the system was formulated.

Kumar and Singh (2016) discussed the reliability analysis of a system made up of two serially-connected repairable subsystems A (linear consecutive 2 out of 3:F) and B (1 out of n: F system). Two types of failure namely deliberate and critical failure were considered incorporating reboot delay. Sensitivity analysis, reliability, transition state probabilities, cost analysis, availability, and MTTF were obtained in addition to the long-run behavior of the system.

Li (2016) introduced a dormant *k*-out-of-*n* system incorporating periodic maintenance so as to reduce and prevent the undesired dormant failures, or costly repairs. Methodology on calculating the reliability parameter such as MTBF for the aforementioned redundant systems was introduced. The mathematical relationship between the effective MTBF and the periodic inspection/maintenance period was also elaborated. Case studies were presented so as to illustrate the application of developed model in the mass transit train reliability and safety design.

Zhang *et al.* (2016) proposed an optimal policy for three-state mechanical units prone to competing failures. In order to describe the operation states, a double-Wiener-process degradation model was set up. A PM policy comprising of degradation threshold, degradation control limit and age threshold were adopted. The effect of delayed detection of state-transition and degradation-level on PM policy was also modeled. Sensitivity analysis was carried out in the end based on numerical examples. The superiority of the proposed policy over two-state maintenance policy was also demonstrated.

Zheng *et al.* (2016) investigated a maintenance model possessing two different failures, namely, repairable and unrepairable. They proposed a maintenance policy under which if the successive operating time was reached, preventive repair was done and if the *n*th repairable failure or an unrepairable failure occurred, the system was replaced. An algorithm was given to attain the optimal policy of the proposed model and the optimal policy introduced was such that the rate of average cost was minimized.

Alaswad and Xiang (2017) provided an overview of the literature on condition-based maintenance (CBM), with a focus on optimization techniques and mathematical modeling. The study focused on key features of the CBM, such as degree of maintenance, inspection frequency, optimization criteria, solution technique, and so on. The research on CBM models was classified based on the fundamental degradation processes, such as continuous and discrete-state deterioration, as well as the proportional hazard model. The CBM models for multi-unit systems were also looked upon in the study.

Mehta *et al.* (2017) determined the reliability/availability of a casting system. By changing the repair rates and maintaining the failure rates constant, the reliability of the system was calculated using Supplementary Variable Technique. Chapman-Kolmogorov differential equations were generated from the transition diagram using the mnemonic rule, and then solved using Lagrange's technique. The system's transient state availability was calculated using the Runge-Kutta fourth order technique in MATLAB and the MTBF was estimated quantitatively.

Mendes and Ribeiro (2017) presented a model to establish the optimal inspection period for a two-unit cold standby system subject to periodic inspection. Authors defined possible states, transition probabilities and MTTF in terms of inspection period using a Markov chain. The limiting availability was also determined; also the cost function was developed and optimized. Besides optimizing, the effect of repair time on availability and MTTF was revealed.

Qiu *et al.* (2017) evaluated the availability and optimal maintenance policies of a system undergoing periodic inspections. System was considered having a working state and *M* modes of failure. Failure and repair times were considered to be random. Corrective repair was performed on each failure. Results on the limiting as well as point availability and cost rate were derived. At the end, optimal inspection period, maximizing the availability or minimizing the cost rate was obtained.

Sharma and Kumar (2017) dealt with the availability evaluation of *K*-out-of-*N* system based on standby and multiple working vacations. The system consisted of 'S' standby and 'O' operating machines, each machine being characterized by its own repair and failures. For increasing the availability in the case of failure of any operating machine, the standby support was provided, i.e., on failure of an operating unit, it was instantly replaced by a standby one. Moreover, the transient state equations were provided by means of state transition diagram. The problem was analyzed using Runge-Kutta method.

Qiu *et al.* (2018) developed maintenance and availability models for one-unit systems prone to dependent soft and hard failures. A soft failure reduced the systems performance ability while, hard failure stopped the system immediately. Dependence between the two failures was reflected based on the conviction that the failure rate of hard failure was increased directly by each soft failure. Recursive equations for the availability and reliability functions of the system were derived. For the failure detection, inspections were executed periodically. Henceforth, the optimal inspection policy was investigated via the minimal expected cost. The models applicability was validated using an electrical distribution system.

Yang *et al.* (2018) investigated a multi-level preventive maintenance approach for a three-state two-failure mode processes industrial system. The first was a continuous deterioration process that followed the general path model, while the second was a shock process that followed a non-homogeneous Poisson process. The system's degradation rate rose quickly once it entered the faulty condition, and the size of the damage produced by a shock load was proportional to two factors: degradation speed and operating age. A predetermined operating age was reached before the system was replaced, and a finite number of inspections were performed based on a two-stage interval partition. Their goal was to optimize the control limit, the replacement age, and two inspection intervals in order to reduce the projected cost. The use of the maintenance model was demonstrated by a study of a peristaltic pump.

Li *et al.* (2019) proposed an availability model for periodical inspection system. Limiting average and point availability of aforementioned model under arbitrary lifetime and repair-time distributions were achieved. Three examples were presented, considering systems lifetime (repair-time) distribution to be exponential (exponential), Weibull (normal) and Weibull (lognormal). The relationship between inspection period and availability were analyzed.

Park *et al.* (2019) developed an optimal policy for a k-out-of-n: G system undergoing both PM and CM. Authors investigated the optimal age of replacement for the case where detection of unit failure was not done until the system fails. The optimal policy to minimize the expected overall system cost was determined when a generalized block replacement model using downtime period was developed. The downtime of each failed unit was investigated using order statistics.

Qiu and Cui (2019a) investigated the repairable systems availability with repair time threshold. The system was considered working, if the repair period was less than a predefined threshold. Meanwhile, in the case of repair time being longer than the threshold, the system was considered working till the repair time exceeds the threshold. Both constant and random repair time thresholds were considered. The user-perceived availability was evaluated and demonstrated using example of a ventilator system.

Qiu and Cui (2019b) studied the point and limiting availability of a competing-risk system incorporating periodic inspections. System was considered having a working state and M modes of failure. Failure and repair times were considered to be random. Corrective repair was performed on each failure. Two models were formulated; Model 1: System was restored to as good as new condition on each inspection. Model 2: No maintenance done on unfailed system. Results on the point and limiting availability of both models were derived.

Qiu *et al.* (2019a) studied the optimal upkeep policy and availability for a two-component failure interactive system. Component 1 had soft failure and was detected by inspections only while component 2 had hard and self-announcing failure. Each hard failure acted as a shock to the first unit and resulted in its increased hazard rate. Component 1 failures were revealed using opportunistic and periodic inspections (given by component 2 failures) succeeded by replacement decisions. Henceforth, for the component 1 preventive age-based replacement were performed. A recursive method was developed to obtain availability measures and total maintenance cost of the component 1. The aim was to determine the optimal upkeep policy for the component 1 minimizing the total cost. The adopted approach was validated using an electrical distribution system.

Qiu *et al.* (2019b) examined Markov systems with a downtime threshold, and the availability and best maintenance policy were explored. A down time threshold was developed based on real usage. If the system's downtime was smaller than a certain threshold, the system was regarded to be operational throughout the downtime, and the downtime was ignored. Otherwise, if a down time exceeded the specified threshold, the system was deemed to remain operational from the start of the system failure until the down time exceeding the specified threshold, i.e. the down time was postponed. The system's immediate and limiting availabilities were calculated using the down time threshold. In addition, a maintenance model was developed to determine the best inspection interval, T*, for minimizing the cost rate. To show the applicability of the established technique, a numerical example for a ventilator system was provided.

Gahlot *et al.* (2020) dealt with a complex system consisting of subsystem 1 (2-out-of-3: G) and subsystem 2 (1-out-of-2: G) connected in series with a human operator. It was assumed that human failure damages the entire system; all failure followed exponential distribution, and repairs followed general and Gumbel–Hougaard family copula distribution. Supplementary variable technique was employed to examine the system, and discussing various reliability measures. The availability, MTTF, reliability, and profit benefit was computed for distinct values of repair and failure rates.

Hu *et al.* (2020) investigated the limiting availability of a repairable redundant dependence series-parallel system. The failure rate of the operational unit changes with the number of other failed units, whereas the repair rate of the failed unit remains constant in each parallel redundant subsystem. To assess the failure rate of the units in each subsystem, a modified failure dependence function is developed to quantify the redundant reliance. The steady-state probability vector of each subsystem and the limiting availability of the entire system were calculated using Markov theory and the matrix analysis technique. A numerical example was given to demonstrate the findings and to investigate the impact of duplicate dependence classes on system availability.

Nautiyal *et al.* (2020) evaluated the reliability, MTTF and sensitivity of a *k*-out-of-*n* network by first calculating the minimal cuts, and repairing of the failed nodes is done using Gumbel–Hougaard copula.

Ruiz-Castro (2020) modeled a multi-state *k-out-of-n: G* system in an algorithmic form. External shocks and internal failures with many repercussions were assumed. PM was introduced as a result of random inspections and unit was removed if a non-repairable internal and/or external failure occurred. Marked Markovian Arrival Processes was used to analyze the performance-profitable.

Tian and Wang (2020) developed a method for condition-based maintenance optimization and reliability assessments of a wind power system taking into account both wind and turbine uncertainty. Optimization was done for minimizing cost or maximizing availability. In order to investigate optimal number of joint repairs, optimization was also done for minor repair activities.

2.2 Review of literature highlighting particle swarm optimization (PSO)

Kennedy and Eberhart (1995) introduced the notion of employing PSO to optimize nonlinear functions. The evolution of many paradigms was described, as well as one of the paradigm’s application was elaborated. The paradigm's benchmark testing was detailed, as well as applications such as neural network training and nonlinear function optimization was proposed. The connections between PSO, artificial life, and genetic algorithms were explored.

Shi and Eberhart (1999) investigated the PSO's performance. As testing functions, four distinct benchmark functions with unequal beginning range settings were chosen. The experimental findings demonstrated the PSO's drawbacks and benefits. The PSO always converged fast towards the optimal locations in all of the testing scenarios. Nonetheless, the experimental findings demonstrated that the PSO is a promising optimization method, and a new strategy, such as utilizing an adjustable inertia weight, was recommended to increase PSO's performance towards the optima.

Parsopoulos and Vrahatis (2002) investigated the PSO techniques performance in coping with constrained optimization problems. A non-stationary multi-stage assignment penalty function was utilized in the proposed technique, and numerous experiments were carried out on well-known and frequently used benchmark issues. The findings were presented and compared to those produced using other evolutionary algorithms, including genetic algorithms and evolution strategies.

He *et al.* (2004) presented a PSO with passive congregation (PSOPC) for improving the performance of standard PSO (SPSO). Using passive congregation (vital biological force maintaining swarm integrity), the swarm's members can share information. PSOPC was compared against a global version of SPSO (GSPSO), PSO with a constriction factor (CPSO) and a local version of SPSO (LSPSO), using a set of 10 standard functions with 30 dimensions. The PSOPC considerably increased the search performance on the standard functions, according to the findings of the experiments.

Jarboui *et al.* (2007) proposed a novel clustering method based on the combinatorial PSO (CPSO) technique. Each particle was denoted as a string of length *n*, *n* representing number of data points. The i^{th} element of the string represented the group number given to item *i*. A possible solution to the clustering issue was represented by an integer vector. A swarm of particles was launched, which flied through the solution space in search of the best solution. Comparisons with genetic algorithm were used to test the effectiveness of the proposed CPSO method. The suggested CPSO method was extremely competitive and beaten the genetic algorithm, according to computational findings.

Yin *et al.* (2007) presented a hybrid PSO technique for determining the near-optimal task allocation in an acceptable amount of time. The hybrid PSO was robust to a variety of task interaction density, issue sizes, and network structure, according to the findings of the experiments. For the test-cases examined, the suggested technique was also more efficient and effective than a genetic algorithm. Both empirical and theoretical analysis were used to address the worst-case characteristics and hybrid PSO's convergence.

Hsieh *et al.* (2008) presented the efficient population utilization strategy for PSO (EPUS-PSO), a variant on the conventional PSO algorithm that uses a population manager to greatly increase PSO efficiency. This was accomplished by employing varied particles in swarms to improve the seeking capabilities and more effectively drive particles. Furthermore, sharing concepts were designed to prevent particles from slipping into the local minimum and to make it simpler for particles to find the global optimal solution. Experiments were done with and without coordinate rotation on multimodal and unimodal test functions like ackley, griewanks, quadric, rastrigin, and weierstrass. When compared to other contemporary PSO versions, the EPUS-PSO performed well in most benchmark issues.

Ai and Kachitvichyanukul (2009) presented two solution representations and their decoding methods to solve the capacitated vehicle routing problem (CVRP) using PSO. For CVRP with *m* vehicles and *n* customers, the first solution was depicted as a ($n + 2m$)-dimensional particle. This representation's decoding technique began with the translation of each particle into a customer’s priority list to enter route and a priority matrix of vehicles to service each client. The vehicle priority matrix and customer priority list was then used to build the vehicle routes. The second representation was a $3m$-dimensional particle. This representation's decoding technique began with the translation of particles into vehicle coverage radius and vehicle orientation points. Then, the vehicle routes were constructed relying on these radius and points. The suggested representations were tested using benchmark problems and GLNPSO, a PSO technique with various social learning components. The computational results demonstrate that the second representation was superior to the first one and moderate with other techniques for solving CVRP.

Jiang *et al.* (2010) proposed a novel master–slave swarms shuffling evolution technique relying on PSO (MSSE-PSO). The feasible space was first sampled randomly for a population of points, which was then partitioned into one master swarm and other slave swarms. PSO or its variations were executed individually by each slave swarm, including the updating of particle velocity and position. The particles in the master swarm improved themselves depending on the slave and master swarm's social knowledge. All the swarms were pushed to mingle at regular intervals throughout the evolution, and points were then redistributed to numerous sub-swarms to guarantee information sharing. The procedure was repeated until the user specified a stop condition. The case study on a hydrological model and numerical simulation experiments indicated that MSSE-PSO significantly lowered computation time, increased calibration accuracy, and improved stability performance.

Wang and Li (2011) proposed a hybrid technique of PSO and LS (local search) for solving the redundancy allotment problem for series-parallel multi-state systems. The suggested algorithm was capable of designing the system structure at a low cost while maintaining the necessary degree of availability. Unlike most prior research, which only considered homogeneous redundancy, the suggested method considered heterogeneous redundancy. To assess system availability, the universal generating function technique was used. To respond to the redundancy allocation problem, the conventional PSO was changed and innovative LS methods were included. Case studies were presented to aid comparisons between the proposed technique and various non-hybrid and meta-heuristics. The findings showed that the suggested technique had advantages in terms of efficiency and solution quality.

Pant *et al.* (2015) presented a particle swarm optimization algorithm. The proposed algorithm's performance was tested on three optimization problems. The results obtained were compared with several well-known methods. In terms of solution quality and computation time, numerical experiments showed that the proposed method was promising, and the results obtained by the proposed algorithm were either comparable or superior to the previously best known ones.

Yao *et al.* (2016) presented a carton heterogeneous vehicle routing problem with a collection depot. PSO was utilized to solve the issue with a collecting depot. A local search technique and self-adaptive inertia weight were employed to increase the PSO's performance. Finally, two test instances were used to demonstrate the method. The findings demonstrated that the suggested PSO can handle the multi-depot as well as the carton heterogeneous vehicle routing problem with a collecting depot effectively.

Zhang and Chen (2016) dealt with the difficulties of reliability redundancy allocation in an interval context. From the basic crisp problem, an interval multi-objective optimization problem was constructed, in which system cost and reliability were both evaluated. A dominance relation for interval-valued functions was developed using newly suggested order relations of interval-valued numbers to make the multi-objective PSO method capable of dealing with interval multi-objective optimization issues. The crowding distance was then applied to the situation of multi-objective interval-valued data. Finally, two numerical examples and a case study of a supervisory control and data acquisition system in water resource management were used to show the efficacy of the suggested technique.

Kumar *et al.* (2017) investigated the applicability of multi-objective PSO incorporating crowding distance (MOPSO-CD) to solve a reliability optimization issue with the goals of maximizing reliability and lowering cost. They gave a thorough overview of multi-objective reliability optimization, PSO, and MOPSO-CD. The MOPSO-CD was applied to a series system at the end. The simulation findings demonstrated that MOPSO-CD can create a well-distributed Pareto optimum set for a decision maker to pick from in a single run.

Sebt *et al.* (2017) used fully informed particle swarm (FIPS) and genetic algorithm to solve the MRCPSP (multi-mode resource-constrained project scheduling problem) with the goal of minimizing project makespan while taking into account resource and priority restrictions. FIPS was a common variation of the PSO technique in the proposed hybrid technique. Encoding techniques included associated mode list and random key representation schemes with the decoding procedure being the MSSGS (multi-modal serial schedule generation system). The findings revealed that the proposed mode enhancement process significantly reduces the project timeline. The efficacy of the proposed technique to solve the MRCPSP is validated by comparing its results to those of other methods using the well-known benchmark sets.

Xu *et al.* (2018) presented intracluster cohesion (ICC), a new metric that measured the similarity of data inside a cluster. Authors presented an ATPSO (accelerated two-stage PSO), which used *K*-means to speed up particle convergence during population initialization. There were two steps to its clustering process. The first goal of reducing ICD (intracluster distance), by preliminary clustering; second, improving ICC to improve clustering accuracy. Extensive tests in diverse geometric distributions are carried out using 17 open-source clustering sets. In terms of accuracy, ATPSO surpassed PSO, chaotic PSO (CPSO), K-means PSO (KPSO), and accelerated CPSO, but its efficiency is similar to that of KPSO. Its convergence trend suggested that using the intended ICC improved clustering accuracy. Surprisingly, when compared to the Pareto-based multi-objective PSO, ATPSO's suggested two-stage search can locate clusters more correctly and rapidly.

Sadeghi *et al.* (2020) used multi-objective PSO technique to explore the optimal size problem of the micro-resources grid's in two different modes in the presence of an electric car. Monte Carlo Simulation was used to represent the electric vehicle's unpredictable behavior. The optimal number of components and cost at various degrees of reliability were established in the first instance, referred to as PV/wind/battery. The electric car was then added to the system, and the chance of losing power was computed in both stochastic and deterministic modes. The findings suggested that using an electric car improved system reliability. The influence of stochastic and deterministic behavior of electric vehicles on the number of units and the possibility of power supply failure was examined for the first time in the second system (PV/wind/battery/EV). The findings showed that both systems were viable to construct, although the first was more efficient than the second. Furthermore, a sensitivity analysis was carried out to demonstrate the impact of load factors and wind speed on choice variables.

Materials and Methods

Chapter 3 MATERIALS AND METHODS

This chapter is divided into eight sections in which we examine the following models:

Model [1]: Availability of systems subject to multiple failure modes under calendar-based inspection

Model [2]: Availability analysis and inspection optimization for a competing-risk *k*-out-of-*n*:G system

Model [3]: Modeling periodically inspected *k*/*r*-out-of-*n* system

Model [4]: Availability and cost assessment of systems with dormant failure undergoing sequential inspections

Model [5]: Modeling sequentially inspected system prone to degradation and shocks

Model [6]: Modeling systems with revealing and non-revealing failures undergoing periodic inspection

Model [7]: Markov process approach for analyzing periodically inspected competing-risk system embodying downtime threshold

Model [8]: Particle swarm optimization strategy for design optimization of a series-parallel system incorporating failure dependencies and multiple repair teams

3.1 Model [1]: Availability of systems subject to multiple failure modes under calendar-based inspection

Traditionally, emphasis is being made on designing of systems incorporating multiple failure modes as configuration of the systems and failure states of components are becoming more variant (can be seen in Ref. **Dhillon, 1978**; **Klutke and Yang, 2002**; **Levitin *et al.*, 2006**; **Zhang *et al.*, 2016**; **Zheng *et al.*, 2016**). Competing failure may take place in the systems suffering with the multiple failure modes, and any of these failures can result in the failure of the system.

Availability has consistently remained a burning issue on the subject of reliability engineering as it is the principal characteristic of operation and design of all engineering systems. A lot of research is carried out on the availability of systems subject to single failure mode. Relevant writings can be accessed in Ref. **Cui and Xie (2001)**, **Cui and Xie (2005)**, **Xu and Hu (2008)**, **Tang *et al.* (2013)** and **Khatab *et al.* (2014)**. Despite the fact that a complex system may fail in lots of different ways, current availability models seldom take into account multiple failure modes. This encourages us to develop a good quality model in order to scrutinize the availability of competing-risk systems.

It is supposed in large number of the existent maintenance models that the failures are identified in no time. Meanwhile, in some realistic systems like Integrated digital communication system (**Liu *et al.*, 2013**), Remote power feeding systems (**Qiu *et al.*, 2017**) and safety valves in protection systems (**Tang *et al.*, 2013**) failures are hidden/ unrevealed. When the system experiences a hidden failure, inspection policy in general is employed to figure out if a failure has taken place or not (see Ref. **Cui *et al.*, 2010**; **Cheng and Li, 2012**; **Berrade *et al.*, 2013**; **Qiu *et al.*, 2017**). Two kinds of inspections are mentioned in the literatures so far, namely: periodic and non-periodic inspection. The customary exercise is to employ periodic inspection in applications since it is easier to schedule and more feasible.

Two kinds of periodic inspection policies have been discussed in the literatures till now, namely: age-based and calendar-based inspection policy. Some researchers studied the point and steady-state availability for a system inspected using calendar-based inspection policy (see Ref. **Sarkar and Sarkar, 2000**). **Cui and Xie (2005)** calculated the point and steady-state availability of a system inspected using age-based inspection policy by taking into account random time for repair. **Tang *et al.* (2013)** examined the limiting average and point availability of a system inspected using both the periodic inspection policies. **Qiu and Cui (2019b)** considered systems encountering multiple failure modes and studied their steady-state and point availability using age-based inspection policy.

Systems performance declines after successive usage, hence proper and adequate maintenance is required to prolong its operational duration. Thus, PR is used to enhance system performance/availability (see Ref. **Qiu *et al.*, 2017** and **Qiu and Cui, 2019b**). No study has yet been conducted to investigate the availability of competing-risk system under calendar-based inspection policy. Here, we propose to study the point and limiting average availability of a system encountering multiple failure modes undergoing inspections and perfect preventive repair (PR) at fixed calendar intervals.

The outcomes on availability of the system extracted in this study can be implemented simply to most of the systems facing multiple failure modes. We consider wind turbine systems to demonstrate our established availability model of the systems experiencing multiple failure modes. As an important characteristic of the system performance, the availability of wind turbine system is examined and it plays a major role in many crucial applications. An important issue that wind turbine system has encountered is of multiple failure modes. Very little research has been carried out to examine the availability and maintenance policy for wind turbine system. Early studies have been indicated in **Ozturk *et al.* (2018)**, wherein various results for direct-drive and geared wind turbines were compared.

3.1.1 Notations

N Total failure modes

X_k Failure time of k^{th} failure mode, $k = 1,2,\ldots,N$

$F_k(x)$ Distribution function of X_k, $k = 1,2,\ldots,N$

$f_k(x)$ Density function of X_k, $k = 1,2,\ldots,N$

$\lambda_k(\tau)$ Hazard rate function of X_k, $k = 1,2,\ldots,N$

$R(\tau)$ Survivor function of the system

Y_k Repair time of k^{th} failure mode, $k = 1,2,\ldots,N$

$G_k(y)$ Distribution function of Y_k, $k = 1,2,\ldots,N$

$S(\tau)$ Systems status at time τ

$A(\tau)$ Systems Point availability

$\bar{A}$ Systems Limiting average availability

T Inspection interval

3.1.2 System description

The precise assumptions used in determining the availability of our proposed model are summarized below.

(1) Periodic inspection: Suppose a system is brought into operation at $\tau = 0$ and inspected regularly at times $T, 2T, 3T, \dots$. While inspecting a system, if it is found working then a PR is carried out and the system is brought back almost to a new state. The time for PR is assumed to be negligible.

(2) The system failure is categorized into N failure modes (FM) independent of each other. The failure time of each failure mode is denoted by X_k $(k = 1,2,\dots,N)$ with distribution function $F_k(\tau)$, density function $f_k(\tau)$ and hazard rate function $\lambda_k(\tau)$.

(3) If the system failure occurs because of the k^{th} failure mode then a corrective repair (CR) is performed taking time Y_k $(k = 1,2,\dots,N)$ with distribution function $G_k(y)$. Y_k $(k = 1,2,\dots,N)$ are random variables independent of each other.

Figure 3.1.1 gives a probable specimen of the system, where the inspection interval is of length T. As shown in the Figure 3.1.1, there is no system failure in the time interval $[0,2T]$; so two perfect PRs are carried out at each of the inspections. FM 1 occurs between inspections second and third, resulting in the system failure and from then till the third inspection, system is at rest and no CR is performed and then at third inspection a CR is carried out taking time Y_1. After completion of CR, the system is renewed.

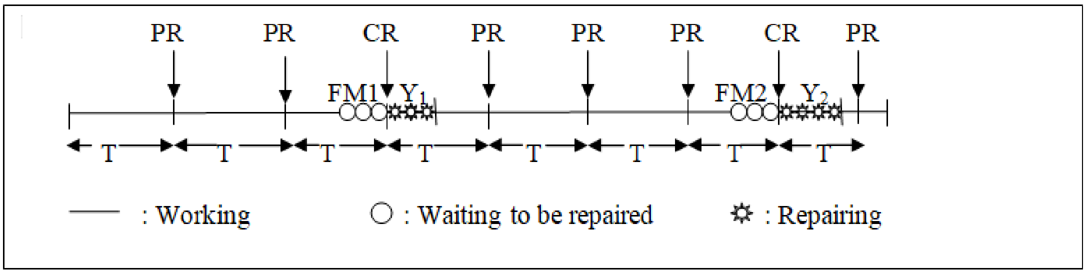


Figure 3.1.1: A probable specimen of the system under calendar-based inspection

3.1.3 Systems point availability

From Assumptions (1) and (2), the lifetime of the system, S, is clearly demonstrated as the minimum of $(X_1, X_2, \dots, X_N)$. As X_k $(k = 1,2,3,\dots,N)$ are random variables independent of each other, hence, we get $R(\tau)$ as

$$R(\tau) = \prod_{k=1}^{N} P(X_k > \tau) = \prod_{k=1}^{N} R_k(\tau)$$

Proposition 1: The point availability of the proposed system is obtained as

$$A(\tau) = \begin{cases} R(\tau), & \text{if} \quad 0 \le \tau \le T \\ R(\tau - sT)A(sT) + \big(1 - A(sT)\big) \sum_{k=1}^{N} G_k(\tau - sT) \int_{0}^{\infty} R(\tau)\lambda_k(\tau)d\tau, & \text{if } sT < \tau \le (s+1)T \text{ for } s = 1,2,3,\dots \end{cases}$$

Proof: We define a Markov process as follows

$$S(\tau) = \begin{cases} 0, & \text{the system is traced failed at time } \tau. \\ 1, & \text{the system is found working at time } \tau. \end{cases}$$

(i) When $0 \le \tau \le T$.

Since availability simply equals reliability for a non-repairable system. As no inspection has taken place till time T i.e. system is not maintained till time T, it is obvious that

$$A(\tau) = R(\tau) \tag{3.1.1}$$

(ii) When $T < \tau \leq 2T$.

During the first inspection the system is either in a working condition or has failed because of the $k^{th}(k = 1,2,3,\dots,N)$ failure mode and using the fact that all failure modes are independent, $A(\tau)$ can be given by

$$A(\tau) = P(S(\tau) = 1)$$

$$\begin{aligned} A(\tau) &= P(S(\tau) = 1, S(T) = 1) + P(S(\tau) = 1, S(T) = 0) \\ &= P(S(\tau) = 1|S(T) = 1)P(S(T) = 1) + P(S(\tau) = 1, S(T) = 0) \\ &= R(\tau - T)A(T) + \sum_{k=1}^{N} P(S(\tau) = 1, S(T) = 0, I = k) \end{aligned}$$

$$= R(\tau - T)A(T)$$
$$+ \sum_{k=1}^{N} P(S(\tau) = 1|S(T) = 0, I = k)\, P(S(T) = 0, I = k)$$
$$= R(\tau - T)A(T) + \sum_{k=1}^{N} G_k(\tau - T)P(S(T) = 0|I = k)P(I = k)$$

where I indicate the failure type and $P(I = k)$ indicate the probability of $k^{th}(k = 1,2,3,\dots,N)$ failure mode.

$$A(\tau) = R(\tau - T)A(T) + \sum_{k=1}^{N} G_k(\tau - T)(1 - A(T)) \int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau$$

$$= R(\tau - T)A(T) + (1 - A(T)) \textstyle\sum_{k=1}^{N} G_k(\tau - T) \int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau$$

(iii)When $sT < \tau \leq (s + 1)T$.

By the similar observation as above, during the s^{th} inspection the system is either in a working condition or has failed because of the $k^{th}(k = 1,2,3,\dots,N)$ failure mode

Hence, $A(\tau)$ can be given by

$$\begin{aligned} A(\tau) &= P(S(\tau) = 1, S(sT) = 1) + P(S(\tau) = 1, S(sT) = 0) \\ &= P(S(\tau) = 1|S(sT) = 1)P(S(sT) = 1) + P(S(\tau) = 1, S(sT) = 0) \end{aligned}$$

$$= R(\tau - sT)A(sT) + \sum_{k=1}^{N} P(S(\tau) = 1, S(sT) = 0, I = k)$$

$$A(\tau) = R(\tau - sT)A(sT) + \sum_{k=1}^{N} P(S(\tau) = 1|S(sT) = 0, I = k)\, P(S(sT) = 0, I = k)$$

$$= R(\tau - sT)A(sT) + \sum_{k=1}^{N} G_k(\tau - sT)P(S(sT) = 0|I = k)P(I = k)$$

$$= R(\tau - sT)A(sT) + \sum_{k=1}^{N} G_k(\tau - sT)(1 - A(sT)) \int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau$$

$$= R(\tau - sT)A(sT) + (1 - A(sT)) \sum_{k=1}^{N} G_k(\tau - sT) \int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau \quad (3.1.2)$$

Using equations (3.1.1) and (3.1.2), we get

$$A(\tau) = \begin{cases} R(\tau), & \text{if } \ 0 \le \tau \le T \\ R(\tau - sT)A(sT) + \big(1 - A(sT)\big) \sum_{k=1}^{N} G_k(\tau - sT) \int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau\,, & \\ & \text{if } sT < \tau \le (s+1)T \ \text{ for } s = 1,2,3,\ldots \end{cases} \quad (3.1.3)$$

Remark 1: Clearly, our model is an extension of the models taking single failure mode into account. If we consider single failure mode, i.e. when $N = 1$, the resultant in equation (3.1.3) is reduced to

$$A(\tau) = \begin{cases} R(\tau), & \text{if } \ 0 \le \tau \le T \\ R(\tau - sT)A(sT) + \big(1 - A(sT)\big)G(\tau - sT) \int_0^{\infty} R(\tau)\lambda(\tau)d\tau, & \\ & \text{if } sT < \tau \le (s+1)T \ \text{ for } s = 1,2,3,\ldots \end{cases}$$

By considering the fact that each system has some lifespan i.e. probability of occurrence of system failure after certain amount of time is equal to 1. Hence, the above expression is equivalent to

$$A(\tau) = \begin{cases} R(\tau), & \text{if } \ 0 \le \tau \le T \\ R(\tau - sT)A(sT) + \big(1 - A(sT)\big)G(\tau - sT), & \text{if } \ sT < \tau \le (s+1)T \ \text{ for } s = 1,2,3,\ldots \end{cases}$$

3.1.4 Systems limiting average availability

The limiting average availability is also a critical performance index of system and can be obtained by the means of the formula $\lim_{t\to\infty}\frac{1}{t}\int_0^t A(u)du$

Proposition 2: The limiting average availability of the proposed system is expressed by

$$\bar{A} = \frac{1}{T}\left(\Psi\int_0^T R(v)dv + (1-\Psi)\sum_{k=1}^{N}\int_0^\infty R(\tau)\lambda_k(\tau)d\tau\int_0^T G_k(v)\,dv\right) \quad (3.1.4)$$

where

$$\Psi = \frac{\sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau}{1-R(T)+\sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau}$$

Proof: Letting $\tau = (s+1)T$ in equation (3.1.2), we have

$A\big((s+1)T\big) = R(T)A(sT) + (1-A(sT))\sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau$ for $s = 1,2,3,\ldots$

$\lim_{s\to\infty} A(sT)$ exists since, $0 < R(T) - \sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau < 1$

Letting $n \to \infty$, we have

$$\Psi = \lim_{s\to\infty} A(sT) = \frac{\sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau}{1-R(T)+\sum_{k=1}^{N} G_k(T)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau}$$

For any $v \in (0,T)$ and applying equation (3.1.2), we get

$$A(v+nT) = A(sT)R(v) + (1-A(sT))\sum_{k=1}^{N} G_k(v)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau$$

Letting $s \to \infty$, we have

$$\lim_{n\to\infty} A(v+sT) = \Psi R(v) + (1-\Psi)\sum_{k=1}^{N} G_k(v)\int_0^\infty R(\tau)\lambda_k(\tau)d\tau$$

Hence, limiting average availability is equal to

$$\bar{A} = \frac{1}{T}\left(\int_0^T \left(\Psi R(v) + (1-\Psi)\sum_{k=1}^{N} G_k(v)\int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau\right)dv\right)$$

$$= \frac{1}{T}\left(\Psi\int_0^T R(v)dv + (1-\Psi)\sum_{k=1}^{N}\int_0^{\infty} R(\tau)\lambda_k(\tau)d\tau\int_0^T G_k(v)\,dv\right)$$

3.2 Model [2]: Availability analysis and inspection optimization for a competing-risk *k*-out-of-*n*:G system

Redundancy is a procedure through which systems availability and reliability can be improved. *k*-out-of-*n*: G system is most frequently used redundant system in which out of *n* components at least *k* must be active for working of the system. Because of its error tolerance potential, the *k*-out-of-*n* system is extensively employed in the many fields such as in nuclear and process industry, in hydroelectric plant, in hardware and software engineering, in hydraulic control system. Many studies are done on reliability of redundant systems. **Barlow and Proschan (1996)** deduced many results on reliability of a *k*-out-of-*n*:G system. **Munjal and Singh (2014)** studied reliability of a system embodying parallel-connected pair of 2-out-of-3:G subsystems. **Negi and Singh (2015)** studied reliability of a system with weighted subsystems coupled in series. **Nautiyal *et al.* (2020)** assessed reliability and traits of *k*-out-of-*n* mesh.

The reliability of a *k*-out-of-*n*: G system with similar units is given by:

$$\sum_{i=k}^{n}\binom{n}{i}(R_c(\tau))^i\,(1-R_c(\tau))^{n-i} \tag{3.2.1}$$

where $R_c(\tau)$ is the reliability of each component.

Availability has consistently remained a burning issue in the subject of reliability engineering as it is the principal characteristic of operation and design of all engineering systems. A lot of research has been carried out for determining the availability of *k*-out-of-*n* systems. Related writings could be seen in **Kenyon and**

Newell (1983); **McGrady (1985)**; **Moustafa (2001)**; **Frostig and Levikson (2002)**; **Sharma and Kumar (2017)**. A lot of research is carried out on the availability of systems subject to single FM. **Sarkar and Sarkar (2000)** and **Cui and Xie (2005)** calculated point and limiting availability for a system with single FM under different policies. However, one can see that the systems can fail due to many mutually exclusive reasons, for example electrical components can fail because of short and open circuit. A fire alarm may fail due to a dead battery, defective detector or faulty wiring. **Moustafa (1996a)** analyzed a k-out-of-n:G system with multiple failure modes (FMs) and found its limiting availability. In **Moustafa (1996b)**, author calculated the transient reliability for k-out-of-n:G systems encountering two FMs.

In several existent maintenance models, it is supposed that the failures are identified in no time. Meanwhile, in some realistic systems like safety valves in protection systems (**Tang *et al.*, 2013**), failures are found to be hidden/ unrevealed. When the system experiences a hidden failure, inspection policy in general is employed to figure out if a failure has taken place or not. Different types of inspections are mentioned in the literature like continuous monitoring (**Liao *et al.*, 2006**), periodic inspections (**Li, 2016**) and non-periodic inspections (**Berrade *et al.*, 2013**). The customary exercise is to employ periodic inspection in applications since it is easier to schedule and more feasible.

Maintenance policy plays important role in minimizing systems expected total costs or maximizing its reliability. If system's performance is not found up to mark then, preventive repair (PR) is performed to achieve satisfactory reliability performance. In most of the systems like power plants, the cost of the system downtime/penalty cost might be significantly higher as compared to its maintenance costs. So, optimal inspection interval is chosen, which helps in reducing the total cost of maintenance. **Pham and Wang (2000)** gave maintenance policy which may help in reducing the frequency of unexpected CR activities at relatively low costs. Recently, maintenance service of k-out-of-n system is thoroughly taken into account by many investigators. **Babishin and Taghipour (2016)** provided the inspection and maintenance optimization of k-out-of-n system. **Park *et al.* (2019)** gave optimality rule for a maintained k-out-of-n system.

The earlier investigations on *k*-out-of-*n*:G system either considered a completely noticeable system or took negligible repair time. No research has yet been undertaken to figure out the availability and optimal inspection interval for a *k*-out-of-*n*:G system with hidden failures and taking non-negligible time for repair of failures. Here, we propose to study the point and limiting availability of a periodically inspected *k*-out-of-*n*:G system encountering hidden failures and taking random time for their repairs. The long-run average cost rate (LRACR) is also obtained and a condition for an optimal inspection interval in order to reduce the expense of system maintenance is given.

3.2.1 Notations

N	Total FMs
I	Inspection period
M	Total inspections till first failure in a renewal cycle
X_s	Failure time of s^{th} FM, $s = 1,2,\dots,N$
$\lambda_s(\tau)$	Hazard rate function of X_s , $s = 1,2,\dots,N$
$F_{c_s}(x)$	Distribution function of each component for s^{th} FM, $s = 1,2,\dots,N$
$R_c(\tau)$	Reliability function of X_s , $s = 1,2,\dots,N$
$R(\tau)$	Survivor function of the system
$F(\tau)$	Cumulative distribution function
Y_s	Repair time of s^{th} FM, $s = 1,2,\dots,N$
$G_s(y)$	Distribution function of Y_s , $s = 1,2,\dots,N$
$g_s(y)$	Density function of Y_s , $s = 1,2,\dots,N$
$K(\tau)$	Systems status at time τ
$A(\tau)$	Systems Point availability
A	Systems Limiting availability
U	Systems uptime in a renewal cycle

D	Systems downtime in a renewal cycle
L	Overall length of a renewal cycle
L_c	LRACR
C	Overall expense in a renewal cycle
C_{ins}	Inspection cost
C_{R_s}	Cost of repair of s^{th} FM, $s = 1,2,\ldots,N$
C_p	Penalty cost

3.2.2 System description

The precise assumptions employed in our proposed work are summarized below:

(i) System is k-out-of-n:G with all the n units being identical.

(ii) All units of system are either operational or are in down state.

(iii) System fails as soon as (n-k+1) units out of n units fail.

(iv) Failures are detected through inspections.

(v) Inspection interval is taken to be I.

(vi) Inspection policy is age-based, i.e. the time for repair is not included in I.

(vii) While inspecting a system, if it is found working then a PR is carried out and the PR is assumed to be instantaneous.

(viii) The system failure is categorized into N FMs independent of each other. The failure time of each FM is denoted by $X_s\,(s = 1,2,\ldots,N)$ with distribution function of each component being $F_{c_s}(t)$.

(ix) If the system failure occurs because of the s^{th} failure mode then a corrective repair (CR) is performed taking time $Y_s\,(s = 1,2,\ldots,N)$ with distribution function $G_s(y)$, where $Y_s\,(s = 1,2,\ldots,N)$ are random variables independent of each other.

(x) The repairs are assumed to be perfect.

(xi) Time interval from setting up of a new system till first CRs termination or duration in the midst of two successive terminations of CRs is defined a renew cycle.

3.2.3 Systems point availability

From Assumptions (viii), the lifespan of the system, *K*, is clearly demonstrated as minimum of $(X_1, X_2, \dots, X_N)$. As $X_s\,(s = 1,2,3, \dots, N)$ are random variables independent of each other, hence, we get $R(\tau)$ as:

$$R(\tau) = \prod_{s=1}^{N} P(X_s > \tau) = \prod_{s=1}^{N} R_s(\tau) \tag{3.2.2}$$

where, $R_s(\tau)$ is obtained using equation (3.2.1) and given by

$$R_s(\tau) = \sum_{i=k}^{n} \binom{n}{i} (R_{c_s}(\tau))^i \, (1 - R_{c_s}(\tau))^{n-i}$$

Proposition 1: The proposed systems point availability is obtained as

$$A(\tau) = \big(R(\tau)\big)^{\left\lfloor \frac{\tau}{I} \right\rfloor} R\left(\tau - \left\lfloor \frac{\tau}{I} \right\rfloor I\right) + \sum_{m=0}^{\left\lfloor \frac{\tau}{I} \right\rfloor - 1} (R(I))^m \left[\sum_{s=1}^{N} \int_0^I R(\tau)\, \lambda_s(\tau) d\tau \int_0^{\tau-(m+1)I} A(\tau - (m+1)I - y) g_s(y)\, dy \right]$$

Proof: We define a Markov process accordingly as

$$K(\tau) = \begin{cases} 0, & \text{when the system is under failure at time } \tau \\ 1, & \text{when the system is operational at time } \tau \end{cases}$$

Based on Assumptions (v), (vii) and (x), the probability of working of system at every planned inspection is $R(I)$. Hence, Probability mass function of *M* satisfies

$$P(M = m) = \big(R(I)\big)^m F(I) \tag{3.2.3}$$

Point availability is the probability that a system will be operational at a specific time, τ. Hence, it may be given as

$$A(\tau) = P(K(\tau) = 1)$$

By using total probability decomposition method, point availability could also be written as

$$A(\tau) = \sum_{m=0}^{\infty} P(K(\tau) = 1, M = m)$$

$$= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau) = 1, M = m) + \sum_{m=\left\lfloor\frac{\tau}{I}\right\rfloor}^{\infty} P(K(\tau) = 1, M = m) \qquad (3.2.4)$$

The first term of equation (3.2.4), represents the occurrence of first failure before time τ. The second term represents that no failure takes place before time τ and may be given as

$$P\left(K(\tau) = 1, M \geq \left\lfloor\frac{\tau}{I}\right\rfloor\right) = P\left(K(\tau) = 1 | M \geq \left\lfloor\frac{\tau}{I}\right\rfloor\right) P\left(M \geq \left\lfloor\frac{\tau}{I}\right\rfloor\right)$$

$$= P\left(K > \tau - \left\lfloor\frac{\tau}{I}\right\rfloor I\right) \left(R(\tau)\right)^{\left\lfloor\frac{\tau}{I}\right\rfloor}$$

$$= R\left(\tau - \left\lfloor\frac{\tau}{I}\right\rfloor I\right) \left(R(\tau)\right)^{\left\lfloor\frac{\tau}{I}\right\rfloor} \qquad (3.2.5)$$

Using the independence of FMs, equation (3.2.3) could also be expressed as

$$P(M = m) = \sum_{s=1}^{N} P(M = m, S = s)$$

$$= (R(I))^m \sum_{s=1}^{N} \int_0^I R(\tau)\, \lambda_k(\tau) d\tau \qquad (3.2.6)$$

Again applying independence of FMs and using equation (3.2.6), first term of equation (3.2.4) can be rewritten as

$$\sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau)=1, M=m) = \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1}\sum_{s=1}^{N} P(K(\tau)=1, M=m, S=s)$$

$$= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1}\sum_{s=1}^{N} P(K(\tau)=1|M=m, S=s)P(M=m, S=s)$$

$$= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1}\sum_{s=1}^{N} P(K(\tau)=1|M=m, S = s)(R(I))^m \int_0^I R(\tau)\,\lambda_s(\tau)d\tau \tag{3.2.7}$$

As type s failure occurs, respective CR taking random time Y_s $(s = 1,2,\dots,N)$ takes place. Therefore,

$$P(K(\tau)=1|M=m, S=s) = \int_0^{\tau-(m+1)I} P(K(\tau)=1|M=m, S=s, Y_s=y)\, dG_s(y)$$

$$= \int_0^{\tau-(m+1)I} P(K(\tau-(m+1)I-y)=1)\, dG_s(y)$$

$$= \int_0^{\tau-(m+1)I} A(\tau-(m+1)I-y)\, dG_s(y)$$

$$= \int_0^{\tau-(m+1)I} A(\tau-(m+1)I-y) g_s(y)\, dy \tag{3.2.8}$$

Hence, putting equation (3.2.8) in equation (3.2.7), we get

$$\sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau)=1, M=m)$$

$$= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} (R(I))^m \left[\sum_{s=1}^{N}\int_0^I R(\tau)\,\lambda_s(\tau)d\tau \int_0^{\tau-(m+1)I} A(\tau-(m+1)I - y)g_s(y)\,dy\right] \tag{3.2.9}$$

Substituting equations (3.2.5) and (3.2.9) in equation (3.2.4), we get the point availability of the system to be

$$A(\tau) = \left(R(\tau)\right)^{\left\lfloor\frac{\tau}{I}\right\rfloor} R\left(\tau - \left\lfloor\frac{\tau}{I}\right\rfloor I\right) + \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} (R(I))^m \left[\sum_{s=1}^{N}\int_0^I R(\tau)\,\lambda_s(\tau)d\tau \int_0^{\tau-(m+1)I} A(\tau-(m+1)I - y)g_s(y)\,dy\right] \tag{3.2.10}$$

3.2.4 Systems limiting availability

Limiting availability is defined as the ratio of systems expected uptime in a renewal cycle to the expected overall length of a renewal cycle.

Proposition 2: The limiting availability of the proposed system is obtained as:

$$A = \frac{\int_0^I R(\tau)d\tau}{I + \sum_{s=1}^{N} E(Y_s)\int_0^I R(\tau)\lambda_s(\tau)d\tau}$$

Proof: Let U demonstrate the systems uptime in a renewal cycle while L represent the total length of a renewal cycle. Then,

$$A = \frac{E(U)}{E(L)} \tag{3.2.11}$$

Let system fails in the time period $[MI, (M+1)I]$ and assume Z to be the time of system operation after M^{th} inspection in a renewal cycle. Then, Z lies between 0 and I.

Hence, U can be given by

$$U = MI + Z$$

Using equation (3.2.6), we can find $E(U)$ as

$$E(U) = \sum_{m=0}^{\infty} E(U|M = m)P(M = m)$$

$$= \sum_{m=0}^{\infty} E(mI + Z)P(M = m)$$

$$= I\sum_{m=0}^{\infty} m\big(R(I)\big)^m F(I) + \sum_{m=0}^{\infty} E(Z)\big(R(I)\big)^m F(I) \tag{3.2.12}$$

Now, we find $E(Z)$

$$E(Z) = \int_0^I \left(1 - \frac{F(\tau)}{F(I)}\right) d\tau$$

$$= \frac{IF(I) - \int_0^I F(\tau)d\tau}{F(I)} \tag{3.2.13}$$

Putting equation (3.2.13) in equation (3.2.12), we get

$$E(U) = \frac{\int_0^I R(\tau)d\tau}{F(I)} \tag{3.2.14}$$

Now, expected overall length of renewal cycle is presented by

$$E(L) = \sum_{m=0}^{\infty}\sum_{s=1}^{N}(E(L)|M = m, S = s)P(M = m, S = s)$$

$$= \sum_{m=0}^{\infty}\sum_{s=1}^{N}E\big((m+1)I + Y_s\big)P(M = m, S = s)$$

$$= \sum_{m=0}^{\infty}\sum_{s=1}^{N}E\big((m+1)I + Y_s\big)(R(I))^m\int_0^I R(\tau)\lambda_s(\tau)d\tau$$

$$= \sum_{m=0}^{\infty}(m+1)I\big(R(I)\big)^m\sum_{s=1}^{N}\int_0^I R(\tau)\lambda_s(\tau)d\tau$$

$$+\sum_{s=1}^{N}E(Y_s)\int_0^I R(\tau)\lambda_s(\tau)d\tau\sum_{m=0}^{\infty}\big(R(I)\big)^m$$

$$= \sum_{m=0}^{\infty}I(m+1)\big(R(I)\big)^m F(I) + \frac{1}{F(I)}\sum_{s=1}^{N}E(Y_s)\int_0^I R(\tau)\lambda_s(\tau)d\tau$$

Hence, $E(L)$ equals

$$E(L) = \frac{I + \sum_{s=1}^{N}E(Y_s)\int_0^I R(\tau)\lambda_s(\tau)d\tau}{F(I)} \tag{3.2.15}$$

Hence, using equations (3.2.11), (3.2.14) and (3.2.15), we get limiting availability in this case as

$$A = \frac{\int_0^I R(\tau)d\tau}{I + \sum_{s=1}^{N}E(Y_s)\int_0^I R(\tau)\lambda_s(\tau)d\tau} \tag{3.2.16}$$

3.2.5 Systems long-run average cost rate

The LRACR is expressed as the ratio of expected total expense in a renewal cycle to that of the expected length of a renewal cycle.

Proposition 3: The LRACR of the proposed system is obtained as:

$$L_c = \frac{C_{ins} + \sum_{s=1}^{N} C_{R_s} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau + C_p(I - \int_0^I R(\tau)d\tau)}{I + \sum_{s=1}^{N} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau}$$

Proof: Let C be the overall expense in a renewal cycle which includes the inspection cost, cost of CR and penalty cost at the time of system down time.

Then, LRACR, L_c equals

$$L_c = \frac{E(C)}{E(L)}$$

Since, it is presumed that $M + 1$ inspections are conducted in a renewal cycle. Hence, $E(C)$ will be given as

$$E(C) = \sum_{m=0}^{\infty} \sum_{s=1}^{N} (E(C)|M = m, S = s)P(M = m, S = s)$$

$$= \sum_{m=0}^{\infty} \sum_{s=1}^{N} \left(C_{ins}(m+1) + C_{R_s}E(Y_s) + C_pE(D)\right)P(M = m, S = s)$$

$$= \sum_{m=0}^{\infty} \sum_{s=1}^{N} \left(C_{ins}(m+1) + C_{R_s}E(Y_s)\right.$$

$$\left. + C_pE(D)\right)\left(R(I)\right)^m \int_0^I R(\tau)\lambda_s(\tau)d\tau$$

$$= \sum_{m=0}^{\infty} C_{ins}(m+1)\left(R(I)\right)^m \sum_{s=1}^{N} \int_0^I R(\tau)\lambda_s(\tau)d\tau$$

$$+ \sum_{s=1}^{N} C_{R_s}E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau \sum_{m=0}^{\infty} \left(R(I)\right)^m$$

$$+ C_pE(D) \sum_{m=0}^{\infty} \left(R(I)\right)^m \sum_{s=1}^{N} \int_0^I R(\tau)\lambda_s(\tau)d\tau$$

$$= \frac{C_{ins}}{F(I)} + \frac{\sum_{s=1}^{N} C_{R_s} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau}{F(I)} + C_p E(D) \tag{3.2.17}$$

The expected downtime by using equations (3.2.14) and (3.2.15) is obtained as

$$E(D) = \frac{I - \int_0^I R(\tau)d\tau}{F(I)} \tag{3.2.18}$$

Hence, using equations (3.2.17) and (3.2.18), we have

$$E(C) = \frac{C_{ins} + \sum_{s=1}^{N} C_{R_s} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau + C_p(I - \int_0^I R(\tau)d\tau)}{F(I)} \tag{3.2.19}$$

Hence, using equations (3.2.15) and (3.2.19), LRACR is obtained to be

$$L_c = \frac{C_{ins} + \sum_{s=1}^{N} C_{R_s} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau + C_p(I - \int_0^I R(\tau)d\tau)}{I + \sum_{s=1}^{N} E(Y_s) \int_0^I R(\tau)\lambda_s(\tau)d\tau} \tag{3.2.20}$$

For obtaining the optimal inspection period, we solve for distinct values of I, the LRACR and estimate the value of I for which the cost rate is minimum. This can be done numerically or graphically using an online tool.

3.3 Model [3]: Modeling periodically inspected *k*/*r*-out-of-*n* system

Redundancy is a procedure through which systems availability and reliability can be improved. *k*-out-of-*n:G* system is most frequently used redundant system, which is active if at least *k* of its *n* units are working. Many studies are done on *k*-out-of-*n:G* system. **Munjal and Singh (2014)** studied reliability of a system embodying parallel-connected pair of 2-out-of-3:G subsystems. **Negi and Singh (2015)** studied reliability of a system with weighted subsystems coupled in series. Here, we introduce a new redundant *k*/*r*-out-of-*n* configured system.

The proposed system is based on the realistic condition: Some units of a system being unavailable at some point do not mandatorily mean that the system is down at that point. Even if some units fail the system could fulfill some functions/missions to certain extent. To illustrate, we take an example of a cell phone, carrying out numerous operations such as, making voice and video calls, internet surfing, clicking pictures and many more. For instance, the calling operation doesn't work but, at the same time the internet surfing is responding, in such case the phone is said to be in a defected state. If both the mentioned operations are non-functioning, then the cell phone is said to be completely failed. Other examples are- motor vehicles are drivable with one punctured tire, a large structure propped by welded joints; the structure fails only after the failure of a series of supporting joints, pumps in boiler feed system, gear system (**Li *et al.*, 2017**).

Availability has consistently remained a burning issue on the subject of reliability engineering as it is the principal characteristic of operation and design of all engineering systems. Maintenance cost also plays an important role in system engineering. A lot of research is carried out on the analysis of cost and availability of systems with multiple states. Related literatures could be viewed in **Gupta and Gupta (1986)** where both expected profit and availability of the system composed of two subsystems coupled in series was calculated. **Moustafa (2002)** discussed minimization of unavailability of multistage degraded system. **El-Damcese and Shama (2013)** studied availability and reliability of a system incorporating degradation.

In several existent maintenance models, it is supposed that the failures or system states are identified in no time. Moreover, in some realistic systems like safety valves in protection systems (**Tang *et al.*, 2013**), failures/system states are found to be hidden/unrevealed. To figure out the state of the system, inspection policy in general is employed in such cases. Different types of inspections are mentioned in the literature like continuous monitoring (**Liao *et al.*, 2006**), periodic inspections (**Li, 2016**) and non-periodic inspections (**Berrade *et al.*, 2013**). Out of several inspection policies the customary exercise is to employ periodic inspection in applications since it is more convenient and feasible. In this study, periodic inspection is proposed to disclose the state of the system.

Maintenance policy plays important role in minimizing systems expected total costs or maximizing its reliability. It is worth noting that optimal maintenance modeling has been considered in many multiple failure states systems (**Xu and Hu, 2008**; **Golmakani and Moakedi, 2012**; **Liu *et al.*, 2013**; **Zhang *et al.*, 2016**). In most of the systems like power plants, the cost of the system downtime/penalty cost might be significantly higher as compared to its maintenance costs. So, optimal inspection interval is chosen, which helps in reducing the total cost of maintenance. Related studies could be accessed in **Nourelfath and Ait-Kadi (2007)**, **Peng *et al.* (2009)** and **Li and Peng (2014)**.Whereas, for systems like satellite systems and nuclear powerhouse, availability plays important role compared to the service cost. Optimal maintenance policy based on availability can be accessed in **Berenguer *et al.* (2003)** and **Khatab *et al.* (2014)**. Cost and availability analysis have been conducted on multi-state system with its units having multiple states (**Ruiz-Castro, 2020**) but no such analysis has been performed on systems having multiple states based on number of its units failed. In this study, we consider optimal inspection based on both availability and cost.

The current study introduces a *k*/*r*-out-of-*n* configured system having three states and defined as: The system works normally till failure of units is less than *r* whereas the system degrades upon failure of *r* units and it fails completely as soon as *k* units fail.

In particular system has three states defined as:

1. Normal state: System works properly if number of failed units < *r*.
2. Degraded state: System works/fails partially if *r* ≤ number of failed units < *k*.
3. Complete failure state: System fails completely if *k* ≤ number of failed units.

If system has *n* identical units then, the probability of occurrence of partial/complete failure $F_D(\tau)/F_F(\tau)$ are given by

$$F_D(\tau) = \sum_{x=r}^{k-1} \binom{n}{x} (1-p)^x p^{n-x} \tag{3.3.1}$$

$$F_F(\tau) = \sum_{x=k}^{n} \binom{n}{x} (1-p)^x p^{n-x} \tag{3.3.2}$$

where *p* is the reliability of each unit.

Here, we propose to study the point and limiting availability of periodically inspected *k/r*-out-of-*n* system and find its long-run average cost rate (LRACR) and thus give a condition for an optimal inspection interval so as to reduce the expense of system maintenance and maximize its availability.

3.3.1 Notations

I	Inspection period
μ	Total inspections till first failure in a renewal cycle
λ	Hazard rate of each unit
μ_D/μ_F	Repair rate if system is in degraded/completely failed state
$F_D(\tau)/F_F(\tau)$	Probability of occurrence of partial/complete failure
$g_D(y)/g_F(y)$	Density function for repair of degradation/complete failure
Y_D/Y_F	Repair time of degraded/completely failed state
$K(\tau)$	Systems status at time τ
$A(\tau)$	Systems point availability
$R(\tau)$	Systems reliability function
$R'(\tau)$	Probability of the system being in normal state
A	Systems limiting availability

U	Systems uptime in a renewal cycle
D	Systems downtime in a renewal cycle
L	Total length of a renewal cycle
L_c	LRACR
C	Overall expense in a renewal cycle
C_{ins}	Inspection cost
$C_{R_{D/F}}$	Cost of repair of a degradation/complete failure
C_p	Penalty cost

3.3.2 System description

The precise assumptions employed in our proposed work are summarized below:

(1) System is *k/r*-out-of-*n* with all the *n* units being identical.

(2) All units of system are either operational or are in a down state.

(3) System has three states: Normal, Degraded and Completely failed.

(4) Failure of each unit is independent and follows exponential distribution.

(5) Failures are detected through inspections only.

(6) Inspection interval is taken to be *I*.

(7) Inspection policy is age-based i.e., the time for repair is not included in *I*.

(8) If the system is found degraded or completely failed then a perfect corrective repair (CR) is performed following exponential distribution.

(9) Time interval from setting up of a new system till first CRs termination or duration in the midst of two successive terminations of CRs is defined a renew cycle.

3.3.3 Systems point availability

Proposition 1: The proposed systems point availability is obtained as

$$A(\tau) = R(\tau) + \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu = m, D) \int_{0}^{\tau-(m+1)I} A(\tau - (m+1)I - y)\mu_D e^{-\mu_D y} dy$$
$$+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu = m, F) \int_{0}^{\tau-(m+1)I} A(\tau - (m+1)I - y)\mu_F e^{-\mu_F y} dy$$

where

$$P(\mu = m, D) = F_D\big((m+1)I\big) - F_D(mI)$$

and

$$P(\mu = m, F) = F_F\big((m+1)I\big) - F_F(mI)$$

Proof: We define a Markov process as

$$K(\tau) = \begin{cases} 0, & \text{when the system is under complete failure at time } \tau. \\ 1, & \text{when the system is in degraded state at time } \tau. \\ 2, & \text{when the system is in normal state at time } \tau. \end{cases}$$

The systems point availability for the case $\tau \leq I$ will be clearly equal to reliability since no maintenance is taking place till time I.

i.e., $A(\tau) = R(\tau)$ for $\tau \leq I$

The systems point availability may be given as

$$A(\tau) = P(K(\tau) = 1) + P(K(\tau) = 2)$$
$$= A_1(\tau) + A_2(\tau) \quad (3.3.3)$$

Now, firstly we derive $A_2(\tau)$

$$A_2(\tau) = P(K(\tau) = 2)$$
$$= \sum_{m=0}^{\infty} P(K(\tau) = 2, \mu = m)$$

$$= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau)=2,\mu=m) + \sum_{m=\left\lfloor\frac{\tau}{I}\right\rfloor}^{\infty} P(K(\tau)=2,\mu=m)$$

$$\begin{aligned}
&= \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau)=2|\mu=m,D)\,P(\mu=m,D) \\
&+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(K(\tau)=2|\mu=m,F)\,P(\mu=m,F) \\
&+ \sum_{m=\left\lfloor\frac{\tau}{I}\right\rfloor}^{\infty} P(K(\tau)=2,\mu=m)
\end{aligned} \tag{3.3.4}$$

The third term of equation (3.3.4) represents that no degradation or complete failure takes place before time τ and hence it is equivalent to

$$\sum_{m=\left\lfloor\frac{\tau}{I}\right\rfloor}^{\infty} P(K(\tau)=2,\mu=m) \;= 1 - F_D(\tau) - F_F(\tau) \tag{3.3.5}$$

Since, $P(\mu=m,D)/P(\mu=m,F)$ denotes the probability of the event that the system is in the normal state before time mI and a degradation/complete failure occurred during time interval $[mI,(m+1)I]$ hence, we have

$$P(\mu=m,F) = F_F\big((m+1)I\big) - F_F(mI)$$

where $F_F(mI)$ denotes the probability of occurrence of complete failure before time mI, and

$$P(\mu=m,D) = R'(mI) - R'\big((m+1)I\big) - P(\mu=m,F)$$

where $R'(mI)$ is probability that system is in normal state till time mI. So, $R'(mI) - R'\big((m+1)I\big)$ denotes that a degradation/complete failure occurred in the time interval $[mI,(m+1)I]$. So, $R'(mI) - R'\big((m+1)I\big) - P(\mu=m,F)$ gives the probability that system was in a normal state till time mI and degradation occurred in the time interval $[mI,(m+1)I]$.

Also, $R'(\tau) = 1 - F_D(\tau) - F_F(\tau)$. Hence,

$$P(\mu = m, D) = F_D\big((m+1)I\big) - F_D(mI)$$

Now,

$$P(K(\tau) = 2|\mu = m, D) = \int_0^{\tau-(m+1)I} P(K(\tau) = 2|\mu = m, D, Y_D = y)\, g_D(y)dy$$

$$= \int_0^{\tau-(m+1)I} P(K(\tau - (m+1)I - y) = 2)\, \mu_D e^{-\mu_D y} dy$$

$$= \int_0^{\tau-(m+1)I} A_2(\tau - (m+1)I - y)\mu_D e^{-\mu_D y} dy \qquad (3.3.6)$$

and

$$P(K(\tau) = 2|\mu = m, F) = \int_0^{\tau-(m+1)I} P(K(\tau) = 2|\mu = m, D, Y_F = y)\, g_F(y)dy$$

$$= \int_0^{\tau-(m+1)I} P(K(\tau - (m+1)I - y) = 2)\, \mu_F e^{-\mu_F y} dy$$

$$= \int_0^{\tau-(m+1)I} A_2(\tau - (m+1)I - y)\mu_F e^{-\mu_F y} dy \qquad (3.3.7)$$

Using equations (3.3.5), (3.3.6) and (3.3.7), we have

$$A_2(\tau) = 1 - F_D(\tau) - F_F(\tau)$$

$$+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu$$

$$= m, D) \int_0^{\tau-(m+1)I} A_2(\tau - (m+1)I - y)\mu_D e^{-\mu_D y} dy$$

$$+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu \tag{3.3.8}$$

$$= m, F) \int_0^{\tau-(m+1)I} A_2(\tau - (m+1)I - y)\mu_F e^{-\mu_F y} dy$$

Similarly,

$$A_1(\tau) = F_D(\tau)$$

$$+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu$$

$$= m, D) \int_0^{\tau-(m+1)I} A_1(\tau - (m+1)I - y)\mu_D e^{-\mu_D y} dy$$

$$+ \sum_{m=0}^{\left\lfloor\frac{\tau}{I}\right\rfloor-1} P(\mu \tag{3.3.9}$$

$$= m, F) \int_0^{\tau-(m+1)I} A_1(\tau - (m+1)I - y)\mu_F e^{-\mu_F y} dy$$

Hence, putting equations (3.3.8) and (3.3.9) in equation (3.3.3), we get

$$A(\tau) = R(\tau) + \sum_{m=0}^{\left\lfloor \frac{\tau}{I} \right\rfloor - 1} P(\mu = m, D) \int_0^{\tau-(m+1)I} A(\tau - (m+1)I - y)\mu_D e^{-\mu_D y} dy + \sum_{m=0}^{\left\lfloor \frac{\tau}{I} \right\rfloor - 1} P(\mu = m, F) \int_0^{\tau-(m+1)I} A(\tau - (m+1)I - y)\mu_F e^{-\mu_F y} dy \qquad (3.3.10)$$

3.3.4 Systems limiting availability

The limiting availability is a critical performance index of any system and can be defined as the ratio of systems expected uptime in a renewal cycle to the expected total length of a renewal cycle.

Proposition 2: The limiting availability of the proposed system is obtained as

$$A = \frac{I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D)+\sum_{m=0}^{\infty}\int_{mI}^{(m+1)I}R(\tau)d\tau P(\mu=m,F)}{I\sum_{m=0}^{\infty}(m+1)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

Proof: Let T_0 and T_1 be time for first inspected complete failure/ degradation in a renewal cycle and $T_0{}^*$and $T_1{}^*$ be the exact time of first complete failure/ degradation in a cycle.

Let U be the uptime then,

$$U = \begin{cases} T_1, & \mu I < T_1{}^* \leq (\mu+1)I \\ T_0{}^*, & \mu I < T_0{}^* \leq (\mu+1)I \end{cases}$$

Clearly, $T_1 = (\mu + 1)I$ so,

$$E(U) = \sum_{m=0}^{\infty} E(U|\mu = m, \mu I < T_1{}^* \leq (\mu+1)I)P(\mu = m, \mu I < T_1{}^* \leq (\mu+1)I)$$

$$+ \sum_{m=0}^{\infty} E(U|\mu = m, \mu I < T_0{}^* \leq (\mu+1)I)P(\mu = m, \mu I < T_0{}^* \leq (\mu+1)I)$$

$$= I\sum_{m=0}^{\infty}(m+1)P(\mu = m, D) + \sum_{m=0}^{\infty} \int_{mI}^{(m+1)I} R(\tau)d\tau\, P(\mu = m, F) \qquad (3.3.11)$$

Now,

$$L = \begin{cases} (\mu+1)I + Y_D, & \mu I < T_1{}^* \leq (\mu+1)I \\ (\mu+1)I + Y_F, & \mu I < T_0{}^* \leq (\mu+1)I \end{cases}$$

$$E(L) = \sum_{m=0}^{\infty} E(L|\mu = m, \mu I < T_1 \leq (\mu+1)I)P(\mu = m, \mu I < T_1 \leq (\mu+1)I)$$

$$+ \sum_{m=0}^{\infty} E(L|\mu = m, \mu I < T_0{}^* \leq (\mu+1)I)P(\mu = m, \mu I < T_0{}^* \leq (\mu+1)I)$$

$$= \sum_{m=0}^{\infty} E\big((m+1)I + Y_D\big)P(\mu = m, D) + \sum_{m=0}^{\infty} E\big((m+1)I + Y_F\big)P(\mu = m, F)$$

$$= I\sum_{m=0}^{\infty}(m+1)(P(\mu = m, D) + P(\mu = m, F))$$

$$+ \sum_{m=0}^{\infty} E(Y_D)P(\mu = m, D) + \sum_{m=0}^{\infty} E(Y_F)P(\mu = m, F) \qquad (3.3.12)$$

Using equations (3.3.11) and (3.3.12), limiting availability equals

$$A = \frac{I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D) + \sum_{m=0}^{\infty}\int_{mI}^{(m+1)I} R(\tau)d\tau P(\mu=m,F)}{I\sum_{m=0}^{\infty}(m+1)(P(\mu=m,D)+P(\mu=m,F)) + \sum_{m=0}^{\infty} E(Y_D)P(\mu=m,D) + \sum_{m=0}^{\infty} E(Y_F)P(\mu=m,F)} \qquad (3.3.13)$$

Remark 1: If A of equation (3.3.13) be expressed as

$$A = \frac{U(I)}{U(I) + D(I)}$$

which could also be rewritten as

$$A = \frac{1}{1 + D(I)/U(I)}$$

So, in order to maximize the limiting availability we need to minimize $H(I)$, where

$$H(I) = \frac{D(I)}{U(I)} \tag{14}$$

i.e., we have to find I^* such that *H(I)* is minimum where I^* denotes the optimal interval.

3.3.5 Systems long-run average cost rate

The LRACR is expressed as the ratio of expected total expense in a renewal cycle to that of the expected length of a renewal cycle.

Proposition 3: The LRACR of the proposed system is obtained as

$$L_c = \frac{C_{ins}\sum_{m=0}^{\infty}(m+1)\big(P(\mu=m,D)+P(\mu=m,F)\big)+C_{R_D}\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+C_{R_F}\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F) + C_p\Big(I\sum_{m=0}^{\infty}P(\mu=m,F)(m+1)-\sum_{m=0}^{\infty}\int_{mI}^{(m+1)I}R(\tau)d\tau P(\mu=m,F)\Big)}{I\sum_{m=0}^{\infty}(m+1)\big(P(\mu=m,D)+P(\mu=m,F)\big)+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

Proof: Let C be the overall expense in a renewal cycle which includes the inspection cost, cost of CR and penalty cost at the time of system down time.

Then, LRACR, L_c equals

$$L_c = \frac{E(C)}{E(L)} \tag{3.3.15}$$

Here, the expected total cost is presented as

$$E(C) = C_{ins}E(\mu + 1) + E(C_R) + C_pE(D) \tag{3.3.16}$$

Since, it is considered that $\mu + 1$ inspections are conducted in a cycle, so

$$E(\mu + 1) = \sum_{m=0}^{\infty} E(m+1)P(\mu = m, D) + \sum_{m=0}^{\infty} E(m+1)P(\mu = m, F)$$

$$= \sum_{m=0}^{\infty} (m+1)\big(P(\mu = m, D) + P(\mu = m, F)\big) \qquad (3.3.17)$$

The repair takes place in case of degraded and failed state hence the expected CR cost can be given by

$$E(C_R) = C_{R_D} \sum_{m=0}^{\infty} E(Y_D)P(\mu = m, D) + C_{R_F} \sum_{m=0}^{\infty} E(Y_F)P(\mu = m, F) \qquad (3.3.18)$$

The expected downtime using equations (3.3.11) and (3.3.12) is obtained as

$$E(D) = I \sum_{m=0}^{\infty} P(\mu = m, F)(m+1) - \sum_{m=0}^{\infty} \int_{mI}^{(m+1)I} R(\tau)d\tau \, P(\mu = m, F) \qquad (3.3.19)$$

Hence, using equations (3.3.12), (3.3.16), (3.3.17), (3.3.18), (3.3.19) and (3.3.15), LRACR is obtained to be

$$L_c = \frac{C_{ins} \sum_{m=0}^{\infty}(m+1)\big(P(\mu=m,D)+P(\mu=m,F)\big)+C_{R_D} \sum_{m=0}^{\infty} E(Y_D)P(\mu=m,D)+C_{R_F} \sum_{m=0}^{\infty} E(Y_F)P(\mu=m \atop +C_p\left(I \sum_{m=0}^{\infty} P(\mu=m,F)(m+1)-\sum_{m=0}^{\infty} \int_{mI}^{(m+1)I} R(\tau)d\tau P(\mu=m,F)\right)}{I \sum_{m=0}^{\infty}(m+1)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty} E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty} E(Y_F)P(\mu=m,F)} \qquad (3.3.20)$$

Remark 2: For obtaining the optimal inspection period (I^*), we solve for distinct values of I, the LRACR and estimate the value of I for which the cost rate is minimum. This can be done either numerically or graphically using some online tool.

3.4 Model [4]: Availability and cost assessment of systems with dormant failure undergoing sequential inspections

In large number of the existent maintenance models, the failures are identified in no time. Meanwhile, in some realistic systems like Integrated digital communication system (**Liu *et al.*, 2013**), Remote power feeding systems (**Kojima and Asakawa, 2004**) and safety valves in protection systems (**Tang *et al.*, 2013**), failures are not self-announcing or are unrevealed. Such failures are termed dormant/hidden failures. Dormant failures mainly occur in the systems which rarely operate or in the systems with one or more units aligned in parallel with no indication of failure of each unit. Like, the identical boiler feed water pumps connected in parallel in order to receive a steady discharge.

When the system experiences dormant failures, inspection policy in general is employed to figure out if a failure has taken place or not. Different types of inspections are mentioned in the literature like continuous monitoring (**Liao *et al.*, 2006**; **Zhou *et al.*, 2006**), periodic inspections (**Chelbi *et al.*, 2008**) and sequential/non-periodic inspections (**Berrade *et al.*, 2013**; **Zhao and Nakagawa, 2015**). For the systems which require less inspections in early stage of working and more inspections as the system ages, continuous and periodic inspections generally result in higher inspection cost. Sequential inspection is feasible in such cases.

Availability has consistently remained a burning issue on the subject of reliability engineering as it is the principal characteristic of operation and design of all engineering systems. A lot of research is conducted on the availability of the systems subject to dormant failures undergoing inspections. **Sarkar and Sarkar (2000)** calculated limiting and point availability for a system with hidden failure inspected at determined calendar times. **Cui and Xie (2005)** also did identical work under age-based inspection policy by taking into account random time for repair. **Xu and Hu (2008)** investigated the limiting availability of the system undergoing both condition and time based maintenance. **Qiu and Cui (2019b)** also derived the limiting and point availability of a system with dormant failures inspected at constant age intervals. In our study sequential inspection is applied, i.e. it could be applied to the systems which are likely to fail with aging like systems which are subject to consumption (brakes, tyres), corrosion (pipelines) or erosion (hydraulic structures).

Most literatures on maintenance modeling assume negligible downtime due to repairs, replacements or inspections (**Ram *et al.*, 2013**; **Qiu *et al.*, 2019b**). However, in some cases, the time for repair and maintenance is non-negligible. Since, most systems are kept inoperable for various maintenance actions and repairs (**Qiu and Cui, 2019a**). In this model we assume the time for repair to be non-negligible.

Maintenance cost has been vastly used in government agencies and industries varying from simple tools to complex designs like large-scale telecommunication networks. For these applications, some cost parameters are customary while others are not. For example, repairable systems generally include the repair cost. Maintenance cost is considered as a significant element by many authors in their work (**Qiu *et al.*, 2018**). **Wang and Pham (2006)** investigated the maintenance cost of a series system incorporating imperfect repair. **Nourelfath and Ait-Kadi (2007)** found optimal cost of multi-state series–parallel system, in accordance with the reliability constraints. **Tian and Liao (2011)** calculated cost for a conditionally monitored system inspected at fixed intervals. **Golmakani and Moakedi (2012)** found overall cost of a system with soft and hard failure. **Singh *et al.* (2013)** calculated the cost rate of a series-parallel system. **Qiu *et al.* (2017)** found the LRACR of the system subject to dormant failure undergoing inspection at constant time differences. **Tian and Wang (2020)** found the optimal repairs of a wind turbine device on the basis of cost or availability.

Although, a lot of investigation is being done to evaluate the availability and cost rate of systems with dormant failure; undergoing periodic inspections. But no such research has yet been conducted on such systems undergoing sequential inspections. So, here we consider a perfectly repaired system with dormant failure and undergoing inspections at time $T, T + aT, T + aT + a^2T, \ldots$ where $0 < a \leq 1$, in each cycle. Since perfect repair results to a good/new state and again inspection in new cycle is conducted in a similar manner, i.e. initial inspection is after time T then at time $T + aT$ and so on. Here, we propose to study the point and limiting availability of the system undergoing aforementioned inspection and find its long-run average cost rate (LRACR). The outcome on systems cost rate and availability extracted in this study can be implemented simply to most of the systems facing dormant failures.

3.4.1 Notations

T	Initial inspection time
μ	Total inspections till first failure in a renewal cycle
$g(y)$	Repair density function
$R(\tau)$	Reliability
$S(\tau)$	Systems status at time τ
$A(\tau)$	Systems point availability
A	Systems limiting availability
U	Systems uptime in a renewal cycle
D	Systems downtime in a renewal cycle
L	Overall length of a renewal cycle
L_c	LRACR
C	Overall expense in a renewal cycle
C_{ins}	Inspection cost
C_R	Repair cost
C_p	Penalty cost due to system downtime

3.4.2 System Description

A single-unit system subject to dormant failures is considered in the present study. The considered system may be either operational or in down state. Since, failures are dormant or we can say failures are not self-revealing, so inspections are conducted to reveal the failure. Inspections are considered to be perfect, i.e. they correctly reveal whether a failure occurred or not. The system is modeled under following presumptions:

(i) Inspections are assumed to be conducted at time $T, T+aT, T+aT+a^2T, \ldots$ till its failure detection, and after repairs again it is inspected in the same manner.

(ii) Inspections are supposed to take negligible/very less time.

(iii) The time for repair is not included in the inspection interval.

(iv) At inspection if the system is found failed then a perfect corrective repair (CR) is performed otherwise no action is performed.

(v) Time interval from setting up of a new system till first CRs termination or duration in the midst of two successive terminations of CRs is defined a renew cycle.

Figure 3.4.1 gives a probable specimen of the system, where the initial inspection time is T. As shown in the Figure 3.4.1, there is no system failure in the time interval $[0, T+aT]$. First failure occurs between inspections second and third, and from then till the third inspection, system is at rest and no CR is performed and then at third inspection a CR is carried out taking some random time. After completion of the CR, the system is renewed and new cycle started.

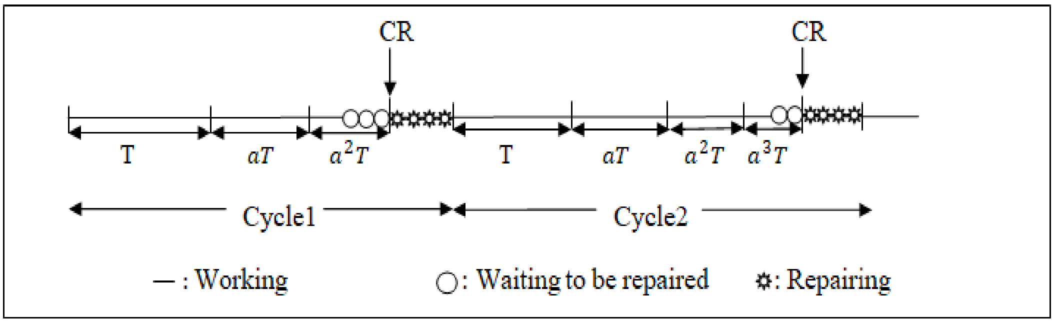


Figure 3.4.1: A probable specimen of the system under sequential inspection

3.4.3 Systems point availability

Proposition 1: The proposed systems point availability is obtained as

$$A(\tau) = \begin{cases} R(\tau), & \text{for } 0 \leq \tau \leq T \\ R(\tau) + \sum_{m=0}^{k-1} \left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] \int_0^{\tau - T\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - T\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g(y)dy, & \\ \qquad \text{for } T\left(\frac{1-a^k}{1-a}\right) \leq \tau \leq T\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2,\ldots & \end{cases}$$

Proof: We define a Markov process as follows

$$S(\tau) = \begin{cases} 0, & \text{the system is traced failed at time } \tau. \\ 1, & \text{the system is traced working at time } \tau. \end{cases}$$

Then, the point availability will be given by

$$A(\tau) = P(S(\tau) = 1) \tag{3.4.1}$$

Since, no inspection has taken place till time T. Hence,

$$A(\tau) = R(\tau) \text{ for } 0 \leq \tau \leq T \tag{3.4.2}$$

Now, for any τ in a cycle such that

$$T + aT + a^2T + \cdots + a^{k-1}T \leq \tau \leq T + aT + a^2T + \cdots + a^kT \text{ where } k = 1,2,\ldots$$

i.e., for

$$T\left(\frac{1-a^k}{1-a}\right) \leq \tau \leq T\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2,\ldots$$

There are two cases either no failure would have occurred till time τ or it would have been failed before time τ. Hence, equation (3.4.1) becomes

$$A(\tau) = R(\tau) + \sum_{m=0}^{k-1} P(S(\tau) = 1, \mu = m)$$

$$= R(\tau) + \sum_{m=0}^{k-1} P(S(\tau) = 1 | \mu = m)\, P(\mu = m) \tag{3.4.3}$$

$\{\mu = m\}$ represents that first failure occurred during $\left[T\left(\frac{1-a^m}{1-a}\right), T\left(\frac{1-a^{m+1}}{1-a}\right)\right]$

Let S denotes time for system failure. Then, the frequency function of μ satisfies

$$P(\mu = m) = P\left(S \geq T\left(\frac{1-a^m}{1-a}\right)\right) - P\left(S \geq T\left(\frac{1-a^{m+1}}{1-a}\right)\right)$$

$$= R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right) \qquad (3.4.4)$$

Also, as the failure occurs, CR takes place with repair density function $g(y)$.

$$P(S(\tau) = 1|\mu = m) = \int_0^{\tau - T\left(\frac{1-a^{m+1}}{1-a}\right)} P(S(\tau) = 1|\mu = m, Y = y)\, g(y)dy$$

$$= \int_0^{\tau - T\left(\frac{1-a^{m+1}}{1-a}\right)} P\left(S\left(\tau - T\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) = 1\right) g(y)dy$$

$$= \int_0^{\tau - T\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - T\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g(y)dy \qquad (3.4.5)$$

Hence, on substituting equations (3.4.4) and (3.4.5) in equation (3.4.3), we get

$$A(\tau) = R(\tau) + \sum_{m=0}^{k-1}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] \int_0^{\tau - T\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - T\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g(y)dy \qquad (3.4.6)$$

$$\text{for } T\left(\frac{1-a^k}{1-a}\right) \leq \tau \leq T\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2, \ldots$$

Remark 1: Evidently, our model is the extension of model considering system with dormant failure undergoing periodic inspections. Since $a = 1$ implies inspections are carried out at time $T, 2T, 3T, \ldots$ which is the case of periodic inspection.

For $a = 1$, we get the point availability as

$$A(\tau) = \begin{cases} R(\tau), & \text{for } 0 \le \tau \le T \\ R(\tau) + \sum_{m=0}^{k-1}\left[R(mT) - R\big((m+1)T\big)\right] \int_0^{\tau-(m+1)T} A(\tau - (m+1)T - y)g(y)dy, & \text{for } kT \le \tau \le (k+1)T, k = 1,2, \ldots \end{cases}$$

Special case: In the case of exponential repair times, i.e. putting $g(y) = \alpha e^{-\alpha y}$ in the above remark, we get availability for periodic inspections subject to exponential repair time as

$$A(\tau) = \begin{cases} R(\tau), & \text{for } 0 \le \tau \le T \\ R(\tau) + \sum_{m=0}^{k-1}\left[R(mT) - R\big((m+1)T\big)\right] \int_0^{\tau-(m+1)T} A(\tau - (m+1)T - y)\alpha e^{-\alpha y}dy, & \text{for } kT \le \tau \le (k+1)T, k = 1,2, \ldots \end{cases}$$

3.4.4 Systems limiting availability

The limiting availability is defined as the ratio of systems expected uptime in a renewal cycle to the expected overall length of a renewal cycle.

Proposition 2: The limiting availability of the proposed system is obtained as

$$A = \frac{\int_0^k R(\tau)d\tau}{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]}$$

where $k = \lim_{m\to\infty}\left(T\left(\frac{1-a^m}{1-a}\right)\right)$.

Proof: Let U demonstrate the systems uptime in a renewal cycle while L represent the overall cycle length. Then,

$$A = \frac{E(U)}{E(L)} \tag{3.4.7}$$

It is evident that, the systems expected uptime in a cycle equals

$$E(U) = \int_0^k R(\tau)d\tau \qquad (3.4.8)$$

where $k = \lim_{m\to\infty}\left(T\left(\frac{1-a^m}{1-a}\right)\right)$.

Now, expected total length of renewal cycle is presented by

$$E(L) = \sum_{m=0}^{\infty} E(L|\mu = m)P(\mu = m)$$

$$E(L) = \sum_{m=0}^{\infty} E(T + aT + a^2T + \cdots + a^mT + Y)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]$$

$$= \sum_{m=0}^{\infty}(T + aT + a^2T + \cdots + a^mT)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]$$

$$= T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] \qquad (3.4.9)$$

Hence, using equations (3.4.7), (3.4.8) and (3.4.9), we get limiting availability in this case as

$$A = \frac{\int_0^k R(\tau)d\tau}{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]+E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]} \qquad (3.4.10)$$

Remark 2: For periodic inspection i.e. when $a = 1$, limiting availability equals

$$A = \frac{\int_0^\infty R(\tau)d\tau}{T\sum_{m=0}^{\infty}(m+1)[R(mT) - R((m+1)T)] + E(Y)}$$

4. Systems long-run average cost rate

The LRACR is expressed as the ratio of expected overall expense in a renewal cycle to that of the expected overall length of a renewal cycle.

Proposition 3: The LRACR of the proposed system is obtained as

$$L_c = \frac{C_{ins}\left\{\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]\right\}+C_R E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + C_p\left\{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]-\int_0^k R(\tau)d\tau\right\}}{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]+E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right)-R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]}$$

Proof: Let C be the overall expense in a renewal cycle which includes the inspection cost, cost of CR and penalty cost at the time of system down time.

Then, LRACR, L_c equals

$$L_c = \frac{E(C)}{E(L)} \tag{3.4.11}$$

Here, the expected total cost is presented as

$$E(C) = C_{ins}E(N_{ins}) + E(C_R) + C_p E(D) \tag{3.4.12}$$

Since, it was assumed that $\mu + 1$ inspections were conducted in a cycle. So,

$$E(N_{ins}) = \sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] \tag{3.4.13}$$

The expected CR cost can be given by

$$E(C_R) = C_R\sum_{m=0}^{\infty}E(Y)P(\mu = m)$$

$$= C_R E(Y) \sum_{m=0}^{\infty} \left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] \tag{3.4.14}$$

The expected downtime using equations (3.4.8) and (3.4.9) is obtained as

$$E(D) = T \sum_{m=0}^{\infty} \left(\frac{1-a^{m+1}}{1-a}\right) \left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] - \int_0^k R(\tau)d\tau \tag{3.4.15}$$

Therefore, putting equations (3.4.13), (3.4.14) and (3.4.15) in equation (3.4.12), we get

$$E(C) = C_{ins}\left\{\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]\right\} + C_R E(Y) \sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + C_p\left\{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] - \int_0^k R(\tau)d\tau\right\} \tag{3.4.16}$$

Hence, using equations (3.4.9), (3.4.11) and (3.4.16), LRACR is obtained to be

$$L_c = \frac{C_{ins}\left\{\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]\right\} + C_R E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + C_p\left\{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] - \int_0^k R(\tau)d\tau\right\}}{T\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right] + E(Y)\sum_{m=0}^{\infty}\left[R\left(T\left(\frac{1-a^m}{1-a}\right)\right) - R\left(T\left(\frac{1-a^{m+1}}{1-a}\right)\right)\right]} \tag{3.4.17}$$

Remark 3: For periodic inspection ($a = 1$), the LRACR is reduced to

$$L_c = \frac{C_{ins}\{\sum_{m=0}^{\infty}(m+1)[R(mT) - R((m+1)T)]\} + C_R E(Y) + C_p\left\{T\sum_{m=0}^{\infty}(m+1)[R(mT) - R((m+1)T)] - \int_0^{\infty} R(\tau)d\tau\right\}}{T\sum_{m=0}^{\infty}(m+1)[R(mT) - R((m+1)T)] + E(Y)}$$

3.5 Model [5]: Modeling sequentially inspected system prone to degradation and shocks

With the complexity of systems/machines, the possibility of economic loss due to failures/degradation of the system/machine increases. For most of the systems, like aircrafts, medical instruments, oil/gas pipeline systems, power generating systems, their quality generally undergoes continuous degradation with time owing to various factors like, corrosion, erosion, fatigue, wear etc. (**Shafiee and Finkelstein, 2015**). Meanwhile, in several practical cases, the degradation rates are not constant, but depend on the condition/state of the system. A standard example is the propagation of cracks in metal components, where there is sharp rise in the rate of crack propagation on crack length reaching a definite level (and then the system/unit becomes defective) (**Zhang *et al.*, 2016**). **Zhang *et al.* (2016)** developed a maintenance model, where maintenance is triggered based on the degradation level and age-threshold.

Industrial systems generally experiences two modes of failure: Degradation-based failure and abrupt/sudden failure (**Qiu *et al.*, 2018**; **Zhai and Ye, 2018**). Numerous systems undergo failure on receiving a shock. For example, optical fiber could break through a transient strain (**Castilone *et al.*, 2000**). While modeling health of human, a hip might fracture by falling (**He *et al.*, 2015**). **Jiang *et al.* (2015)**, **Peng *et al.* (2010)** and **Rafiee *et al.* (2015)** studied micro-electro-mechanical system prone to sudden debris and progressive wear. **Zhou *et al.* (2016)** proposed imperfect maintenance of a rented device undergoing shocks and degradation.

However, most of the complex systems like oil pipelines and wind turbines are subjected to haphazard environmental pressures such as severe climate, stress, temperature and voltages (**Nakagawa, 2007**), which simultaneously affect their transition rates (**Levitin and Finkelstein, 2018**; **Zhao *et al.*, 2013**). Less significance has been made on the effect on degraded state by deadly shocks. This encouraged us to examine the effect of deadly shocks and degradation on system states.

Most complex systems experience hidden failures, i.e. system states are unrevealed and can be uncovered only through inspections. Inspection policies generally used are: continuous inspection (**Liao *et al.*, 2006**), periodic inspection (**Chelbi *et al.*, 2008**) and sequential/non-periodic inspection (**Berrade *et al.*, 2013**). While discussing inspection policy, considering only normal and failed state is not enough. Since inspections not only identifies the working or failed state but also determines if the system is degraded or not (**Okumura *et al.*, 1996**).

For the systems which require less inspections in early stage of working and more inspections as the system ages, continuous and periodic inspections generally result in higher inspection cost. However, sequential inspection is feasible in such cases (**Alaswad and Xiang, 2017**). **Lam and Yeh (1994)** reviewed and contrasted a sequential strategy to several continuous methods based on a Markov model. **Jiang (2010)** developed a sequential inspection system for identifying an item's status in order to avoid functional failure, in which the alert threshold inspection intervals were optimized utilizing two cost models. **Zhao *et al.* (2015)** created various approximation models for optimum strategies for inspection, maintenance and replacement; sequential maintenance plans were also created. **Zhu *et al.* (2017)** proposed a sequential inspection approach for stochastically degraded systems. **Zhao *et al.* (2020)** considered the sequential and periodic inspection optimization with mission failure probabilities.

Meanwhile, inspections should be carried out carefully; since cost usually depends on inspection, i.e. upkeep cost plays a major role when modeling a system (**Mahmoodi *et al.*, 2020**). **Wang and Pham (2006)** investigated the maintenance cost of a series system incorporating imperfect repair. **Singh *et al.* (2013)** calculated the cost rate of a series-parallel system. **Qiu *et al.* (2017)** examined the long-run average cost rate (LRACR) of the system subject to dormant failure undergoing inspection at constant time differences.

The current model is engrossed on a single-unit system prone to degradation and deadly shocks. System can be in any of the following three states: Normal, degraded and failed. Nonetheless, the systems multistate structure complicates reliability assessment and system modeling. **Lisnianski (2007)** utilized the block diagram strategy for reliability assessment of a multi-state system. **Dui *et al.* (2015)**

employed stochastic process for evaluating multi-state systems reliability. **Jafary and Fiondella (2016)** used universal generating function for assessing the reliability of multi-state system with correlated failures. Meanwhile, multilevel inspection data was used by **Liu and Chen (2017)** for reliability assessment of a non-repairable multistate system.

A newer inspection policy is proposed for the above defined three-state single-unit system with the purpose of minimizing the loss incurred by degradation and deadly shocks. Inspections are done at times $I, I + aI, I + aI + a^2I, \ldots$ where $0 < a \leq 1$.

The purposes of inspections are:

a) Checking the state in which system lies.

b) Disclosing the failure-type (owing to shock/degradation).

If system is found partially/completely failed at inspection, the system is immediately renewed. The theoretical/research contribution of the current model involves:

(iv) Newer model of maintenance/inspection policy is developed based on minimizing the loss due to system degradation and deadly shocks.

(v) The effect of degradation along with shocks on the system state is examined.

(vi) Concise results on reliability, availabilities (point and limiting) and LRACR of the proposed model are presented considering both the cases of random and constant repair time.

3.5.1 Notations

I	Initial inspection period
μ	Total inspections till first failure in a renewal cycle
$F_D(\tau)/F_F(\tau)$	Probability of occurrence of partial/complete failure
y_D/y_F	Constant repair time of degraded/completely failed state

Y_D/Y_F	Random repair time of degraded/completely failed state
$g_D(y)/g_F(y)$	Density function of Y_D/Y_F
$R_{nor}(\tau)$	Systems reliability of staying in normal state
$R_F(\tau)$	Systems reliability
$K(\tau)$	Systems status at time τ
$A(\tau)$	Systems point availability
A	Systems limiting availability
U	Systems uptime in a renewal cycle
D	Systems downtime in a renewal cycle
L	Overall length of a renewal cycle
C	Overall expense in a renewal cycle
C_{ins}	Cost per inspection
$C_{R_{D/F}}$	Cost of repair of a partial/complete failure
C_p	Penalty cost

3.5.2 Model Description

(i) A repairable single-unit system is considered.

(ii) System is prone to both degradation and shocks.

(iii) The system can be in any of three states: Normal, Degraded and failed.

(iv) If system enters degraded state it is said to be partially failed.

(v) System states are detected through inspections only.

(vi) Inspections are presumed to be perfect and considered to take negligible/very less time.

(vii) The system is inspected at times $I, I + aI, I + aI + a^2I, \ldots$ where a is assumed to lie in the interval (0,1].

(viii) System is repaired when it is found degraded or failed, i.e., in the case of partial or complete failure.

(ix) Repairs are assumed to be perfect i.e. system becomes new on repair.

(x) The time for repair is not included in the inspection interval.

(xi) Time interval from setting up of a new system till first repair termination or duration in the midst of two successive terminations of repair is defined a renew cycle.

(xii) *Failure owing to degradation*:

Let the system enters the degraded state from normal state with rate λ_1. Since, the degraded state generally quickens the speed of deterioration so the rate with which system enters failed state from degraded state is assumed to be $\lambda_2(\lambda_2 > \lambda_1)$.

(xiii) *Failure owing to shock*:

The system is also prone to deadly shocks, which may lead to the failure of the system. Deadly shocks means system fails instantly on arrival of shock. Let the rate at which system enters failed state from normal state be λ_3. Since, the degraded system is more prone to shocks hence the system is assumed to enter the failed state from degraded state due to shocks at rate $\lambda_4(\lambda_4 > \lambda_3)$.

3.5.3 Reliability Modeling

Proposition1: The reliability of above defined system is obtained to be

$$R_F(\tau) = \left(1 - e^{-\lambda_1 \tau}\right)\left(e^{-\lambda_2 \tau} + e^{-\lambda_4 \tau}\right) + e^{-\lambda_1 \tau} e^{-\lambda_3 \tau}$$

Proof: We define a Markov process as

$$K(\tau)=\begin{cases}0, & \text{when the system is in normal state at time } \tau.\\ 1, & \text{when the system is in degraded state at time } \tau.\\ 2, & \text{when the system is under complete failure at time } \tau.\end{cases}$$

Reliability of the system staying in normal state, denoted by $R_{nor}(\tau)$ is equal to the probability that system has neither degraded nor failed till time τ.

i.e. $R_{nor}(\tau) = P(\text{system has neither entered state 1 nor state 2 from state 0})$.

Since, the event of entering state 1 and state 2 from state 0 are mutually exclusive. Therefore,

$$R_{nor}(\tau) = P(\text{system has not entered state 1 from state 0 till time } \tau)$$
$$\times P(\text{system has not entered state 2 from state 0 till time } \tau\ )$$
$$= \int_{\tau}^{\infty} \lambda_1 e^{-\lambda_1 t} dt \int_{\tau}^{\infty} \lambda_3 e^{-\lambda_3 t} dt$$

$$= e^{-\lambda_1 \tau} e^{-\lambda_3 \tau} \tag{3.5.1}$$

The probability of occurrence of partial failure before time τ denoted by $F_D(\tau)$ is given by

$$F_D(\tau)$$
$$= P(\text{system has entered state 1 from state 0 till time } \tau)$$
$$\times P(\text{system has not entered state 2 from state 1 due to degradation till time } \tau)$$
$$+ P(\text{system has entered state 1 from state 0 till time } \tau)$$
$$\times P(\text{system has not entered state 2 from state 1 due to shock till time } \tau)$$
$$= \int_{0}^{\tau} \lambda_1 e^{-\lambda_1 t} dt \int_{\tau}^{\infty} \lambda_2 e^{-\lambda_2 t} dt + \int_{0}^{\tau} \lambda_1 e^{-\lambda_1 t} dt \int_{\tau}^{\infty} \lambda_4 e^{-\lambda_4 t} dt$$
$$= \left(1 - e^{-\lambda_1 \tau}\right) e^{-\lambda_2 \tau} + \left(1 - e^{-\lambda_1 \tau}\right) e^{-\lambda_4 \tau}$$

$$= \left(1 - e^{-\lambda_1 \tau}\right)\left(e^{-\lambda_2 \tau} + e^{-\lambda_4 \tau}\right) \tag{3.5.2}$$

Let probability of occurrence of complete failure before time τ be given by $F_F(\tau)$.

Then,

$$R_{nor}(\tau) = 1 - F_D(\tau) - F_F(\tau)$$

So, $F_F(\tau)$ becomes

$$F_F(\tau) = 1 - F_D(\tau) - R_{nor}(\tau)$$

$$= 1 - \left(1 - e^{-\lambda_1\tau}\right)\left(e^{-\lambda_2\tau} + e^{-\lambda_4\tau}\right) - e^{-\lambda_1\tau}e^{-\lambda_3\tau} \tag{3.5.3}$$

Hence, Reliability of the system, i.e. probability that system is operational till time τ equals

$$R_F(\tau) = 1 - F_F(\tau)$$

$$= \left(1 - e^{-\lambda_1\tau}\right)\left(e^{-\lambda_2\tau} + e^{-\lambda_4\tau}\right) + e^{-\lambda_1\tau}e^{-\lambda_3\tau} \tag{3.5.4}$$

3.5.4 Systems Point-availability ($A(\tau)$)

3.5.4.1 Systems point-availability under constant repair times

Proposition 2: The point availability of the aforementioned system given that it takes constant time y_D/y_F if system is found partially/completely failed is obtained as

$A(\tau) = R_F(\tau)$ for $0 \le \tau \le I$ and

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) A\left(\tau - I\left(\frac{1 - a^{m+1}}{1 - a}\right) - y_D\right)$$
$$+ \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F) A\left(\tau - I\left(\frac{1 - a^{m+1}}{1 - a}\right) - y_F\right)$$
$$\text{for } I\left(\frac{1 - a^k}{1 - a}\right) \le \tau \le I\left(\frac{1 - a^{k+1}}{1 - a}\right), k = 1,2, \ldots$$

where

$$P(\mu = m, D) = F_D\left(I\left(\frac{1 - a^{m+1}}{1 - a}\right)\right) - F_D\left(I\left(\frac{1 - a^m}{1 - a}\right)\right)$$

and

$$P(\mu = m, F) = F_F\left(I\left(\frac{1-a^{m+1}}{1-a}\right)\right) - F_F\left(I\left(\frac{1-a^{m}}{1-a}\right)\right).$$

Proof: System is operational at any time τ means either it is in normal state or degraded state. Hence, the systems point availability may be given as

$$A(\tau) = \mathrm{P}(K(\tau) = 0 \text{ or } K(\tau) = 1)$$

Since, the events $\{ K(\tau) = 0\}$ and $\{ K(\tau) = 1\}$ are mutually exclusive so,

$$A(\tau) = P(K(\tau) = 0) + P(K(\tau) = 1)$$

Let us denote $P(K(\tau) = 0)$ by $A_0(\tau)$ and $P(K(\tau) = 1)$ by $A_1(\tau)$. So,

$$A(\tau) = A_0(\tau) + A_1(\tau) \tag{3.5.5}$$

Since, no inspection has taken place till time I. Hence,

$$A(\tau) = R_F(\tau) \text{ for } 0 \leq \tau \leq I \tag{3.5.6}$$

Now, for any τ in a cycle such that

$$I + aI + a^2 I + \cdots + a^{k-1} I \leq \tau \leq I + aI + a^2 I + \cdots + a^k I, \text{where } k = 1,2,\ldots$$

i.e., for

$$I\left(\frac{1-a^k}{1-a}\right) \leq \tau \leq I\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2,\ldots$$

Firstly we derive $A_0(\tau)$

$$\begin{aligned} A_0(\tau) &= P(K(\tau) = 0) \\ &= \sum_{m=0}^{\infty} P(K(\tau) = 0, \mu = m) \end{aligned}$$

Here, $\{\mu = m\}$ represents that total m inspections were conducted before a failure occurred and first failure (complete/partial) occurred during the time $\left[I\left(\frac{1-a^m}{1-a}\right), I\left(\frac{1-a^{m+1}}{1-a}\right)\right]$

$$A_0(\tau) = \sum_{m=0}^{\mathrm{k}-1} P(K(\tau) = 0, \mu = m) + \sum_{m=\mathrm{k}}^{\infty} P(K(\tau) = 0, \mu = m)$$

$$= \sum_{m=0}^{\mathrm{k}-1} P(K(\tau) = 0 | \mu = m, D)\, P(\mu = m, D)$$

$$+ \sum_{m=0}^{\mathrm{k}-1} P(K(\tau) = 0 | \mu = m, F)\, P(\mu = m, F)$$

$$+ \sum_{m=\mathrm{k}}^{\infty} P(K(\tau) = 0, \mu = m) \tag{3.5.7}$$

The third term of equation (3.5.7) represents that no degradation/complete failure takes place before time τ and hence it is equivalent to

$$\sum_{m=\mathrm{k}}^{\infty} P(K(\tau) = 0, \mu = m) = 1 - F_D(\tau) - F_F(\tau) \tag{3.5.8}$$

Now we derive the first and second term of equation (3.5.7). First failure can be a partial/ complete failure. Hence, we have

$$P(\mu = m, D) = F_D\left(I\left(\frac{1 - a^{m+1}}{1 - a}\right)\right) - F_D\left(I\left(\frac{1 - a^{m}}{1 - a}\right)\right)$$

and

$$P(\mu = m, F) = F_F\left(I\left(\frac{1 - a^{m+1}}{1 - a}\right)\right) - F_F\left(I\left(\frac{1 - a^{m}}{1 - a}\right)\right)$$

Since, perfect repair taking constant time y_D/y_F is carried out if system is found partially/completely failed. Hence, we have

$$P(K(\tau) = 0 | \mu = m, D) = A_0\left(\tau - I\left(\frac{1 - a^{m+1}}{1 - a}\right) - y_D\right) \tag{3.5.9}$$

and

$$P(K(\tau) = 0 | \mu = m, D) = A_0\left(\tau - I\left(\frac{1 - a^{m+1}}{1 - a}\right) - y_F\right) \tag{3.5.10}$$

Therefore, using equations (3.5.7), (3.5.8), (3.5.9) and (3.5.10), we have

$$A_0(\tau) = 1 - F_D(\tau) - F_F(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_D\right) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F)\, A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_F\right) \tag{3.5.11}$$

Similarly, $A_1(\tau)$ is obtained to be

$$A_1(\tau) = F_D(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_D\right) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F)\, A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_F\right) \tag{3.5.12}$$

Hence, putting equations (3.5.11) and (3.5.12) in equation (3.5.5), we get the point availability as

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_D\right) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F) A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y_F\right) \tag{3.5.13}$$

$$\text{for } I\left(\frac{1-a^k}{1-a}\right) \le \tau \le I\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2, \ldots$$

3.5.4.2 Systems point-availability under random repair times

Proposition 3: The point availability of the above defined system given that it takes random time Y_D/Y_F if system is found partially/completely failed is obtained as

$A(\tau) = R_F(\tau)$ for $0 \le \tau \le I$ and

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{k-1} P(\mu = m, D) \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_D(y)dy$$

$$+ \sum_{m=0}^{k-1} P(\mu = m, F) \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_F(y)dy$$

$$\text{for } I\left(\frac{1-a^{k}}{1-a}\right) \le \tau \le I\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2,\ldots$$

Proof: Since, perfect repair taking random time Y_D/Y_F is carried out if system is found partially/completely failed. Hence, equation (3.5.9) is modified to

$$P(K(\tau) = 0|\mu = m, D) = \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} P(K(\tau) = 0|\mu = m, D, Y_D = y)\, g_D(y)dy$$

$$= \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} P\left(K\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) = 0\right) g_D(y)dy$$

$$= \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_D(y)dy \qquad (3.5.14)$$

Similarly, equation (3.5.10) becomes

$$P(K(\tau) = 0|\mu = m, F) = \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} P(K(\tau) = 0|\mu = m, D, Y_F = y)\, g_F(y)dy$$

$$= \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} P\left(K\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) = 0\right) g_F(y)dy$$

$$= \int_{0}^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_F(y)dy \qquad (3.5.15)$$

Hence, using equations (3.5.7), (3.5.8), (3.5.14) and (3.5.15), we have

$$\begin{aligned} A_0(\tau) = 1 - F_D(\tau) - F_F(\tau) \\ + \sum_{m=0}^{\mathrm{k}-1} P(\mu \\ = m, D) \int_{0}^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_D(y)dy \\ + \sum_{m=0}^{\mathrm{k}-1} P(\mu \\ = m, F) \int_{0}^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_0\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_F(y)dy \end{aligned} \qquad (3.5.16)$$

Similarly, $A_1(\tau)$ is obtained to be

$$\begin{aligned} A_1(\tau) = F_D(\tau) \\ + \sum_{m=0}^{\mathrm{k}-1} P(\mu \\ = m, D) \int_{0}^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_1\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_D(y)dy \\ + \sum_{m=0}^{\mathrm{k}-1} P(\mu \\ = m, F) \int_{0}^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A_1\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_F(y)dy \end{aligned} \qquad (3.5.17)$$

Now, putting equations (3.5.16) and (3.5.17) in equation (3.5.5), we get the point availability as

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{k-1} P(\mu = m, D) \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_D(y)dy + \sum_{m=0}^{k-1} P(\mu = m, F) \int_0^{\tau - I\left(\frac{1-a^{m+1}}{1-a}\right)} A\left(\tau - I\left(\frac{1-a^{m+1}}{1-a}\right) - y\right) g_F(y)dy \qquad (3.5.18)$$

$$\text{for } I\left(\frac{1-a^k}{1-a}\right) \le \tau \le I\left(\frac{1-a^{k+1}}{1-a}\right), k = 1,2,\ldots$$

Remark 1: Evidently, our model is the extension of model considering periodically inspected single-unit system prone to degradation and shock. Since $a = 1$ implies inspections are carried out at time $T, 2T, 3T, \ldots$ which is the case of periodic inspection.

For $a = 1$,

$$P(\mu = m, D) = F_D\big((m+1)I\big) - F_D(mI)$$

and

$$P(\mu = m, F) = F_F\big((m+1)I\big) - F_F(mI)$$

Hence, we get the point availability as:

- In case of constant repair time, point-availability equals

$A(\tau) = R_F(\tau)$ for $0 \le \tau \le I$ and

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) A(\tau - (m+1)\mathrm{I} - y_D)$$
$$+ \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F) A(\tau - (m+1)\mathrm{I} - y_F)$$
$$\text{for } kI \le \tau \le (k+1)I, k = 1,2, \ldots$$

- In case of random repair time, point-availability equals

$A(\tau) = R_F(\tau)$ for $0 \le \tau \le I$ and

$$A(\tau) = R_F(\tau) + \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, D) \int_0^{\tau-(m+1)I} A(\tau - (m+1)I - y) g_D(y) dy$$
$$+ \sum_{m=0}^{\mathrm{k}-1} P(\mu = m, F) \int_0^{\tau-(m+1)I} A(\tau - (m+1)I - y) g_F(y) dy$$
$$\text{for } kI \le \tau \le (k+1)I, k = 1,2, \ldots$$

3.5.5 Systems Limiting availability (A)

3.5.5.1 Systems limiting availability under constant repair times

Proposition 4: The limiting availability of the proposed system given that system takes constant time y_D/y_F if system is found partially/completely failed is obtained as

$$A = \frac{I \sum_{m=0}^{\infty} \left(\frac{1-a^{m+1}}{1-a}\right) P(\mu=m,D) + \sum_{m=0}^{\infty} P(\mu=m,F) \int_0^k R_F(\tau) d\tau}{I \sum_{m=0}^{\infty} \left(\frac{1-a^{m+1}}{1-a}\right) (P(\mu=m,D) + P(\mu=m,F)) + y_D \sum_{m=0}^{\infty} P(\mu=m,D) + y_F \sum_{m=0}^{\infty} P(\mu=m,F)}$$

where $k = \lim_{m\to\infty} \left(I \left(\frac{1-a^m}{1-a} \right) \right)$.

Proof: Limiting availability is the ratio of expected uptime to expected overall length of a cycle. So, we find both the length and uptime expectation.

Let T_1 and T_2 be time for first inspected degradation/complete failure in a renewal cycle and $T_1{}^* \; and \; T_2{}^*$ be the exact time of first degradation/complete failure in a cycle.

Clearly, $T_1 = I \left(\frac{1-a^{m+1}}{1-a} \right)$.

Then, uptime(U) of the system equals

$$U = \begin{cases} I\left(\frac{1-a^{m+1}}{1-a}\right), & I\left(\frac{1-a^{m}}{1-a}\right) < T_1{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right) \\ T_2{}^*, & I\left(\frac{1-a^{m}}{1-a}\right) < T_2{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right) \end{cases}$$

So, expected uptime equals

$$E(U) = \sum_{m=0}^{\infty} E\left(U \middle| \mu = m, I\left(\frac{1-a^{m}}{1-a}\right) < T_1{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right) P\left(\mu = m, I\left(\frac{1-a^{m}}{1-a}\right) < T_1{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)$$
$$+ \sum_{m=0}^{\infty} E\left(U \middle| \mu = m, I\left(\frac{1-a^{m}}{1-a}\right) < T_2{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right) P\left(\mu = m, I\left(\frac{1-a^{m}}{1-a}\right) < T_2{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)$$
$$= \sum_{m=0}^{\infty} E\left(I\left(\frac{1-a^{m+1}}{1-a}\right)\right) P(\mu = m, D) + \sum_{m=0}^{\infty} E(\ T_2{}^*) P(\mu = m, F)$$

$$= I \sum_{m=0}^{\infty} \left(\frac{1-a^{m+1}}{1-a}\right) P(\mu = m, D) + \sum_{m=0}^{\infty} P(\mu = m, F) \int_0^k R_F(\tau) d\tau \qquad (3.5.19)$$

where $k = \lim_{m\to\infty} \left(I\left(\frac{1-a^{m}}{1-a}\right)\right)$.

Now, we derive the expression for expected length of a cycle. Let L denotes the cycle length. Then,

$$L = \begin{cases} I\left(\frac{1-a^{m+1}}{1-a}\right) + y_D, & I\left(\frac{1-a^{m}}{1-a}\right) < T_1{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right) \\ I\left(\frac{1-a^{m+1}}{1-a}\right) + y_F, & I\left(\frac{1-a^{m}}{1-a}\right) < T_1{}^* \leq I\left(\frac{1-a^{m+1}}{1-a}\right) \end{cases}$$

Hence, expected length equals

$$E(L)=\sum_{m=0}^{\infty}E\left(L\middle|\mu=m,I\left(\frac{1-a^{m}}{1-a}\right)<T_1^{*}\leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)P\left(\mu\right.$$

$$\left.=m,I\left(\frac{1-a^{m}}{1-a}\right)<T_1^{*}\leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)$$

$$+\sum_{m=0}^{\infty}E\left(L\middle|\mu=m,I\left(\frac{1-a^{m}}{1-a}\right)<T_2^{*}\leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)P\left(\mu\right.$$

$$\left.=m,I\left(\frac{1-a^{m}}{1-a}\right)<T_2^{*}\leq I\left(\frac{1-a^{m+1}}{1-a}\right)\right)$$

$$=\sum_{m=0}^{\infty}E\left(I\left(\frac{1-a^{m+1}}{1-a}\right)+y_D\right)P(\mu=m,D)$$

$$+\sum_{m=0}^{\infty}E\left(I\left(\frac{1-a^{m+1}}{1-a}\right)+y_F\right)P(\mu=m,F)$$

$$=I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))$$

$$+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F) \qquad (3.5.20)$$

Now, using equations (3.5.19) and (3.5.20), limiting availability equals

$$A=\frac{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)+\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^k R_F(\tau)d\tau}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F)} \qquad (3.5.21)$$

3.5.5.2 Systems limiting availability under random repair times

Proposition 5: The limiting availability of the proposed system given that system takes random time Y_D/Y_F if system is found partially/completely failed is obtained as

$$A=\frac{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)+\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^k R_F(\tau)d\tau}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

where $k = \lim_{m\to\infty}\left(I\left(\frac{1-a^m}{1-a}\right)\right)$.

Proof: The expression for length of a cycle changes in this case and is equivalent to

$$L = \begin{cases} I\left(\dfrac{1-a^{m+1}}{1-a}\right) + Y_D, & I\left(\dfrac{1-a^m}{1-a}\right) < {T_1}^* \le I\left(\dfrac{1-a^{m+1}}{1-a}\right) \\ I\left(\dfrac{1-a^{m+1}}{1-a}\right) + Y_F, & I\left(\dfrac{1-a^m}{1-a}\right) < {T_1}^* \le I\left(\dfrac{1-a^{m+1}}{1-a}\right) \end{cases}$$

Therefore, expected length equals

$$\begin{aligned} E(L) = \sum_{m=0}^{\infty} E&\left(L \middle| \mu = m, I\left(\frac{1-a^m}{1-a}\right) < {T_1}^* \le I\left(\frac{1-a^{m+1}}{1-a}\right)\right) P\left(\mu \right.\\ &\left.= m, I\left(\frac{1-a^m}{1-a}\right) < {T_1}^* \le I\left(\frac{1-a^{m+1}}{1-a}\right)\right) \\ &+ \sum_{m=0}^{\infty} E\left(L \middle| \mu = m, I\left(\frac{1-a^m}{1-a}\right) < {T_2}^* \le I\left(\frac{1-a^{m+1}}{1-a}\right)\right) P\left(\mu \right.\\ &\left.= m, I\left(\frac{1-a^m}{1-a}\right) < {T_2}^* \le I\left(\frac{1-a^{m+1}}{1-a}\right)\right) \\ = \sum_{m=0}^{\infty} E&\left(I\left(\frac{1-a^{m+1}}{1-a}\right) + Y_D\right) P(\mu = m, D) \\ &+ \sum_{m=0}^{\infty} E\left(I\left(\frac{1-a^{m+1}}{1-a}\right) + Y_F\right) P(\mu = m, F) \end{aligned}$$

$$\begin{aligned} = I \sum_{m=0}^{\infty} &\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu = m, D) + P(\mu = m, F)) \\ &+ \sum_{m=0}^{\infty} E(Y_D) P(\mu = m, D) + \sum_{m=0}^{\infty} E(Y_F) P(\mu = m, F) \qquad (3.5.22) \end{aligned}$$

Hence, using equations (3.5.19) and (3.5.22), limiting availability for random repair time equals

$$A = \frac{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)+\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^k R_F(\tau)d\tau}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)} \quad (3.5.23)$$

Remark 2: For periodic inspection i.e., when $a = 1$, $k = \lim_{m\to\infty}\left(I\left(\frac{1-a^m}{1-a}\right)\right)$ tends to ∞.

Hence, limiting availability is obtained as

- In case of constant repair time, limiting availability of the system using equation (3.5.21) equals

$$A = \frac{I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D)+\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{\infty}R_F(\tau)d\tau}{I\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F)}$$

- In case of random repair time, systems limiting availability using equation (3.5.23) is equal to

$$A = \frac{I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D)+\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{\infty}R_F(\tau)d\tau}{I\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

3.5.6 Systems LRACR (L_c)

3.5.6.1 Systems LRACR under constant repair times

Proposition 6: The LRACR of the proposed system under the condition that system undergoes repair of constant times is obtained as

$$L_c = \frac{\begin{gathered}C_{ins}\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+C_{R_D}y_D\sum_{m=0}^{\infty}P(\mu=m,D)+C_{R_F}y_F\sum_{m=0}^{\infty}P(\mu=m,F)\\ +C_p\left(I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))-I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^k R_F(\tau)d\tau\right)\end{gathered}}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F)}$$

Proof: The LRACR is the ratio of expected overall expense in a cycle to expected overall length of a cycle.

Let C be the overall expense in a renewal cycle which includes the inspection cost, cost of repair and penalty cost due to system down time.

Then, LRACR (L_c) equals

$$L_c = \frac{E(C)}{E(L)} \tag{3.5.24}$$

Here, the expected total cost is expressed as

$$E(C) = C_{ins}E(\mu + 1) + E(C_R) + C_p E(D) \tag{3.5.25}$$

Since, it is considered that $\mu + 1$ inspections are conducted in a cycle, so

$$E(\mu + 1) = \sum_{m=0}^{\infty} \left(\frac{1 - a^{m+1}}{1 - a}\right) (P(\mu = m, D) + P(\mu = m, F)) \tag{3.5.26}$$

The repair takes place in case of degraded and failed state taking times y_D and y_F respectively. Hence, the expected repair cost can be given by

$$E(C_R) = C_{R_D} y_D \sum_{m=0}^{\infty} P(\mu = m, D) + C_{R_F} y_F \sum_{m=0}^{\infty} P(\mu = m, F) \tag{3.5.27}$$

The expected downtime using equations (3.5.19) and (3.5.20) is obtained as

$$\begin{aligned} E(D) = I \sum_{m=0}^{\infty} \left(\frac{1 - a^{m+1}}{1 - a}\right) \big(P(\mu = m, D) + P(\mu = m, F)\big) \\ - I \sum_{m=0}^{\infty} \left(\frac{1 - a^{m+1}}{1 - a}\right) P(\mu = m, D) \\ - \sum_{m=0}^{\infty} P(\mu = m, F) \int_0^k R_F(\tau) d\tau \end{aligned} \tag{3.5.28}$$

Finally, using equations (3.5.15), (3.5.17), (3.5.18), (3.5.19), (3.5.20) and (3.5.21), LRACR is obtained to be

$$L_c = \frac{C_{ins}\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+C_{R_D}y_D\sum_{m=0}^{\infty}P(\mu=m,D)+C_{R_F}y_F\sum_{m=0}^{\infty}P(\mu= \\ +C_p\left(I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))-I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{k}\right.}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F)} \quad (3.5.29)$$

3.5.6.2 Systems LRACR under random repair times

Proposition 7: The LRACR of the proposed system undergoing repairs of random times is expressed as

$$L_c = \frac{C_{ins}\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+C_{R_D}\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+C_{R_F}\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F) \\ +C_p\left(I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))-I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{k}R_F(\tau)d\tau\right)}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

Proof: Since, the repair taking random time Y_D and Y_F is conducted in case of degraded and failed state. Hence, the expected repair cost can be given by

$$E(C_R) = C_{R_D}\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+C_{R_F}\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F) \quad (3.5.30)$$

The expression for expected downtime and $E(\mu+1)$ are same as obtained in above case. Hence, LRACR using equations (3.5.22), (3.5.24), (3.5.25), (3.5.26), (3.5.28) and (3.5.30) is obtained to be

$$L_c = \frac{C_{ins}\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+C_{R_D}\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+C_{R_F}\sum_{m=0}^{\infty}E(Y_F)P(\\ +C_p\left(I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))-I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{k}\right.}{I\sum_{m=0}^{\infty}\left(\frac{1-a^{m+1}}{1-a}\right)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F} \quad (3.5.31)$$

Remark 3: For periodic inspection ($a = 1$), the LRACR is reduced to the following

- For the case of constant repair times.

LRACR using equation (3.5.29) will be obtained as

$$L_c = \frac{\begin{array}{c} C_{ins}\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]+C_{R_D}y_D\sum_{m=0}^{\infty}P(\mu=m,D)+C_{R_F}y_F\sum_{m=0}^{\infty}P(\mu=m,F) \\ +C_p\left(I\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]-I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{\infty}R_F(\tau)d\tau\right) \end{array}}{I\sum_{m=0}^{\infty}(m+1)(P(\mu=m,D)+P(\mu=m,F))+y_D\sum_{m=0}^{\infty}P(\mu=m,D)+y_F\sum_{m=0}^{\infty}P(\mu=m,F)}$$

- In case of random repair times.

LRACR is obtained using equation (3.5.31) as follows

$$L_c = \frac{\begin{array}{c} C_{ins}\{\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]\}+C_{R_D}\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+C_{R_F}\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F) \\ +C_p\left\{I\sum_{m=0}^{\infty}(m+1)[P(\mu=m,D)+P(\mu=m,F)]-I\sum_{m=0}^{\infty}(m+1)P(\mu=m,D)-\sum_{m=0}^{\infty}P(\mu=m,F)\int_0^{\infty}R_F(\tau)d\tau\right\} \end{array}}{I\sum_{m=0}^{\infty}(m+1)(P(\mu=m,D)+P(\mu=m,F))+\sum_{m=0}^{\infty}E(Y_D)P(\mu=m,D)+\sum_{m=0}^{\infty}E(Y_F)P(\mu=m,F)}$$

3.6 Model [6]: Modeling systems with revealing and non-revealing failures undergoing periodic inspection

Numerous studies are being conducted on randomly failing systems undergoing inspections (**Valdez-Flores and Feldman, 1989**; **Sarkar and Sarkar, 2000**). A vast number of inspection models aim at finding the inspection times during which the systems need to be inspected, optimizing the overall cost or the availability of the system (**Tian and Liao, 2011**; **Qiu *et al.*, 2017**).

Inspection models are classified based on: Effect of inspection on system (whether inspection affects the degradation process of unit or not (**Chou and Butler, 1983**)), inspection quality (whether inspections are perfect or imperfect), inspection frequency (whether inspections are periodic or non-periodic), the system structure (whether system is single-unit or multi-unit) and systems failure type (whether failure is revealing or non-revealing) and lastly the evaluation criteria chosen (overall expected cost or availability).

The type of failure plays a crucial role while dealing with the inspection models. There are situations where the unit may have some invisible malfunction but the same is not revealed to the server during the operation. These are called non-revealing failures. While, the revealing failures are abundantly clear. Speaking in terms of the performance, the system might seem to work correctly in case of non-revealing failure but the system is having some hidden defect which is discovered by some special testing only. For example, in the event of an instrument measuring high pressure of a security monitoring system, sometimes the pressure might exceed its limit due to the hidden failure of the instrument, which will be disclosed only by some proof-test (**Smith, 2017**).

For revealing failures, i.e. for failures that are detected on their occurrence; in such cases repair is initiated immediately. While for non-revealing failure, the failure is not discovered unless the system is inspected and repair is initiated on their detection only. The non-revealing failure usually occurs in standby units or those equipments which rarely operate like the flat backup tire in a truck is noticed only when it is needed for operation (**Keles *et al.*, 2017**). Another example is of a pressure switch, which is used to spot if oil pressure reaches a hazardous point. In order to detect the failure of switch, oil pressure is dropped below unsafe level and then it is detected there may be the case that pressure switch is failed (revealing failure) or might not show correct result (non-revealing failure). Most authors took systems with non-revealing failures into account in their studies (**Wortman *et al.*, 1994**). **Chelbi and Ait-Kadi (2000)** calculated the limiting availability of the system with non-revealing failures inspected at some predetermined intervals. **Tang *et al.* (2013)** evaluated the availability for systems with non-revealing failures undergoing inspections periodically.

Meanwhile, some authors examined both revealing and non-revealing failures in their work (**Adachi and Nishida, 1981**; **Phillips, 1981**). **Gertsbakh (1977)** considered a situation where failures are at times revealing and at some instances failures are non-revealing. As in the case of a computer, the computational results might seem reasonable, despite the fact that they are wrong because of the hidden defect, which is revealed through special checking. **Goel and Mumtaz (1994)** analyzed a two-unit system undergoing revealing and non-revealing failures with correlated failure and repair times. **Baohe (2002)** considered a multi-mode system undergoing periodic inspections having both revealing and non-revealing failures.

Several works have been done on imperfect inspections resulting in undetected failure or false alerts (**Kaio and Osaki, 1986**; **Sahraoui *et al.*, 2013**). **Badia *et al.* (2001)** presented an inspection policy for a one-unit system with non-revealing failures undergoing imperfect inspections. **Bukowski (2001)** considered both perfect and imperfect repairs and inspections in his study. Closed-form solutions for average availability, MTTF, and mean probability of failing dangerously were derived. **Gertsbakh (2013)** presented a fault sensing device that may result in imperfect signaling.

An efficient inspection/maintenance policy holds a very important role in reducing the chances of system failure. For various complex systems, like satellite systems, availability holds larger importance than the upkeep cost (**Berenguer *et al.*, 2003**; **Khatab *et al.*, 2014**). In many cases, achieving the extra availability could result in extra spending. However, it should be noted that over-spending in achieving higher availability may not always be beneficial and can also result in net losses. So, for such cases the optimization is done based on cost. The optimization work based on the cost could be viewed in **Peng *et al.* (2009)** and **Pant *et al.* (2015)**. **Badia *et al.* (2002)** determined an imperfect inspection policy for a one-unit system undergoing both revealing and non-revealing failures incorporating maintenance irrespective of failure status. In this study, authors dealt only with the cost function of the system. **Chelbi *et al.* (2008)** evaluated the availability function of the one-unit system undergoing both revealing and non-revealing failures under the imperfect inspection policy incorporating maintenance at each inspection.

As done in **Badia *et al.* (2002)** and **Chelbi *et al.* (2008)**, the repair was conducted even if the system was working, such type of actions result in utilization of time and resources resulting in greater cost. Above discussions reveal that no work has yet been undertaken to calculate both the availability and cost function of one-unit system undergoing both revealing and non-revealing failure, subject to imperfect inspections including maintenance only when system is detected failed. To bridge this gap, an inspection policy for a one-unit system under aforementioned conditions is proposed in this model. An optimal inspection strategy based on minimizing the cost or maximizing the availability for such systems is also developed here.

The key contribution of the current model involves:

- Newer inspection model is developed based on revealing and non-revealing failures of single-unit system.
- Concise results on availability (limiting) and long-run average cost rate (LRACR) of the proposed model are presented considering significant inspection and repair time.
- Optimal inspection scheme is also developed relative to availability and LRACR.
- A descriptive example of an electric motor is considered to explain the derived results.

3.6.1 *Notations*

X	Systems failure time
$f(u)$	Density function of X
$R(u)$	Systems survivor function
a	Probability of the non-revealing failure
α	Probability of getting a false alert
β	Probability of unobserved failure
T_i	Mean inspection duration
T_c	Mean corrective maintenance duration
T_f	Mean system downtime duration because of false alerts
$\mathcal{N}_1$	Number of inspections prior to failure
$\mathcal{N}_2$	Number of inspections post-failure until detection
$\mathcal{N}_f$	Number of false alerts

I	Periodic inspection period
I^*	Optimal inspection period
$A(I)$	Systems limiting availability
$U(I)/D(I)$	Systems mean uptime/downtime in a renewal cycle
$U_r(I)/D_r(I)$	Systems mean uptime/downtime in case of revealing failure
$U_{nr}(I)/D_{nr}(I)$	Systems mean uptime/downtime in case of non-revealing failure
H	Duration between occurrence and detection of failure
$C(I)$	Mean overall expense in a cycle
$C_r(I)/C_{nr}(I)$	Mean overall expense in a cycle for revealing/non-revealing failure
C_i	Cost per inspection
C_c	Cost of corrective maintenance
C_f	Cost due to false alarm
C_d	Cost due to down-time
$L(I)$	LRACR

3.6.2 Model Description

(1) A single-unit system is considered which has either revealing or non-revealing failure.

(2) Non-revealing failure occurs with probability *a* and are detected by inspections only.

(3) Inspections are conducted at times NI, N=1,2,3,… i.e. periodically.

(4) Inspections need not be perfect, i.e. erroneous results may be given: it may wrongly claim that failure has taken place or, contrarily, a failure may persist undetected.

(5) If the system breakdown is discovered at inspection, corrective maintenance is performed. It brings the system back to a new one.

(6) Time of inspection and maintenance is assumed to be non-negligible and known.

(7) The lifetime probability distribution of the system is supposedly known.

(8) Renewal cycle is the time in the midst of two consecutive corrective repairs.

Figure 3.6.1 and Figure 3.6.2 describe the considered model. Figure 3.6.1 is plotted for the case when the system is prone to revealing failures, i.e. as soon as failure occurs corrective repair is conducted. However, Figure 3.6.2 illustrates that in the case of non-revealing failure, repair action is conducted only when the failure is revealed by the inspection.

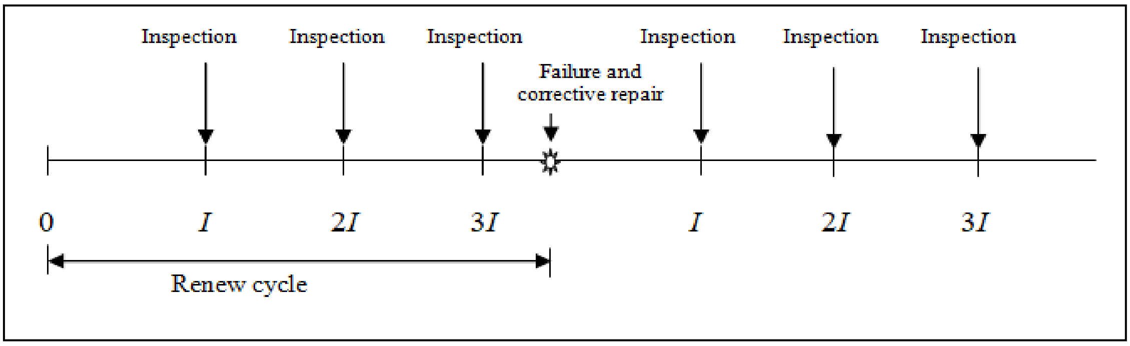


Figure 3.6.1: Renewal cycle in the case of revealing failure

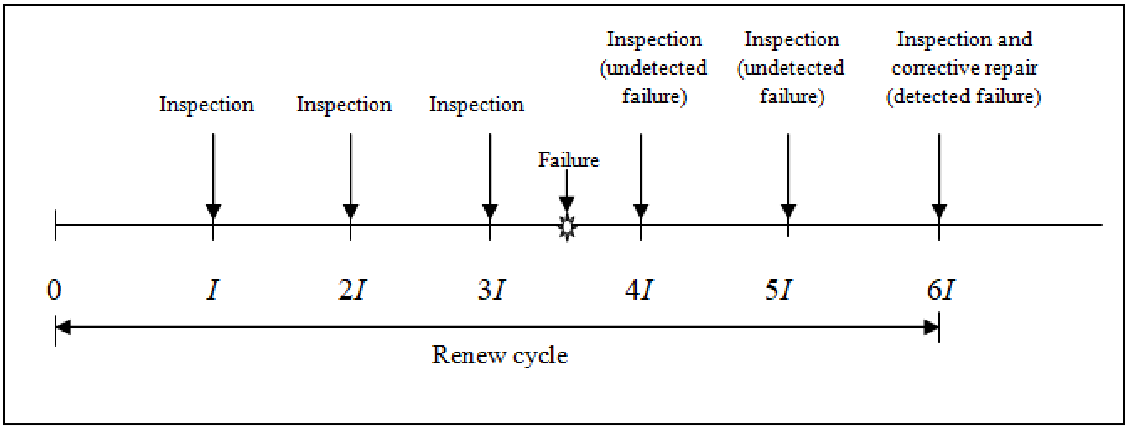


Figure 3.6.2: Renewal cycle in the case of non-revealing failure

3.6.3 Availability modeling

Here the objective is to obtain the general expression for limiting availability. Limiting availability, $A(I)$, is defined as the proportion of expected up time to expected overall cycle length.

i.e., $A(I)$ is given by

$$A(I) = \frac{U(I)}{U(I) + D(I)}$$

Considering two failure classes: non-revealing failure with probability a and revealing failure with probability $1 - a$, $A(I)$ is obtained to be

$$A(I) = \frac{(1-a)U_r(I) + aU_{nr}(I)}{[(1-a)U_r(I) + aU_{nr}(I)] + [(1-a)D_r(I) + aD_{nr}(I)]} \tag{3.6.1}$$

Case of revealing failure

In this case, the system is subject $\mathcal{N}$ times to inspection and fails between $\mathcal{N}I$ and $(\mathcal{N}+1)I$ for $\mathcal{N}$=1,2,3,… Hence, mean uptime in this case ($U_r(I)$) is given by

$$U_r(I) = \sum_{\mathcal{N}=0}^{\infty} \int_{\mathcal{N}I}^{(\mathcal{N}+1)I} R(u)du = \int_0^{\infty} R(u)du \tag{3.6.2}$$

The mean downtime for this case ($D_r(I)$) is given by

$$D_r(I) = T_i E(\mathcal{N}_1) + T_c + T_f E(\mathcal{N}_f)$$

where

$$E(\mathcal{N}_1) = \frac{1}{I}\sum_{\mathcal{N}=0}^{\infty} \mathcal{N}IP(\mathcal{N}I \le X \le (\mathcal{N}+1)I) = \sum_{\mathcal{N}=0}^{\infty} \mathcal{N}[R(\mathcal{N}I) - R((\mathcal{N}+1)I)] \tag{3.6.3}$$

The number of false alerts, corresponding to the case when $\mathcal{N}_1$ inspections are performed, comply a binomial distribution having $\mathcal{N}_1$ and α as the parameters. Hence, we have

$$E(\mathcal{N}_f) = \alpha E(\mathcal{N}_1) \tag{3.6.4}$$

Thus, $D_r(I)$ is obtained to be

$$D_r(I) = (T_i + \alpha T_f)\sum_{\mathcal{N}=0}^{\infty} \mathcal{N}[R(\mathcal{N}I) - R((\mathcal{N}+1)I)] + T_c \tag{3.6.5}$$

Case of non-revealing failure

For the instant case, the system fails between $\mathcal{N}I$ and $(\mathcal{N}+1)I$ for $\mathcal{N}$=1,2,3,… (Figure 3.6.2). Hence, the mean up-time ($U_{nr}(I)$) is equivalent to

$$U_{nr}(I) = \int_0^{\infty} R(u)du \tag{3.6.6}$$

In addition to down-time because of inspections, repair and false alerts, the system stays down for the duration H, i.e. duration post-failure until detection after $(\mathcal{N}_1 + \mathcal{N}_2)$ inspections. Hence, the mean down-time ($D_{nr}(I)$) in this case is given by

$$D_{nr}(I) = T_i\big(E(\mathcal{N}_1) + E(\mathcal{N}_2)\big) + T_c + T_f E(\mathcal{N}_f) + E(H) \tag{3.6.7}$$

where

$$E(H) = I\big(E(\mathcal{N}_1) + E(\mathcal{N}_2)\big) - U_{nr}(I) \tag{3.6.8}$$

As, the number of inspections post-failure to detection $(\mathcal{N}_2)$ is a random geometric variable. Therefore, the expected value of $\mathcal{N}_2$ is given by

$$E(\mathcal{N}_2) = \frac{1}{1-\beta} \tag{3.6.9}$$

Putting equations (3.6.3), (3.6.4), (3.6.8) and (3.6.9), in equation (3.6.7), we obtain $D_{nr}(I)$ as

$$D_{nr}(I) = \left(T_i + \alpha T_f + I\right)\sum_{\text{Ŋ}=0}^{\infty} \text{Ŋ}\left[R(\text{Ŋ}I) - R\left((\text{Ŋ}+1)I\right)\right] + T_c + \frac{1}{1-\beta}(I + T_i) - U_{nr}(I) \tag{3.6.10}$$

Substituting equations (3.6.2), (3.6.5), (3.6.6) and (3.6.10) in equation (3.6.1) results the expression of the limiting availability as

$$A(I) = \frac{\int_0^{\infty} R(u)du}{(1-a)\int_0^{\infty} R(u)du + \left(T_i + \alpha T_f + aI\right)\sum_{\text{Ŋ}=0}^{\infty} \text{Ŋ}\left[R(\text{Ŋ}I) - R\left((\text{Ŋ}+1)I\right)\right] + \frac{a}{1-\beta}(I+T_i) + T_c} \tag{3.6.11}$$

3.6.4 Cost modeling

General expression for LRACR is obtained here. LRACR is defined as the proportion of overall expense in a cycle to overall cycle length.

$$L(I) = \frac{C(I)}{U(I) + D(I)}$$

Again on considering two failure classes: non-revealing failure and revealing failure with probability a and 1-a respectively, $L(I)$ is obtained to be

$$L(I) = \frac{(1-a)C_r(I) + aC_{nr}(I)}{[(1-a)U_r(I) + aU_{nr}(I)] + [(1-a)D_r(I) + aD_{nr}(I)]} \tag{3.6.12}$$

Case of revealing failure

For the case of revealing failure, the cost of a cycle is based on cost due to inspections, cost due to false alerts and the corrective maintenance cost. Hence, the expected overall expense in a cycle for revealing failure is given by

$$C_r(I) = C_i E(\text{Ŋ}_1) + C_c + C_f E(\text{Ŋ}_f)$$

Using equation (3.6.3) and equation (3.6.4), $C_r(I)$ is obtained to be

$$C_r(I) = (C_i + \alpha C_f)\sum_{\text{Ŋ}=0}^{\infty} \text{Ŋ}[R(\text{Ŋ}I) - R((\text{Ŋ}+1)I)] + C_c \tag{3.6.13}$$

Case of non-revealing failure

For the present case, the costs of a cycle is based on cost due to inspections, cost due to false alerts, cost of corrective maintenance and down-time cost while the failure remains undiscovered. Hence, the expected overall expense for this case is given by

$$C_{nr}(I) = C_i(E(N_1) + E(N_2)) + C_c + C_f E(N_f) + C_d E(H)$$

Using equations (3.6.3), (3.6.4), (3.6.8) and (3.6.9) we get $C_{nr}(I)$ as

$$\begin{aligned} C_{nr}(I) = {} & \left(C_i + \alpha C_f + C_d I\right) \sum_{N=0}^{\infty} N\left[R(NI) - R\left((N+1)I\right)\right] \\ & + (C_i + C_d I)\frac{1}{1-\beta} + C_c - C_d \int_0^{\infty} R(u)du \end{aligned} \qquad (3.6.14)$$

Both cases result in the expression of the LRACR as

$$L(I) = \frac{\left(C_i+\alpha C_f+C_d I a\right)\sum_{N=0}^{\infty} N\left[R(NI)-R\left((N+1)I\right)\right]+\frac{a}{1-\beta}(C_i+C_d I)+C_c-aC_d\int_0^{\infty} R(u)du}{(1-a)\int_0^{\infty} R(u)du+\left(T_i+\alpha T_f+aI\right)\sum_{N=0}^{\infty} N\left[R(NI)-R\left((N+1)I\right)\right]+\frac{a}{1-\beta}(I+T_i)+T_c} \qquad (3.6.15)$$

3.6.5 Optimization Strategy

Maximizing the availability

The systems limiting availability has been established in equation (3.6.11). As periodic inspection is performed and limiting availability is obtained in terms of *I*, we consider the inspection interval (*I*) to be the decision variable.

In real-life applications, a minimum inspection period (*i*) exists as a result of the practical issues and cost concerns, i.e. the inspection period shall not be smaller than *i*. For instance, in **Taghipour and Banjevic (2012)**, authors presumed that the inspection period of a medical device will not be lower than one month.

Therefore, the optimal problem is framed as follows:

$$\begin{cases} \max\limits_{I} \dfrac{\int_0^\infty R(u)du}{(1-a)\int_0^\infty R(u)du + \left(T_i + \alpha T_f + aI\right)\sum_{\mathrm{N}=0}^{\infty}\mathrm{N}\left[R(\mathrm{N}I) - R\left((\mathrm{N}+1)I\right)\right] + \dfrac{a}{1-\beta}(I+T_i) + T_c}, \\ \text{s.t. } I \geq i \end{cases}$$

To maximize the above term we only need to minimize the denominator. This is done by calculating $A(I)$ for certain values of *I* and then determining the value of I^*.

Minimizing the cost

Evidently, on one hand, the smaller value of *I* suggests that the inspections are conducted more often, which helps to minimize the number of failures whereas it increases the cost of inspections. On the contrary, high value of *I* means inspections are conducted less often, resulting in the lesser cost of inspections but will increase the chances of failure and consequently, the cost induced by corrective repair increases. In order to maintain the tradeoff between the two, the purpose of optimizing inspection here is minimizing the overall cost rate.

The optimization problem formulated in the aforementioned situation is:

$$\begin{cases} \min\limits_{I} \dfrac{\left(C_i + \alpha C_f + C_d I a\right)\sum_{\mathrm{N}=0}^{\infty}\mathrm{N}\left[R(\mathrm{N}I) - R\left((\mathrm{N}+1)I\right)\right] + \dfrac{a}{1-\beta}(C_i + C_d I) + C_c - aC_d\int_0^\infty R(u)du}{(1-a)\int_0^\infty R(u)du + \left(T_i + \alpha T_f + aI\right)\sum_{\mathrm{N}=0}^{\infty}\mathrm{N}\left[R(\mathrm{N}I) - R\left((\mathrm{N}+1)I\right)\right] + \dfrac{a}{1-\beta}(I+T_i) + T_c}, \\ \text{s.t. } I \geq i \end{cases}$$

With the purpose of achieving the optimal inspection period, the LRACR is calculated for certain values of *I* and the value of I^* is determined.

3.7 Model [7]: Markov process approach for analyzing periodically inspected competing-risk system embodying downtime threshold

In practice, a repairable system working under unpredictable situation might be subject to deterioration, breakdown or certain repairs. In most instances, the states of repairable systems could be categorized as functional or up state and breakdown or down state, on the basis of their condition. A Markov process holds an important role in the analysis of repairable systems. Studies suggesting the application of Markov process in repairable systems could be found in **Arnold (1973)**, **Cao (1989)**, **Wu *et al.* (2007)** and **Cui *et al.* (2014)**.

In various studies, the investigated models concentrate more on single failure mode (FM) (**Sarkar and Sarkar, 2000**; **Cui and Xie, 2005**; **Li *et al.*, 2019**). However, practically as the system structure and failure of units are turning increasingly complex and diverse; the need for investigating models with multiple FMs is gaining more popularity right now. Reviews of models with multiple FM can be seen in **Moustafa (1996)**, **Takamori *et al.* (2005)**, **Zhang *et al.* (2006)** and **Hajeeh (2011)**. For systems experiencing multiple FMs, competing failure may happen and any of the possible breakdowns would result in the overall breakdown of the system. As in the case of an electronic unit (**Zhang, 2004**), which might fail due to open or closed circuit failure, the time distribution for relative corrective repair (CR) may not be same. Many systems also undergo soft and hard failure, in such cases the repair implication of these FMs are distinct (**Qiu *et al.*, 2018**).

Availability analysis has always been a crucial index in terms of the systems performance and hence in reliability engineering it is significant so much and for so long. Numerous works have been published to evaluate the availability of single FM system. Associated works could be noted in **Cui and Xie (2001)**, **Xu and Hu (2008)**, **Tang *et al.* (2013)** and **Khatab *et al.* (2014)**. However, the system may fail because of more than one possible ways, which is rarely considered in the presently existing models. Upshot of the above discussion and disparity between existing and the proposed models motivated us to analyze a competing-risk system.

In the majority of the existent models, the breakdown of the system is supposed to be noted instantly. Meanwhile for the systems like valves in protection devices (**Tsai *et al.*, 2017**), power feeding systems (**Kojima and Asakawa, 2004**), micro engine systems (**Peng *et al.*, 2010**) and wind turbines (**Liu and Zhang, 2020**), breakdown is unrevealed. So, with the motive of revealing the breakdown, improving the availability and preventing the unwanted breakdown, inspection policies are generally adopted. One of the most common policies to do so is to employ a periodic inspection because it is easily implementable.

In some practical instances, the breakdown of the system is either ignored or deferred. For example, as in the case of a water supply system (**Zheng *et al.*, 2006**;

Bao and Cui, 2010), if any kind of breakdown is revealed and repaired during a short time span, the system may be considered functional during that period since the water in the reserve tank will be enough for the usage. However if the downtime due to the breakdown is more than a given pre-specified threshold, the system may be thought functional from commencement of the breakdown till that threshold limit, i.e. till the water in the reservoir clear ups. Furthermore, after that time till the termination of repair, the system is considered to be in the breakdown state. Consider another example of an electricity supply system (**Du *et al.*, 2017**); if the repairing of the system takes time no longer than a preset limit, the breakdown has lesser effect on the system, i.e. the breakdown can be neglected while estimating its availability.

A more comprehensive model is proposed in this research article by taking into account all the three aforementioned conditions, i.e. considering multiple FM, downtime threshold and periodic inspections. To be more specific, a repairable system undergoing periodic inspections and experiencing M modes of failures, in which the breakdown can be either deferred or ignored, is proposed. Corresponding CR is carried out if the system is discovered failed because of the any FM. If the downtime, i.e. time from start of the breakdown till its repair takes time lesser than τ, the system is assumed functional. If the downtime takes more time than τ, then the system is assumed functional from commencement of breakdown till downtime exceeding time τ and after that time till the completion of repair, the system is taken in the breakdown state.

The major contributions of the ongoing article are:

- Newer model is developed based on periodic inspection, multiple FM and ignored/deferred failures.
- The model is divided into two types namely the initial one and the new one based on ignoring or postponing the failures.
- Concise results on point and limiting availability and long-run average cost RATE (LRACR) of the proposed model are presented.
- A descriptive example of a protection device is considered to explain the derived results.

3.7.1 Notations

L	Systems lifetime
$L_i(t)$ $/L_n(t)$	Markov process of the initial/new system
$R(t)$	Systems reliability function
M	Total FMs
I	Inspection time
N	Inspections till first failure in a cycle
F_m	Failure time of m^{th} FM, $m = 1,2,\ldots,M$
λ_m	Hazard rate of $F_m, m = 1,2,\ldots,M$
$F_m(t)$	Distribution function of $F_m, m = 1,2,\ldots,M$
$R_m(t)$	Reliability function of $F_m, m = 1,2,\ldots,M$
G_m	Repair time of m^{th} FM, $m = 1,2,\ldots,M$
$G_m(u)$	Distribution function of $G_m(m = 1,2,\ldots,M)$
$g_m(u)$	Density function of $G_m(m = 1,2,\ldots,M)$
τ	Downtime threshold
$A_i(t)/A_i$	Point/Limiting availability of the initial system
$A_n(t)/A_n$	Point/Limiting availability of the new system
D_i^m	Downtime of initial system in a cycle for the case of m^{th} FM, $m = 1,2,\ldots,M$
$R_{D_i^m}(x)$	Reliability function of D_i^m, $m = 1,2,\ldots,M$
U_n/D_n	Uptime/Downtime of the new system in a cycle
D	Duration of a cycle
L_{AC}/L'_{AC}	LRACR of the new/initial system
C	Overall cost in a cycle

C_I	Cost per inspection
C_{R_m}	CR cost of the m^{th} FM, $m = 1,2,\ldots,M$
C_p	Penalty cost

3.7.2 Model Description

The current model analyzes a scrutiny-based competing-risk system with ignored/deferred failures. The precise assumptions used in this study are:

***Assumption* 1:** At the outset, a fully operable system is brought into use. The breakdown of the system can be characterized into M distinct FMs. As, in the case of a wind turbine system, blade failure and generator failure are its common FMs (Pant *et al.* (2021)).

***Assumption* 2:** Each FM is assumed to have independent breakdown time designated by $F_m(m = 1,2,\ldots,M)$ with distribution function $F_m(t) = 1 - e^{-\lambda_m t}, t > 0$.

***Assumption* 3:** Failures are presumed to be revealed at inspection only, which is conducted at every I unit. Furthermore, inspections are supposed to be non-detrimental, perfect and take insignificant time.

***Assumption* 4:** When the breakdown due to m^{th} FM is revealed, the perfect CR taking random time $G_m(m = 1,2,\ldots,M)$ is conducted having distribution function $G_m(u)$ and density function $g_m(u)$.

***Assumption* 5:** The cost of each inspection is denoted by C_I. Moreover, the CR cost of m^{th} FM is assumed to be $C_{R_m}(m = 1,2,\ldots,M)$ and penalty cost as a result of downtime is expressed as C_P.

***Assumption* 6:** A renewal cycle is defined as the duration between the commencement of system till the termination of first CR or duration in between the termination of two successive CRs.

***Assumption* 7:** Relying on the practical utilization, a downtime threshold is preset, denoted by τ. Based on this limit, the actual system is classified as the initial system and the new system. Where, the so called new system is demonstrated as follows:

- The new system will be in functional state if the initial system is functional.
- On the failure of the initial system, if the downtime (the duration between the outset of failure till its repair) is not more than τ, then the new system will be considered having functional state.
- Meanwhile, if the downtime of the initial system is more than τ, then the new system shall be regarded in functional state from commencement of failure till the downtime reaches τ units.

A possible specimen of the initial and new system is demonstrated in Figure 3.7.1. As visible in Figure 3.7.1, system experiences a breakdown in the time period $(I, 2I)$ because of the FM1, which is revealed at time $2I$ and a CR is conducted immediately. Since, the total downtime was not more than τ hence, the new system was considered being functional for the time $[0,2I + G_1]$. After that, the system breakdown happens in the time period $(2I + G_1, 6I + G_1)$ due to FM2 and a CR was conducted on its revelation. In this case, the downtime exceeded τ units as a result of which, the new system was considered to be functional during the time $(2I + G_1, 2I + G_1 + F_2 + \tau)$ and in the breakdown state in $(2I + G_1 + F_2 + \tau, 6I + G_1 + G_2)$. Additionally, Cycle 1 is the time from where the system was commenced till first CRs termination and Cycle 2 is the time in the midst of completion of first CR and second CR.

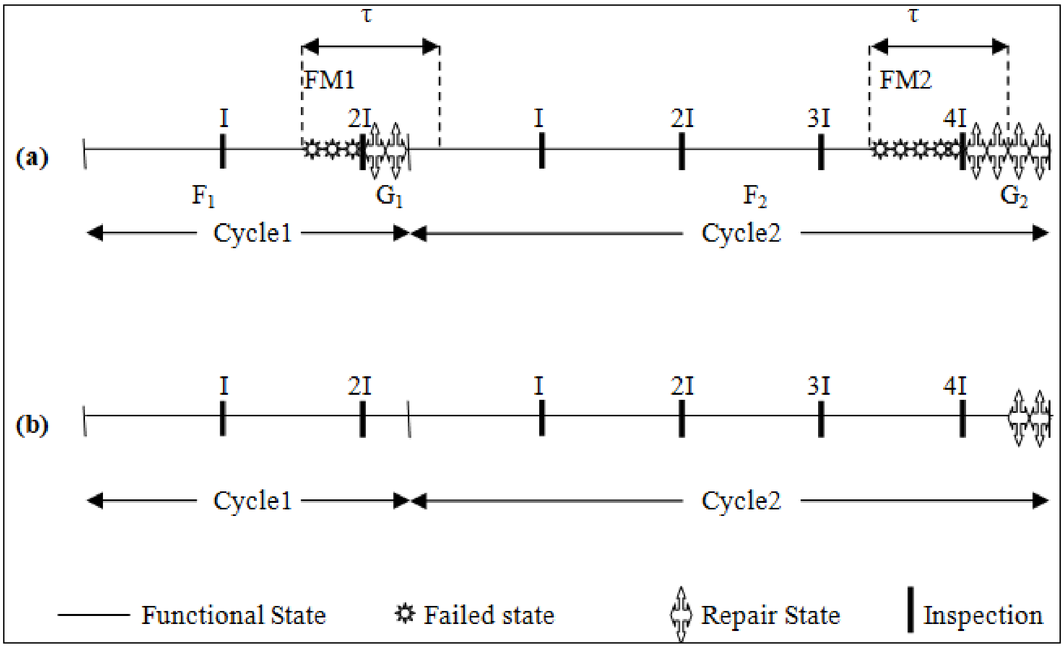


Figure 3.7.1: Possible specimen of (a) Initial system (b) New system

3.7.3 Point availability analysis

For discussing the point availability of the new system we firstly need to find the point availability of the initial system.

3.7.3.1 Point availability of the initial system

In the present section, the point availability of the initial system will be obtained. Based on Assumptions 1 and 2, the lifespan of the system (L) will naturally be minimum of $(F_1, F_2, \dots, F_M)$. Using the independence of $F_m's$, systems reliability function is obtained as

$$R(t) = \prod_{m=1}^{M} P(F_m > t) = e^{-\alpha t} \tag{3.7.1}$$

where $\alpha = \lambda_1 + \lambda_2 + \cdots + \lambda_M$.

Proposition 1: The point availability of the initial system is given by

$$A_i(t) = e^{-\alpha t} + \sum_{n=0}^{\lfloor t/I \rfloor - 1} \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} (e^{-\alpha n I} - e^{-\alpha(n+1)I}) \int_0^{t-(n+1)I} A_i(t-(n+1)I-u) g_m(u) du$$

where $\lfloor a \rfloor$ provides the greatest integer less than a.

Proof: For deriving the point availability let us define a stochastic process as

$$L_i(t) = \begin{cases} 0, \text{when the initial system is in breakdown state at time t} \\ 1, \text{when the initial system is in functional state at time t} \end{cases}$$

Since, Markov analysis provides an interface between possible states of a system where the transition rates between these states are determined by the failure and repair rates. Clearly, any transition can only occur from the current state of the system so the transition rates are only effective from the current state at any given time, i.e. Markovian property is satisfied.

Clearly, the point availability of the initial system will be expressed as

$$A_i(t) = P(L_i(t) = 1)$$

The systems lifespan (L) and the number of inspections in a cycle (N) are related as

$$(N-1)I < L < NI$$

This relation results in the frequency function of N as given below

$$P(N = n) = R(nI) - R\big((n+1)I\big) = e^{-\alpha nI} - e^{-\alpha(n+1)I} \quad (3.7.2)$$

Furthermore, on utilizing the decomposition method of probability, availability could also be given as

$$A_i(t) = \sum_{n=0}^{\infty} P(L_i(t) = 1, N = n)$$

$$= \sum_{n=0}^{\lfloor t/I \rfloor - 1} P(L_i(t) = 1, N = n) + \sum_{n=\lfloor t/I \rfloor}^{\infty} P(L_i(t) = 1, N = n) \quad (3.7.3)$$

The former and later terms of equation (3.7.3) relatively represents that first breakdown occurred ahead of time *t* and no breakdown is experienced before time *t*.

Clearly, the later term of equation (3.7.3) can be written as

$$\sum_{n=\lfloor t/I \rfloor}^{\infty} P(L_i(t) = 1, N = n) = R(t) = e^{-\alpha t} \quad (3.7.4)$$

Utilizing the independence of FMs, former term of equation (3.7.3) could also be expressed as

$$\sum_{n=0}^{\lfloor t/I \rfloor - 1} P(L_i(t) = 1, N = n)$$
$$= \sum_{n=0}^{\lfloor t/I \rfloor - 1} \sum_{m=1}^{M} P(L_i(t) = 1 | N = n, K = m) P(N = n, K = m) \quad (3.7.5)$$

Here, the event $\{ N = n, K = m \}$ means that the first system breakdown in a cycle happened due to m^{th} FM in the interval $[nI, (n+1)I]$, whose probability is given as

$$P(N = n, K = m) = \int_{nI}^{(n+1)I} \lambda_m R(t)dt = \frac{\lambda_m}{\alpha}(e^{-\alpha nI} - e^{-\alpha(n+1)I}) \qquad (3.7.6)$$

Now, as the m^{th} FM is experienced, the CR is performed on system taking random time $G_m, m = 1,2, \dots, M$. Hence, we obtain

$$P(L_i(t) = 1 | N = n, K = m)$$
$$= \int_{0}^{t-(n+1)I} P(L_i(t) = 1 | N = n, K = m, G_m = u) dG_m(u) du$$

$$= \int_{0}^{t-(n+1)I} A_i(t - (n+1)I - u) g_m(u) du \qquad (3.7.7)$$

On utilizing the obtained results of equations (3.7.6) and (3.7.7), equation (3.7.5) is modified to

$$\sum_{n=0}^{\left\lfloor \frac{t}{I} \right\rfloor - 1} P(L_i(t) = 1, N = n)$$
$$= \sum_{n=0}^{\lfloor t/I \rfloor - 1} \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} (e^{-\alpha nI} \qquad (3.7.8)$$
$$- e^{-\alpha(n+1)I}) \int_{0}^{t-(n+1)I} A_i(t - (n+1)I - u) g_m(u) du$$

Finally, on putting equations (3.7.4) and (3.7.8) in equation (3.7.3), the point availability of the initial system is obtained as

$$A_i(t) = e^{-\alpha t} + \sum_{n=0}^{\lfloor t/I \rfloor - 1} \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} (e^{-\alpha n I} - e^{-\alpha(n+1)I}) \int_0^{t-(n+1)I} A_i(t-(n+1)I-u) g_m(u) du \tag{3.7.9}$$

Remark 1: Clearly, this model is an extension of works taking single FM into consideration. Taking only single FM, i.e. M=1, the point availability as acquired in equation (3.7.9) will be simplified as

$$A_i(t) = e^{-\alpha t} + \sum_{n=0}^{\lfloor t/I \rfloor - 1} (e^{-\alpha n I} - e^{-\alpha(n+1)I}) \int_0^{t-(n+1)I} A_i(t-(n+1)I-u) g(u) du$$

3.7.3.2 Point availability of the new system

The current section aims at finding the point availability expression for the new system by utilizing the results of previous section.

Proposition 2: The point availability of the new system is given by

$$A_n(t) = \begin{cases} 1, t \leq \tau \\ A_i(t) + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} A_i(t-x) \lambda_m R_{D_i^m}(x) dx \,, t > \tau \end{cases}$$

where $R_{D_i^m}(x) = \int_{\max(0,(x-I))}^{x} \sum_{n=1}^{\infty} (e^{-\lambda_m (n-1)I} - e^{-\lambda_m (nI-x+u)})\, dG_m(u) + \left(1 - G_m(x)\right)$.

Proof: As done in the previous section, let us define the Markov process in the similar manner as

$$L_n(t) = \begin{cases} 0, \text{when the new system is in breakdown state at time t} \\ 1, \text{when the new system is in functional state at time t} \end{cases}$$

Then, the point availability of the new system will be

$$A_n(t) = P(L_n(t) = 1)$$

$$= P(L_n(t) = 1, L_i(t) = 1) + P(L_n(t) = 1, L_i(t) = 0) \qquad (3.7.10)$$

Since, the new system will be functional if the initial system is functional. Hence, we have

$$P(L_n(t) = 1, L_i(t) = 1) = A_i(t) \qquad (3.7.11)$$

Now, the later term of equation (3.7.10) demonstrates that at time t, the new system is in a functional state and the initial system is in a breakdown state. Let us denote this term by $A(t)$, i.e.

$$A(t) = P(L_n(t) = 1, L_i(t) = 0)$$

When $t < \tau$

$$A(t) = P(L_n(t) = 1 | L_i(t) = 0) P(L_i(t) = 0)$$

$$= 1 - A_i(t) \qquad (3.7.12)$$

When $t > \tau$

$$A(t) = \sum_{m=1}^{M} p_m \int_0^{\tau} A_i(t - x) \lambda_m R_{D_i^m}(x) dx \qquad (3.7.13)$$

The term, $p_m A_i(t - x) \lambda_m dx$ gives the probability of movement of the initial system from functional state to the breakdown state in the time $(t - x, t - x + dx)$ due to m^{th} FM. Here, p_m is the probability of system breakdown due to m^{th} FM in each cycle, i.e., p_m is given by

$$p_m = \int_0^{\infty} \lambda_m R(t) dt = \frac{\lambda_m}{\alpha} \qquad (3.7.14)$$

Since, the initial system undergoes repair so, the downtime D_i^m should be more than x but not more than τ. Hence, $R_{D_i^m}(x)$ will be given by

$$\begin{aligned}
R_{D_i^m}(x) &= P(D_i^m > x) \\
&= P(NI - F_m + G_m > x) \\
&= \int_0^x P(NI - F_m > x - u)dG_m(u) + \int_x^\infty P(NI - F_m > x - u)dG_m(u) \\
&= \int_0^x P(NI - F_m > x - u)dG_m(u) + (1 - G_m(x)) \\
&= \int_0^x P((N-1)I < F_m < NI - x + u)dG_m(u) + \big(1 - G_m(x)\big) \\
&= \int_{\max(0,(x-I))}^x \sum_{n=1}^\infty (R_m\big((n-1)I\big) - R_m(nI - x + u))\, dG_m(u) + (1 \\
&\quad - G_m(x))
\end{aligned}$$

$$= \int_{\max(0,(x-I))}^x \sum_{n=1}^\infty (e^{-\lambda_m(n-1)I} - e^{-\lambda_m(nI-x+u)})\, dG_m(u) + \big(1 - G_m(x)\big) \tag{3.7.15}$$

Using equation (3.7.10) to equation (3.7.15), $A_n(t)$ is obtained to be

$$A_n(t) = \begin{cases} 1, t \leq \tau \\ A_i(t) + \displaystyle\sum_{m=1}^M \frac{\lambda_m}{\alpha} \int_0^\tau A_i(t-x)\lambda_m R_{D_i^m}(x)dx\,, t > \tau \end{cases} \tag{3.7.16}$$

Remark 2: As we can observe from equation (3.7.16), the probability that new system is functional in the time $t \leq \tau$ is 1 because of the delayed failure effect. If we take $\tau = 0$, the point availability of the new system will be equal to that of the initial system (i.e., $A_n(t) = A_i(t)$), which matches our insight.

Remark 3: In the case of single FM, the point availability of the new system given in equation (3.7.16) will be simplified to

$$A_n(t) = \begin{cases} 1, t \le \tau \\ A_i(t) + \int_0^{\tau} A_i(t-x)\alpha R_{D_i}(x)dx\,, t > \tau \end{cases}$$

where $R_{D_i}(x) = \int_{\max(0,(x-I))}^{x} \sum_{n=1}^{\infty}\left(e^{-\alpha(n-1)I} - e^{-\alpha(nI-x+u)}\right) dG(u) + \left(1 - G(x)\right)$.

3.7.4 Limiting availability analysis

In this section, the limiting availability of the new system (A_n) is given, which is stated as the ratio of mean up-time in a cycle ($E(U_n)$) to mean duration of a cycle ($E(D)$), i.e.

$$A_n = \frac{E(U_n)}{E(D)} \tag{3.7.17}$$

Proposition 3: The limiting availability of the new system is given by

$$A_n = \frac{\frac{1}{\alpha} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx}{\frac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} E(G_m)}$$

Proof: The up-time in a cycle for the case of new system is given as

$$U_n = L + \sum_{m=1}^{M} p_m \min(D_i^m, \tau)$$

Then, the mean uptime equals:

$$E(U_n) = E(L) + \sum_{m=1}^{M} p_m \int_0^{\tau} R_{D_i^m}(x)dx$$

$$= \frac{1}{\alpha} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx \tag{3.7.18}$$

Obviously, the mean duration of a cycle may be given as

$$E(D) = E((N+1)I) + \sum_{m=1}^{M} p_m E(G_m)$$

$$= \sum_{n=0}^{\infty}(n+1)I[R(nI) - R((n+1)I)] + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m)$$

$$= \sum_{n=0}^{\infty}(n+1)I[e^{-\alpha nI} - e^{-\alpha(n+1)I}] + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m)$$

$$= \frac{I}{1 - e^{-\alpha I}} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m) \qquad (3.7.19)$$

Utilizing equations (3.7.17), (3.7.18) and (3.7.19), limiting availability of the new system equals

$$A_n = \frac{\frac{1}{\alpha} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}\int_0^{\tau} R_{D_i^m}(x)dx}{\frac{I}{1 - e^{-\alpha I}} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m)} \qquad (3.7.20)$$

Remark 4: For the case of single FM, i.e. $M = 1$, the limiting availability expression becomes

$$A_n = \frac{\frac{1}{\alpha} + \int_0^{\tau} R_{R_{D_i}}(x)dx}{\frac{I}{1 - e^{-\alpha I}} + E(G)}$$

Remark 5: Clearly, large value of τ comply a low mean downtime resulting in high limiting availability. This suggests that A_n is a growing function of τ. The same could also be viewed from equation (3.7.20). Hence, the lower and upper bounds of A_n can relatively be given as

$$(A_n)_{low} = (A_n|\tau \to 0) = \frac{\frac{1}{\alpha}}{\frac{I}{1 - e^{-\alpha I}} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m)}$$

$$(A_n)_{up} = (A_n|\tau \to \infty) = \frac{\frac{1}{\alpha} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}\int_0^{\infty} R_{D_i^m}(x)dx}{\frac{I}{1 - e^{-\alpha I}} + \sum_{m=1}^{M}\frac{\lambda_m}{\alpha}E(G_m)}$$

Also, according to our intuition, the value of A_n at $\tau = 0$ corresponds to the point availability of the initial system. Hence, the point availability of the initial system (A_i) becomes

$$A_i = \frac{\frac{1}{\alpha}}{\frac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} E(G_m)} \tag{3.7.21}$$

3.7.5 Cost analysis

The LRACR, denoted by L_{AC} for the new system is calculated in this section, given by the proportion of mean overall cost incurred in a cycle ($E(C)$) to mean duration of a cycle ($E(D)$), i.e.

$$L_{AC} = \frac{E(C)}{E(D)}$$

Proposition 4: The LRACR of the new system is given by

$$L_{AC} = \frac{C_I \frac{1}{1-e^{-\alpha I}} + \sum_{m=1}^{M} C_{R_m} \frac{\lambda_m}{\alpha} E(G_m) + C_P\left(\frac{I}{1-e^{-\alpha I}} - \frac{1}{\alpha} - \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx\right)}{\frac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} E(G_m)}$$

Proof: In a cycle, the mean overall cost is given as

$$E(C) = C_I E(N+1) + \sum_{m=1}^{M} C_{R_m} p_m E(G_m) + C_P E(D_n) \tag{3.7.22}$$

where, $E(N+1)$ is written as

$$E(N+1) = \sum_{n=0}^{\infty} (n+1)[R(nI) - R((n+1)I)]$$

$$= \sum_{n=0}^{\infty} (n+1)[e^{-\alpha n I} - e^{-\alpha(n+1)I}]$$

$$= \frac{1}{1 - e^{-\alpha I}} \tag{3.7.23}$$

and $E(D_n)$ denotes mean downtime for the new system (excluding the downtime of repair) and is given using equation (3.7.18) and equation (3.7.19) as

$$E(D_n) = \frac{I}{1 - e^{-\alpha I}} - \frac{1}{\alpha} - \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx \tag{3.7.24}$$

Based on equations (3.7.22), (3.7.23) and (3.7.24), mean overall cost in a cycle is given as

$$E(C) = \begin{matrix} C_I \dfrac{1}{1 - e^{-\alpha I}} + \displaystyle\sum_{m=1}^{M} C_{R_m} \frac{\lambda_m}{\alpha} E(G_m) \\ +C_P \left(\dfrac{I}{1 - e^{-\alpha I}} - \dfrac{1}{\alpha} - \displaystyle\sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx \right) \end{matrix} \tag{3.7.25}$$

Using equation (3.7.19) and equation (3.7.25), LRACR is found to be

$$L_{AC} = \frac{\begin{matrix} C_I \frac{1}{1 - e^{-\alpha I}} + \sum_{m=1}^{M} C_{R_m} \frac{\lambda_m}{\alpha} E(G_m) \\ +C_P \left(\frac{I}{1 - e^{-\alpha I}} - \frac{1}{\alpha} - \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} \int_0^{\tau} R_{D_i^m}(x)dx \right) \end{matrix}}{\frac{I}{1 - e^{-\alpha I}} + \sum_{m=1}^{M} \frac{\lambda_m}{\alpha} E(G_m)} \tag{3.7.26}$$

Remark 6: In the instance of single FM ($M = 1$), the LRACR of the new system will be

$$L_{AC} = \frac{\begin{matrix} C_I \frac{1}{1 - e^{-\alpha I}} + C_R E(G) \\ +C_P \left(\frac{I}{1 - e^{-\alpha I}} - \frac{1}{\alpha} - \int_0^{\tau} R_{D_i}(x)dx \right) \end{matrix}}{\frac{I}{1 - e^{-\alpha I}} + E(G)}$$

Remark 7: Obviously, large value of τ results in greater penalty cost. Hence, L_{AC} is a decreasing function of τ. Using equation (3.7.26), we can get both the lower and upper bounds of L_{AC} as

$$(L_{AC})_{low} = (L_{AC}|\tau \to \infty) = \frac{\begin{array}{c} C_I \dfrac{1}{1-e^{-\alpha I}} + \sum_{m=1}^{M} C_{R_m} \dfrac{\lambda_m}{\alpha} E(G_m) \\ +C_P\left(\dfrac{I}{1-e^{-\alpha I}} - \dfrac{1}{\alpha} - \sum_{m=1}^{M} \dfrac{\lambda_m}{\alpha} \int_0^\infty R_{D_i^m}(x)dx\right) \end{array}}{\dfrac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \dfrac{\lambda_m}{\alpha} E(G_m)}$$

$$(L_{AC})_{up} = (L_{AC}|\tau \to 0) = \frac{\begin{array}{c} C_I \dfrac{1}{1-e^{-\alpha I}} + \sum_{m=1}^{M} C_{R_m} \dfrac{\lambda_m}{\alpha} E(G_m) \\ +C_P\left(\dfrac{I}{1-e^{-\alpha I}} - \dfrac{1}{\alpha}\right) \end{array}}{\dfrac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \dfrac{\lambda_m}{\alpha} E(G_m)}$$

Moreover, the value of L_{AC} at $\tau = 0$ corresponds to the LRACR of the initial system. Hence, the LRACR of the initial system (L'_{AC}) equals

$$L'_{AC} = \frac{\begin{array}{c} C_I \dfrac{1}{1-e^{-\alpha I}} + \sum_{m=1}^{M} C_{R_m} \dfrac{\lambda_m}{\alpha} E(G_m) \\ +C_P\left(\dfrac{I}{1-e^{-\alpha I}} - \dfrac{1}{\alpha}\right) \end{array}}{\dfrac{I}{1-e^{-\alpha I}} + \sum_{m=1}^{M} \dfrac{\lambda_m}{\alpha} E(G_m)} \tag{3.7.27}$$

3.8 Model [8]: Particle swarm optimization strategy for design optimization of a series-parallel system incorporating failure dependencies and multiple repair teams

Systems optimal design and reliability are an important aspects of design engineering and successfully been implemented to improve performance (**Wang and Watada, 2009**). A repairable system indicates that in any instance of failure, the system can be fixed such that it can continue to function normally. System availability is a term highly connected to reliability and cites to the range of possibilities for accessing the repairable systems reliability (**Juang *et al.*, 2008**). Availability is a particularly significant measure for repairable systems, and attaining a high or necessary degree of availability is a must. Generally, repair teams and redundant units are employed to ensure that the system operates at the appropriate degree of availability. Also, as the repair teams and redundant units, continues to grow, so does the expense. As a result, decision-makers and system designers generally attempt to decide how many units and repair personnel's must be utilized so as to reduce system cost while meeting the system availability requirement.

A series-parallel configuration is made up of a few serially-connected subsystems, and each subsystem is made up of a few components that are connected in parallel. If all of the subsystem's components fail, the subsystem is considered to be failed. The breakdown of any subsystem leads to the failure of the entire system. As an effective design technique, increasing redundant components in parallel can improve the availability or reliability of the series-parallel system. As a result, redundancy allocation must be addressed at the original design phase. The redundancy allocation problem (RAP) entails selecting components or system configuration/design in order to concurrently optimize various objective functions given certain design restrictions (**Zhang and Chen, 2016**). Lot of research has been conducted in the realm of RAP for series-parallel systems appertaining to numerous assumptions. According to **Ramirez-Marquez and Coit (2004)**, optimization techniques for determining optimum or excellent solutions include integer, dynamic, mixed and non-mixed programming and heuristics. **Kuo and Prasad (2000)** and **Gen and Yun (2006)** provided comprehensive overviews on this issue.

Repairable series-parallel configurations are widely utilized in reality, such as power systems, production systems, telecommunication, industrial systems etc. For repairable series-parallel systems, redundancy optimization has gotten a lot of attention recently, both in terms of problem and solution approaches (**Levitin and Lisnianski, 1999**). For a repairable series-parallel configuration, the repair resources are considered to be infinite in conventional RAP. In **Nourelfath and Ait-Kadi (2007)**, the RAP of the system study suggested the more general situation when repair facilities were restricted. Because it takes into consideration a restricted amount of maintenance teams, this issue is more practical than the traditional RAP. It was assumed in the research of **Mehta *et al.* (2017)** that in each subsystem, the component breakdowns happen independently. This independence assumption, however, is not applicable in a number of scenarios.

Because of the increased loading caused by the other failed units, the hazard rate of the running unit may rise in some systems. **Pecht (2009)** suggested three kinds of failure dependencies: common-mode, multi-mode, and additional failure dependencies. **Li *et al.* (2010)** addressed the optimization challenge for series-parallel configuration with common-mode failures. **Levitin (2002)** explored the optimization issue for multi-state two failure mode series-parallel system. **Barros *et al.* (2003)** investigated systems comprising of two units, in which the hazard rate of the operational unit rises as a result of the increased stress caused by the other failed unit. **Yu *et al.* (2007)** looked into a redundant system embodying N components and devised a dependency function to measure the reliance. **Hu *et al.* (2020)** examined the long-run availability of a repairable series-parallel configuration, taking into consideration the various types of units and repairmen. The hazard rate of the operational component varied with the failed components and a dependence function was also introduced.

Despite the fact that the RAP of repairable series-parallel configuration and the failure dependence problem for certain systems were documented in the preceding studies, relatively few intellectuals investigated the topic of allotting redundant units and repair teams in a repairable series-parallel configuration with failure reliance. Due to its complexity, failure dependence is frequently overlooked, and repair teams are frequently believed to be infinite in some models (**Malik and Anand, 2010**). This drives and motivates us to examine a repairable series-parallel system with restricted number of repair teams and failure dependencies. Furthermore, an optimum allocation problem is outlined, with the goal of reducing the system cost, which includes expenses associated with components as well as with repair teams, subject to the availability restriction. Because of its versatility in expressing discrete design variables and its strong/rapid optimization capacity, the PSO strategy is utilized to identify the best solutions.

PSO techniques support the fundamental processes of fish schooling and bird flocking. This approach evolved from Kennedy’s and Eberhart's modeling of social behavior in 1995. Because of its ease of implementation, low computing costs and minimal memory needs, PSO is a useful approach for optimization issues. Unlike other methods, conventional PSO does not employ crossover or mutation. Numerous applications of PSO in the field of combinatorial optimization issues, such as shop scheduling problems (**Shao *et al.*, 2018**), project scheduling problems (**Sebt *et al.*, 2017**), partitional clustering problem (**Jarboui *et al.*, 2007**; **Xu *et al.*, 2018**), optimization problems (**Pant *et al.*, 2015**; **Kumar *et al.*, 2017**) and vehicle routing problem (**Yao *et al.*, 2016**) have been presented in the past.

In this work, a PSO strategy with dynamic parameters is used to tackle the optimal design problem discussed in **Hu *et al.* (2012)**. PSO algorithm replaced the GA in quest for the optimal computation of the components and repair teams, additionally reducing the systems cost of a series-parallel configuration having failure dependency. A comparison is made between the results provided by both algorithms (PSO in the proposed model and GA in **Hu *et al.* (2012)**) based on cost evaluation.

3.8.1 Notations

N	:	Total subsystems
n_k	:	Total components in subsystem k
r_k	:	Total repair teams in subsystem k
C_k^c	:	Unit component cost in subsystem k
C_k^r	:	Unit repair team cost in subsystem k
C_s	:	Systems cost
C_p	:	Penalty Cost
A_k	:	Availability of subsystem k
A_s	:	Systems availability
A_0	:	Availability constraint value
λ_k	:	Inherent hazard rate of components of subsystem k
μ_k	:	Repair rate of components of subsystem k

$g(.)$	:	Dependence function
i_{max}/i_c	:	Maximum/Current iteration
v_i/x_i	:	Velocity/Position of the i^{th} particle
P	:	Population size
w	:	Inertia weight
w_i/w_f	:	Initial/Final value of interia weight
c_1,c_2	:	Acceleration coefficients
c_{1i}/c_{1f}	:	Initial/Final value of c_1
c_{2i}/c_{2f}	:	Initial/Final value of c_2

3.8.2 Problem description

The model is built upon the work of **Hu *et al.* (2012)** as follows

A series-parallel system of N serially-connected subsystems with each subsystem having n_k $(k = 1,2, \dots, N)$components connected in parallel is considered. Clearly, the system works on the successful operation of all of its subsystem and a subsystem works if even one of its component work. An illustration of the configuration is given in Figure 3.8.1.

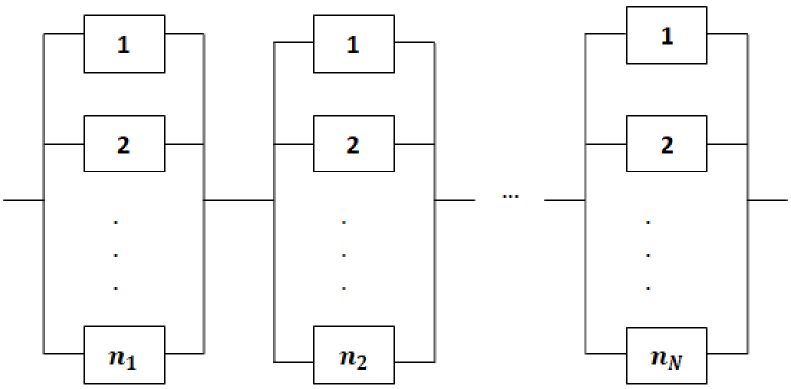


Figure 3.8.1: An illustration of the series-parallel configuration

The suppositions of the model are specified as under

(1) The system and all the units possess two states, either fully working or completely failed.

(2) All the components of a subsystem are assumed to be alike.

(3) *Failure dependency*: The hazard rate of the component in each subsystem relies on the load i.e., the hazard rates of the functioning components rises with the increase of failed units.

The hazard rate of the component in subsystem k is given by

$$\frac{\lambda_k}{g(i)}$$

where, λ_k is the inherent hazard rate, $g(i)$ suggests the dependence function and i is the total functioning components in subsystem k.

Then, the transition rate from state i to $i+1$ for the subsystem k is given by

$$\lambda_k^i = i\frac{\lambda_k}{g(i)}$$

(4) r_k repair teams are supposed to be available in the subsystem $k, (k = 1,2, \ldots, N)$.

(5) The repair rate of each unit is constant in any subsystem and, given by μ_k. Then, the transition rate from state $i-1$ to i for the subsystem k will be given as

$$\mu_k^i = \begin{cases} r_k\mu_k, & i = 0,1, \ldots, n_k - r_k \\ (n_k - i)\mu_k, & i = n_k - r_k + 1, n_k - r_k + 2, \ldots, n_k - 1 \end{cases}$$

(6) A failed unit is restored to as good as a new one.

(7) Each repair team repairs only one failed unit at a time.

Now, our goal is to minimize the systems construction cost while meeting the availability requirements of the system. The construction cost comprises of the repair teams costs and the components costs. Hence, the problem of design optimization to be debated is as follows

$$\text{Minimize } C_s = \sum_{k=1}^{N} (n_k C_k^c + r_k C_k^r)$$

$$\text{subject to } A_s \geq A_0 \tag{3.8.1}$$

$$n_k, r_k \in \{1,2,3, \dots\}$$

$$r_k \leq n_k, k = 1,2, \dots, N$$

where, the systems availability (A_s) is given as

$$A_s = \prod_{k=1}^{N} A_k$$

$$= \prod_{k=1}^{N} \left\{ 1 - \left[1 + \sum_{j=1}^{n_k - r_k} \frac{{r_k}^j \prod_{i=1}^{j} g(i)}{j!} \left(\frac{\mu_k}{\lambda_k}\right)^j + \sum_{j=n_k-r_k+1}^{n_k} \frac{{r_k}^{n_k - r_k} r_k! \prod_{i=1}^{j} g(i)}{j!\,(n_k - j)!} \left(\frac{\mu_k}{\lambda_k}\right)^j \right]^{-1} \right\} \tag{3.8.2}$$

The detailed proof of A_s could be found in **Hu *et al.* (2012)**.

Clearly, the availability A_s relies on the following elements: Total number of subsystems (N); Each subsystem's total units (n_k) and repair teams (r_k); Each components inherent hazard rate (λ_k) and repair rate (μ_k) ; Dependence function, $g(i)$.

Total subsystems, N, is mainly decided depending on the system function required (**Liu *et al.*, 2003**). Each components inherent hazard and repair rates, λ_k and μ_k respectively, are supposed to be known. The four types of the dependencies are considered: Independence ($g(i) = 1$); Linear Dependence ($g(i) = i$); Weak Dependence ($1 < g(i) < i$); Strong Dependence ($g(i) > i$).

Additionally, to solve the optimization problem given by equation (3.8.1), the dependence function $g(i)$ is determined using particular forms. Subsequently, the availability expressions are given as follows

1) Independence ($g(i) = 1$)

$$A_s = \prod_{k=1}^{N} \left\{ 1 - \left[1 + \sum_{j=1}^{n_k - r_k} \frac{{r_k}^j}{j!} \left(\frac{\mu_k}{\lambda_k} \right)^j + \sum_{j=n_k - r_k + 1}^{n_k} \frac{{r_k}^{n_k - r_k} r_k!}{j!\,(n_k - j)!} \left(\frac{\mu_k}{\lambda_k} \right)^j \right]^{-1} \right\} \quad (3.8.3)$$

2) Linear Dependence ($g(i) = i$)

$$A_s = \prod_{k=1}^{N} \left\{ 1 - \left[1 + \sum_{j=1}^{n_k - r_k} {r_k}^j \left(\frac{\mu_k}{\lambda_k} \right)^j + \sum_{j=n_k - r_k + 1}^{n_k} \frac{{r_k}^{n_k - r_k} r_k!}{(n_k - j)!} \left(\frac{\mu_k}{\lambda_k} \right)^j \right]^{-1} \right\} \quad (3.8.4)$$

3) Weak and Strong dependence ($g(i) = i^l, 0 < l < 1 \; for \; weak \; and \; l > 1 \; for \; strong$)

$$A_s = \prod_{k=1}^{N} \left\{ 1 - \left[1 + \sum_{j=1}^{n_k - r_k} {r_k}^j (j!)^{l-1} \left(\frac{\mu_k}{\lambda_k} \right)^j + \sum_{j=n_k - r_k + 1}^{n_k} \frac{{r_k}^{n_k - r_k} r_k!\,(j!)^{l-1}}{(n_k - j)!} \left(\frac{\mu_k}{\lambda_k} \right)^j \right]^{-1} \right\} \quad (3.8.5)$$

Thus, the design optimization problem is focused on determining the optimal values of $n_1, n_2, \ldots, n_N, r_1, r_2, \ldots, r_N$ so as to minimize the systems cost conditional on the systems availability constraint.

3.8.3 PSO Strategy

With the intention of solving the optimization problem given by equation (3.8.1), we implement PSO. In typical PSO, the initial population is produced randomly. Meanwhile, the velocity and position factors describe the particle status in the space, as follows

$$v_i^{a+1} = w * v_i^a + c_1 * r_1 * (pbest_i^a - x_i^a) + c_2 * r_2 * (gbest^a - x_i^a) \quad (3.8.6)$$

$$x_i^{a+1} = x_i^a + v_i^{a+1} \quad (3.8.7)$$

where, v_i^a / x_i^a is the velocity/position vector of the i^{th} particle at the a^{th} iteration.

v_i^{a+1}/ x_i^{a+1} denotes the velocity/position vector of the i^{th} particle at the $(a+1)^{th}$ iteration.

$pbest_i^a$ denotes the personal best of the i^{th} particle at the a^{th} iteration.

$gbest^a$ is the global best of the all the particles at the a^{th} iteration.

c_1 and c_2 are acceleration coefficients, which control movement of particles.

w is an inertia weight, which along with c_1 and c_2 controls the effect of prior velocities on the new one.

r_1 and r_2 are arbitary numbers between 0 and 1.

3.8.3.1 Penalization

PSO has been used extensively to solve problems like optimization, neural network training and scheduling (**He *et al.*, 2004**; **Yin *et al.*, 2007**; **Ozcift *et al.*, 2009**). In spite of its advantages, PSO has some drawbacks that need to overcome. While tackling optimization problem, the severe difficulty is that the solution might reach an infeasible space, which causes the PSO's inefficiency. When the next position reaches an infeasible location, **El-Gallad *et al.* (2001)** developed an approach in which particles stay behind. This technique works well for a variety of problems, but it severely limits particle search space, reducing the PSO's ability to find solutions. To address this obstacle, **Parsopoulos and Vrahatis (2002)** used a penalty function for transforming a constrained optimization problem into an unconstrained one. This method necessitates the subjective predetermination of a penalty coefficient. To be brief, the problem of particles moving into an infeasible area is solvable.

In this model, a penalty cost is introduced for ensuring that the ultimate optimal solution is feasible while providing a competent search across the infeasible region. In case of reaching an infeasible region, the cost will be updated as

$$C_s = \sum_{k=1}^{N} (n_k C_k^c + r_k C_k^r) + C_p \tag{3.8.8}$$

where, C_p is the penalty cost.

3.8.3.2 Dynamic weight and acceleration coefficients

Furthermore, the PSO parameters might have a significant impact on the optimum solution. Despite the fact that **Kennedy and Eberhart (1995)** advocated an optimum fixation of 2 for learning variables, other researchers (**Hseih *et al.*, 2008**; **Ai and Kachitvichyanukul, 2009**; **Jiang *et al.*, 2010**) have chosen a best setting of 0.5–2. Conclusively, the optimum PSO parameter setting may change depending on the problem attributes. Dynamic weight and acceleration coefficients are chosen here to deal with this problem.

It was discovered that linearly varying the inertia weight (w) improves the performance of a PSO algorithm (**Shi and Eberhart, 1999**). A time-varying inertia weight, given by **Shi and Eberhart (1999)** is adopted here, which is defined as follows

$$w = (w_i - w_f)\frac{i_{max} - i_c}{i_{max}} + w_f \tag{3.8.9}$$

where, w_i/w_f denotes the initial/final value of the inertia weight.

i_c is the present iteration number.

i_{max} is the maximum number of iterations.

In addition to this inertia weighting method, time-dependent acceleration factors are used to balance exploitation and exploration abilities. As described by **Limbourg & Aponte (2005)**, this may be accomplished by altering the coefficients c_1 and c_2 linearly over time

$$c_1 = \left(c_{1f} - c_{1i}\right)\frac{i_c}{i_{max}} + c_{1i} \tag{3.8.10}$$

$$c_2 = \left(c_{2f} - c_{2i}\right)\frac{i_c}{i_{max}} + c_{2i} \tag{3.8.11}$$

where, c_{1i} /c_{2i} stands for the initial value of c_1 / c_2.

c_{1f} / c_{2f} is the final values of c_1 / c_2.

3.8.3.3 Discretization

In addition to all the above-mentioned changes, the position and velocity vector must be upgraded at each step/iteration to integer domain (**Wang and Li, 2011**) as follows

$$x_i^{a+1} = round|x_i^{a+1}| \tag{3.8.12}$$

$$v_i^{a+1} = \begin{cases} -1, & if\ v_i^{a+1} \leq -0.5 \\ 0, & if -0.5 < v_i^{a+1} < 0.5 \\ 1, & if\ v_i^{a+1} \geq 0.5 \end{cases} \tag{3.8.13}$$

This velocity restriction ensures that the search step size is modest enough to avoid sudden jumps in the solution space.

3.8.3.4 Algorithm

Figure 3.8.2 shows a flow chart describing the PSO algorithm comprehensively.

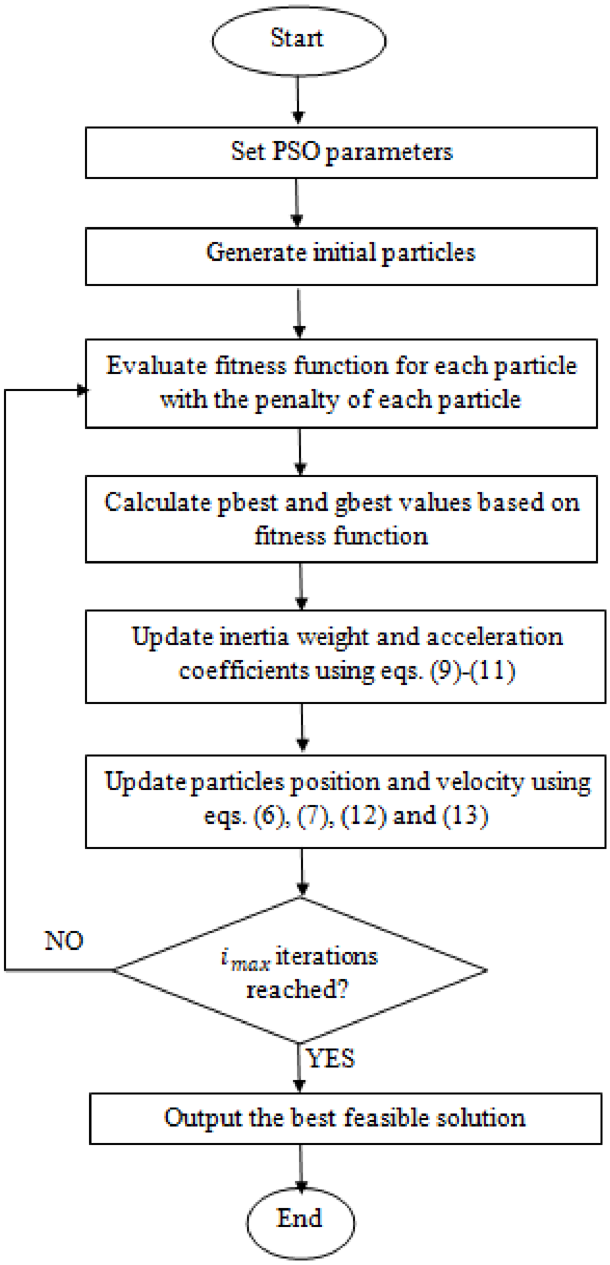


Figure 3.8.2: PSO Algorithm's Flowchart

Results and Discussion

Chapter 4 RESULTS AND DISCUSSION

The various outcomes suggested in Chapter 3 are illustrated in this chapter. This chapter's major goal is to discuss the outcomes of the proposed models.

4.1 Model [1]: Availability of systems subject to multiple failure modes under calendar-based inspection

To illustrate the model with real life, an example of a wind turbine system is considered. Wind turbine systems are used to convert kinetic energy in the wind into electrical energy. It can be used in many practical fields such as in making contribution in domestic power supply.Wind turbine system can fail because of multiple factors like blade failure (FM 1) and generator failure (FM 2). It is not economical to inspect the wind turbine system continuously, so to investigate the failure, periodic inspections are performed at regular intervals. If the wind turbine is found working during the inspection, a perfect PR is carried out. If the system is found failed during the inspection because of any of the above defined failure modes, respective CR is carried out taking a random time. The distributions of repair time for these FM may be dissimilar. Thus, the model proposed in this study is employed to determine the availability of wind turbine system.

Let us assume that FM 1 and FM 2 have failure times expressed by $F_1(\tau) = 1 - e^{-0.01\tau}$ and $F_2(\tau) = 1 - e^{-0.02\tau}$ respectively. Let X_1 and X_2 be mutually independent. Suppose that the CR times for blade failure and generator failure are Y_1 and Y_2 respectively with distribution function $G_1(y) = G_2(y) = 1 - e^{-0.5y}$. Let the inspection time in this case be 50 unit.

4.1.1 Point availability

By using equation (3.1.3), we can get the point availability of the system, $A(\tau)$ which is graphed in Figure 4.1.1.

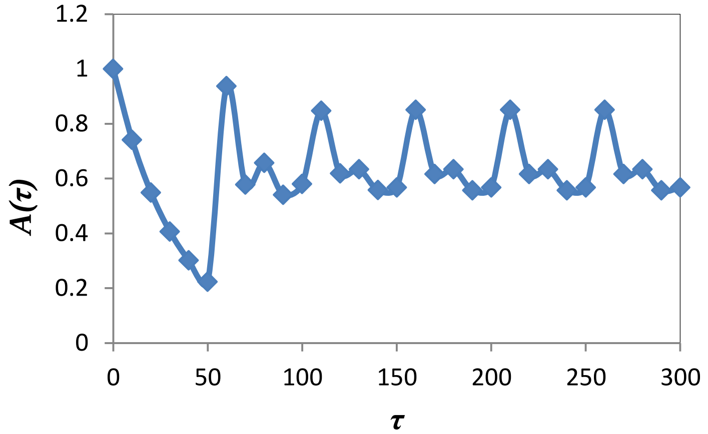


Figure 4.1.1: Point availability of the wind turbine system

4.1.2 Limiting average availability

The relation amongst the systems limiting average availability and the inspection interval is given by the Table 4.1.1 and Figure 4.1.2.

Table 4.1.1: Limiting average availability versus inspection time

T	$\bar{A}$
50	0.71091
100	0.63974
150	0.60096
200	0.57753
250	0.56244
300	0.55213
350	0.54471
400	0.53912
450	0.53478
500	0.5313

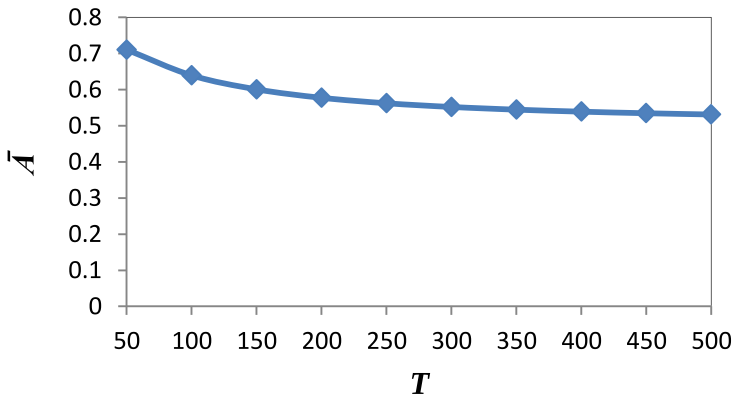


Figure 4.1.2: Limiting average availability versus inspection time for wind turbine system

4.1.3 Sensitivity analysis

The systems point availability differs for various values of the inspection intervals. The point availability for $T = 50, T = 100$ and $T = 150$ respectively are shown in Figure 4.1.3.

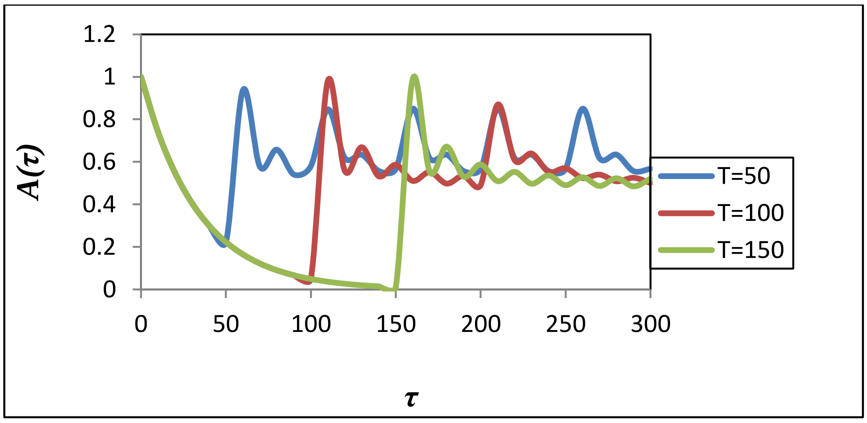


Figure 4.1.3: Point availability of the wind turbine system for different values of *T*

4.1.4 Interpretation of results and discussions

As we can see from the Figure 4.1.1 that in the time period $[0,50]$, $A(\tau)$ decreases as system is not maintained during this time. At $\tau = 50$, $A(\tau)$ increases since either a perfect PR or a CR will be carried out if the system is found to be working or failed respectively.

It could be viewed from Figure 4.1.2, that limiting average availability reduces speedily with the increase in inspection period (T). Since larger value of T mean lesser inspections are performed hence resulting in worse availability.

Also, from Figure 4.1.3 we can observe that despite the fact that the distribution of each failure modes and the repair time distributions are same, the availability fluctuates in different life stages for different values of the inspection time (T). So it is very crucial to make a choice of reasonable inspection time for the sake of improvement of availability and lowering the maintenance cost of the system. For the systems with short life, we can choose the reasonable time for inspection in accordance with the point availability and for the systems with long life, we can decide on the basis of limiting-average availability.

4.2 Model [2]: Availability analysis and inspection optimization for a competing-risk k-out-of-n:G system

To demonstrate the proposed model, example of a Boiler feed water pumps, which is used in steam power plants is considered. These pumps control the amount of water fed to the boiler and then boilers convert heat energy into steam under pressure to produce power. Let for receiving a steady discharge, 2 identical boiler feed water pumps are connected in parallel (1-out-of-2:G system, i.e. Assumption (i) is enforced) into a common header. Excessive corrosive effect of fluid (FM 1) and excessive pressure on the shaft (FM 2) cause failure of the boiler feed water pumps (i.e. Assumption (viii) can be enforced). If any of the pump is found working during the inspection, a perfect PR is carried out (i.e. Assumptions (vii) and (x) can be enforced). If both the pumps are found failed during the inspection because of any of the above defined FMs, respective perfect CR is carried out taking a random time. The distributions of repair time for these FMs may be dissimilar (i.e. Assumption (ix) can be enforced).Thus, the model proposed in this study is utilized to determine the availability and LRACR of the system composed of 2 identical boiler feed water pumps connected in parallel.

Let us assume that failure distribution for each component corresponding to FM 1 and FM 2 be $F_{c_1}(t) = 1 - e^{-0.1t}$ and $F_{c_2}(t) = 1 - e^{-0.2t}$ respectively. Let, failure time X_1 and X_2 are mutually independent. Suppose that the CR times for FM 1 and FM 2 are Y_1 and Y_2 respectively with distribution function $G_1(y) = G_2(y) = 1 - e^{-y}$. Let the inspection time in this case be 5.

4.2.1 Reliability

The reliability of this example can be calculated using equation (3.2.2) and is plotted in Figure 4.2.1.

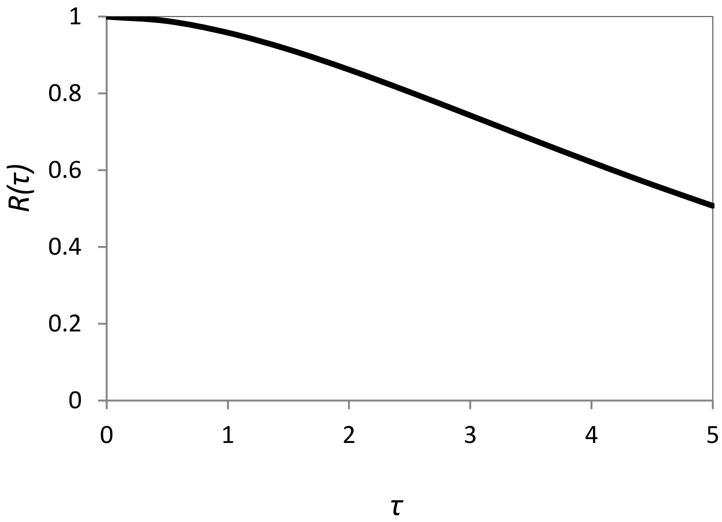


Figure 4.2.1: Reliability versus time

4.2.2 Point availability

By using equation (3.2.10), we can get the point availability of the system, $A(\tau)$ which is graphed in Figure 4.2.2.

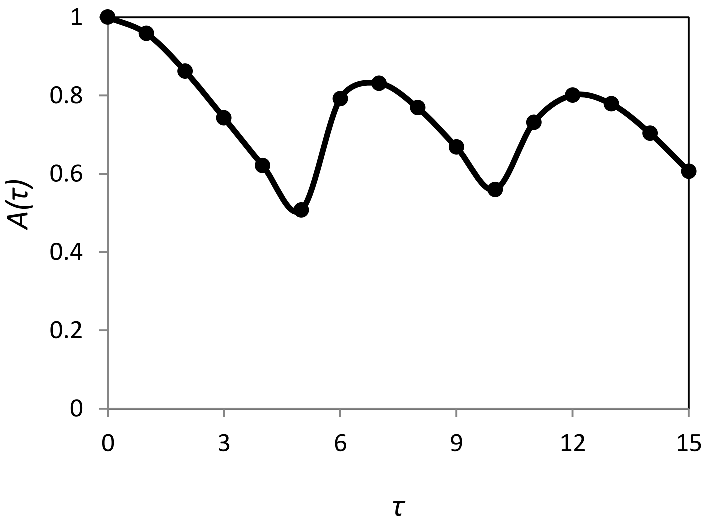


Figure 4.2.2: Systems point availability

4.2.3 Sensitivity analysis

In order to observe the consequence of I on $A(\tau)$, sensitivity study is carried out, which is given in Figure 4.2.3.

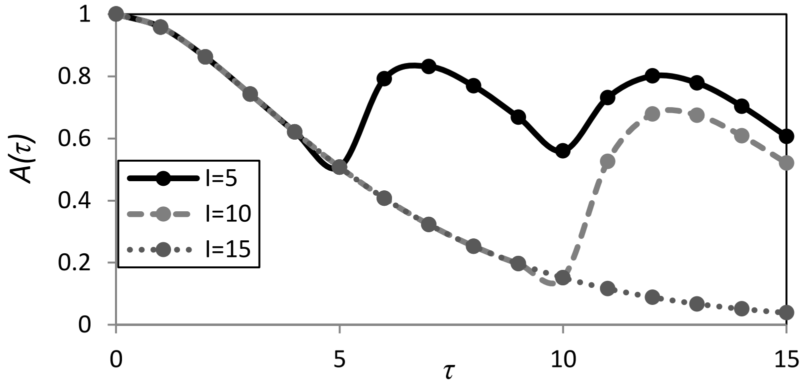


Figure 4.2.3: System point availability for distinct values of *I*

4.2.4 Limiting availability

Evidently, inspections help to reduce the system failure and consequently help to improve the system availability. Using equation (3.2.16), we calculate the limiting availability, which is plotted in Figure 4.2.4.

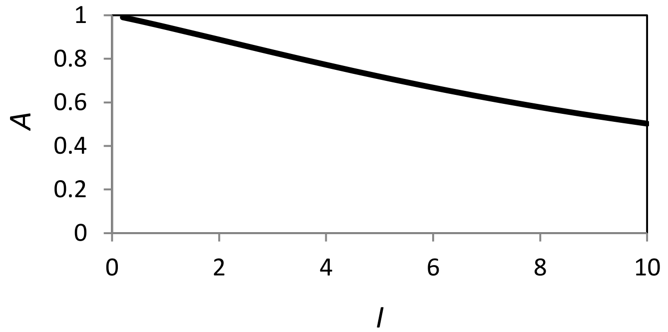


Figure 4.2.4: Limiting availability versus inspection time

4.2.5 LRACR

Let us estimate the cost parameters to be $C_{ins} = 1, C_{R_1} = 5, C_{R_2} = 5$ and $C_p = 10$. The LRACR is calculated using equation (3.2.20) and given by Figure 4.2.5.

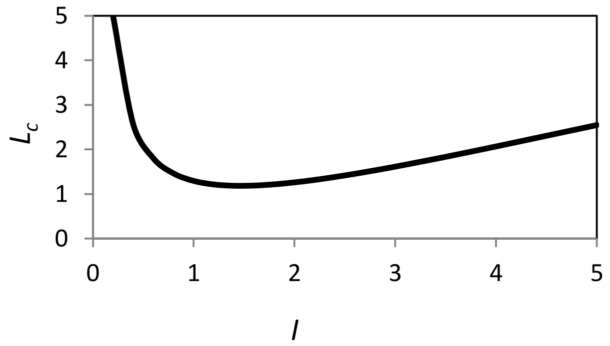


Figure 4.2.5: LRACR against inspection period

4.2.6 Interpretation of results and discussions

It is observable from Figure 4.2.1 that system becomes less reliable with increasing time. As we can see from the Figure 4.2.2 that in the time period [0,5], $A(\tau)$ decreases as system is not maintained during this time. $A(\tau)$ increases in the interval [5,6.9] since either a perfect PR/CR will be carried out if system is found to be working/failed. Further, the point availability is found to be decreasing in the time interval [6.9,10]. Again at $\tau=$10 as the repair action is taken here so the availability started increasing.

More often the system is inspected; sooner the failures are resolved and hence surging systems availability, i.e. a higher value of I results in worst availability of the system. Taking every other parameters same, I is steadily increased from 5 to 15 with step size 5 and the availability is calculated for each I respectively which is then plotted in Figure 4.2.3.

It could be viewed from Figure 4.2.4, that limiting availability reduces speedily with the increase in inspection period. Since larger value of I mean lesser inspections and hence worse availability.

Generally, smaller inspection interval increases the inspection cost and large value of inspection interval results in greater penalty cost on systems down time. Hence, an optimal period must be selected in order to balance the spending on inspections and the penalty cost. As we can see from Figure 4.2.5, increasing the value of I first compels the LRACR to deplete and then it increases.

Also, we see that minimum cost is obtained at I=1.464. Hence, optimal inspection period for minimum LRACR is I=1.464.

4.3 Model [3]: Modeling periodically inspected *k*/*r*-out-of-*n* system

Consider a power supply used to supply electric power of specified current, voltage and frequency to an electrical load. Power supplies consist of a power input terminal, receiving energy in form of electric current and at least one power output terminal, delivering current to the load. Let there are 4 power output terminals. In order to get the specific output, it is considered that, if two terminals fail then the system is considered to enter a degraded state whereas if three fail then system is said to be failed. That means $n = 4, k = 3, r = 2$, i.e. we considered a 3/2-out-of-4 system. Let hazard rate (λ) of each terminal be 0.2.

Now, we demonstrate the numeric results for reliability, point and limiting availability and LRACR for the power supply system.

4.3.1 Reliability

Systems reliability($R(\tau)$) is given by $R(\tau) = 1 - F_F(\tau)$ which is calculated using equation (3.3.2) and is plotted in Figure 4.3.1.

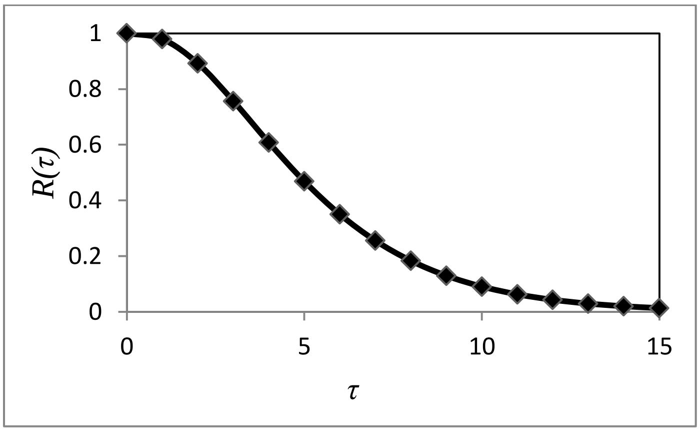


Figure 4.3.1: Reliability for power supply system

Reliability of the system being in normal state ($R'(\tau)$) is given by $R'(\tau) = 1 - F_D(\tau) - F_F(\tau)$ and can be calculated using equations (3.3.1) and (3.3.2). Figure 4.3.2 is the graph of $R'(\tau)$.

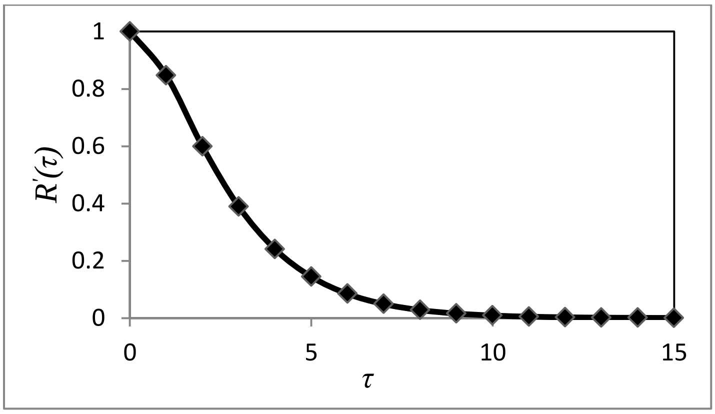


Figure 4.3.2: Reliability of the system being in normal state

4.3.2 Point Availability

Let, $\mu_D = 1, \mu_F = 1 \ and \ I = 5.$ Using equation (3.3.10), we can get the point availability of the system, $A(\tau)$ which is graphed in Figure 4.3.3.

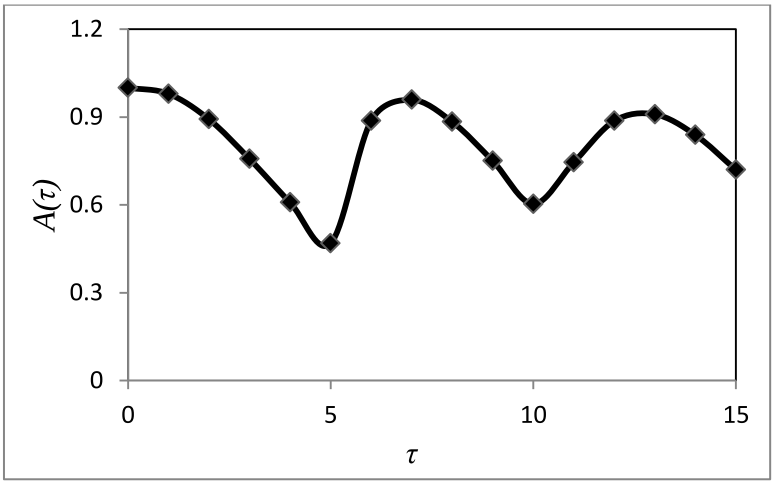


Figure 4.3.3: Point availability of the power supply system

4.3.3 Sensitivity analysis

In order to observe the consequence of I on $A(\tau)$, sensitivity study is carried out. Taking every other parameters same, I is steadily increased from 5 to 15 with step size 5 and the availability is calculated for each I respectively and plotted in Figure 4.3.4.

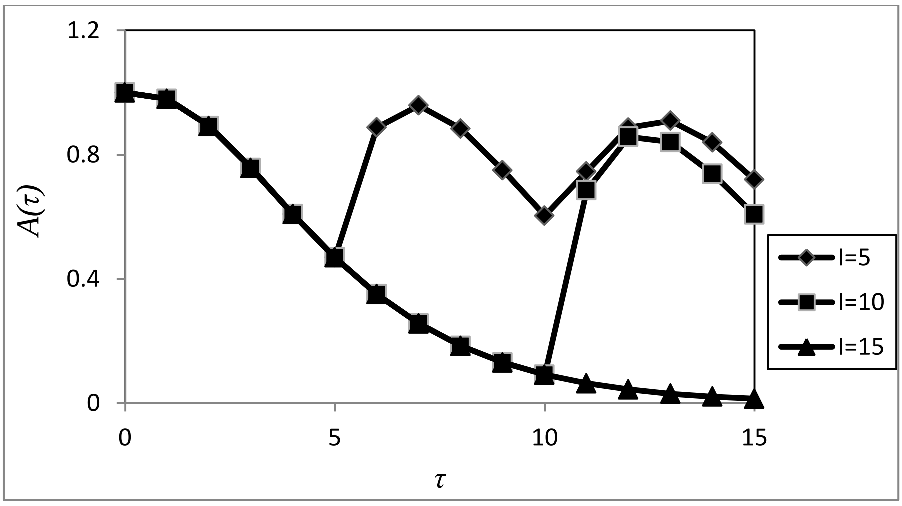


Figure 4.3.4: Point availability for distinct values of *I*

4.3.4 Limiting Availability

Using equation (3.3.13), one can calculate the limiting availability, which is plotted in Figure 4.3.5.

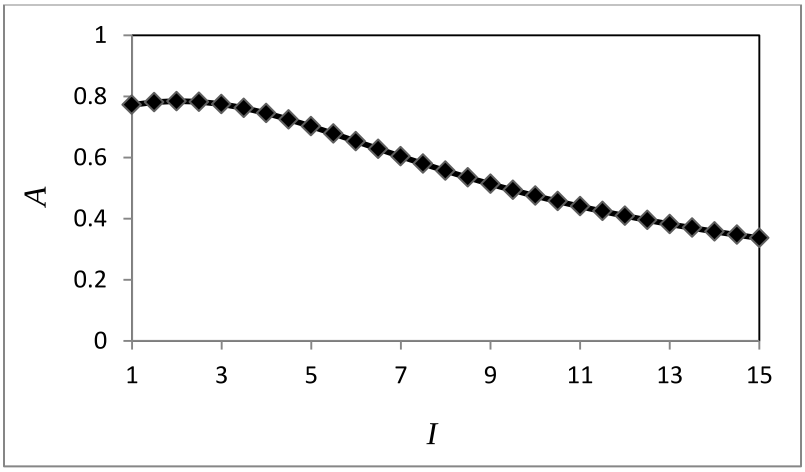


Figure 4.3.5: Limiting availability versus inspection time

Also, we plot the function $H(I)$, calculated using equation (3.3.14) and given by Figure 4.3.6.

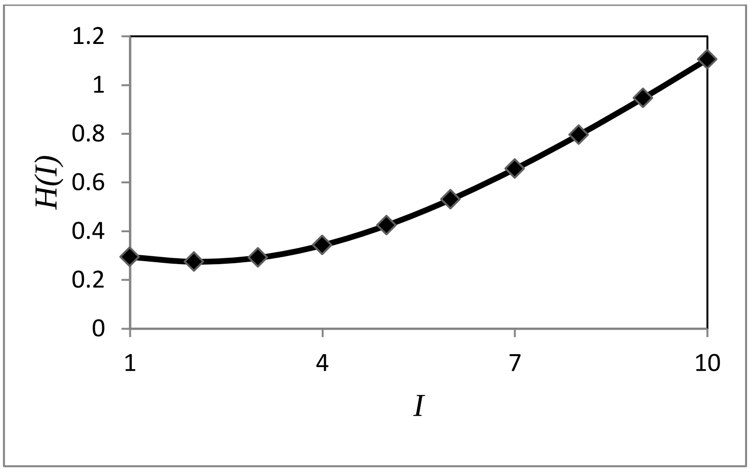


Figure 4.3.6: Plot of $H(I)$

4.3.5 LRACR

To estimate the cost, consider $C_{ins} = 1, C_{R_1} = 4, C_{R_2} = 4$ and $C_p = 10$. The LRACR is calculated using equation (3.3.21) and given by Figure 4.3.7.

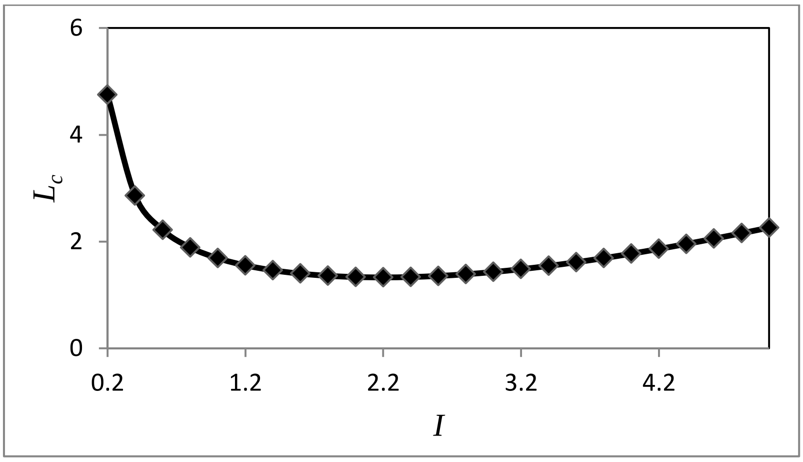


Figure 4.3.7: LRACR against inspection period

4.3.6 Interpretation of results and discussions

The objective here is to discuss the numerical results and observe the impact of inspection period on both the availabilities and the cost rate, thus obtaining optimal inspection period.

From Figure 4.3.1 and Figure 4.3.2 we can clearly observe that system becomes less reliable with increasing time.

From Figure 4.3.3, we can see that in the time period [0,5], $A(\tau)$ decreases as system is not maintained during this time. $A(\tau)$ increases in the interval [5,6.9] since a CR would have been conducted if system is found to be partially/completely failed. Further, the point availability is found to be decreasing in the time interval [6.9,10]. Again at $\tau = 10$ as the repair action is taken here so the availability started increasing.

Evidently, inspections help to reduce the system failure and consequently help to improve the system availability. Frequently the system is inspected; sooner the failures are detected/resolved and hence systems availability surges, i.e. a higher value of I means lesser inspections; leading to worse availability. From Figure 4.3.4 we observe that availability is worse when $I = 15$ compared to $I = 5$ and 10.

It could be viewed from Figure 4.3.5, that limiting availability firstly increases attains maximum at I=2.054 then reduces speedily with the increase in inspection period. Also, from Figure 4.3.6 we can see that minimum of $H(I)$ is obtained at

I=2.054. Hence, optimal inspection time which maximizes availability is obtained to be *I*=2.054 corresponding to maximum limiting availability of 0.7843.

Figure 4.3.7 reveals that increasing the value of I first compels the LRACR to deplete and then allows it to increase. Generally, smaller inspection interval increases the inspection cost and large value of inspection interval results in greater penalty cost on systems down time. Hence, an optimal period must be selected in order to balance the spending on inspections and the penalty cost. From Figure 4.3.7, one can observe that minimum cost is obtained at *I*=2.19.

Hence, on the basis of our analysis (maximizing availability and minimizing cost) the optimal inspection period can be considered to be 2.

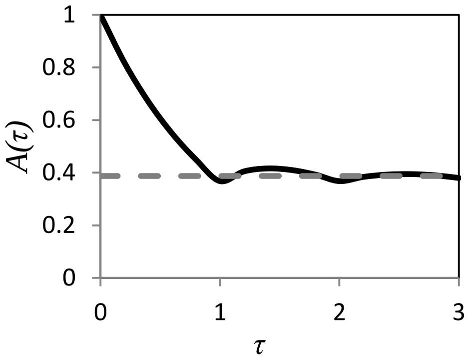
4.4 Model [4]: Availability and cost assessment of systems with dormant failure undergoing sequential inspections

Consider a pressure switch, employed in various technical and industrial processes to demonstrate the proposed model. Pressure switch helps automatically control the switch contact, if a preset pressure is reached. For example, a self-adhesive mat is used to automatically open/close doors on commercial buildings; Hydraulic pressure switches are used in vehicles, to alert if the engine's oil pressure reaches an unsafe level. Here, we consider a pressure switch, which is used to spot if oil pressure reaches a hazardous point. In order to detect the failure of switch, oil pressure is dropped below unsafe level and then it is detected if it responds accurately or not. Here, dropping the pressure and its detection corresponds to an inspection.

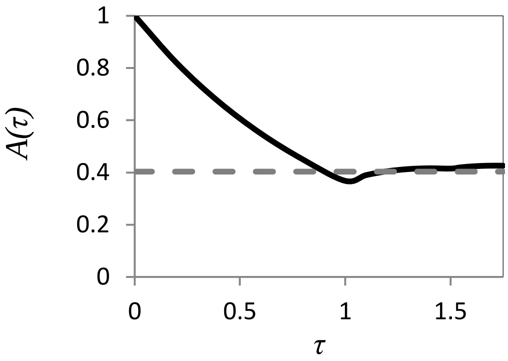
Let reliability of the switch be $R(\tau) = e^{-\tau}$. Let its repair density function be given by $g(y) = e^{-y}$. Let initially inspection be conducted at 1 time unit, i.e. $T = 1$. Now, we acquire the numeric results for point and limiting availability and LRACR for the proposed example. The objective here is to observe graphically the impact of inspection period on both the availabilities and the cost rate.

4.4.1 Availability analysis

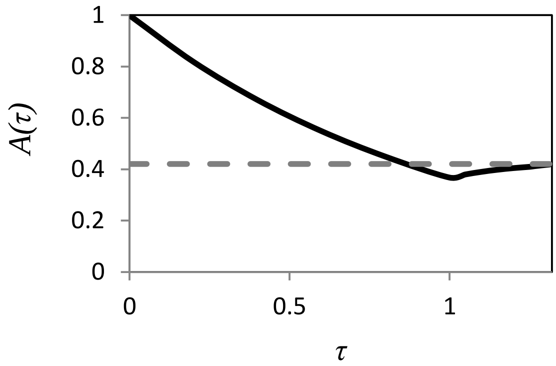
By using equation (3.4.6), we can get the point availability of the system, $A(\tau)$ which is graphed for different values of 'a' in Figure 4.4.1. Also, the limiting availability is also graphed, which was found on the basis of equation (3.4.10).



(a)



(b)



(c)

Figure 4.4.1: Systems point availability
(a) For *a*=1 (b) For *a*=1/2 (c) For *a*=1/4

The limiting availability calculated using equation (3.4.10) for different values of '*a*' is plotted in Figure 4.4.2.

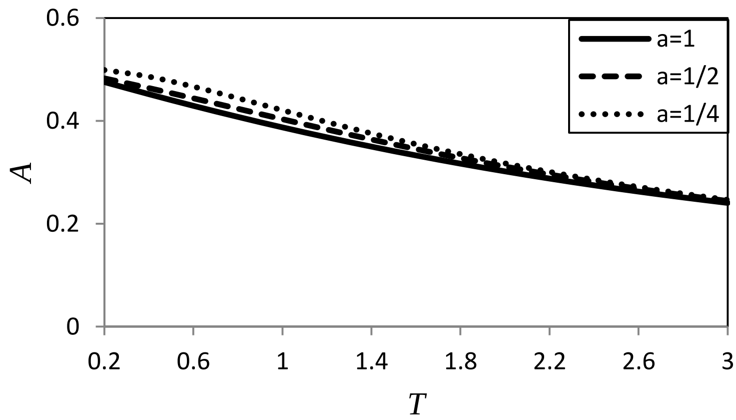
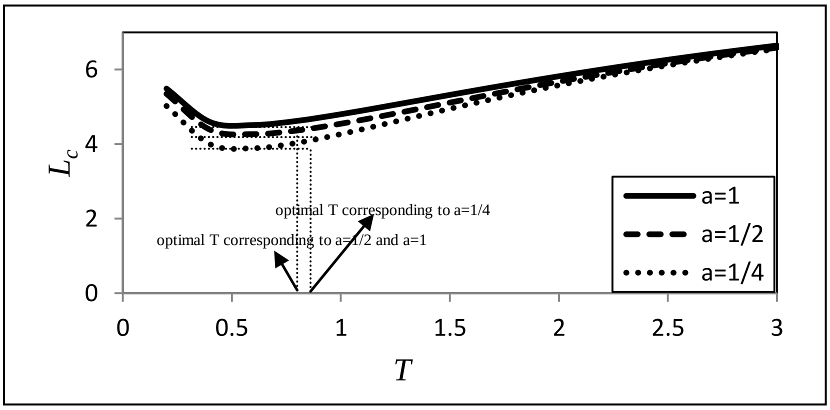


Figure 4.4.2: Limiting availability versus inspection time for different values of '*a*'

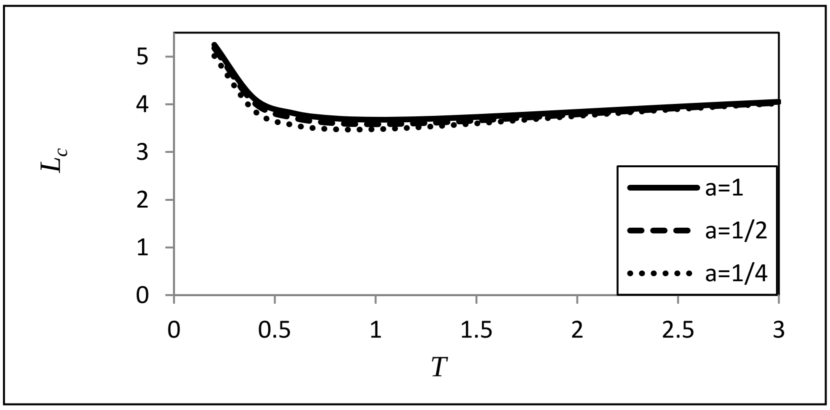
4.4.2 Cost analysis

To estimate the cost rate of the system, consider $C_{ins} = 1, C_R = 5$ and $C_p = 10$. The LRACR is calculated for a =1, 1/2 and 1/4 using equation (3.4.17) and given by Figure 4.4.3(a).

Figure 4.4.3(b) is plotted by keeping the inspection cost same and decreasing the penalty cost to 5.



(a)



(b)

Figure 4.4.3: LRACR against inspection period
(a) For $C_p = 10$ (b) For $C_p = 5$

4.4.3 Interpretation of results and discussions

As we can see from Figure 4.4.1 that for each 'a', $A(\tau)$ decreases in the time period $[0,1]$ as the system is not maintained during this time. At $\tau = 1$, $A(\tau)$ increases since a CR would have been conducted if system is found to be failed. This process of failure and repair results in the decrease and increase in value of $A(\tau)$ respectively and after some time the availability becomes steady/constant which is the limiting availability. It can be seen from Figure 4.4.1 that limiting availability corresponding to $T = 1$ for $a = 1, a = 1/2$ and $a = 1/4$ are 39%, 40% and 42% respectively.

It could be viewed from Figure 4.4.2, that limiting availability reduces speedily with the increase in inspection period. Since, inspections help to improve the availability so it is evident that more often the system is inspected; sooner the failures are resolved and hence systems availability increases, i.e. a higher value of T results in worse availability of the system. Also low value of 'a' suggest that inspections are conducted frequently compared to higher value of 'a' thus increasing availability. It is also evident from the Figure3, that availability increases as 'a' decreases; availability is highest when a =1/4, lowest when $a = 1$ and lies somewhere in between for $a = 1/2$.

Generally, smaller inspection interval increases the inspection cost and large value of inspection interval results in greater penalty cost on systems down time. Hence, an optimal period must be selected in order to balance the spending on inspections and the penalty cost. As we can see from the Figure 4.4.3(a), increasing the value of T first compels the LRACR to deplete and then it increases hence, we get optimal value of T.

It is evident from Figure 4.4.3(b) that on keeping the inspection cost same, as the penalty cost is decreased, the cost rate for a =1, 1/2 and 1/4 becomes almost similar. Since, downtime was more for higher value of 'a' but low penalty cost resulted in small variation between the cost rates. But if the penalty cost will be higher then there will be large variation between the cost rates for different values of 'a' as seen in the Figure 4.4.3(a).

4.5 Model [5]: Modeling sequentially inspected system prone to degradation and shocks

The practical contribution of this model is motivated by the descriptive example of an oil pipeline system, which is subject to leakages and sudden burst. Oil pipelines are extremely economical, energy-efficient, environment friendly and safest way to transport refined/crude oil across great distances from oil fields/terminals to an oil device/refinery. During the transportation process, pipelines may undergo several failures, like leakages (owing to corrosion or crack) and sudden shock/burst/fracture (owing to earthquake, thunder or wind). As the crack defects emerge because of corrosion, pipelines are more subjected to shock damages and the possibility of occurrence of burst increases sharply. So, as per our model, as the crack arises in pipeline, it enters degraded state.

Let the pipeline corrosion rate be 0.1 i.e. $\lambda_1 = 0.1$ and after a crack let it be equivalent to 0.2, i.e. $\lambda_2 = 0.2$. Let the rate at which pipeline burst being at normal state, i.e. without any cracks be 0.2 i.e. $\lambda_3 = 0.2$ and with the cracks it burst with rate 0.3, i.e. $\lambda_4 = 0.3$. Let the pipeline in degraded and failed state be repaired with density function $g_D(y) = g_F(y) = 1 - \mathrm{e}^{-\mathrm{y}}$.

4.5.1 Reliability Analysis

The reliability of the oil pipeline and reliability of oil pipeline being in normal state is obtained using equation (3.5.4) and equation (3.5.1) respectively and is given by Figure 4.5.1.

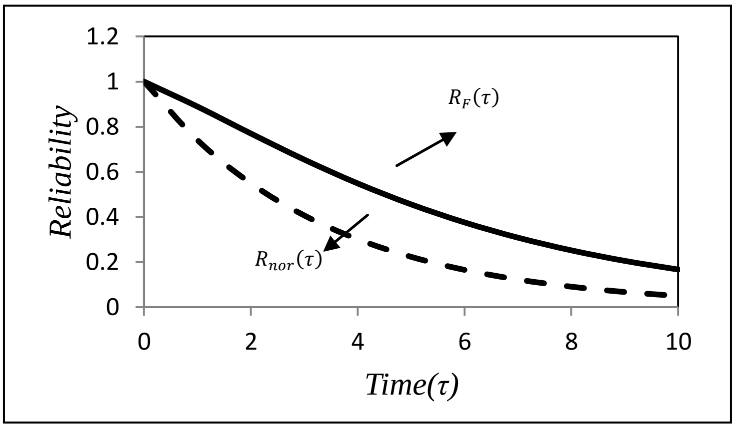


Figure 4.5.1: Different Reliabilities of the system versus time

4.5.2 Availability Analysis

Let our system be inspected initially at time $I = 2$, and let $a = 1/2$. Then, the point availability of our sequentially inspected oil pipeline system could be obtained using equation (3.5.18) and is plotted in Figure 4.5.2.

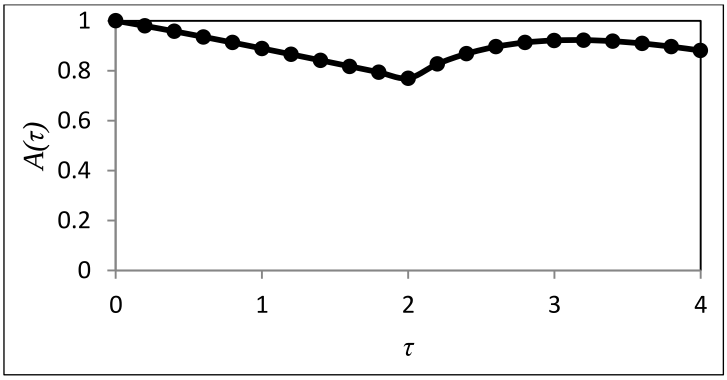


Figure 4.5.2: Point availability of the system versus time

Limiting availability of the oil pipeline for different values of I is calculated using equation (3.5.23) and plotted in Figure 4.5.3.

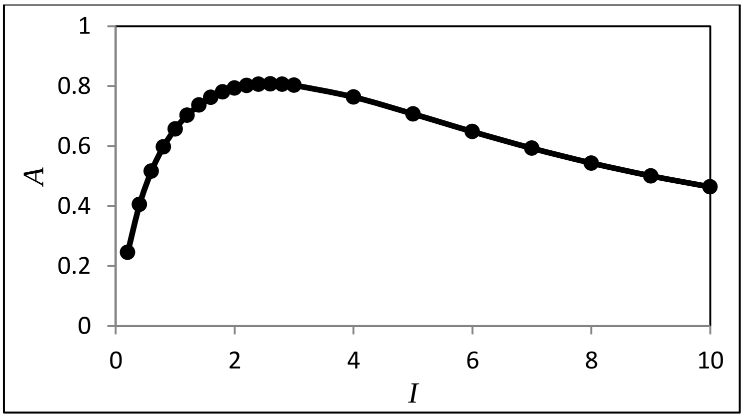


Figure 4.5.3: Limiting availability of the system for different values of *I*

4.5.3 Cost Analysis

Let us assume the cost parameters as $C_{ins} = 1, C_{R_D} = 4, C_{R_F} = 4$ and $C_p = 8$. The LRACR of the pipeline system is calculated for different values of I using equation (3.5.31) and is plotted in Figure 4.5.4.

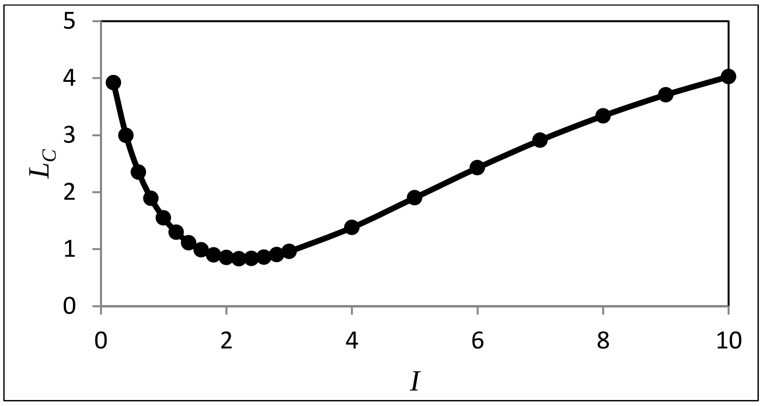


Figure 4.5.4: LRACR of the system for different values of *I*

4.5.4 Interpretation of results and discussions

Clearly, as we can see from the Figure 4.5.1, the reliability of oil pipeline is greater than reliability of it being in normal state. Since, probability of degradation is excluded from the reliability of the pipeline for finding reliability of it being in normal state.

As we can see from the Figure 4.5.2, $A(\tau)$ decreases in the time period [0,2] as the system is not maintained during this time. $A(\tau)$ increases in the interval [2,3.2] since a repair would have been conducted if system is found to be failed (partially/completely). Further, the point availability is found to be decreasing in the time interval [3.2, 4]. This process of failure and repair will repeat and result in the decrease and increase in value of $A(\tau)$ respectively. After some time, the availability will become constant/steady called the limiting availability.

As we can see from the Figure 4.5.3, limiting availability firstly increases attains maximum at $I = 2.6$ then reduces speedily with the increase in value of *I*. Hence, the optimal value of *I* corresponding to $a = 1/2$ is $I = 2.6$.

Figure 4.5.4 reveals that increasing the value of *I* first compels the LRACR to deplete and then allows it to increase. Generally, smaller inspection interval increases the inspection cost and large value of inspection interval results in greater penalty cost on systems down time. Hence, an optimal period must be selected in order to balance the spending on inspections and the penalty cost. From the Figure 4.5.4, one can observe that minimum cost is obtained at $I = 2.4$. Hence, optimal value of I corresponding to *a=1/2* is obtained to be *I=2.4.*

4.6 Model [6]: Modeling systems with revealing and non-revealing failures undergoing periodic inspection

The proposed approach could be applied to systems like electric motors wherein Megger test helps in revealing the age and health of the motor. During inspection sometime it may happen that while applying Megger test, the actual existence failure is not detected, i.e. a motor may show positive result on applying the Megger test even when though motor itself is not visibly properly functioning.

Some distributions corresponding to systems life are considered in the numeric examples below to demonstrate the proposed approach. In the considered system (electric motor), two life distributions (Normal and Weibull) with rising hazard rates and one life distribution (Exponential) with constant hazard rate along with three probability values of non-revealing failures ($a = 0.1, a = 0.5$ and $a = 0.9$) are taken into account.

Meanwhile, the input parameters are considered to be $\alpha = 0.003, \beta = 0.005, T_i = 0.1, T_f = 0.6$ and $T_c = 0.6$. Additionally, the costs parameters are assumed to be: $C_i = 1, C_c = 8, C_f = 3$ and $C_d = 5$.

The availability and cost is calculated using equation (3.6.11) and equation (3.6.15) respectively, corresponding to each value of p. The optimal I^* is determined in each case corresponding to maximal availability and minimal cost.

(a) *Normal Distribution* $(\boldsymbol{\mu}, \boldsymbol{\sigma})$**:** $\boldsymbol{f(u) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{u-\mu}{\sigma})^2}}$

The availability and cost is calculated for $\mu = 2$ and $\sigma = 1$, which is graphed in Figure 4.6.1 and Figure 4.6.2 respectively. Corresponding optimal values are given in Table 4.6.1 and Table 4.6.2.

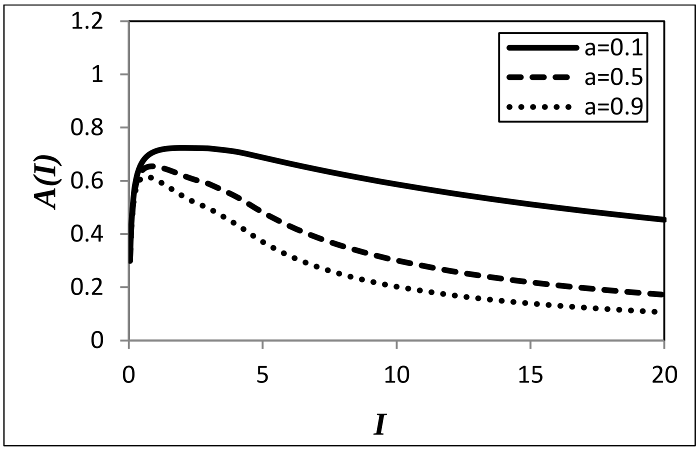


Figure 4.6.1: $A(I)$ for normally distributed lifetime with $\mu = 2$ and $\sigma = 1$

Table 4.6.1: Optimal strategy based on availability for normal distribution with $\mu = 2$ and $\sigma = 1$

a	**0.1**	**0.5**	**0.9**
I^*	2	0.9	0.7
$A(I^*)$	0.723519	0.654417	0.614832

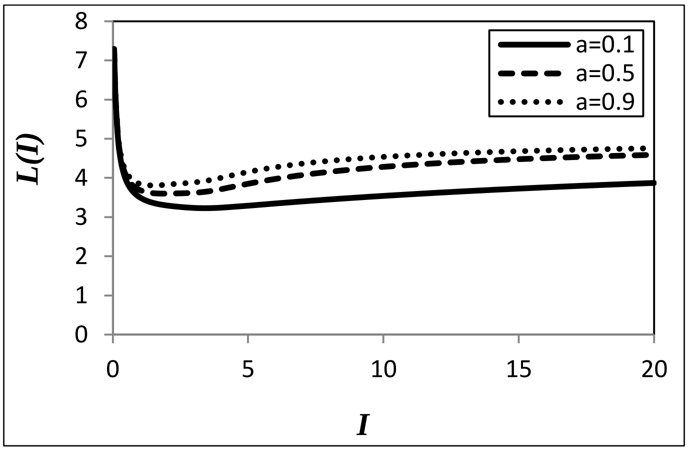


Figure 4.6.2: $L(I)$ for normally distributed lifetime with $\mu = 2$ and $\sigma = 1$

Table 4.6.2: Optimal strategy based on cost for normal distribution with $\mu = 2$ and $\sigma = 1$

a	**0.1**	**0.5**	**0.9**
I^*	3.4	1.9	1.5
$L(I^*)$	3.228463	3.599862	3.808749

(b) ***Weibull Distribution*** $(\boldsymbol{\lambda}, \boldsymbol{\gamma})$: $\boldsymbol{f(u) = \gamma\lambda^{\gamma} t^{\gamma-1} e^{-(\lambda t)^{\gamma}}}$

The availability and cost is evaluated for parameters $\lambda = 0.2$ and $\gamma = 2$, which is further plotted in Figure 4.6.3 and Figure 4.6.4 respectively, with optimal values given in Table 4.6.3 and Table 4.6.4 correspondingly.

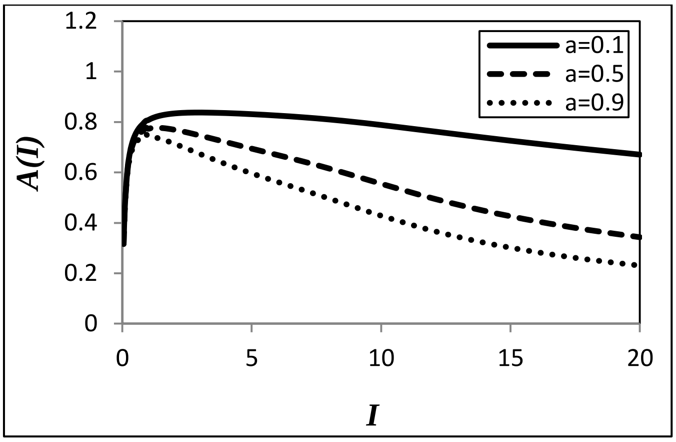


Figure 4.6.3: $\boldsymbol{A(I)}$ **for lifetime following Weibull distribution with** $\boldsymbol{\lambda = 0.2}$ **and** $\boldsymbol{\gamma = 2}$

Table 4.6.3: Optimal strategy based on availability for weibull distribution with $\boldsymbol{\lambda = 0.2}$ **and** $\boldsymbol{\gamma = 2}$

$\boldsymbol{a}$	**0.1**	**0.5**	**0.9**
$\boldsymbol{I^*}$	3	0.9	0.9
$\boldsymbol{A(I^*)}$	0.837441	0.777407	0.752839

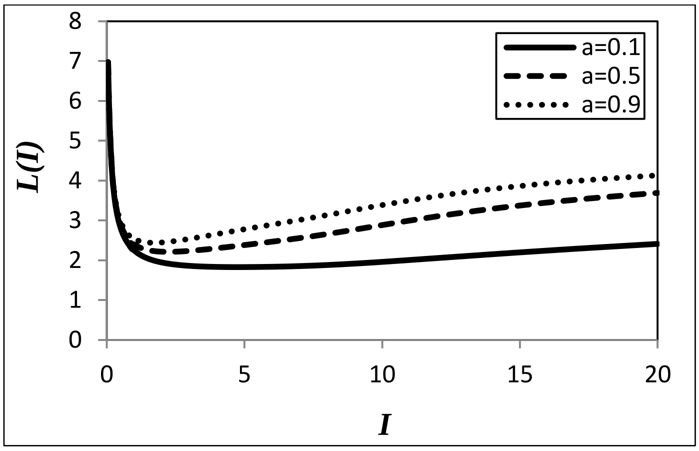


Figure 4.6.4: $\boldsymbol{L(I)}$ **for lifetime following Weibull distribution with** $\boldsymbol{\lambda = 0.2}$ **and** $\boldsymbol{\gamma = 2}$

Table 4.6.4: Optimal strategy based on cost for weibull distribution with $\boldsymbol{\lambda = 0.2}$ and $\boldsymbol{\gamma = 2}$

a	**0.1**	**0.5**	**0.9**
I^*	4.8	2.2	1.7
$L(I^*)$	1.828595	2.212197	2.438828

(c) *Exponential distribution* (λ): $f(u) = \lambda e^{-\lambda u}$

The availability and cost is obtained corresponding to $\lambda = 0.2$ and is given by Figure 4.6.5 and Figure 4.6.6 with optimal values in Table 4.6.5 and Table 4.6.6 respectively.

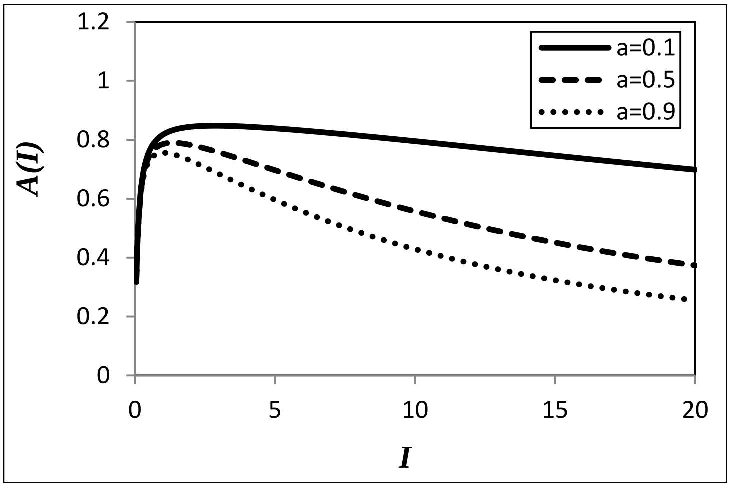


Figure 4.6.5: $A(I)$ for exponentially distributed lifetime with $\boldsymbol{\lambda = 0.2}$

Table 4.6.5: Optimal strategy based on availability for exponential distribution with $\boldsymbol{\lambda = 0.2}$

a	**0.1**	**0.5**	**0.9**
I^*	2.9	1.4	1
$A(I^*)$	0.847628	0.789598	0.755778

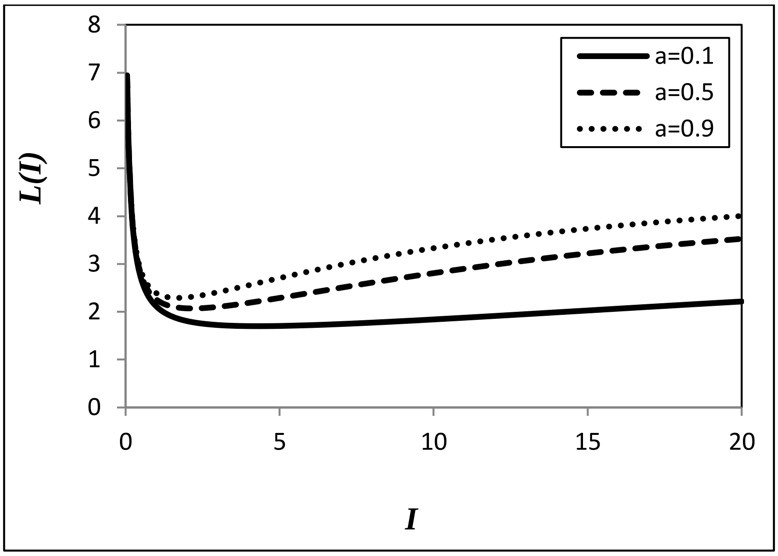


Figure 4.6.6: $L(I)$ for exponentially distributed lifetime with $\lambda = 0.2$

Table 4.6.6: Optimal strategy based on cost for exponential distribution with $\lambda = 0.2$

a	**0.1**	**0.5**	**0.9**
I^*	4.3	2.2	1.7
$L(I^*)$	1.699135	2.069946	2.290845

4.6.1 Interpretation of results and discussions

Generally, smaller inspection interval (i.e. more number of inspections) results in the higher inspection cost and large value of inspection interval (i.e. lesser number of inspections) results in greater penalty cost due to systems down time. Hence, an optimal period must be selected in order to balance the inspection and the penalty cost. As seen from Figure 4.6.2, Figure 4.6.4 and Figure 4.6.6, increasing the value of I first compels the LRACR to deplete and then it increases providing us the optimal inspection interval in each case.

Initially, the system availability increases monotonically since the system failure rate is low but after some time, the larger value of I (i.e. lesser number of inspections) would result in worse availability. As we can see from Figure 4.6.1,

Figure 4.6.3 and Figure 4.6.5, the availability firstly increases and then keeps on decreasing with increasing value of I providing us the maximal availability.

It is worthwhile to see from the above cases that as the probability a of non-revealing failure increases, the system needs to be inspected more often, since I^* decreases with the increasing value of a (See Table 4.6.1 to 4.6.6).

4.7 Model [7]: Markov process approach for analyzing periodically inspected competing-risk system embodying downtime threshold

Numerical example of a protection device is considered here to illustrate the model. Protection devices hold a very vital role in power systems. In order to safeguard the power system from breakdown, protection devices are required. Protection devices protect the power system by disconnecting the faulty parts from the remaining electric network. The typical failures of protection devices are: Circuit breaker failure (FM1) and Protection relays failure (FM2). In the case of failure of protection devices, back-up protection is used to remove the faults of the power system. The protection device is believed to be in functional state if the device is repaired within a critical time, i.e. the failure is ignored or deferred. Assume that periodic inspection is conducted on the protection device system. Definitely, enormous loss is brought up on the failure of protection devices. In the following section, protection device under the assumption of delayed failure is used to illustrate the findings of prior sections.

Let us consider the hazard rate of FM1 and FM2 to be 1 and 2 respectively. Let the distribution function corresponding to CR time of FM1 and FM2 be $G_1(u) = G_2(u) = 1 - e^{-u}$. Furthermore assume $\tau = 0.1$ and $I = 1$.

4.7.1 Point availability analysis for protection device

Using equation (3.7.9) and equation (3.7.16), the point availability for the initial and new system is evaluated, which is given by Figure 4.7.1.

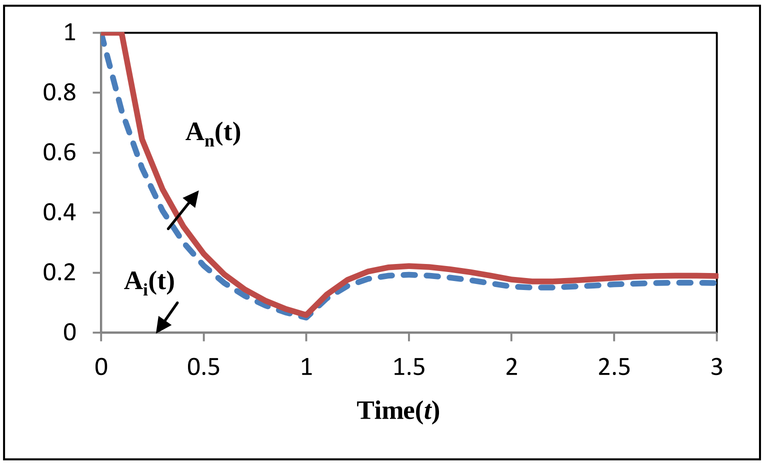


Figure 4.7.1: Point availability of the initial and new system corresponding to $\tau = 0.1$

4.7.2 Limiting availability analysis for protection device

From equation (3.7.20) and equation (3.7.21), the limiting availability of the new and the initial system is obtained and further graphed in Figure 4.7.2.

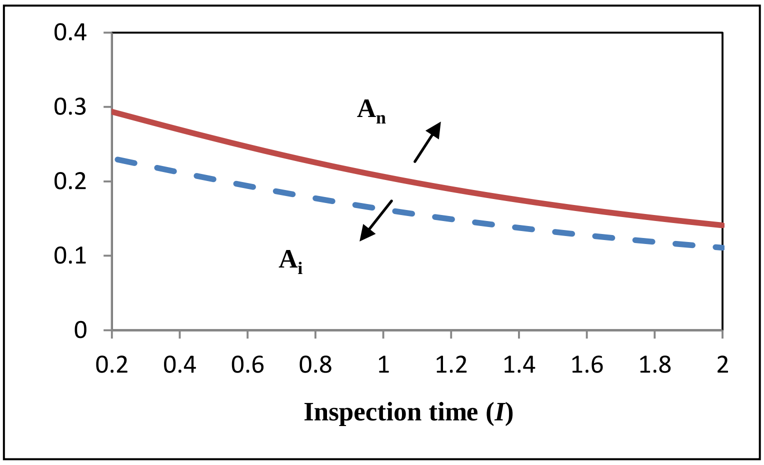


Figure 4.7.2: Limiting availability of the initial and new system for $\tau = 0.1$

A sensitivity analysis is being done to see the consequence of downtime threshold on the limiting availability of the new system. Figure 4.7.3 gives the limiting availability for different values of τ.

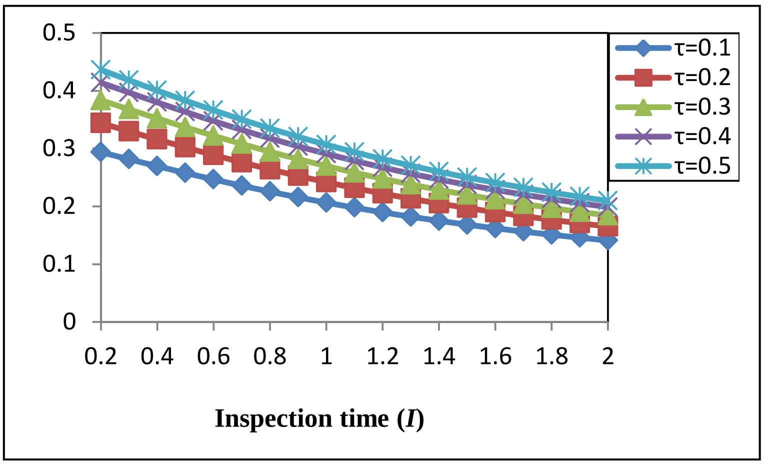


Figure 4.7.3: Limiting availability of the new system for distinct values of τ

4.7.3 Cost availability for protection device

For performing the cost analysis, cost parameters are assumed to be $C_I = 1, C_{R_1} = 4, C_{R_2} = 4$ and $C_P = 8$. The LRACR of the new and the initial system is estimated using equation (3.7.26) and equation (3.7.27) respectively. Figure 4.7.4 shows the LRACR of the new and the initial system.

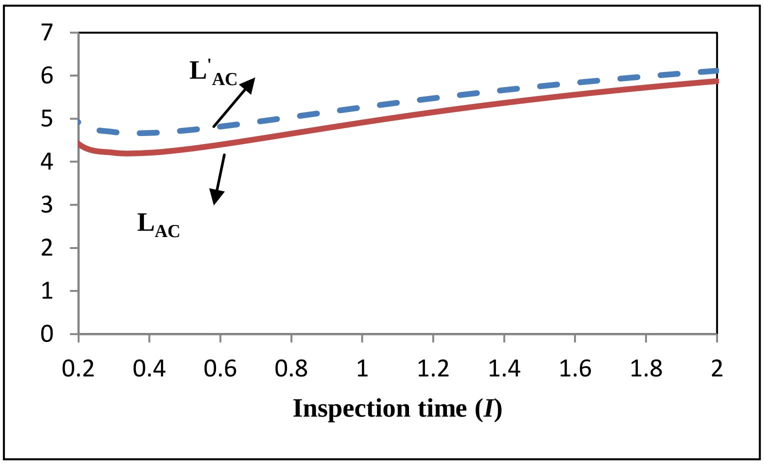


Figure 4.7.4: LRACR of the initial and new system for $\tau = 0.1$

A sensitivity analysis is being conducted for analyzing the effect of downtime threshold on the LRACR of the new system and plotted in Figure 4.7.5.

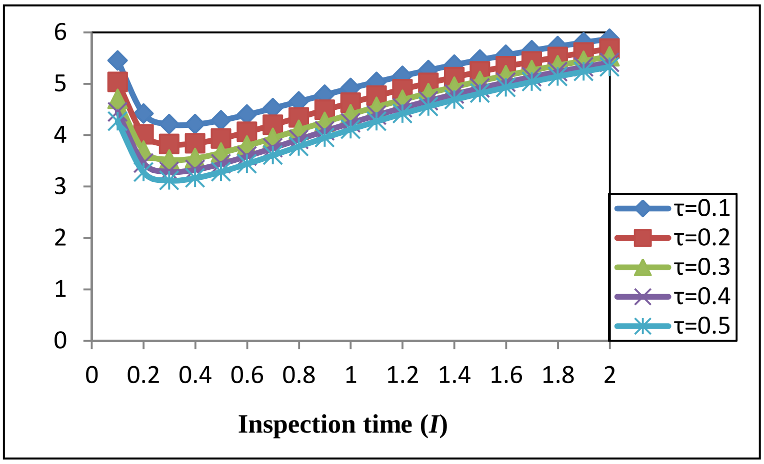


Figure 4.7.5: LRACR of the new system for distinct values of τ

4.7.4 Interpretation of results and discussions

From the Figure 4.7.1, it is clearly observable that the point availability of the new system for $t \leq 0.1$ equals 1 due to delayed breakdown effect. Evidently, because of downtime threshold, availability for the new system is more than that of the initial system. However, the availability for both the initial and new systems goes down in the time period $[0,1]$ as no inspection or maintenance is conducted during that period. Then, at time $t = 1$, availability goes up as CR may have been done if system is found in breakdown state.

It is evident that if the system is inspected more often, the availability will improve, i.e., if *I* is large the availability will be worse but if *I* is small, the availability will be more. The same is observable from Figure 4.7.2. Also, from Figure 4.7.3, we can see that for a fixed inspection time, the limiting availability of the system surges with the surge in value of τ.

If the frequency of inspections is more, the addressing and rectification of the failure will be done earlier consequently lowering the downtime/penalty cost and incrementing the inspection cost. However, low frequency of inspections mean lesser inspection cost but greater penalty cost. Henceforth, an optimal period is obtained which corresponds to the balance of the inspection and the penalty cost simultaneously. Additionally, because of the delayed breakdown effect, the downtime of the new system is reduced as compared to that of the initial system (can be viewed from Figure 4.7.4) thereby resulting in lesser LRACR.

As depicted in Figure 4.7.5, as τ increases, the LRACR decreases due to decreased penalty cost.

4.8 Model [8]: Particle swarm optimization strategy for design optimization of a series-parallel system incorporating failure dependencies and multiple repair teams

The PSO technique mentioned in section 3.8 is applied at the same data set as considered by **Hu *et al.* (2012)** for the purpose of checking the feasibility of the proposed approach. A system with serially-connected six parallel subsystems is considered, where each subsystem can have a maximum of 15 components. The inherent hazard rate, repair rate and cost of unit components and repair teams of each subsystem are given in Table 4.8.1.

Table 4.8.1: Each subsystems data

Subsystem(k)	**1**	**2**	**3**	**4**	**5**	**6**
λ_k	0.03	0.04	0.05	0.06	0.07	0.09
μ_k	0.10	0.13	0.14	0.20	0.18	0.27
C_k^c	40	50	30	70	65	80
C_k^r	15	20	10	30	25	35

Furthermore, l is fixed to be 0.5 and 1.5 for weak and strong dependence case, i.e. the dependence function for weak dependence will be $g(i) = i^{0.5}$ and for strong it will be $g(i) = i^{1.5}$.

Moreover, the PSO parameters are set as follows:

$w_i = 0.1, w_f = 1, i_{max} = 100, c_{1i} = 0.5, c_{1f} = 2, c_{2i} = 0.5, c_{2f} = 2$ and P=1000.

Our goal is minimizing the construction cost of the system while also meeting the availability requirement as defined in equation (3.8.1). Allocation strategies i.e. the vector $(n_1, n_2, \dots, n_N, r_1, r_2, \dots, r_N)$ are then investigated considering various dependencies (independence, linear, weak and strong) and availability requirements.

4.8.1 Working of PSO

The working of PSO is explained considering following assumptions:

Redundant dependency is taken to be strong. The availability constraint of $A_0 = 0.70$ is considered. Furthermore, only 5 particles were taken so as to understand and scrutinize the PSO's functioning.

Now, we initialize the particle position randomly and calculate the corresponding fitness value as illustrated in Table 4.8.2.

Table 4.8.2: Initial positions/personal best positions of the particles

	Positions	Fitness value
Particle 1	**[8, 10, 10, 3, 1, 6, 5, 10, 10, 3, 1, 6]**	**2575**
Particle 2	[3, 7, 3, 14, 2, 12, 2, 7, 3, 7, 2, 1]	3125
Particle 3	[2, 14, 8, 2, 15, 14, 2, 7, 3, 2, 10, 3]	3870
Particle 4	[11, 2, 3, 12, 7, 7, 9, 2, 3, 1, 7, 7]	3140
Particle 5	[15, 10, 6, 6, 11, 10, 3, 4, 6, 6, 11, 8]	4135

Now, we move to the first iteration. Firstly, the particle velocity is updated and new position vectors are calculated for each particle using equation (3.8.6) and equation (3.8.7). Next, the fitness value is calculated corresponding to each particle, which could be seen from Table 4.8.3.

Table 4.8.3: Second positions of the particles

	Positions	Fitness value
Particle 1	[8, 10, 10, 3, 1, 6, 5, 10, 10, 3, 1, 6]	2575
Particle 2	[4, 8, 4, 13, 2, 11, 3, 8, 4, 6, 2, 2]	3145
Particle 3	[3, 13, 8, 2, 14, 13, 3, 8, 4, 2, 9, 4]	3770
Particle 4	[10, 3, 4, 11, 6, 7, 8, 3, 4, 2, 6, 7]	3065
Particle 5	[14, 10, 7, 5, 10, 9, 4, 5, 7, 5, 10, 7]	3865

Further, each particle's present position is compared with the particle's previous best position and based on the best fitness value, the best position of the particle is updated. The second personal best of the particle is given in Table 4.8.4.

Table 4.8.4: Second personal best positions of the particles

	Positions	Fitness value
Particle 1	**[8, 10, 10, 3, 1, 6, 5, 10, 10, 3, 1, 6]**	**2575**
Particle 2	[3, 7, 3, 14, 2, 12, 2, 7, 3, 7, 2, 1]	3125
Particle 3	[3, 13, 8, 2, 14, 13, 3, 8, 4, 2, 9, 4]	3770
Particle 4	[10, 3, 4, 11, 6, 7, 8, 3, 4, 2, 6, 7]	3065
Particle 5	[14, 10, 7, 5, 10, 9, 4, 5, 7, 5, 10, 7]	3865

4.8.2 Best Solution

Firstly, the availability constraint is set to $A_0 = 0.90$ and several allocations with various forms of redundant dependencies are obtained on the basis of 20 PSO algorithm runs. Table 4.8.5 shows the best optimal allocations for each of the dependencies, where number of units (n_k) and repair teams (r_k) are presented for each subsystem k.

Table 4.8.5: Best optimal solution for $A_0 = 0.90$

	Subsystem(k)	**1**	**2**	**3**	**4**	**5**	**6**
Independence	n_k	3	3	4	3	3	3
	r_k	3	2	3	2	3	2
	C_s			1355			
	A_s			0.9025			
Linear Dependence	n_k	3	3	3	2	3	2
	r_k	2	2	2	2	2	2
	C_s			1125			
	A_s			0.9020			
Weak Dependence	n_k	3	3	4	2	3	3
	r_k	2	2	2	2	2	2
	C_s			1230			
	A_s			0.9012			
Strong Dependence	n_k	3	2	2	2	2	2
	r_k	1	2	2	2	2	2
	C_s			1060			
	A_s			0.9031			

Then, availability constraint value is raised to 0.95 and 0.99, and the optimal problem is investigated for each redundant dependencies. The respective best optimal findings are listed in Table 4.8.6 and Table 4.8.7.

Table 4.8.6: Best optimal solution for $A_0 = 0.95$

	Subsystem(k)	1	2	3	4	5	6
Independence	n_k	4	4	4	3	4	3
	r_k	3	3	3	3	3	3
	C_s	1595					
	A_s	0.9506					
Linear Dependence	n_k	3	3	3	3	3	3
	r_k	3	2	3	1	2	2
	C_s	1270					
	A_s	0.9513					
Weak Dependence	n_k	4	3	4	3	3	3
	r_k	2	3	2	2	3	2
	C_s	1390					
	A_s	0.9503					
Strong Dependence	n_k	3	3	3	2	3	3
	r_k	2	2	3	2	2	1
	C_s	1180					
	A_s	0.9511					

Table 4.8.7: Best optimal solution for $A_0 = 0.99$

	Subsystem(k)	1	2	3	4	5	6
Independence	n_k	5	5	6	4	5	5
	r_k	4	3	5	4	4	3
	C_s	2130					
	A_s	0.9901					
Linear Dependence	n_k	4	4	4	3	4	4
	r_k	3	2	3	2	2	2
	C_s	1565					
	A_s	0.9902					
Weak Dependence	n_k	4	4	5	4	4	4
	r_k	3	3	3	2	4	3
	C_s	1770					
	A_s	0.9901					
Strong Dependence	n_k	3	3	4	3	4	3
	r_k	2	2	2	2	2	3
	C_s	1405					
	A_s	0.9901					

Henceforth, in Table 4.8.8 the outcomes of this model are compared with those obtained by **Hu *et al.* (2012)**.

Table 4.8.8: Results of the proposed study and Hu *et al.* (2012)

	Independence	Linear Dependence	Weak Dependence	Strong Dependence
$A_0 = 0.9$				
Proposed	1355	1125	1230	1060
Hu *et al.* (2012)	1355	1125	1285	1060
$A_0 = 0.95$				
	Independence	**Linear Dependence**	**Weak Dependence**	**Strong Dependence**
Proposed	1595	1270	1390	1180
Hu *et al.* (2012)	1615	1275	1410	1185
$A_0 = 0.99$				
	Independence	**Linear Dependence**	**Weak Dependence**	**Strong Dependence**
Proposed	2130	1565	1770	1405
Hu *et al.* (2012)	2135	1590	1810	1410

4.8.3 Solution Deviations

The optimal solution is found for each proposed dependencies corresponding to availability constraint values 0.75, 0.80 and 0.85, as given in Table 4.8.9.

Table 4.8.9: 20 PSO runs for different availability constraints corresponding to each kind of dependency

$A_0 = 0.75$				$A_0 = 0.80$				$A_0 = 0.85$			
Independence	**Linear**	**Weak**	**Strong**	**Independence**	**Linear**	**Weak**	**Strong**	**Independence**	**Linear**	**Weak**	**Strong**
1050	905	975	860	1125	975	1040	905	1230	1035	1125	970
1045	905	975	855	1150	985	1040	905	1230	1055	1125	980
1050	905	990	855	1125	970	1035	895	1235	1035	1125	975
1050	915	970	860	1150	970	1035	915	1230	1035	1125	985
1045	905	970	850	1125	975	1035	925	1230	1040	1155	970
1045	925	980	850	1125	970	1035	905	1230	1045	1125	970
1050	925	970	850	1125	995	1040	925	1230	1050	1180	975
1050	910	970	865	1140	970	1035	905	1230	1035	1125	970
1045	905	970	860	1150	990	1035	895	1230	1035	1125	975
1050	905	970	850	1145	970	1050	910	1230	1055	1125	980
1045	905	975	865	1125	970	1035	895	1240	1055	1125	985
1045	915	970	850	1125	985	1040	915	1230	1035	1125	975
1045	905	975	850	1150	975	1035	910	1240	1045	1125	975
1050	905	975	855	1125	980	1040	895	1230	1050	1170	975
1045	905	970	860	1125	970	1035	905	1230	1035	1125	975
1050	910	975	850	1150	970	1040	905	1240	1040	1125	970
1045	905	970	850	1125	975	1040	905	1230	1055	1140	970
1050	905	970	850	1150	970	1055	895	1230	1035	1125	985
1050	905	970	850	1155	970	1035	895	1230	1035	1125	975
1045	930	970	855	1125	970	1040	930	1230	1045	1125	995

The fundamental statistical indexes of the above given data is presented in Figure 4.8.1–4.8.3 in order to assist the algorithms performance evaluation. For the 20 PSO runs of each kind of dependency (labeled on the horizontal axis), the lowest, average, and highest costs are represented as the bottom bound, in between circle and the top bound respectively.

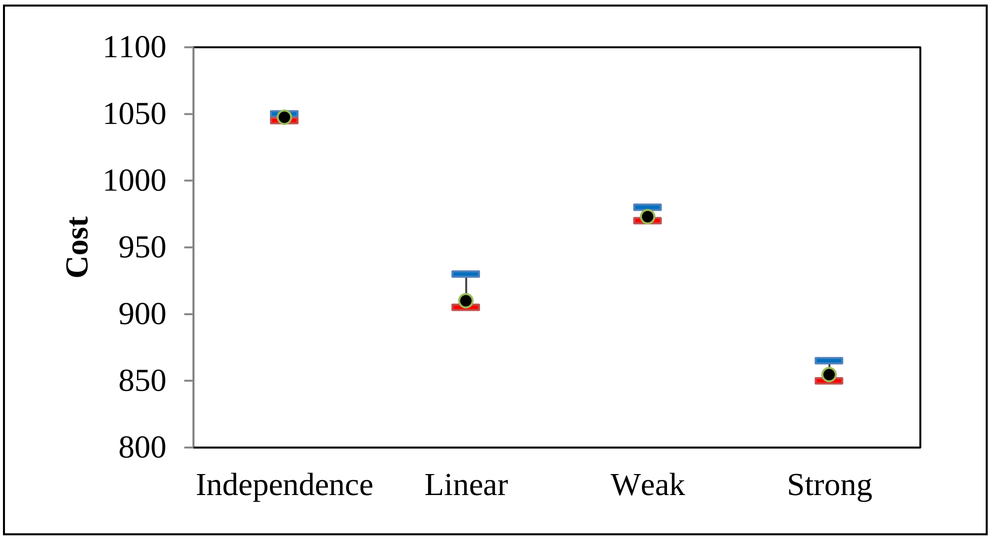


Figure 4.8.1: Statistical results of 20 PSO runs for $A_0 = 0.75$

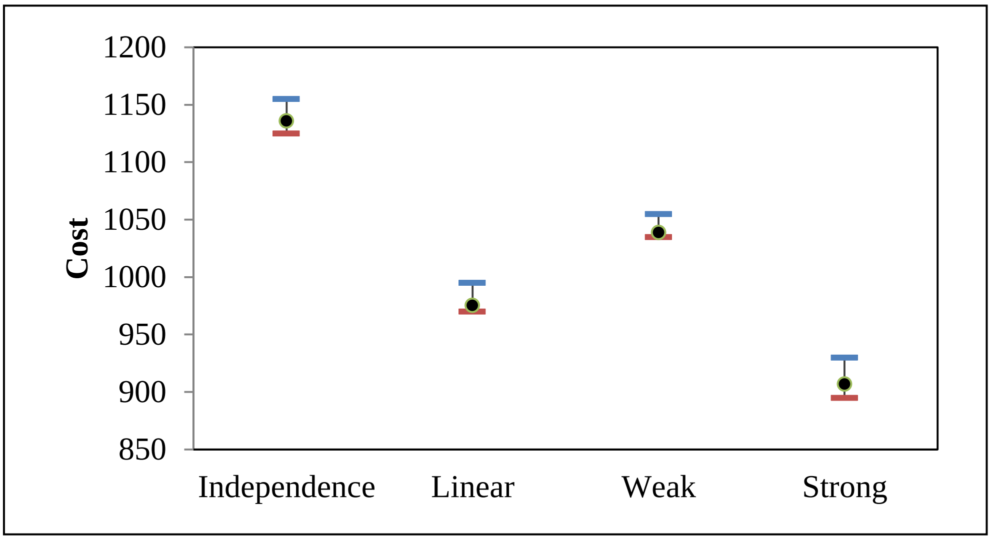


Figure 4.8.2: Statistical results of 20 PSO runs for $A_0 = 0.80$

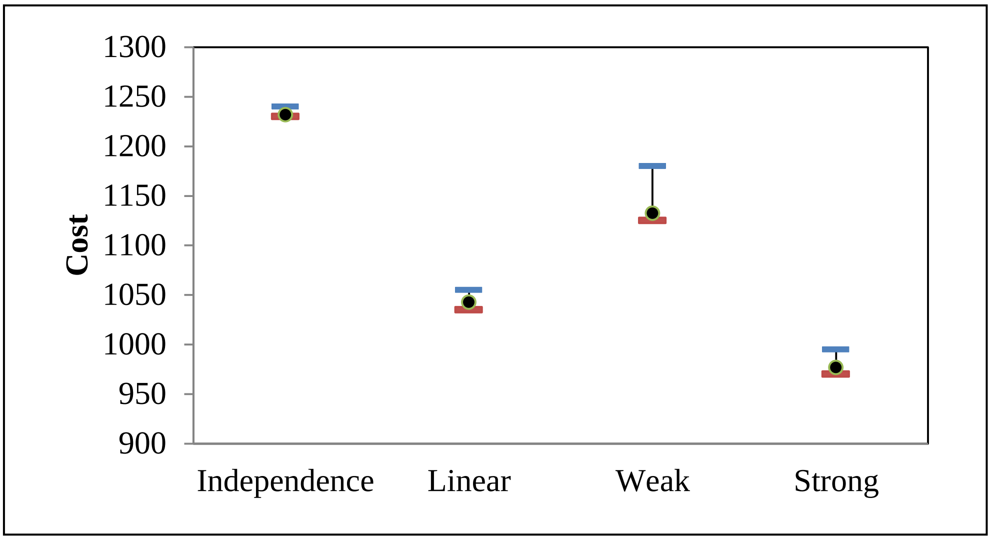


Figure 4.8.3: Statistical results of 20 PSO runs for $A_0 = 0.85$

4.8.4 Interpretation of results and discussions

For the initial case, the particle position is the personal best position. As seen from Table 4.8.2, the minimum fitness value is obtained corresponding to particle 1. So, particle 1 will be set as the global best particle.

From Table 4.8.2 and Table 4.8.3, we can see that the particle 1 has same fitness. So, particle 1 will not be updated and remains the same. But, for particle 2 the fitness value corresponding to particle's current position is less than the previous best position. Hence, particle 2 is updated. In the same way, we can update each particle. From Table 4.8.4, we can see that the minimum fitness value is again for particle 1. Hence, it will be set as the global best particle.

In the similar manner, the personal and global best of each particle could be obtained corresponding to different assumptions, i.e. different availability constraint and redundancies.

It can also be viewed from Table 4.8.5 that when different forms of dependencies are taken into account, the optimum solutions change. The ideal system comprises 19 (16, 18 and 13) units and 15 (12, 12 and 11) repair teams for the independence (linear dependence, weak dependence and strong dependence) type, and the cost employed is 1355 (1125, 1230 and 1060). The findings reveal that the strong dependence type produces the most cost-effective system and requires the fewest units and repair teams.

From the findings listed in Tables 4.8.5-4.8.7, it's clear that when the availability constraint value rises, more units and repair teams need to be utilized, and the system cost rises as well. Also, for stronger dependency types, fewer units and repair teams will be needed, and a more cost-effective system will be built. As a result, dependent characters must be included so as to obtain a more economic system.

From the Table 4.8.8, it could be clearly viewed that the findings of the current model are slightly better than those obtained by **Hu *et al.* (2012)**.

Sometimes meta-heuristics might experience premature convergence because of their stochastic character due to the usage of random operators. As a result, the outcome of a single run might not fully demonstrate the achievement of a meta-heuristic algorithm. To avoid the aforementioned situation, several trials must be carried out so that the assessed computed findings are reliable. The PSO was run 20 times and the corresponding findings are given in Table 4.8.9.

Based on the Table 4.8.9 and Figures 4.8.1-4.8.3, it is clear that in most situations, more than 10 PSO runs out of 20 intersected with their best solutions. This suggests that the PSO algorithm in this work may requires just two runs to approach the best solution.

Summary and Conclusions

Chapter 5 SUMMARY AND CONCLUSION

The unsafe failure of the majority of complex systems has given rise to the Reliability theory and its analysis. In recent years, technology has advanced at a rapid speed, increasing the complexity of systems and, as a result, the susceptibility of breakdowns. Due to this, the importance of reliability assessment has got new ground and has become an integral part for achieving high performance and life cycle cost of the product. As a result, it is critical to develop items in such a manner that they result in the ideal combination of reliability and cost-effective system. The current study covers specific challenges for complex systems in general, includes an analysis of reliability characteristics, and shows some results in the context of reliability and availability to explain and address the overall consequences.

The present work is focused to derive propositions to estimate reliability characteristics of systems incorporating maintenance and inspection policies. It is anticipated that the proposed research study shall be of great significance to deduce optimal availabilities and feasible period of inspection through various analysis and comparatives. The concept of Markov processes has been applied to discuss and solve the real time models mathematically. Also, various case studies and numerical examples have been presented to show the practical implementation of the developed models.

Present research work comprises of following four chapters.

Chapter 1 presents the general overview of reliability theory and includes elementary point of concepts and techniques which are used in this study. It gives the brief description on various maintenance and inspection policies along with Markov process.

Chapter 2 entitled “Review of Literature” encapsulates the wide amount of research work that has been done in the past related to reliability theory, multi-component system availabilities, inspection optimization policy and particle swarm optimization.

Chapter 3 is "Materials and Methods", involving analytical propositions and complete methodology used to evaluate the reliability and availability indices of the considered models and their mathematical formulation. The models considered in this research work are as follows:

Model [1]: Availability of systems subject to multiple failure modes under calendar-based inspection

Model [2]: Availability analysis and inspection optimization for a competing-risk *k*-out-of-*n*:G system

Model [3]: Modeling periodically inspected *k*/*r*-out-of-*n* system

Model [4]: Availability and cost assessment of systems with dormant failure undergoing sequential inspections

Model [5]: Modeling sequentially inspected system prone to degradation and shocks

Model [6]: Modeling systems with revealing and non-revealing failures undergoing periodic inspection

Model [7]: Markov process approach for analyzing periodically inspected competing-risk system embodying downtime threshold

Model [8]: Particle swarm optimization strategy for design optimization of a series-parallel system incorporating failure dependencies and multiple repair teams

Model 1 analyses an availability model for a maintained system encountering multiple failure modes undergoing periodic inspection incorporating calendar-based inspection policy. If the system is found to be working during inspection, the system is renewed. The theorems on the limiting average and point availability for the proposed model are derived. Theorems derived will be useful in finding the probability of system being available at any point (point availability) and the availability after it becomes steady (Limiting-average availability) of any system subject to multiple failures undergoing inspections at fixed calendar-intervals. Analysis is being made on the relations amongst availability and inspection period. Different inspection period can affect the systems limiting average and point availability. The model comprises the results obtained in **Li *et al.* (2019)** as the special case of our model. The derived results are demonstrated with the help of the example of a wind turbine system.

Model 2 proposes an availability and cost model for a maintained *k*-out-of-*n*:G system encountering multiple FMs undergoing periodic inspection. When the system is found to be working/failed during inspection, the renewal of system takes place. The theorems on the limiting and point availability for a competing-risk *k*-out-of-*n*:G system are derived and analysis is being made on the relations amongst availability and inspection period. Different inspection period can affect the systems limiting and point availability. Derived theorems could be used to find the availability at any instant of time (point availability), availability for the long-run could also be evaluated (limiting availability). For systems with short life, point availability is reasonable and for systems with long life, limiting availability is more concise. The theorem on LRACR is also derived in this model and an optimality condition for inspection period based on LRACR is stated. A numerical example of boiler feed water pump is taken into consideration with a view to explain the application of the derived results.

Model 3 presents an availability and cost model for a periodically inspected three-state (viz. normal, degraded and completely failed) and newly introduced *k*/*r*-out-of-*n* system with a repair policy. The theorems on LRACR, limiting and point availability for the proposed model are derived. Optimality conditions for inspection period based on availability and LRACR are also discussed. A numerical illustration of a power supply system is taken into consideration with a view to explain the applications of the derived results. The effect of inspection period on the systems limiting and point availability is also analyzed graphically.

Model 4 studies a system encountering hidden failure undergoing sequential inspection. Perfect repairs taking random times are carried out on detection of system failure. Inspections are conducted at time $T, T + aT, T + aT + a^2T, \ldots$ where $0 < a \leq 1$, till its failure detection, and repairs result in a new state so again it is inspected in the same manner. It is seen that $a = 1$ results into periodic inspection. Propositions on point availability, limiting availability and LRACR of proposed system are obtained. Example of a pressure switch is considered to explain the results. Analysis is being made on the relations amongst availability and inspection period for different values of 'a'. Effect of inspection time on cost rate is also analyzed.

Model 5 considers a single-unit system prone to degradation and shocks. The system is expected to have three states viz. normal, degraded and failed. The degraded state incurs higher degradation and is more prone to shocks in contrast to normal state. In order to determine the state and failure-type of the system, inspections are conducted sequentially at time $I, I + aI, I + aI + a^2I, \ldots$ where $0 < a \leq 1$, till the detection of degradation/failure. Perfect repairs are conducted immediately after the complete/partial failure is detected. Reliability, availability (both point and limiting availability) and LRACR of a sequentially inspected single-unit system prone to degradation and shocks is modeled herein subject to cases that repair takes constant times and random times. Numerical example of an oil pipeline system is given so as to justify the obtained results.

Model 6 proposes the inspection policy for a single-unit randomly failing system with alternating operating and inactivity periods. Failures are detected instantly for the former case whereas in the latter case inspections are required for failure detection. This study focuses on evaluating the general expressions for limiting availability and LRACR of the system undergoing periodic inspections. The distinctive characteristic of the proposed model is that the inspections are not perfect. Furthermore, it is supposed that inspection and maintenance time is non-negligible and corrective maintenance results in as good as new unit. The optimal inspection problem is developed predicated on maximizing the availability and minimizing the cost. Numerical example of electric motor corresponding to different life distributions is also presented to justify the obtained results.

Model 7 conducts availability and cost analysis on a periodically surveyed system incorporating multiple failure modes and downtime limit concept. Investigation is being done by classifying the system on the grounds of downtime limit as the initial and the new one. Precisely, analytical results on point and limiting availability are proposed for the aforesaid model. Furthermore, the LRACR is also investigated for the current model. This model consists of the results procured in **Qiu *et al.* (2019b)** as particular case of this study. Finally, the acquired study is demonstrated using a protection system. The influence of the inspection time and the downtime limit on the cost rate and limiting availability is also explained graphically.

Model 8 investigates an optimal design problem for the repairable series-parallel system adopted in **Hu *et al.* (2012).** The system's components are subject to failure dependency, with each of its subsystem having multiple repair teams. Four types of dependencies were considered: Independence, Linear, Weak and Strong dependence. The problem focuses on finding the optimal vector comprising of system components and repair teams, $(n_1, n_2, \dots, n_N, r_1, r_2, \dots, r_N)$, such that the system cost (attributed to the components and repair personnel's cost) is minimized. The PSO strategy incorporating dynamic parameters is being utilized to find the optimal vector for each type of the dependence. The calculations were made for three availability constraint values: $A_0 = 0.90, A_0 = 0.95\ and\ A_0 = 0.99$. A comparison between GA (**Hu *et al.* (2012)**) and PSO (proposed study) is made by virtue of the respective costs. It was found that the most economical system is obtained in the case of strong dependency, and utilizes the fewest components and maintenance teams.

Chapter 4 entitled "Results and Discussions" deals with the findings of discussed models along with the reliability measures such as reliability, availability (point availability, limiting average and limiting availability), sensitivity, maintenance cost analysis and optimum interval inspection along with the illustration. The analytical results have been represented in graphical forms.

5.1 Future scope

Obviously, as a preliminary investigation, there are many potential expansions worth investigating. For example, our most of the results have so far been confined to the circumstance where inspections are perfect. It would be interesting to look at several reliability indices and the best maintenance strategy for a multiple failure mode (FM) system with imperfect inspections. It may be important to characterize the nature of FMs in a more realistic scenario in which FMs are reliant. Because most systems feature mutually dependent FMs, such a model would be quite useful. Furthermore, the condition-based inspection schedule should be more effective due to the rapid development of sensing technology. Furthermore, the failure time of various FMs is believed to be exponentially distributed, which may be investigated by looking at general distributions.

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2. Pant, H., and Singh, S. B. (2021). Availability and cost assessment of systems with dormant failure undergoing sequential inspections. *Journal of Quality in Maintenance Engineering*. (Doi: 10.1108/JQME-10-2020-0112)
3. Pant, H., Singh, S. B., Pant, S., and Chantola, N. (2020). Availability analysis and inspection optimisation for a competing-risk k-out-of-n: G system. *International Journal of Reliability and Safety*, *14*(2-3), 168-181. (Doi: 10.1504/IJRS.2020.113315)
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Papers communicated from thesis:

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ABSTRACT

The present research is based on the development of reliability models incorporating maintenance and inspection policies and studying them utilizing a Markov process approach. In this study, eight different models have been developed. **Model 1** examines a maintained system with numerous failure modes that is subjected to periodic inspection using a calendar-based inspection strategy. The system is renewed at each of the inspection. The limiting average and point availability theorems for the model are deduced. **Model 2** provides a model for a maintained k-out-of-n:G system with several failure modes that must be inspected on a periodic basis. During the inspection, the system is renewed if discovered to be operating or failed. Theorems on limiting and point availability and long-run average cost rate (LRACR) for the system are developed. **Model 3** presents a three-state (normal, degraded, and entirely failed) and newly introduced k/r-out-of-n system with a repair strategy that is periodically examined. For the suggested model, theorems on LRACR and limiting and point availability are derived. **Model 4** investigates a system with hidden defects that is subjected to sequential examination. When a system breakdown is detected, perfect repairs are carried out at random times. Propositions on point and limiting availability and system's LRACR are obtained. **Model 5** examines a single-unit system that is susceptible to degradation and shocks. The system has three states: normal, degraded, or failed. Inspections are undertaken progressively at sequential times. When a total or partial failure is identified, perfect repairs are done. The reliability, availability (point and limiting) and the LRACR of the system are computed. **Model 6** looks at broad expressions for limiting availability and LRACR of a single-unit randomly failing system with alternating phases of operation and rest. System undergoes imperfect periodic inspections. Furthermore, it is assumed that inspection and maintenance time is non-negligible, and that corrective repairs are perfect. **Model 7** analyses the availability (point and limiting) and cost of a system examined on a periodic basis and incorporating numerous failure modes and a downtime limit concept. **Model 8** looks into an optimal design problem for a repairable series-parallel system. The system's components are prone to breakdown, with various repair teams assigned to each subsystem. Four types of interdependence are explored. The goal of the task was to identify the ideal number of system components and repair teams that minimises the system cost. The PSO technique with dynamic parameters is used to find the ideal structure.

All the presented models are demonstrated by appropriate illustrative examples.

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सत्र एवं प्रवेश वर्ष	: द्वितीय सेमेस्टर, 2018-2019	उपाधि	: पीएच.डी.
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मुख्य विषय	: गणित	गौण विषय	: संगणक विज्ञान
शोध शीर्षक	: मार्कोव मॉडल रखरखाव और निरीक्षण को शामिल करने वाली मरम्मत योग्य प्रणालियों की विश्वसनीयता और लागत का विश्लेषण		
पृष्ठ संख्या	: 184	सलाहकार	: डॉ. एस.बी. सिंह

सारांश

वर्तमान शोध विश्वसनीयता मॉडल के विकास पर आधारित है जिसमें रखरखाव और निरीक्षण नीतियों को शामिल किया गया है और मार्कोव प्रक्रिया दृष्टिकोण का उपयोग करके उनका अध्ययन किया गया है। इस अध्ययन में आठ अलग-अलग मॉडल विकसित किए गए हैं। मॉडल 1 कई विफलता मोड के साथ एक अनुरक्षित प्रणाली की जांच करता है जिसे कैलेंडर-आधारित निरीक्षण रणनीति का उपयोग करके आवधिक निरीक्षण के अधीन किया जाता है। प्रत्येक निरीक्षण पर सिस्टम का नवीनीकरण किया जाता है। मॉडल के लिए सीमित औसत और तात्कालिक उपलब्धता प्रमेय यहां प्राप्त किए गए हैं। मॉडल 2 कई विफलता मोड के साथ बनाए रखा k-out-of-n:G सिस्टम के लिए एक मॉडल प्रदान करता है जिसका आवधिक आधार पर निरीक्षण किया जाना चाहिए। निरीक्षण के दौरान, ऑपरेटिंग या विफल होने का पता चलने पर सिस्टम को नवीनीकृत किया जाता है। सिस्टम के लिए सीमित और तात्कालिक उपलब्धता और लंबी अवधि की औसत लागत दर (एल.आर.ए.सी.आर.) पर प्रमेय विकसित किए गए हैं। मॉडल 3 तीन- अवस्था (सामान्य, अवक्रमित, और पूरी तरह से विफल) और नई पेश की गई k/r-out-of-n प्रणाली को एक मरम्मत रणनीति के साथ प्रस्तुत करता है जिसकी समय-समय पर जांच की जाती है। सुझाए गए मॉडल के लिए, एल.आर.ए.सी.आर. और सीमित और तात्कालिक उपलब्धता पर प्रमेय व्युत्पन्न किए गए हैं। मॉडल 4 एक ऐसी प्रणाली की जांच करता है जिसमें छिपे हुए दोष हैं जो क्रमिक परीक्षा के अधीन हैं। जब एक सिस्टम ब्रेकडाउन का पता चलता है, तो यादृच्छिक समय पर सही मरम्मत की जाती है। तात्कालिक और सीमित उपलब्धता और सिस्टम के एल.आर.ए.सी.आर. पर प्रस्ताव प्राप्त किए जाते हैं। मॉडल 5 एकल-इकाई प्रणाली की जांच करता है जो गिरावट और शॉक के लिए अतिसंवेदनशील है। सिस्टम में तीन अवस्थाएँ होती हैं: सामान्य, ख़राब या विफल। क्रमिक समय पर उत्तरोत्तर निरीक्षण किया जाता है। जब पूर्ण या आंशिक विफलता की पहचान की जाती है, तो पूर्ण मरम्मत की जाती है। सिस्टम की विश्वसनीयता, उपलब्धता (तात्कालिक और सीमित) और एल.आर.ए.सी.आर. की गणना की जाती है। मॉडल 6 ऑपरेशन और आराम के वैकल्पिक चरणों के साथ एकल-इकाई बेतरतीब ढंग से विफल प्रणाली की उपलब्धता और एल.आर.ए.सी.आर. को सीमित करने के लिए व्यापक अभिव्यक्तियों को देखता है। सिस्टम अपूर्ण आवधिक निरीक्षण से गुजरता है। इसके अलावा, यह माना जाता है कि निरीक्षण और रखरखाव का समय नगण्य है, और सुधारात्मक मरम्मत सही है। मॉडल 7 समय-समय पर जांच की गई प्रणाली की उपलब्धता (तात्कालिक और सीमित) और लागत का विश्लेषण करता है और कई विफलता मोड और डाउनटाइम सीमा अवधारणा को शामिल करता है। मॉडल 8 एक मरम्मत योग्य श्रृंखला-समानांतर प्रणाली के लिए एक इष्टतम डिजाइन समस्या को देखता है। सिस्टम के घटकों के टूटने का खतरा होता है तथा प्रत्येक सबसिस्टम को विभिन्न मरम्मत दल सौंपे जाते हैं। चार प्रकार की अन्योन्याश्रयता का पता लगाया जाता है। कार्य का लक्ष्य सिस्टम घटकों और मरम्मत टीमों की आदर्श संख्या की पहचान करना था जो सिस्टम लागत को कम करता है। गतिशील मापदंडों के साथ पीएसओ तकनीक का उपयोग आदर्श संरचना को खोजने के लिए किया गया है।

सभी प्रस्तुत मॉडलों को उपयुक्त उदाहरण के द्वारा प्रदर्शित किया गया है।

(एस.बी. सिंह)
सलाहकार

Himani Pant
(हिमानी पंत)
लेखिका